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# *Dynamics of Variance Risk Premia, Investors' Sentiment and Return Predictability*

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## Abstract

We develop a joint model for the physical and risk neutral variance implying variance risk premia that are reasonably smooth, appropriately reacting to changes in level and variability of the variance and naturally satisfying the sign constraint.

The model, straightforward to estimate using option market data and high frequency returns, allows identifying agents' sentiment indicators. Our results show that excess returns are to a large extent explained by fear or optimism towards future extreme variance events and only marginally, if at all, by the premium associated with normal price fluctuations. We also find evidence of asymmetry in the response of future excess returns to fear and optimism with respect to the state of the market at the time expectations are formed. Robustness checks show that the documented predictive power is distinct from that contained in several well-known economic predictors and that the general findings extend internationally to eight major markets.



Keywords: Variance risk premium; return predictability; Kalman filter; international stock market returns.

JEL classification: C12, C22, G12, G13.

# 1 Introduction

Financial volatility, or financial assets' price variability, is a crucial ingredient in asset pricing and portfolio management. Understanding its dynamics and economic drivers is of great interest to academics and practitioners. The main reason is that volatility is closely tied to the identification of risk premia. While the evolution and the economic determinants of the equity risk premium have been extensively investigated, the recent literature focuses on the variance risk premium (VRP) that investors require for the well known fact that volatility is stochastic. The VRP is defined as the difference between the risk-neutral and physical expectations of an asset's total return variation. While clear conceptually, the estimation of the VRP requires multiple sources of financial data as well as assumptions on the latent volatility processes, rendering its dynamic properties difficult to pinpoint.

This paper proposes a new flexible approach for retrieving the VRP which delivers precise and realistic estimates of the market price of volatility risk. We advocate the inclusion of interactions and discontinuities, with emphasis on regime shifts, extreme events, uncertainty due to heteroskedasticity, correlations and spillovers, as being essential to replicate dynamics and interdependencies between the physical variance and its risk neutral expectation. We define a class of time series models that isolates as structural components the dynamics of the physical variance and, by embedding its expectations into the model, the price attached by the market to the variance risk. Given the latent nature of the variables of interest of which only imprecise approximations are observable (i.e., high frequency return based variance measures and option implied risk neutral variance expectations), we use signal extraction techniques based on a state-space representation of the model and the Kalman-Hamilton filter. This approach allow us to obtain measurement error free estimates of the VRP, and thus to disentangle, with a high degree of precision, its underlying time series properties as well as potential dependencies with the state of the economy.

Carr and Wu (2009) are among the first to provide evidence on the VRP and show that for the S&P 500 index it is on average strongly positive and time varying. Their VRP can be interpreted as the ex-post payoff of a variance swap rather than a premium that investors require ex-ante. To be in line with the definition of a proper risk premium, the VRP is typically directly computed as the difference from one period to the next between the squared VIX index, a measure of implied variance computed from option prices on the S&P 500 index, and expected realized variance computed with high frequency historical returns and filtered with a particular choice of dynamic model. See Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011) and Bekaert and Hoerova (2014) for examples of models to construct realized variance expectations. While the above approaches are relatively simple to implement, their drawback stands in a resulting VRP time series extremely noisy and violating the positivity constraint too often to be genuine risk premia. Also, because the VRP is computed only period by period, analyzing its dynamic properties, its dependence with the level of volatility, the market conditions and, more generally, the state of the economy, is neglected or possible only ex-post.

The state-space framework used in this paper brings the following advantages. First, it allows to account for possible errors in the measurement of the realized variance and option implied risk neutral variance expectations. Second, it allows to embed the expectations under the physical measure directly into the model. This avoids relying on the typically used multistep estimation approach which is less efficient because subject to the compounding of estimation errors. In contrast to most standard procedures of estimating the VRP which focus uniquely on the construction of expectations under the physical measure, our model also allows to fit and test possible structures for the agents' expectation formation mechanism under the risk neutral measure. The model parameters are straightforwardly estimated by maximum likelihood using the Kalman-Hamilton filter, see for example Kim (1994). See also Egloff, Leippold, and Wu (2010) for a similar estimation technique to fit a VRP term structure model.

We construct a joint dynamic model for the historical variance, its physical and its risk neutral expectations with direct focus on the VRP. We leave the dynamics of the underlying price process unspecified, allowing the direct use of realized and implied variances computed respectively from 5-minute returns and option data on the underlying index. See also Bollerslev, Gibson, and Zhou (2011), Wu (2011) and Gruber, Tebaldi, and Trojani (2015) who directly model risk premia without assumptions on the price dynamics. An alternative to study the dynamic properties of the VRP is the specification of a full parametric model, as introduced for example in Brodie, Chernov, and Johannes (2007), Andersen, Fusari, and Todorov (2015b) and Ait-Sahalia, Karaman, and Mancini (2015). In order to estimate equity and variance risk premia, this particular type of approach requires a complete specification of the price process, along with assumptions about the stochastic discount factor. However, in these models, the tight parameterization often constraints, to different extents, the dynamics of the VRP to mirror that of the variance itself.

Without imposing strong parametric assumptions, our approach allows us to precisely estimate the VRP associated with normal market activity and generates VRPs with appropriate characteristics in terms of variability, level and persistence. Our simple, though effective modeling strategy, also allows us to identify the occurrence of unusual and extreme episodes of market volatility and isolate their contribution to the total VRP. Finally, by exploiting the temporal causality between realization of shocks on the spot market and formation of expectations in the option market, our methodology allows us to gauge agents' reaction to extreme market volatility events and to build risk factors representing their confidence towards future market states.

In our empirical application, we first estimate our model on data from January, 1990, to September, 2015, for the S&P 500 index. Our results document the importance of allowing for interactions, discontinuities and occurrence of extreme events. Our model specification is strongly supported by the data and all structural parameters are estimated with a high degree of precision. Moreover, the filtered variance predictions and the risk neutral variance expectations match adequately the level and the dynamics of their observable counterparts along the entire sample. Most importantly though, the resulting filtered variance risk premium satisfies naturally the positivity constraint, shows a reasonable degree of smoothness and it appropriately reacts to changes in level and variability. For sake of comparison, we show that existing methods, assuming e.g. random walk or autoregressive dynamics for realized variances, lead to variance risk premia that are too volatile and systematically become negative in periods of market distress.

While estimation of the VRP is interesting for its own sake, identification of proper risk premia have important economic applications. For example, as first shown by Bollerslev, Tauchen, and Zhou (2009), it turns out that market index returns become more predictable when including the VRP as a regressor in a future return model. Drechsler and Yaron (2011) provide a theoretical explanation for this return predictability and show that time varying economic uncertainty and a preference for early resolution of uncertainty yields time varying positive VRPs that predict stock market returns. Also, Chabi-Yo (2012) motivate that changes in VRP can be caused by changes in investors' skewness preference. However, as shown by Bekaert and Hoerova (2014), the results from applying the existing models empirically offers an interesting puzzle as models with more realistic dynamics for the variance process fall short in terms of offering predictability compared to a simpler yet unrealistic model that assumes a random walk specification for the physical variance.

The VRP identified by the existing literature in fact contains two distinct components: one that reflects compensation for continuous price moves and one that is related to compensation for disaster risk, see e.g. Bollerslev and Todorov (2011). For example, Bollerslev, Todorov, and Xu (2015) provide evidence that these two components explain to a different extent aggregate return variation. Our approach to estimate the VRP uses a switching model with regimes that become active during events of unusually high volatility. This allows to explicitly account for the occurrence of extreme variance events, agents' fear, uncertainty deriving from heteroskedasticity, changes in the dynamics and dependence between the latent states. Being able to identify the occurrence and measure the intensity of extreme variance shocks, we exploit the causal effect between their realization and next period variance expectations to construct two sentiment indicators based on agents' reaction to such shocks. Fear is defined as a conservative reaction of variance expectations to the size of the observed shock. Conversely, optimism reflects a reaction of variance expectations that is less than proportional with respect to the size of the variance shock.

For the S&P 500 index, our results show that there is essentially no substantial return predictability from the smooth component of the variance risk premium. In fact, the predictability attains a maximum value equal to 1.3% at the five month horizon. Including our sentiment indicators in the predictive regressions $R^2$s in the regressions increase dramatically at every horizon and surpasses 10% at the four month horizon. Furthermore, this predictive power is distinct from that contained in several well-known economic predictors, such as price-earning ratio, dividend yield, term spread, consumption-wealth ratio and output gap among others. This finding indeed shows that much of the return predictability previously ascribed in the literature to the variance risk premium is effectively coming from the part of the premium related to how agents gauge extreme variance events, their prediction and compensation as also advocated by Bollerslev, Todorov, and Xu (2015). Our results also show that allowing for asymmetric effects of the sentiment on future aggregate market returns, obtained by interacting the sentiment with the sign of realized market performance, further increases return predictability especially at those horizons where the latter is known to be difficult to capture. In particular, fear in conjunction with negative market performance represents the prevailing risk factor, especially for long horizons, whereas optimism combined with bearish markets contributes substantially to return predictability in the short term and dominates over medium horizons.

To complement the results for the S&P 500 index, we estimate our model on eight other market indices, finding similar results in terms of parameter estimates and filtered dynamics. Our international evidence confirms that the smooth part of the variance risk premia generally has little or no predictive power on future aggregate market returns at any horizon. However, when our sentiment indicators are added to the regressions the $R^2$s globally increase in the same fashion as observed for the S&P 500 index. This suggests that the information contained in the sentiment indicators is needed to effectively explain international return predictability. However, the heterogeneity of the return predictability suggests that the attitude of agents towards risk, thus the pricing of different risk factors, varies largely across markets. Compared to the standard methods for obtaining the variance risk premium, i.e. the random walk hypothesis for which our results are in line with Bollerslev, Marrone, Xu, and Zhou (2014), the $R^2$s of the predictive return regressions including the sentiment indicators and their interaction with the market conditions largely dominate over all horizons and for all indices.

The rest of the paper is organized as follows. Section 2 states basic definitions, describes the data used to model the VRP and computes the VRP in various ways proposed recently by the literature. Section 3 explains our new approach for estimating the VRP by modelling both physical and riskneutral variances in a joint system and details how to incorporate features like heteroskedasticity and jumps that, when ignored, drastically affect the VRP. Section 4 constructs the sentiment indicators and discusses their implication for return predictability. Section 5 estimates the new model on the S&P 500 index, illustrates the dynamic properties of the filtered VRP time series and extensively discusses the implications of using the estimated VRP series and sentiment indicators on predictive return regressions. Section 6 provides international evidence and shows that the general conclusions extend to eight other market indices. Section 7 concludes.

## 2 Definitions and data

The VRP at time $t$ for a given maturity $\tau$ is defined as

$$\Pi_{t,t+\tau} = E_t^Q[QV_{t,t+\tau}] - E_t^P[QV_{t,t+\tau}], \tag{1}$$

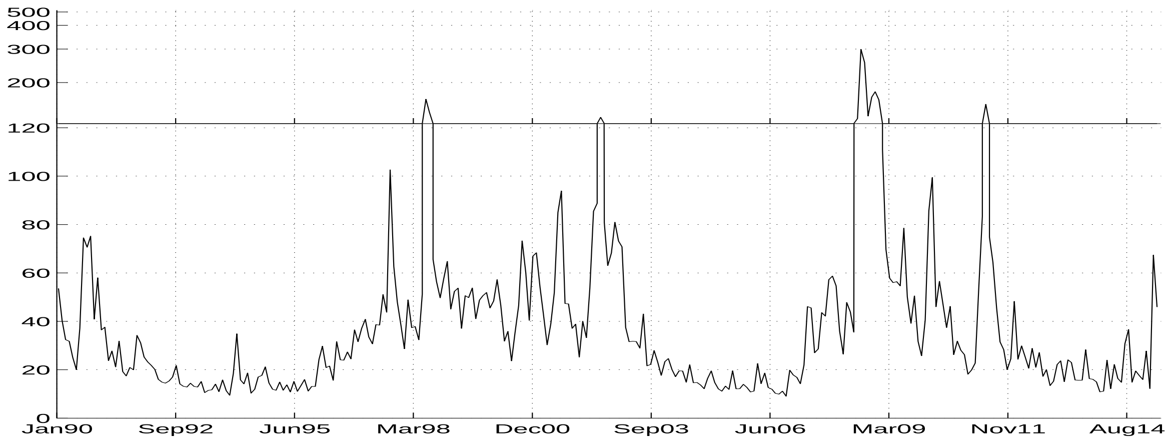
where $QV_{t,t+\tau}$ is the (latent) quadratic variation of the underlying price process and the conditional expectations are under the risk neutral (Q) and physical (P) measures, respectively. In fact, the VRP represents the expected profit to the long side of a variance swap contract, which is entered into at time $t$ and held until maturity $t+\tau$. See Ait-Sahalia, Karaman, and Mancini (2015) for a brief description of variance swaps. Note that in line with the general definition of a risk premium, the VRP could be defined as the negative of $\Pi_{t,t+\tau}$, see for example Carr and Wu (2009). However, for the ease of interpretation we follow the notation used in Bollerslev, Tauchen, and Zhou (2009) so that $\Pi_{t,t+\tau}$ as defined in (1) is expected to be positive.

To compute $\Pi_{t,t+\tau}$ in practice, we thus need to compute conditional expectations of the variance of an asset both under the risk neutral and physical measure. In the VRP literature, $E_t^Q[QV_{t,t+\tau}]$ is typically directly computed using market quantities or non-parametric estimators, i.e. using the variance swap strike or the square of an option market implied volatility index. The expectation under the physical measure, $E_t^P[QV_{t,t+\tau}]$, is, on the other hand, typically obtained as a projection computed from a parametric time-series analysis of a non-parametric estimator of $QV_{t,t+\tau}$. This approach results in a VRP which is no longer model free. We now describes the data typically used to model the VRP. We also compute the VRP in various ways proposed recently by the literature and we assess the return predictability of these.

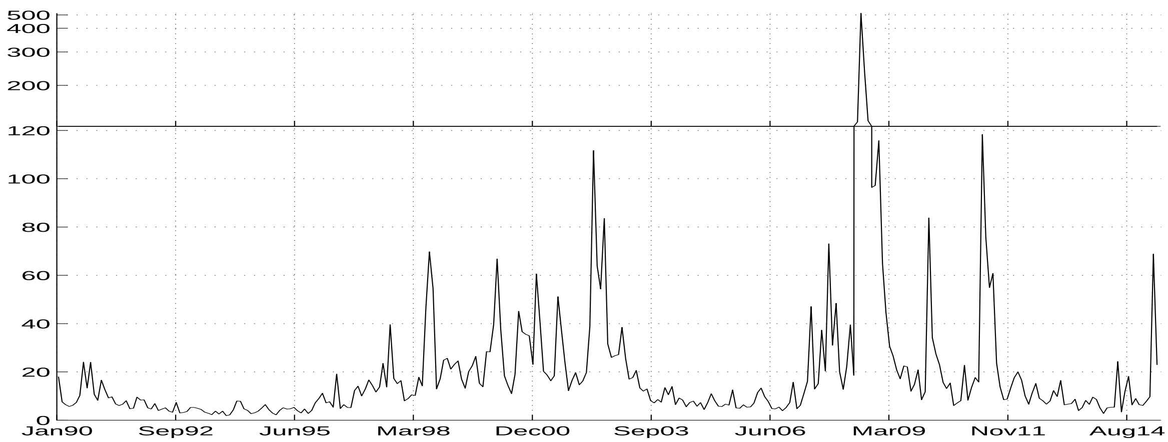
## 2.1 Data

Our monthly sample on the S&P 500 index runs from January, 1990, to September, 2015. The choice of monthly frequency is not coincidental. While on the one hand it allows including a large spectrum of macro-finance variables, as for instance in Bollerslev, Marrone, Xu, and Zhou (2014), on the other hand it solves the usual problem of overlapping forecast horizons that the predictions of the variance under the risk neutral measure is subject to. As a proxy for the one month ahead risk neutral expectation of the variance, we take from Datastream the last day of the month squared value of VIX divided by 12 to make it monthly. Our sample starts in 1990 when the CBOE began to publish the VIX. The realized variance series at monthly frequency, updated from Bollerslev, Tauchen, and Zhou (2009) using Oxford-Man Institute data, is based on 5 minute returns along with the squared close-to-open overnight return. In Section 6 we expand the analysis to eight other markets.

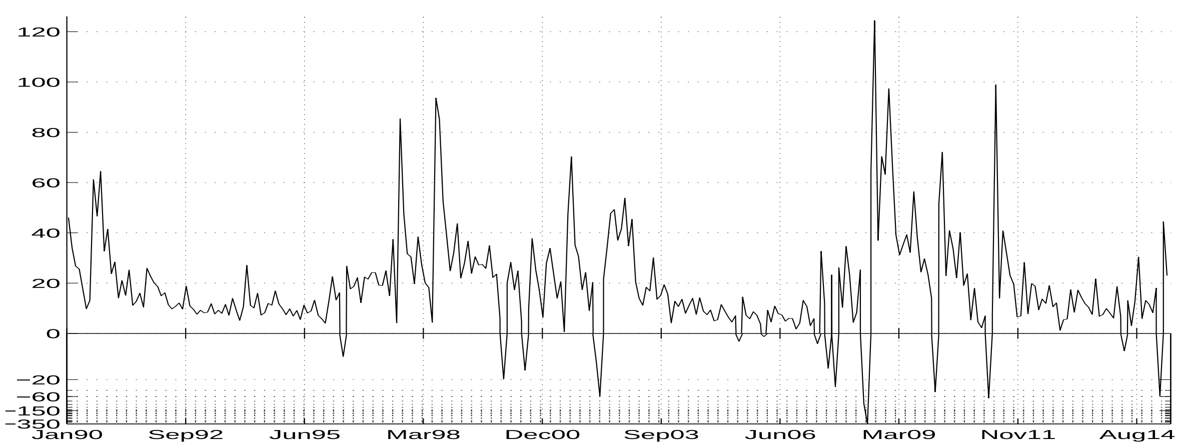
Figure 1 plots the squared VIX and the realized variance in panel (a) and (b), respectively, and shows that both series follow a similar pattern. However, the squared VIX, see Figure 1(a), generally shows smoother and more persistent dynamics than the realized variance (see also Table 1). This is because, over the one-month observation period, shocks are incorporated into the medium-term expectations gradually, as they occur and only to the extent they are perceived to be persistent. As pointed out in Bollerslev, Todorov, and Xu (2015) the VIX represents an approximation to the risk neutral expectation of the total quadratic variation and reflects the compensation for both time-varying diffusive volatility and jump intensity risks, as well as market expectations about future extreme volatility events and their pricing. The realized variance, see Figure 1(b), shows the usual characteristic discontinuous behavior emphasized by large jumps in periods of market instability, i.e. the dot-com bubble burst, the global financial crisis, the Flash crash and the Euro-




(a) Monthly squared VIX




(b) Monthly realized variance




(c) Monthly variance swap payoff

Figure 1: Risk-neutral variance expectation, realized variance and variance swap payoff

Table 1: Sample statistics

| | $VIX^2_{t,t+1}$ | $RV_{t-1,t}$ | Payoff | $\Pi^{RW}_{t,t+1}$ | $\Pi^{AR(1)}_{t,t+1}$ | $\Pi^{AR(2)}_{t,t+1}$ | $\Pi^{ARX1}_{t,t+1}$ | $\Pi^{ARX2}_{t,t+1}$ |
|---|---|---|---|---|---|---|---|---|
| Mean | 38.796 | 20.179 | 17.584 | 17.584 | 17.602 | 17.568 | 17.568 | 17.602 |
| Stddev | 34.331 | 35.755 | 29.918 | 19.502 | 19.231 | 19.651 | 19.574 | 13.642 |
| Skewn. | 3.456 | 8.126 | -5.873 | -2.394 | 2.345 | 2.341 | 2.308 | 2.441 |
| Kurt. | 20.43 | 94.362 | 77.303 | 39.276 | 10.466 | 10.480 | 10.252 | 10.854 |
| Min | 9.048 | 1.865 | -350.28 | -180.68 | -18.830 | -25.428 | -22.755 | 1.652 |
| Max | 298.9 | 479.58 | 124.45 | 116.52 | 125.910 | 125.07 | 125.820 | 91.840 |
| $\rho(1)$ | 0.804 | 0.648 | 0.274 | 0.263 | 0.567 | 0.554 | 0.576 | 0.639 |

Notes: This table reports sample statistics for variables used to estimated the VRP. $\Pi^i_{t,t+1}$, $i = RW, AR(1), AR(2), ARX1$ and $ARX2$, is computed using (1) where the expectation under the physical measure is obtained using model $i$ and the risk neutral expectation is approximated by the squared VIX. The first order sample autocorrelation coefficient is denoted by $\rho(1)$. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

pean sovereign debt crisis, due to the aggregation of shocks over non-overlapping monthly intervals. The effect of the large jumps is reflected in the comparatively small sample autocorrelation coefficient of 0.648. Finally, Figure 1 also shows that the squared VIX generally lies above the realized variance, indicating a positive VRP. In fact, the difference between the average squared VIX and the average realized variance, i.e. the unconditional VRP, amounts to 17.584 as can be seen from Table 1.

Carr and Wu (2009) compute the ex-post payoff from an artificial variance swap contract entered into at time $t$ with maturity $\tau$ and strike equal to $VIX^2_{t,t+1}$ as $\Pi_{t,t+1} = VIX^2_{t,t+1} - RV_{t,t+1}$, where $RV_{t,t+1}$ is the (non-parametric) realized variance estimator of $QV_{t,t+1}$, see Andersen, Bollerslev, Diebold, and Ebens (2001) and Andersen, Bollerslev, Diebold, and Labys (2001), computed at time $t+1$. Carr and Wu (2009) document that $\Pi_{t,t+1}$ is positive on average and time varying. Although the variance swap payoff does not measure the ex ante expectation and thus the variance premium, their analysis is based on sample averages, thus allowing to quantify the VRP level unconditionally. Figure 1 (c), displays the variance swap payoff which shows clustering of calm and highly volatility periods. Although the payoff is generally positive, on several occasions it is negative with strikingly extreme values (up to $-350.28$) around the beginning of the global financial crisis.

## 2.2 Current approaches to estimate the VRP

Several possibilities have been explored already in the literature to estimate the VRP. Bollerslev, Tauchen, and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014) propose a very simple approach to compute the VRP. While maintaining the assumption that the riskneutral expectation of the quadratic variation, $E_t^Q[QV_{t,t+\tau}]$, is approximated by the square of the VIX index, they assume random walk (RW) dynamics for $QV_{t,t+\tau}$ under the physical measure, such that $E_t^P[QV_{t,t+\tau}] = QV_{t-\tau,t}$. When $QV_{t-\tau,t}$ is estimated by the realized variance, $RV_{t-\tau,t}$, this allows to approximate the VRP by

$$\Pi_{t,t+\tau}^{RW} = VIX_{t,t+\tau}^2 - RV_{t-\tau,t}. \tag{2}$$

This way of computing the VRP does not require any additional parameter estimation and is therefore directly usable in financial applications. The random walk assumption, though, is at odds with the recent literature on volatility modelling which advocate instead the need for autoregressive dynamics.

Figure 2 presents the monthly time series of the VRP for the random walk ($\Pi_{t,t+1}^{RW}$) and shows that this time series is very noisy and is highly positive and negative due to the presence of extreme variance events (jumps). This contrasts with financial theory which rules out negative VRP, see Drechsler and Yaron (2011). In fact, $\Pi_{t,t+1}^{RW}$ is negative on multiple occasions with a minimum of $-180$ observed on October 2008 and shows large positive spikes with a maximum of 116 on August 1998. Indeed, it would appear that in periods of market turmoil, when the premium to bear the risk of variance fluctuations should be the highest for risk neutral agents, the variance risk premium sharply reduces

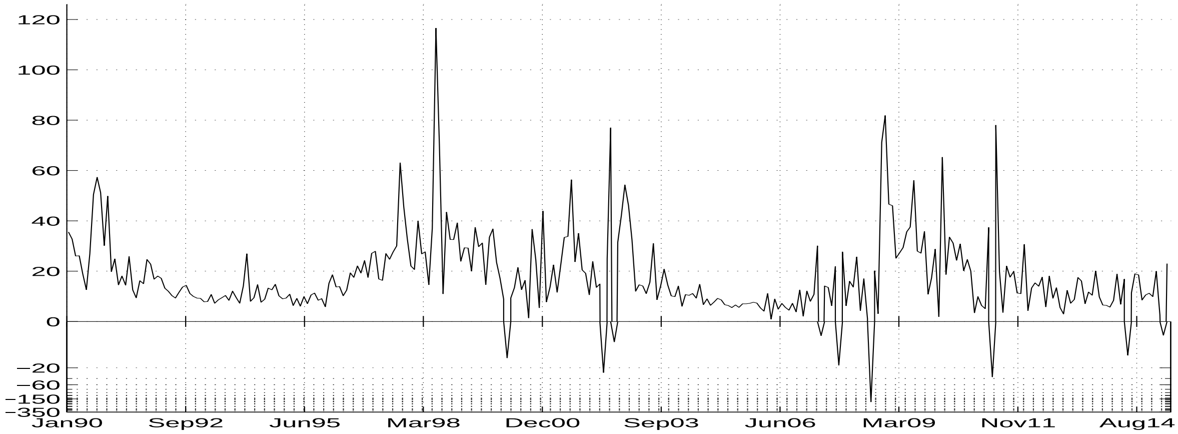
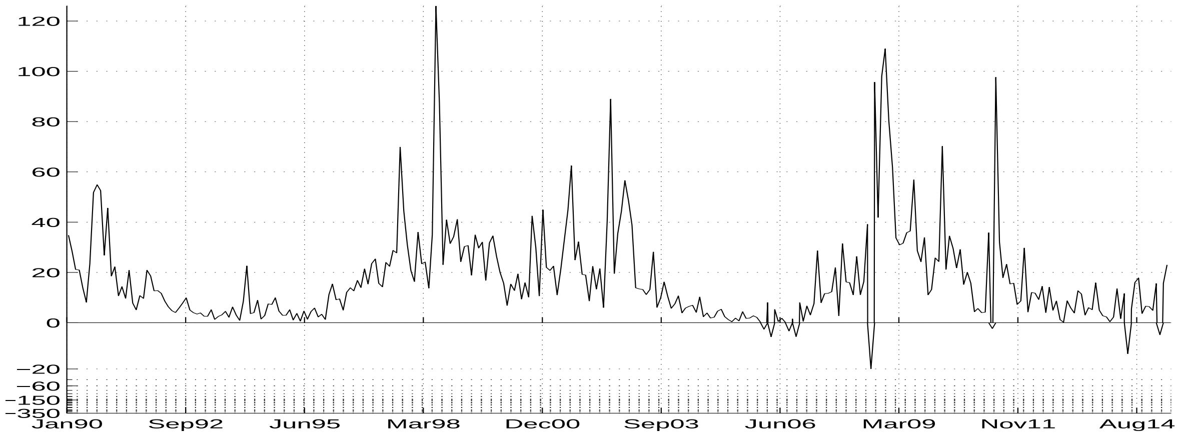



Figure 2: $\Pi^{RW}_{t,t+1}$ – VRP based on random walk assumption for realized variance

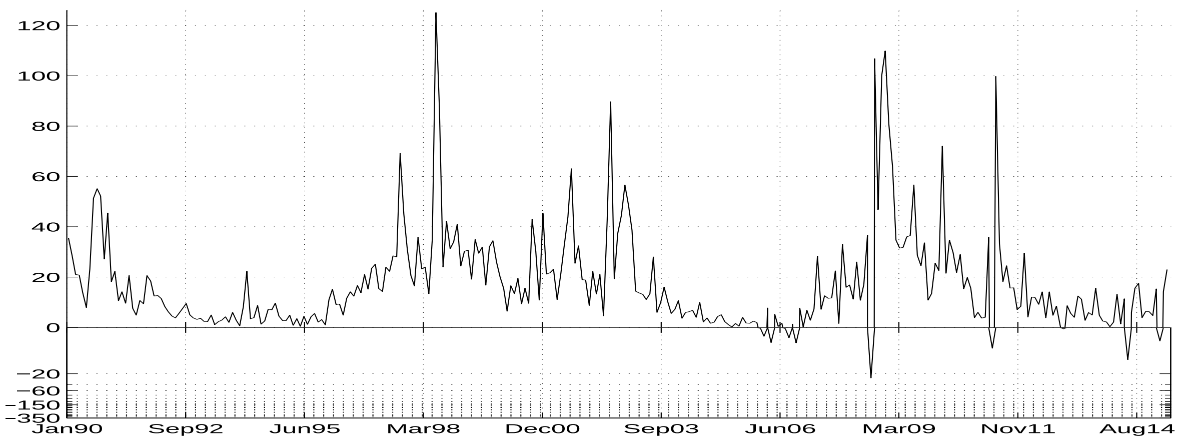
and even becomes highly negative. This latter effect is due to the asynchronicity of the occurrence and/or relative size of extreme observations in $RV_{t-1,t}$ and $VIX_{t,t+1}$.

Relaxing the random walk assumption, Drechsler and Yaron (2011) and Bekaert and Hoerova (2014) compute $E^P_t[QV_{t,t+\tau}]$ by projecting realized variance measures on a set of predictor variables including autoregressive dynamics and implied volatility. Mueller, Vedolin, and Yen (2015) follow a similar approach when studying bond VRP's. To illustrate this methodology we consider the VRP from autoregressive models of order one and two ($\Pi^{AR(1)}_{t,t+1}$ and $\Pi^{AR(2)}_{t,t+1}$ respectively). Furthermore, to explicitly incorporate revisions in expectations and error in expectations, respectively, we also consider two AR(1) specifications that include the ex-ante options implied expectation of the current ($VIX^2_{t,t+1}$) or previous ($VIX^2_{t-1,t}$) period variance level as an additional regressor. The corresponding VRP time series are called $\Pi^{ARX1}_{t,t+1}$ and $\Pi^{ARX2}_{t,t+1}$, respectively.

For the autoregressive models, the implied VRP time series in Figure 3 are as expected smoother than the $\Pi^{RW}_{t,t+1}$ series and the positivity requirement is less often violated. However, due to the usual downward bias of the autoregressive parameters observed in presence of extreme realizations, the resulting dynamics under the physical measure turns out to be



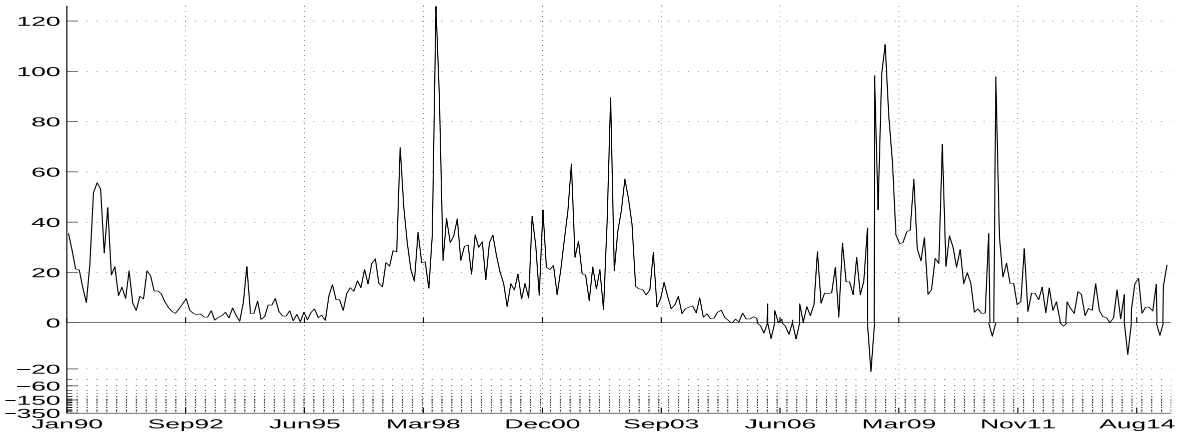

(a) $\Pi_{t,t+1}^{AR(1)}$ – VRP based on AR(1) model for realized variance



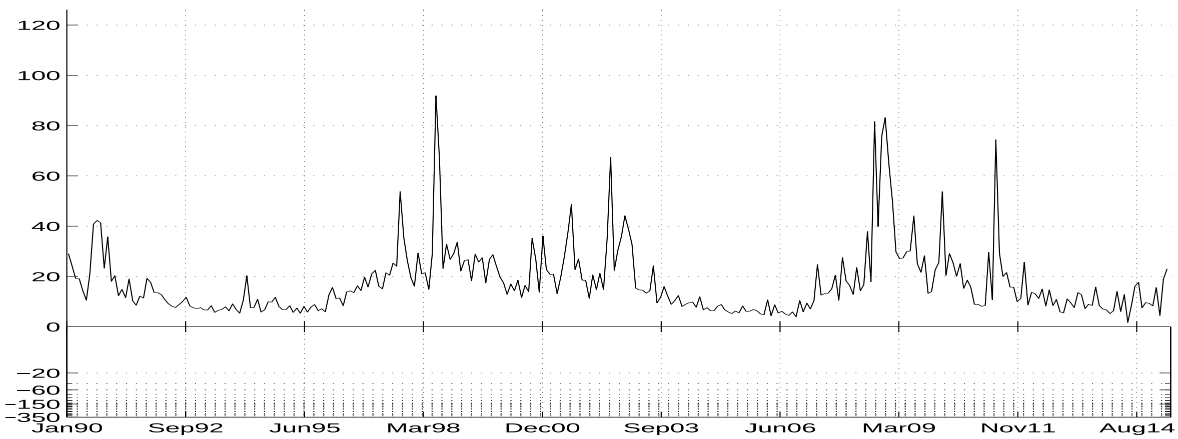

(b) $\Pi_{t,t+1}^{AR(2)}$ – VRP based on AR(2) model for realized variance

Figure 3: VRP based on autoregressive dynamics to forecast realized variance

characterized by a fast rate of mean reversion and upward bias in the unconditional level. This translates into a VRP that shrinks towards, and sometimes crosses, the zero lower bound in periods of calm and upward trending markets, e.g. the period spanning September 2003 and July 2007. Yet, the VRP exhibits extreme positive peaks with a maximum of 125 in periods of market stress. The $\Pi_{t,t+1}^{AR(1)}$ and $\Pi_{t,t+1}^{AR(2)}$ time series almost coincide which is not surprising since the AR(2) specification has an insignificant second lag, see Table 2 which contains parameter estimates for the models fitted to realized variance.




(a) $\Pi^{ARX1}_{t,t+1}$ – VRP based on ARX1 model for realized variance




(b) $\Pi^{ARX2}_{t,t+1}$ – VRP based on ARX2 model for realized variance

Figure 4: VRP based on autoregressive dynamics with VIX to forecast realized variance

The VRP paths for the models including VIX information are given in Figure 4. The $\Pi^{ARX1}_{t,t+1}$ model does not show any substantial difference from the AR(1) model. In fact, the parameter estimates in Table 2 show that the $VIX^2_{t-1,t}$ coefficient is insignificant whereas the other estimated coefficients are very similar to those from the AR(1) model. However, the $\Pi^{ARX2}_{t,t+1}$ model allows to produce variance expectations which are smoother and induce a better behaved VRP. In fact, this is the only model where the VRP is always positive, though still extremely volatile in periods of market turmoil.

Table 2: Times series regressions for $RV_{t,t+1}$

| Model name | Regressors | | $\beta_0$ | $\beta_1$ | $\beta_2$ | Adj.$R^2$ |
|---|---|---|---|---|---|---|
| AR(1) | $RV_{t-1,t}$ | - | 7.122 | 0.647 | | 41.9 |
| | | | (4.272) | (11.522) | | |
| AR(2) | $RV_{t-1,t}$ | $RV_{t-2,t-1}$ | 7.402 | 0.670 | -0.035 | 42.0 |
| | | | (3.877) | (7.440) | (-0.505) | |
| ARX1 | $RV_{t-1,t}$ | $VIX^2_{t-1,t}$ | 7.684 | 0.660 | -0.020 | 42.0 |
| | | | (2.756) | (7.114) | (-0.284) | |
| ARX2 | $RV_{t-1,t}$ | $VIX^2_{t,t+1}$ | 0.111 | 0.381 | 0.328 | 45.5 |
| | | | (0.092) | (4.953) | (4.873) | |

Notes: This table report parameter estimates from time series models for $RV_{t,t+1}$. Newey-West t-statistics are in brackets and adjusted $R^2$s are expressed in percentages. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

Overall, the VRP time series resulting from the existing methods are all too volatile to be proper risk premia and all but one (the $\Pi^{ARX2}_{t,t+1}$) violates the positivity constraint. In fact, all the series show switching phases of persistent versus short memory dynamics, level shifts in the volatility, and characteristics of short lived cluster of spikes. The estimated VRP's are all computed at each point in time using equation (1) without any joint modelling of the risk neutral and physical expectations of the variance. In particular, while dynamics may be used for predicting the expectations under the physical measure the current approaches all ignore modelling explicitly the dynamics under the risk neutral measure. Moreover, because they all estimate $\Pi_{t,t+\tau}$ using a multistep estimation approach the existing methods potentially suffer from inefficiencies and the compounding of estimation errors. In Section 3, we develop a joint model that puts more structure on the relationship between risk neutral and physical variances and as a result yields more refined and realistic dynamics for the VRP.

## 2.3 Return predictability implied by existing VRPs

Being closely tied to the time varying volatility of consumption growth, the VRP has been identified as an important factor in explaining aggregate stock market returns. See for example Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) who demonstrate empirically and theoretically that, in addition to consumption risk, volatility risk plays an important role in generating returns (see also Bonomo, Garcia, Meddahi, and Tédongap (2015) for a framework starting from daily rather than a monthly frequency). The natural question to ask then is whether the different ways of computing the VRP series deliver differences in return predictability?

Table 3 provides several return predictability regressions using as explanatory variable each $\Pi^i_{t,t+1}$, for $i = RW, AR(1), AR(2), ARX1$ and $ARX2$ respectively. Predictability is measured by the adjusted $R^2$ from regressions of the following type

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+j} = a_0(h) + a_1(h)\Pi^i_{t,t+1} + u_{t+h,t}, \tag{3}$$

where $h$ denotes the forecast horizon, $r_t$ denotes the monthly excess return for month $t$, and $\Pi^i_{t,t+1}$ is one of the variance risk premia estimators described above. Using data from Datastream, we construct the monthly future aggregated broad market returns in excess of the three-month T-bill rate over eight horizons from one month up to one year.

In line with Bollerslev, Tauchen, and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014), for $\Pi^{RW}_{t,t+1}$ we find an inverse U-shaped pattern in the $R^2$ over the forecasting horizons, peaking at 4 months with an adjusted $R^2$ of 12.7%. The table also shows that refining the physical expectations of $QV_{t,t+1}$ by an autoregressive model preserves the inverse U-shaped pattern but uniformly lowers the $R^2$ values, e.g. the maximum value being 6.8% with the AR assumption instead of 12.7% with the RW assumption. In fact, while there seems to be no difference in return predictability for the nine and twelve month horizon, the smoothing implied by the AR(1) model halves the $R^2$s up to the six month

Table 3: Return predictability with existing VRPs

| $\Pi^i_{t,t+1}$ | | Horizon (h) 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^{RW}_{t,t+1}$ | $a_0(h)$ | -0.549 | -0.407 | -0.413 | -0.377 | -0.247 | -0.124 | 0.091 | 0.165 |
| | | (-2.033) | (-1.641) | (-1.707) | (-1.428) | (-1.056) | (-0.556) | (0.416) | (0.775) |
| | $a_1(h)$ | 0.050 | 0.042 | 0.043 | 0.041 | 0.034 | 0.027 | 0.015 | 0.012 |
| | | (4.073) | (4.538) | (7.901) | (7.844) | (6.589) | (4.645) | (2.611) | (2.161) |
| | Adj.$R^2$ | 5.08 | 6.91 | 10.7 | 12.7 | 10.3 | 7.46 | 3.37 | 2.48 |
| $\Pi^{AR(1)}_{t,t+1}$ | $a_0(h)$ | -0.240 | -0.219 | -0.200 | -0.192 | -0.146 | -0.093 | 0.073 | 0.148 |
| | | (-0.820) | (-0.805) | (-0.770) | (-0.735) | (-0.541) | (-0.311) | (0.243) | (0.494) |
| | $a_1(h)$ | 0.033 | 0.032 | 0.031 | 0.031 | 0.028 | 0.025 | 0.016 | 0.013 |
| | | (1.941) | (2.843) | (3.282) | (4.001) | (4.008) | (3.633) | (2.378) | (1.975) |
| | Adj.$R^2$ | 1.89 | 3.65 | 5.27 | 6.78 | 6.83 | 6.30 | 3.76 | 2.83 |
| $\Pi^{AR(2)}_{t,t+1}$ | $a_0(h)$ | -0.227 | -0.190 | -0.175 | -0.175 | -0.133 | -0.080 | 0.082 | 0.154 |
| | | (-0.783) | (-0.699) | (-0.684) | (-0.690) | (-0.503) | (-0.273) | (0.277) | (0.526) |
| | $a_1(h)$ | 0.032 | 0.030 | 0.030 | 0.030 | 0.027 | 0.024 | 0.016 | 0.012 |
| | | (1.940) | (2.642) | (3.003) | (3.876) | (3.993) | (3.654) | (2.384) | (1.994) |
| | Adj.$R^2$ | 1.90 | 3.42 | 5.02 | 6.66 | 6.77 | 6.20 | 3.70 | 2.80 |
| $\Pi^{ARX1}_{t,t+1}$ | $a_0(h)$ | -0.331 | -0.344 | -0.312 | -0.312 | -0.277 | -0.228 | -0.027 | 0.070 |
| | | (-0.866) | (-1.024) | (-0.995) | (-0.991) | (-0.875) | (-0.668) | (-0.079) | (0.200) |
| | $a_1(h)$ | 0.038 | 0.039 | 0.037 | 0.038 | 0.036 | 0.033 | 0.022 | 0.017 |
| | | (1.545) | (2.128) | (2.384) | (2.927) | (3.229) | (3.155) | (2.174) | (1.795) |
| | Adj.$R^2$ | 1.17 | 2.68 | 3.78 | 5.02 | 5.43 | 5.36 | 3.41 | 2.56 |
| $\Pi^{ARX2}_{t,t+1}$ | $a_0(h)$ | -0.238 | -0.210 | -0.193 | -0.186 | -0.141 | -0.087 | 0.077 | 0.151 |
| | | (-0.825) | (-0.770) | (-0.744) | (-0.720) | (-0.523) | (-0.291) | (0.259) | (0.506) |
| | $a_1(h)$ | 0.033 | 0.031 | 0.031 | 0.031 | 0.028 | 0.025 | 0.016 | 0.013 |
| | | (1.984) | (2.834) | (3.272) | (4.037) | (4.048) | (3.638) | (2.370) | (1.980) |
| | Adj.$R^2$ | 1.97 | 3.68 | 5.34 | 6.92 | 6.96 | 6.37 | 3.78 | 2.87 |

Notes: This table reports parameter estimates from the model in (3) using various VRPs. Newey-West t-statistics are in brackets and adjusted $R^2$s are in percentages. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

horizon. Enriching further the model for predicting $E^P_t[QV_{t,t+1}]$ by including the VIX index in the autoregressive model, the $R^2$ values are similar to the AR models for all forecasting horizons of the predictive return regressions. The only model that yields higher predictive power than the AR(1) model is the ARX2 model but this model still falls short of the RW model except for the very long horizons.

The large predictability obtained when using $\Pi^{RW}_{t,t+1}$ seems somewhat puzzling. In fact, although based on an uninformative model for the physical variance, which is possibly misspecified and contaminated by noise, the VRP has the highest explanatory power compared to the more sophisticated models. On the one hand, this result could simply be due to a spurious relationship. On the other hand, the expectation of the physical variance computed under the random walk assumption embeds the full extent of extreme shocks on the market. The implied VRP from the RW model thus ends up containing information on both normal and extreme market conditions, the contribution of each not being directly measurable. This result is also pointed out by Bollerslev, Todorov, and Xu (2015) who advocate the inclusion of discontinuities in the variance dynamics to separate the normal component of the VRP from jump tail risk, arguing that the two components explain to a different extent aggregate return variation. The model we propose in the next section allow separating normal dynamics from the impact of extreme variance events and consequently the reaction of the agents when forming expectations in a very simple way.

# 3 A dynamic model for variance risk premia

This section provides a flexible model for the dynamics of the VRP. Before introducing the model, we first define more precisely the variables involved in the computation of the VRP in (1) and the properties of the associated estimators. We next set the theoretical framework for the mechanism of formation of expectations and the linkages between the risk-neutral and physical measures based on pricing variance swap contracts under smooth volatility dynamics. Finally, we account for data heterogeneity and extreme volatility events, two features that are empirically present in the data and that bias the estimation of the VRP if not accounted for explicitly. To do this, we generalize the framework by including regimes specifically designed to capture heteroskedasticity and isolate jumps.

## 3.1 Nonparametric measures of quadratic variation

Assuming the asset price process evolves as $d \ln S_t = \Sigma_t^{1/2} dW_t + \int_{\mathbb{R}} x\mu(dt, dx)$, where $\Sigma_t$ is the instantaneous variance and $\mu(dt, dx)$ is a counting measure for the jump in the price, its quadratic variation is defined as

$$\begin{aligned} QV_{t-\tau,t} = \int_{t-\tau}^{t} d(\ln S_t)^2 &= \int_{t-\tau}^{t} \Sigma_s ds + \int_{t-\tau}^{t} \int_{\mathbb{R}} x^2 \mu(dt, dx) \\ &= \Sigma_{t-\tau,t} + J_{t-1,t}^{P}. \end{aligned} \tag{4}$$

A model free estimator of $QV_{t-\tau,t}$, or in the absence of jumps the integrated variance $\Sigma_{t-\tau,t}$, is given by the realized variance, or $RV_{t-\tau,t}$, see Andersen, Bollerslev, Diebold, and Ebens (2001) and Andersen, Bollerslev, Diebold, and Labys (2001), defined as

$$RV_{t-\tau,t} = \sum_{i=1}^{I} \left( \ln(S_{t+i}/S_{t+i-1}) \right)^2, \tag{5}$$

where $S_t$ is the price level at time $t$, and $I$ is the number of observations between $t - \tau$ and $t$. Thus, $RV_{t-\tau,t}$ is a nonparametric estimator of the sum of $\Sigma_{t-\tau,t}$ and the squared jumps. The expectation under the physical measure, $E_t^P[QV_{t,t+\tau}]$, is obtained by projecting $RV_{t-\tau,t}$ using a parametric model.

As shown by Bakshi and Madan (2000), Britten-Jones and Neuberger (2000) and Jiang and Tian (2005), the term $E_t^Q[QV_{t,t+\tau}]$ in (1) represents the strike of a variance swap entered into at time $t$ with maturity $\tau$. As a variance swap may be replicated using a portfolio of (infinitely many) European call and put options with weights inversely proportional to the square of their strike price, the fair variance swap strike can be written as

$$E_t^Q[QV_{t,t+\tau}] = \frac{2}{B_{t,\tau}} \left( \int_0^{\mathcal{F}_t} \frac{P_{t,\tau}(K)}{K^2} dK + \int_{\mathcal{F}_t}^{\infty} \frac{C_{t,\tau}(K)}{K^2} dK \right) + \mathcal{O}\left( \left( \frac{d\mathcal{F}_t}{\mathcal{F}_{t-}} \right)^3 \right), \tag{6}$$

where $B_{t,\tau}$ is the price of a time $t$ zero-coupon bond maturing at time $t+\tau$, $\mathcal{F}_t$ is the forward price of the underlying asset, and $P_{t,\tau}(K)$ and $C_{t,\tau}(K)$ are, respectively, the European put and call option prices with strike price $K$. The first term in (6) represents a model free estimator of the fair price of a variance swap in the absence of jumps, i.e. $E^Q(\Sigma_{t,t+\tau})$. The second term is the compensator for the discontinuous component (i.e. the jumps), see Carr and Wu (2009) for details. In reality we do not have access to infinitely many options and instead an approximation to (6) is given by

$$SW_{t,t+\tau} = \frac{2}{B_{t,\tau}}\left[\sum_{K_i \le K_0} \frac{\Delta K_i}{K_i^2} P_{t,\tau}(K_i) + \sum_{K_i \ge K_0} \frac{\Delta K_i}{K_i^2} C_{t,\tau}(K_i)\right] - \left[\frac{\mathcal{F}_t}{K_0} - 1\right]^2, \tag{7}$$

where $K_0$ is the strike price immediately below the forward price. Since option contracts are available only at fixed maturities not necessarily coinciding with $\tau$, $SW_{t,t+\tau}$ can be estimated as a linear interpolation of (7) computed using near and next maturity options. Its square root is the well known VIX trademarked by the CBOE in 1993, see CBOE (2015).

Following Bollerslev, Todorov, and Xu (2015), we decompose the VRP in (1) as

$$\begin{aligned}
\Pi_{t,t+1} &= E_t^Q[QV_{t,t+\tau}] - E_t^P[QV_{t,t+\tau}] \\
&= \left(E_t^Q[\Sigma_{t,t+1}] - E_t^P[\Sigma_{t,t+1}]\right) + \left(E_t^Q[J_{t,t+1}^Q] - E_t^P[J_{t,t+1}^P]\right) \\
&= \Pi_{t,t+1}^C + \left(E_t^Q[J_{t,t+1}^Q] - E_t^P[J_{t,t+1}^P]\right). \qquad (8)
\end{aligned}$$

The variable $\Pi_{t,t+1}^C$ represents the smooth component of the VRP, i.e. the part of the variance risk premium attributable to normal sized price fluctuations. The upperscript $C$, which stands for continuous, stresses the absence of discontinuities in the variance process. The term in brackets in (8) refers to the jump risk premium, see for instance Ait-Sahalia, Karaman, and Mancini (2015). The decomposition in (8) finds justification in the fact that, while the no arbitrage condition restricts the smooth component of the volatility process $\Sigma_{t-1,t}$ to be the same under the physical and the risk neutral measures, it puts no restrictions on jump dynamics.

## 3.2 Smooth dynamics

In this section, we focus on the smooth component of the VRP defined as

$$\Pi^C_{t,t+1} = E^Q_t[\Sigma_{t,t+1}] - E^P_t[\Sigma_{t,t+1}]. \tag{9}$$

Discontinuities in the price and volatility process will be explicitly introduced in the next section. In the absence of market microstructure effects $RV_{t-1,t}$ consistently estimates $QV_{t-1,t}$ and in the absence of jumps $\Sigma_{t-1,t}$ and we assume that the realized variance can be written as

$$RV_{t-1,t} = \Sigma_{t-1,t} + \varepsilon^{RV}_t, \tag{10}$$

where $\varepsilon^{RV}_t$ is a homoskedastic measurement error, independently and identically distributed (i.i.d.) with zero mean and with $\text{Var}(\varepsilon^{RV}_t) = \sigma^2_{RV}$. See for instance Barndorff-Nielsen and Shephard (2002). We also assume that $SW_{t,t+1}$ is an unbiased estimator of $E^Q_t[\Sigma_{t,t+1}]$ such that $SW_{t,t+1} = E^Q_t[\Sigma_{t,t+1}] + \varepsilon^{SW}_t$. The estimation error $\varepsilon^{SW}_t$ is also an i.i.d. zero mean noise with $\text{Var}(\varepsilon^{SW}_t) = \sigma^2_{SW}$ originating from e.g. discretisation of (6). Then by using (9) we have that

$$SW_{t,t+1} = \Pi^C_{t,t+1} + E^P_t[\Sigma_{t,t+1}] + \varepsilon^{SW}_t, \tag{11}$$

where the stochastic component $\Pi^C_{t,t+1}$ acts as a wedge between the physical and risk neutral expectations of the variance. The model can be naturally interpreted as a structural model for risk neutral expectations. Given current and past information on the physical market, agents construct expectations about the risk and they price it. The decomposition of $SW_{t,t+1}$ in a stochastic latent factor and measurement error can be also justified using the arguments of Andersen, Bondarenko, and Gonzalez-Perez (2015).

We now specify suitable dynamics for the latent states. First, without loss of generality, we assume that $\Sigma_{t-1,t}$ is a stationary AR(1) process, i.e., $\Sigma_{t-1,t} = b + \phi^\Sigma(\Sigma_{t-2,t-1} - b) + \sigma_\Sigma v^\Sigma_t$

with $v_t^\Sigma \sim i.i.d.\, N(0,1)$, although this can be easily relaxed to allow for more flexible dynamics. As can be cleary seen in the state space formulation below, the autoregressive dynamics of $\Sigma_{t-1,t}$ allow us to structurally embed the variance expectation under the physical measure into the system by substituting $E_t^P[\Sigma_{t,t+1}] = b + \phi^\Sigma(\Sigma_{t-1,t} - b)$ into (11). Second, we specify the latent state $\Pi^C_{t,t+1}$ as

$$\Pi^C_{t,t+1} = F_{t,t+1} + d(\Sigma_{t-1,t} - b), \tag{12}$$

i.e., the sum of a dynamic factor $F_{t,t+1}$, idiosyncratic to $E_t^Q[\Sigma_{t,t+1}]$, and a term that captures the contemporaneous relationship between $\Pi^C_{t,t+1}$ and $\Sigma_{t-1,t}$.

Note that the common latent factor $\Sigma_{t-1,t}$ enters the dynamics of the risk neutral expectation of the variance in two ways: i) directly through the expectation under the physical measure and ii) indirectly through the VRP allowing, as advocated by Bollerslev, Gibson, and Zhou (2011), for correlation between the variance premium and the variance itself. In our empirical application, we assume the idiosyncratic factor $F_{t,t+1}$ to be a stationary AR(1) process, i.e. $F_{t,t+1} = a + \phi^F(F_{t-1,t} - a) + \sigma_F v_t^F$ with $v_t^F \sim i.i.d.\, N(0,1)$. As $E_t^Q[\Sigma_{t,t+1}]$ results from the contemporaneous aggregation of two AR(1) processes, it inherits a longer persistence then $\Sigma_{t-1,t}$, mimicking the behaviour documented in Andersen, Bollerslev, and Diebold (2007) and Andersen, Fusari, and Todorov (2015a).

Concentrating the parameters in the measurement system (10)-(11) and expressing the transition equations system in terms of the latent states $\Sigma_{t-1,t}$ and $F_{t,t+1}$, we can write the model in the following state-space formulation

$$\begin{bmatrix} RV_{t-1,t} \\ SW_{t,t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ h^{(12)} & 1 \end{bmatrix} \begin{bmatrix} \Sigma_{t-1,t} \\ F^*_{t,t+1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{RV} \\ \varepsilon_t^{SW} \end{bmatrix}, \tag{13}$$

with

$$\begin{bmatrix} \Sigma_{t-1,t} \\ F^*_{t,t+1} \end{bmatrix} = \begin{bmatrix} k^{(1)} \\ k^{(2)} \end{bmatrix} + \begin{bmatrix} \phi^\Sigma & 0 \\ 0 & \phi^F \end{bmatrix} \left( \begin{bmatrix} \Sigma_{t-2,t-1} \\ F^*_{t-1,t} \end{bmatrix} - \begin{bmatrix} k^{(1)} \\ k^{(2)} \end{bmatrix} \right) + \begin{bmatrix} \sigma_\Sigma & 0 \\ 0 & \sigma_F \end{bmatrix} \begin{bmatrix} v_t^\Sigma \\ v_t^F \end{bmatrix}, \tag{14}$$

where $F^*_{t,t+1} = F_{t,t+1} + b(1 - \phi^\Sigma - d) = F_{t,t+1} + k^{(2)} - a$, $h^{(12)} = \phi^\Sigma + d$, $k^{(1)} = b$, $k^{(2)} = a + b(1 - \phi^\Sigma - d)$. In general terms, this state-space formulation represents a latent common factor model with $\Sigma_{t-1,t}$ being the factor linking the physical variance dynamics and its risk-neutral expectation. Central to the estimation of the VRP, the latent factor $F_{t,t+1}$ represents the idiosyncratic component of its smooth dynamics. Note that the substitution of $E^P_t[\Sigma_{t,t+1}]$ into (11) is justified by the fact that for a linear state-space system the Kalman filter delivers the best linear predictions of the state vector, conditionally on the observations, for a correctly specified model. Furthermore, in the particular case of Gaussian innovations in both the state and measurement equations, the one-step-ahead prediction of the state vector coincide with the conditional expectations.

### 3.3 Heteroskedasticity and extreme variance events

When confronted with real data, the simple model above should be enriched by accounting for discontinuities, or jumps, in the variance dynamics and heteroskedasticity. In this paper we propose to introduce heteroskedasticity and jumps by means of a regime switching setup making the model parameters regime dependent. As we want to accommodate separately for regime dependence both in the level and in the variance of the variance (both historical and its risk neutral expectation) as well as jumps, we work with a three-regime model with the following characteristics:

-*Regime 1*: Calm markets (low variance level/low variance of variance/high measurement accuracy). The error vector $(\varepsilon^{RV}_t, \varepsilon^{SW}_t)$ represents a pure measurement error.

-*Regime 2*: Volatile markets (high variance level/high variance of variance/low measurement accuracy). The error vector $(\varepsilon^{RV}_t, \varepsilon^{SW}_t)$ can be interpreted as in regime 1. We advocate that market turmoil hampers the measurement accuracy of both the physical variance and its risk neutral expectation, i.e., the measurement error variance increases. We also postulate that higher levels of $\Sigma_{t-1,t}$ and $\Pi^C_{t,t+1}$ correspond to higher variance of the latent state errors. The change in the variance of the latent state errors of $\Sigma_{t-1,t}$ and $\Pi^C_{t,t+1}$ accounts for the heteroskedastic behaviour of the variance.

-*Regime 3*: Jumps (extreme variance events/extreme variance of variance/rare and short lasting episodes). The error vector $(\varepsilon_t^{RV}, \varepsilon_t^{SW})$ is a combination of measurement error and variance of the discontinuous component. The average size of the jump is reflected in the marginal increase in the variance level from regime 2. As the measurement error variance of the observables and the jump variance cannot be individually identified, we assume that the jump component dominates and impose the restriction that the measurement error variance of the observables remains the same between regime 2 and regime 3.

We define a regime indicator $S_t = \{0, 1, 2\}$ and assume that it is a first order Markov Chain with probability transition matrix $P$ given by

$$P = \begin{pmatrix} p_{00} & p_{01} & 0 \\ p_{10} & p_{11} & p_{12} \\ 0 & p_{21} & p_{22} \end{pmatrix}.$$

The zero's indicating the fact that we only allow departures from regime 1 to regime 2 and back and from regime 2 to regime 3 and back. This assumption allows us to clearly identify the particular features of the regimes stressed above. In particular, allowing only departures from regime 2 to regime 3 and back guarantees that the extreme variance regime constitutes systematically a departure from regime 2. In a similar sense, by allowing only departures from regime 1 to regime 2 and back we are able to accommodate heteroskedastic behaviour of the variance. Note that this assumption is in fact supported by the data in our empirical application.

We now generalize the model to account for jumps by including the second term in (8) which quantifies the jump risk premium. Modelling explicitly the jump risk premium requires the expectation of the jump process under the physical measure, which can be attained only by imposing tightly parameterized jump dynamics, see for example Bardgett, Gourier, and Leippold (2015). However, as stressed by Andersen, Fusari, and Todorov (2016), jumps are rare and heterogenous in size leading to imprecise inference. To avoid this problem, we instead model discontinuities in the variance process by considering separately the jump in the historical variance and in the risk-neutral expectation. The measurement system (10)-(11) can be written as

$$RV_{t-1,t} = \Sigma_{t-1,t} + J^P_{t-1,t} + \varepsilon^{RV}_t, \text{ and} \tag{15}$$

$$SW_{t,t+1} = \Pi^C_{t,t+1} + E^P_t[\Sigma_{t,t+1}] + E^Q_t[J^Q_{t,t+1}] + \varepsilon^{SW}_t. \tag{16}$$

The system (15)-(16) allows us to exploit the temporal causality that links the occurrence of extreme shocks in the spot market to the agents' reactions in the option market and thus infer agents' attitude towards risk and their sentiment about future market conditions. We elaborate more on this in Section 4.

The dynamics of the smooth component of the variance takes the form

$$\Sigma_{t-1,t} = b_{S_t} + \phi^{\Sigma}(\Sigma_{t-2,t-1} - b_{S_{t-1}}) + \sigma_{\Sigma,S_t} v^{\Sigma}_t, \tag{17}$$

with $v^{\Sigma}_t \sim i.i.d.\, N(0,1)$. To model the heteroskedasticity in $\Sigma_{t-1,t}$, captured by movements between the base regime ($S_t = 0$) and the high level-high variance regime ($S_t = 1$), we impose the identifying restrictions $b_0 \leq b_1 = b_2$ and $\sigma_{\Sigma,0} < \sigma_{\Sigma,1} = \sigma_{\Sigma,2}$. Similarly, the smooth component of the variance risk premium $\Pi^C_{t,t+1}$ is defined as

$$\Pi^C_{t,t+1} = F_{t,t+1} + d_{S_t}(\Sigma_{t-1,t} - b_{S_t}), \tag{18}$$

with

$$F_{t,t+1} = a_{S_t} + \phi^F(F_{t-1,t} - a_{S_{t-1}}) + \sigma_{F,S_t} v^F_t, \tag{19}$$

where $v^F_t \sim i.i.d.\, N(0,1)$. Since the smooth component moves between regimes 0 and 1, we impose the identifying restrictions $a_0 \leq a_1 = a_2$ and $\sigma_{F,0} < \sigma_{F,1} = \sigma_{F,2}$. Note that the extension of (10)-(11) to a regime dependent model defined in (15)-(16) naturally assumes that physical and risk neutral expectations are driven by the same regime indicator $S_t$ and that agents make expectations given the current regime. This allows direct computation of the physical expectation of $\Sigma_{t-1,t}$ in (16).

The jumps in $RV_{t-1,t}$ and $SW_{t,t+1}$ are captured by the third regime ($S_t = 2$). For the jump process under the physical dynamics, we assume basic dynamics characterized as a random additive shock, i.e., a constant equaling the average jump size, plus noise,

$$J^P_{t-1,t} = j^P_{S_t} + \sigma_{J^P,S_t} u^P_t = \begin{cases} 0 & S_t = 0, 1 \\ j^P + \sigma_{J^P} u^P_t & S_t = 2, \end{cases} \tag{20}$$

with $u^P_t \sim i.i.d.\, N(0, 1)$. The advantage of (20) is that it can adequately fit in a realistic and parsimonious manner rare and short lasting extreme variance episodes. Similarly, for the expectation of the jump process under the risk neutral measure, we assume

$$E^Q_t[J^Q_{t,t+1}] = j^Q_{S_t} + \sigma_{J^Q,S_t} u^Q_t = \begin{cases} 0 & S_t = 0, 1 \\ j^Q + \sigma_{J^Q} u^Q_t & S_t = 2, \end{cases} \tag{21}$$

with $u^Q \sim i.i.d.\, N(0, 1)$. Note that this setup assumes that discontinuities occur simultaneously in the physical and risk neutral variances. That is, if a shock occurs in the physical market at a point in time (of size $J^P_{t-1,t}$), this generates a reaction in the risk neutral market, and prices in the latter will adjust accordingly (by an amount $E^Q_t[J^Q_{t,t+1}]$).

Finally, the regime dependent error terms $\varepsilon^{RV}_t$ and $\varepsilon^{SW}_t$ are expressed as

$$\varepsilon^{RV}_t = \sigma_{RV,S_t} e^{RV}_t, \text{ and} \tag{22}$$

$$\varepsilon^{SW}_t = \sigma_{SW,S_t} e^{SW}_t, \tag{23}$$

with $e^{RV}_t \sim i.i.d.\, N(0, 1)$ and $e^{SW}_t \sim i.i.d.\, N(0, 1)$. As discussed above, in order to identify the variance of the jump processes, we impose the restriction $\sigma_{i,0} < \sigma_{i,1} = \sigma_{i,2}$, $i = RV, SW$. The restriction implies that in periods of instability the accuracy of the estimated proxies,

$RV_{t-1,t}$ and $SW_{t,t+1}$, for their latent counterparts deteriorates. However, the restriction impose the same variance of the measurement error in the high level-high variance regime ($S_t = 1$) and the jump regime ($S_t = 2$), implying that the quality of the measurements remains invariant to the occurrence of extreme shocks.

In summary, by accounting explicitly for the occurrence of extreme variance events and filtering out measurement error noise for the variables of interest, our approach allows for a more precise estimation of the VRP. Also, being relatively simple and flexible, the model can easily account for features advocated by many authors and typically evinced from the data such as uncertainty deriving from heteroskedasticity, dynamic characteristics such as level shifts, persistence as well as time varying correlation structure and dependence between the VRP, the risk neutral and the physical variance. In the Appendix, we explain how the model can be written in state-space form and how it can be estimated with maximum likelihood using filtering techniques.

# 4 Return predictability and investors' sentiment

As discussed in Section 2, the existing VRPs have been shown to be important predictors of future aggregate stock market returns. However, the results from applying these existing models empirically offer an interesting puzzle as models with more realistic dynamics for the variance process fall short in terms of offering additional predictability compared to a simpler yet unrealistic model that assumes a random walk specification for the physical variance. In the literature it has been argued that this finding is caused by the fact that VRP identified by the existing methods in fact contains two distinct components: one that reflects compensation for continuous price moves and one that is related to compensation for disaster risk (see e.g. Bollerslev and Todorov (2011)). For example, Bollerslev, Todorov, and Xu (2015) provide evidence that these two components explain to a different extent aggregate return variation.

Besides estimating the smooth component associated with the price of risk during normal market activity, $\Pi^C_{t,t+1}$ representing the premium associated with normal market activity, the model described in Section 3 allows to identify two other fundamental quantities essential to explicitly account for the occurrence of extreme variance events. The first of these is the occurrence and the size of the extreme variance events under the physical dynamics, $J^P_{t-1,t}$ and the second is the risk neutral expectation of future extreme variance events which represents the reaction of agents to shocks on the market, $E^Q_t[J^Q_{t,t+1}]$. We now explain how these variables can be combined to construct attitude and sentiment indicators based on agents' expectations response to extreme shocks in the market.

While our model yields the path of the smooth component of the variance risk premium $\Pi^C_{t,t+1}$, it also allows to extract the jump indicator $I_{S_t=2}$. Using the notation in Kim and Nelson (1999), the indicator function equals one if $\max\Big(P(S_t = j|\psi_t)\Big) = P(S_t = 2|\psi_t)$, where $P(S_t = j|\psi_t)$ is the posterior probability of state $j = 0, 1, 2$ and $\psi_t$ the information set up to and including $t$. Using the filtered states $\Sigma_{t-1,t}$ and $\Pi_{t,t+1}$ together with the indicator $I_{S_t=2}$, we can estimate, up to some measurement error, the size of the jump components $J^P_{t-1,t}$ and its risk neutral expectation $E^Q_t[J^Q_{t,t+1}]$. To measure the type and size of agents' reaction to the occurrence of extreme variance events, we define the quantity

$$\Delta J_t = E^Q_t[J^Q_{t,t+1}] - J^P_{t-1,t}. \tag{24}$$

The interpretation of $\Delta J_t$ relates to the jump risk premium, i.e. $E^Q_t[J^Q_{t,t+1}] - E^P_t[J^P_{t,t+1}]$. In fact, $\Delta J_t$ can be written as the sum of the expected jump differential, i.e. $E^P_t[J^P_{t,t+1} - J^P_{t-1,t}]$, and the jump risk premium. Since the latter represents the price that agents attach to extreme events, $\Delta J_t$ contains direct information about the response of agents to the current market environment.

The sign of $\Delta J_t$ indicates if a shock in the market is incorporated into the risk neutral expectations more or less than proportionally. We use this quantity to assess agents' different attitude toward risk and thus their sentiment towards future market states. Agents which react more than proportionally to an extreme shock on the market ($\Delta J_t > 0$), i.e. agents which become more conservative when forming variance expectations, signal an increase in risk aversion as the observed shock is expected to produce a long lasting aftermath. We define this as fear and denote it by $\Delta J_t^+$. Conversely, a reaction that is less than proportional to the observed shock ($\Delta J_t < 0$), signals a reduction in risk aversion. More liberal variance expectations, after an abnormally large variance shock occurs, indicate that the impact of the extreme event is expected to die out fast. We define this as optimism and denote it by $\Delta J_t^-$.

Turning to the predictability of future returns on the broad market portfolio, in a related framework Bollerslev, Todorov, and Xu (2015) attribute to a large extent such return predictability to the left jump tail variation that they use as a proxy for the market fear. We argue that not only the direction and the size of the agents' reaction to extreme shock to the market has a relevant effect on future market performances but also that the effect may be asymmetric and likely to be systematically related to the current market performance at the moment the shock occurs. This hypothesis relates to the concept of good vs. bad volatility developed in Patton and Sheppard (2015). To assess whether agents price the fear and optimism risk factors asymmetrically with respect to the state of the market, we combine the two sentiment indicators with the sign of the aggregate market performance realized at the time expectations are formed. A fear episode in conjunction with a bad market performance, i.e. a situation in which the excess market return is negative ($r_t < 0$), is denoted by $\Delta J^+_{t,r_t^-}$, while the mirroring case of fear for future extreme variance events associated to bullish markets ($r_t > 0$) is denoted $\Delta J^+_{t,r_t^+}$. Similarly, optimism in a state of bearish markets is denoted $\Delta J^-_{t,r_t^-}$, while optimism in conjunction with rallying markets is denoted $\Delta J^-_{t,r_t^+}$. Table 4 summarizes the sentiment indicators and their interaction with

Table 4: Attitude and sentiment indicators associated with extreme market events

| Indicator | $SI_j$ | Market performance | Response of Expect. to Jump | Size of the reaction | Construction of $SI_i$ |
|---|---|---|---|---|---|
| Fear | $\Delta J_t^+$ | | $\Delta J_t > 0$ | $\lvert\Delta J_t\rvert$ | $\lvert\Delta J_t\rvert I_{\Delta J_t>0}$ |
| Optimism | $\Delta J_t^-$ | | $\Delta J_t < 0$ | | $\lvert\Delta J_t\rvert I_{\Delta J_t<0}$ |
| Fear&Bear | $\Delta J_{t,r_t^-}^+$ | $r_t < 0$ | $\Delta J_t > 0$ | $\lvert\Delta J_t\rvert$ | $\lvert\Delta J_t\rvert I_{\Delta J_t>0} I_{r_t<0}$ |
| Optim.&Bear | $\Delta J_{t,r_t^-}^-$ | | $\Delta J_t < 0$ | | $\lvert\Delta J_t\rvert I_{\Delta J_t<0} I_{r_t<0}$ |
| Fear&Bull | $\Delta J_{t,r_t^+}^+$ | $r_t > 0$ | $\Delta J_t > 0$ | $\lvert\Delta J_t\rvert$ | $\lvert\Delta J_t\rvert I_{\Delta J_t>0} I_{r_t>0}$ |
| Optim.&Bull | $\Delta J_{t,r_t^+}^-$ | | $\Delta J_t < 0$ | | $\lvert\Delta J_t\rvert I_{\Delta J_t<0} I_{r_t>0}$ |

the state of the market.

Bollerslev, Todorov, and Xu (2015) find evidence that volatility is strongly related to the direction of market performance. Thus, in terms of impact of the sentiment indicators in the predictive regression model, we expect the overreaction of expectations to shocks to strongly correlate with future returns at all horizons, especially in conjunction with negative returns. In contrast, we expect the impact to be less pronounced when the shock is perceived as transitory or it is associated to a positive market performance.

# 5 VRP and the S&P500 index

In our first empirical application we estimate our model on data for the S&P 500 index, the standard index used in the literature and the data for which we reported descriptive statistics and results using previous models in the literature in Section 2. We first provide parameter estimates for our new model and illustrate the filtered states with a comparison to what has been done in the literature as explained in Section 2. We next examine the implied sentiment indicators and their implications for return predictability. Finally, we demonstrate that these indicators are not simply proxying for omitted economic variables.

## 5.1 Parameter estimates and smooth component dynamics

Table 5 reports parameter estimates for our model showing that most parameters are estimated statistically significant and the estimates support our specification. In particular, the change in the level and variance of the latent factors between the base regime and the high level/high variance of the variance regime is large. For example, the variance of the signals increase by a factor of about six for $F_{t,t+1}$ and five for $\Sigma_{t-1,t}$, respectively, and this accounts for the uncertainty deriving from heteroskedasticity. Moreover, the large marginal increase in the measurement error variance between the second and third regime, which identifies the jump processes variances ($\sigma^2_{JP}$ and $\sigma^2_{JQ}$ respectively), suggests that the filter adequately identifies the extreme variance events. Finally, the transition probability matrix reveals a highly persistent base regime, while the other two are relatively short lived, with expected durations equal 7.24, 2.44 and 2.40 months, respectively. The steady state probabilities for the regimes are equal to 60.8%, 29.8% and 9.2%, respectively, showing that jumps, while rare, occur around 10% of the time. We also estimate the model with an unrestricted transition probability matrix but a standard likelihood ratio test supports the restriction we impose on our model in Section 3.

Table 5 also shows that with respect to the dynamics of the latent states, the stationary idiosyncratic component of $\Pi^C_{t,t+1}$, i.e., $F_{t,t+1}$, exhibits a strong persistence, with an autoregressive coefficient of 0.945, as well as the common factor, $\Sigma_{t-1,t}$, with an autoregressive coefficient of 0.875. For the latter, the effect of filtering out the extreme observations, which allows extracting the smooth component of the process and correctly estimating the mean reversion of the physical variance, is particularly striking as the equivalent coefficient of an AR(1) estimated directly on $RV_{t-1,t}$ equals 0.653, see Table 2. The reason for this is that in our model the extreme variance events in the jump regime gets substantially

Table 5: Parameter estimates for our model

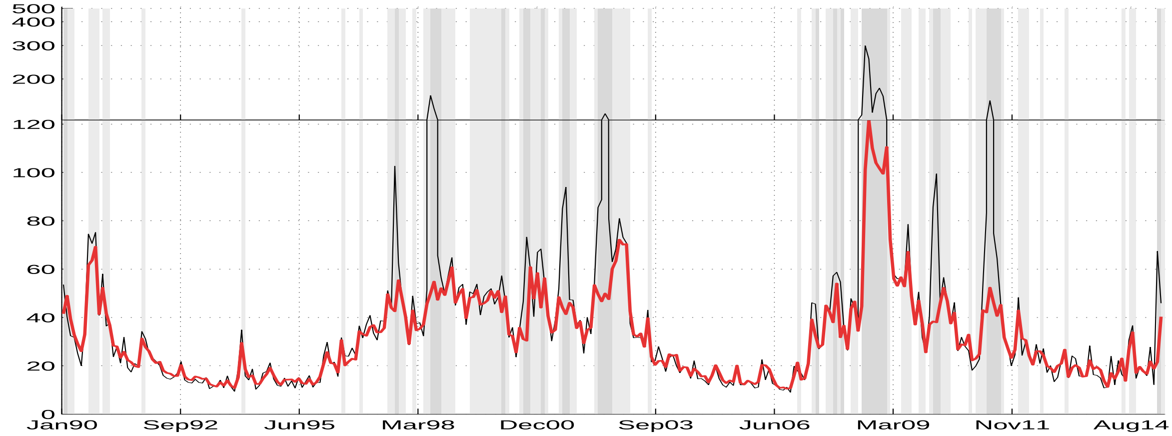
| State Var. | Parameter | Estimate | Standard error | t-statistic |
|---|---|---|---|---|
| $\Sigma_{t-1,t}$ | $b_0$ | 8.184 | 1.165 | 7.023 |
| | $b_1,\ b_2$ | 15.575 | 1.778 | 8.758 |
| | $\phi^{\Sigma}$ | 0.875 | 0.027 | 31.814 |
| | $\sigma^2_{\Sigma,0}$ | 3.034 | 0.861 | 3.521 |
| | $\sigma^2_{\Sigma,1},\ \sigma^2_{\Sigma,2}$ | 14.029 | 4.850 | 2.892 |
| $\Pi^C_{t,t+1}$ | $a_0$ | 12.164 | 1.785 | 6.813 |
| | $a_1,\ a_2$ | 15.426 | 3.084 | 5.001 |
| | $\phi^C$ | 0.945 | 0.026 | 35.358 |
| | $d_0$ | 0.785 | 0.190 | 4.133 |
| | $d_1$ | 1.255 | 0.279 | 4.488 |
| | $d_2$ | 1.131 | 0.283 | 3.990 |
| | $\sigma^2_{C,0}$ | 0.770 | 0.719 | 1.071 |
| | $\sigma^2_{C,1},\ \sigma^2_{C,2}$ | 4.135 | 3.632 | 1.138 |
| $J^P_{t-1,1}$ | $j^P$ | 43.392 | 5.126 | 8.464 |
| | $\sigma^2_{J^P}$ | 502.954 | 163.625 | 3.073 |
| $E^Q_t[J^Q_{t,t+1}]$ | $j^Q$ | 43.541 | 6.417 | 6.784 |
| | $\sigma^2_{J^Q}$ | 750.955 | 233.115 | 3.221 |
| $\varepsilon^{RV}_t$ | $\sigma^2_{RV,0}$ | 1.901 | 0.627 | 3.027 |
| | $\sigma^2_{RV,1},\ \sigma^2_{RV,2}$ | 60.448 | 13.244 | 4.564 |
| $\varepsilon^{SW}_t$ | $\sigma^2_{SW,0}$ | 10.968 | 1.983 | 5.529 |
| | $\sigma^2_{SW,1},\ \sigma^2_{SW,2}$ | 48.197 | 16.859 | 2.858 |
| $P$ | $p_{00}$ | 0.862 | 0.031 | 26.999 |
| | $p_{11}$ | 0.590 | 0.076 | 7.761 |
| | $p_{12}$ | 0.128 | 0.039 | 3.234 |
| | $p_{22}$ | 0.584 | 0.101 | 5.744 |
| Log-likelihood | | | | 1506.321 |

Notes: This table provides maximum likelihood estimates for our model. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

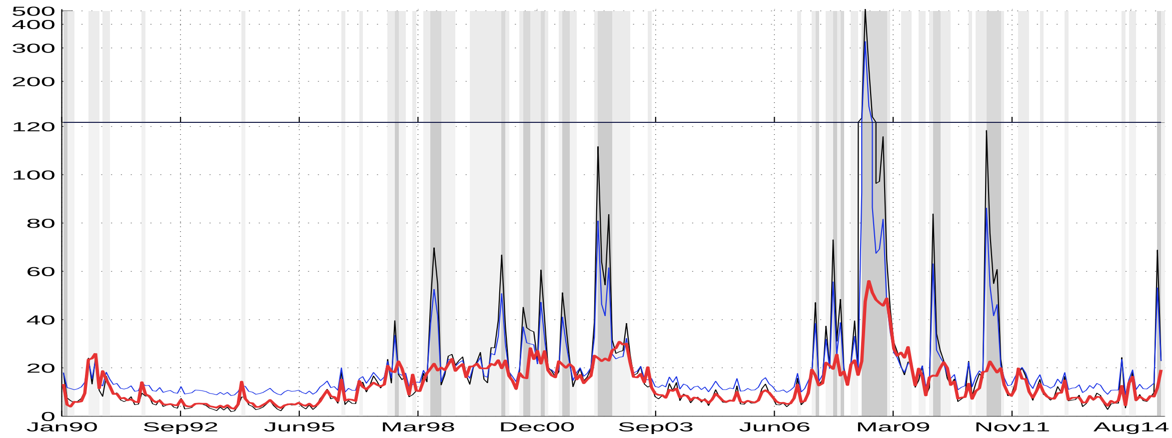
down-weighted in the updating step of the estimation of the latent states $\Pi^C_{t,t+1}$ and $\Sigma_{t-1,t}$. The usual negative bias on the autoregressive coefficient typically due to the presence of extreme realizations appears clearly, and as discussed below, has serious implications on the estimation of the VRP. Also, the link between $\Sigma_{t-1,t}$ and $\Pi^C_{t,t+1}$ shows evidence of time variation especially when shifting between regime 1 and 2, as indicated by the difference $d_1 - d_0 = 0.470$.

Figure 5 plots the filtered latent states, $\Sigma_{t-1,t}$ and $\Pi^C_{t,t+1}$, from our model as well as their combination into the latent risk neutral variance expectation. For comparison, we also consider two alternative models for producing the variance expectations under the physical measure, the random walk and the AR(1), see also Section 2. The first thing to note from the figure is that the implied latent state representing the risk neutral variance expectations, see Figure 5(a) and the variance predictions, see Figure 5(b) (red line), match adequately the level and the dynamics of the variance and its risk neutral expectation along the entire sample. In fact, the variance predictions correctly identify the occurrence of extreme variance episodes and separate the jump from the smooth component. Figure 5(b) also shows that, though providing reasonable predictions in periods of calm markets, when extreme variance realizations occur (indicated by the dark grey shaded areas in the figure), the random walk model (black line) suffers the problem of entirely projecting these extreme realizations into the prediction. The AR(1) model (blue line) on the other hand overestimates the model implied long term variance in calm periods like the periods from February 1991 to December 1996 or from October 2003 to February 2007 and yet still spikes up when extreme variance episodes occur because of the recursive nature of the model.

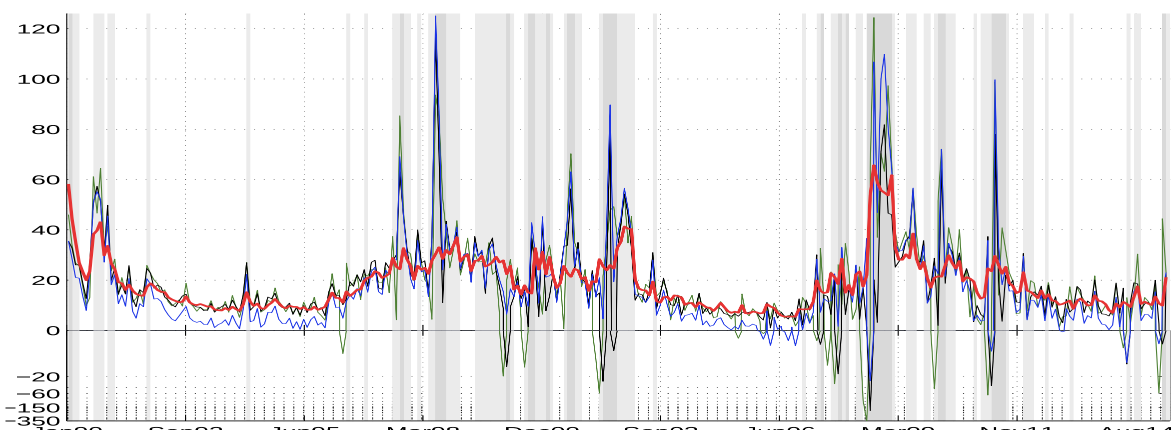
Finally, Figure 5(c) (red line) shows that refining the estimation of the variance expectations under the physical dynamics and that of its risk neutral expectations, by explicitly accounting for heterogeneity and jumps, the smooth component of the variance risk pre-




(a) Squared VIX (black) vs. filtered risk neutral expectations (red)




(b) RV predictions (red) vs. AR(1) (blue) and RW (black)




(c) Smooth component of the VRP, $\Pi^{C}_{t,t+1}$, (red) vs. $\Pi^{AR(1)}_{t,t+1}$ (blue), $\Pi^{RW}_{t,t+1}$ (black) and variance swap payoff (green)

Figure 5: Risk-neutral variance expectations, realized variances and variance risk premia

mium always satisfies the positivity constraint as it ranges from 5.43 to 65.68 with a persistence of 0.95. Furthermore, the variance risk premium increases sensibly, systematically and promptly in periods of market instability, see for instance July and September 2002 (Stock market downturn of 2002), September 2008 (subprime crisis peak), May 2010 (Flash crash) or August 2011 (European sovereign debt crisis peak and US downgrading) among other episodes. Thus, the filtered variance risk premium from our model shows a reasonable degree of smoothness, appropriately reacts to changes in level and variability and, as quantified in Table 5, reflects realistic dynamics and the expected degree of persistence. The RW and AR(1) models on the other hand both deliver variance risk premia estimates which are too imprecise, volatile and in some instances counterintuitive. In particular, the RW model implies a VRP which is sharply reduced and may even become negative in periods of market turmoil when the premium for bearing risk should be the highest and the AR(1) model implies a VRP which shrinks towards zero, eventually crossing the zero lower bound, in periods of calm markets.

## 5.2 Return predictability

The previous section estimates the smooth component of the VRP, i.e. $\Pi^C_{t,t+1}$, the occurrence and the size of the extreme variance events under the physical dynamics, i.e. $J^P_{t-1,t}$, and the risk neutral expectation of future extreme variance events which represents the reaction of agents to shocks on the market, i.e. $E^Q_t[J^Q_{t,t+1}]$. In this section we use the latter two variables to build the set of attitude and sentiment indicators described in Section 4 and we empirically assess their predictive ability for the equity premium. In particular, we argue that the predictability previously attributed to $\Pi^i_{t,t+1}$ stems to a large extent from the effect of rare and extreme events as well as the reaction of agents to those events. To

Table 6: Descriptive statistics for sentiment indicators and their interaction with the market performance

| $SI_j$ | Indicator | Mean | Std. deviation | Minimum | Maximum | N. obs. |
|---|---|---|---|---|---|---|
| $\Delta J_t^+$ | Fear | 36.025 | 21.947 | 2.071 | 86.785 | 12 |
| $\Delta J_t^-$ | Optimism | 34.747 | 48.452 | 2.791 | 229.810 | 20 |
| $\Delta J_{t,r_t^-}^+$ | Fear&Bear | 39.643 | 22.429 | 11.849 | 86.785 | 9 |
| $\Delta J_{t,r_t^-}^-$ | Optim.&Bear | 41.128 | 56.292 | 8.526 | 229.810 | 14 |
| $\Delta J_{t,r_t^+}^+$ | Fear&Bull | 25.172 | 20.036 | 2.071 | 37.809 | 3 |
| $\Delta J_{t,r_t^+}^-$ | Optim.&Bull | 19.860 | 17.378 | 2.791 | 50.709 | 6 |

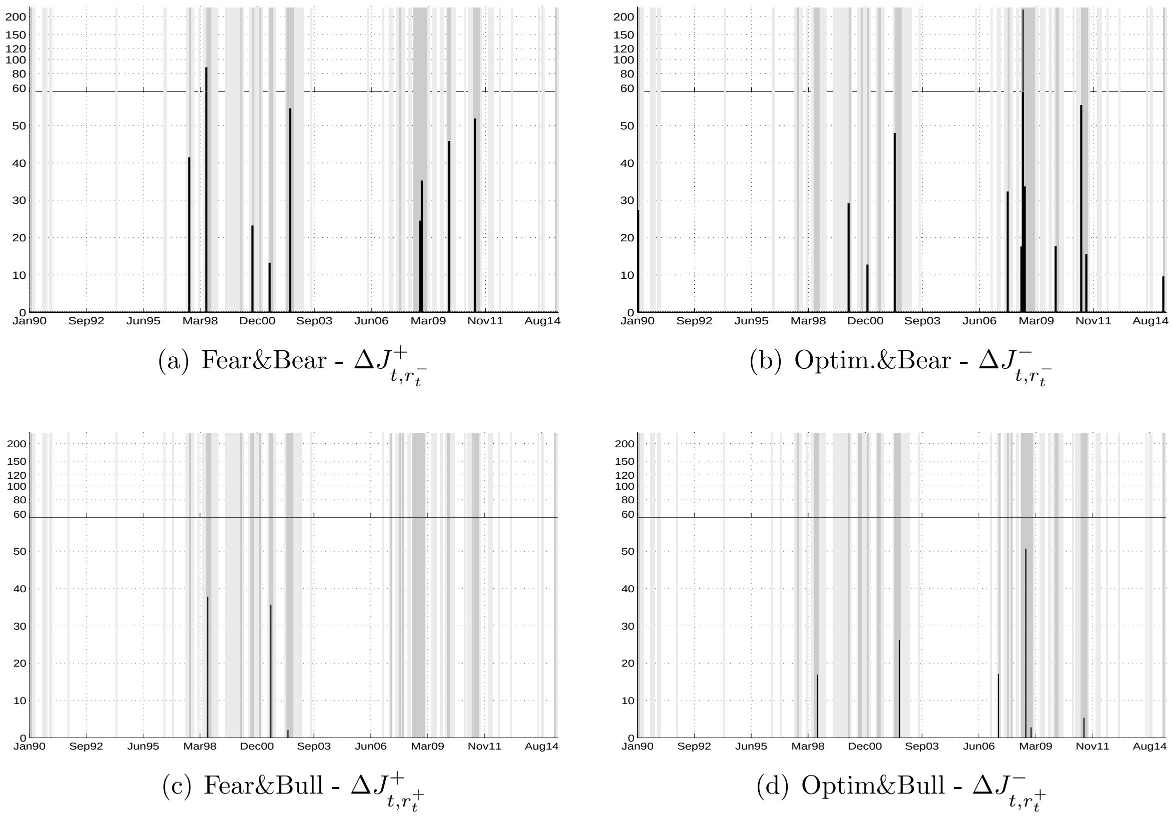
Notes: This table reports descriptive statistics for the attitude and sentiment indicators. The statistics are computed considering only the non-zero values for a given indicator. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

test this hypothesis, we augment the predictive regression model as

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+j} = a_0(h) + a_1(h)\Pi_{t,t+1}^{C} + \sum_{j=1}^{m} b_j(h)SI_{j,t} + u_{t+h,t} \qquad (25)$$

where $SI_j$ is a set of sentiment indicators related to extreme market events. In order to isolate the contribution of the variance premium and the investors' sentiment indexes, we do not consider other explanatory variables $x_i$, $i = 1, .., k$ here. The inclusion of other economic predictors is instead discussed in Section 5.3.

Table 6 reports descriptive statistics for the indicators associated with extreme market events, while Figure 6 shows the occurrence and the size of the sentiment indicators. Out of 32 jump events, we observe 12 fear reactions against 20 optimism episodes, with average sizes of 36.025 and 34.747, respectively. When we associate the size of the reaction to the sign of the market performance, we observe that when returns are negative the average reaction is about twice as large and more than double in occurrence when compared to positive returns. Note that the number of occurrences of the fear indicator when associated to bear markets counts for less than 3% of the total sample size, while the episodes classified as Optimism&Bear represent about 5%. As expected, fear and optimism reactions in




(a) Fear&Bear - $\Delta J^{+}_{t,r_t^-}$

(b) Optim.&Bear - $\Delta J^{-}_{t,r_t^-}$

(c) Fear&Bull - $\Delta J^{+}_{t,r_t^+}$

(d) Optim&Bull - $\Delta J^{-}_{t,r_t^+}$

Figure 6: Sentiment indicators

conjunction with positive shocks in the market, turn out to be much more rare (3 and 6 occurrences, respectively, out of 309 observations).

Tables 7 and 8 report the adjusted $R^2$s and the estimates for the predictive return regression in equation (25) over the same set of horizons used in Section 2. They include combinations of $\Pi^C_{t,t+1}$ and the sentiment indicators as individual regressors as well as their interaction with the state of the market. We test for statistical significance of the parameters using individual $t$ tests. To account for serial dependence in the residuals induced by the overlap in the multi-period return, the $t$ statistics are based on Newey-West heteroskedasticity and autocorrelation consistent standard errors. To compare different combinations of regressors, the overall fit of the regression is measured by the adjusted $R^2$.

Table 7: Adjusted $R^2$s from predictive return regressions

| Regressors $\Pi^i$ $SI_{j,t}$ | Horizon 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|
| $\Pi^C_{t,t+1}$ | -0.30 | -0.17 | 0.16 | 0.84 | 1.32 | 1.28 | 0.65 | 1.01 |
| $\Pi^C_{t,t+1}$ $\Delta J_t$ | 6.23 | 6.25 | 8.31 | 9.98 | 8.16 | 5.43 | 2.39 | 1.91 |
| $\Pi^C_{t,t+1}$ $\Delta J^+_t$ $\Delta J^-_t$ | 7.08 | 7.68 | 8.97 | 10.33 | 8.87 | 6.49 | 3.57 | 2.44 |
| $\Pi^C_{t,t+1}$ $\Delta J^+_{t,r^-_t}$ $\Delta J^-_{t,r^-_t}$ $\Delta J^+_{t,r^+_t}$ $\Delta J^-_{t,r^+_t}$ | 7.37 | 8.93 | 9.27 | 10.38 | 8.89 | 6.54 | 4.23 | 3.26 |
| $\Pi^C_{t,t+1}$ $\Delta J^+_t$ | 4.41 | 5.53 | 5.19 | 5.47 | 5.90 | 5.19 | 3.32 | 2.29 |
| $\Pi^C_{t,t+1}$ $\Delta J^-_t$ | 2.98 | 2.60 | 4.70 | 6.52 | 4.95 | 2.99 | 1.07 | 1.25 |
| $\Pi^C_{t,t+1}$ $\Delta J^+_{t,r^-_t}$ | 3.01 | 4.07 | 3.92 | 4.50 | 5.22 | 4.84 | 3.56 | 2.75 |
| $\Pi^C_{t,t+1}$ $\Delta J^-_{t,r^-_t}$ | 2.68 | 1.62 | 3.95 | 6.10 | 4.76 | 2.93 | 1.23 | 1.37 |
| $\Pi^C_{t,t+1}$ $\Delta J^+_{t,r^+_t}$ | 1.27 | 1.35 | 1.48 | 1.79 | 1.92 | 1.60 | 0.66 | 1.09 |
| $\Pi^C_{t,t+1}$ $\Delta J^-_{t,r^+_t}$ | -0.11 | 1.80 | 0.89 | 1.08 | 1.42 | 1.31 | 0.85 | 1.17 |
| $\Pi^{RW}_{t,t+1}$ | 5.08 | 6.91 | 10.7 | 12.7 | 10.3 | 7.46 | 3.37 | 2.48 |
| $\Pi^{AR(1)}_{t,t+1}$ | 1.89 | 3.65 | 5.27 | 6.78 | 6.83 | 6.30 | 3.76 | 2.83 |
| $\Pi^{AR(2)}_{t,t+1}$ | 1.90 | 3.42 | 5.02 | 6.66 | 6.77 | 6.20 | 3.70 | 2.80 |
| $\Pi^{ARX1}_{t,t+1}$ | 1.97 | 3.68 | 5.34 | 6.92 | 6.96 | 6.37 | 3.78 | 2.87 |
| $\Pi^{ARX2}_{t,t+1}$ | 1.17 | 2.68 | 3.78 | 5.02 | 5.43 | 5.36 | 3.41 | 2.56 |

Notes: This table reports the adjusted $R^2$s in percentage from the predictive return regressions in (25) with VRP, sentiment indicators and their interaction with the state of the market as regressors. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

As a reference, we also add the adjusted $R^2$s of the predictive return regressions where the VRP is computed using the variance expectations under the physical measure based on the models described in Section 2.

When we regress the future aggregate market excess return on $\Pi^C_{t,t+1}$, we find no substantial return predictability at any horizon. In fact, the predictability attains a maximum value equal to 1.3% at the five-month horizon. This size of $R^2$ is in line with Huang, Jiang, Tu, and Zhou (2015), when using the investor sentiment index results of Baker and Wurgler (2006), who size the predictability of the monthly aggregate market excess return in the order of 1.5%. Thus the results of Table 7 suggest that $\Pi^C_{t,t+1}$ adequately measures the economic uncertainty under normal market conditions. More importantly, as also advocated by Bollerslev, Todorov, and Xu (2015), this finding suggests that much of the return predictability previously ascribed in the literature to the variance risk premium is effectively coming from the part of the premium related to how agents gauge extreme variance events, their prediction and compensation. In our framework, the latter effects are measured by the variable $\Delta J_t$ and when including the size of the reaction of agents to unusual variance events, the $R^2$ of the regression sharply increases at every horizon, reaching 10% at the 4 month horizon.

Separating $\Delta J_t$ in its two components $\Delta J_t^+$ and $\Delta J_t^-$, Table 7 shows that the extra degree of freedom allowing for an asymmetric effect on future returns of conservative (fear) versus liberal (optimism) expectations improves the predictability at the short and long horizons. Moreover, Table 8 shows that the marginal impact of $\Delta J_t^+$ is systematically at least double that of $\Delta J_t^-$ and the two consistently carry opposite signs. The null hypothesis of no asymmetry is always rejected at standard significance levels. Therefore, agents effectively price the fear for market instability only when shocks are perceived as long lasting. Although, the parameter estimates of $\Delta J_t^+$ and $\Delta J_t^-$ tend to decrease in absolute value as the horizon increases, Table 8 shows that agents' conservative reactions to large shocks affect returns over longer horizons compared to liberal ones (with the latter insignificant for the 12-month horizon at standard confidence levels).

Interacting the variables $\Delta J_t^+$ and $\Delta J_t^-$ with the market performance indicator further increases the $R^2$s and this especially so at horizons at which return predictability is known to be difficult to capture, i.e. one to three month horizon. This shows that the asymmetry, with respect to the state of the market, in the way each sentiment indicator explains future aggregate market returns cannot be rejected. The inverse U-shaped relationship between horizon and $R^2$ is preserved but the curve that we obtain is somewhat flatter, i.e. starting and ending with higher $R^2$s. For example, the one month horizon $R^2$ is 7.37 against 6.23

Table 8: Parameter estimates from return predictive regressions

| | One-month horizon | | | | Three-month horizon | | | |
|---|---|---|---|---|---|---|---|---|
| $\Pi^C_{t,t+1}$ | 0.006 | 0.024 | 0.011 | 0.011 | 0.017 | 0.029 | 0.022 | 0.024 |
| | (0.187) | (0.854) | (0.038) | (0.402) | (0.829) | (1.719) | (1.215) | (1.276) |
| $\Delta J_t$ | | 0.064 | | | | 0.043 | | |
| | | (4.384) | | | | (7.248) | | |
| $\Delta J^+_t$ | | | 0.107 | | | | 0.065 | |
| | | | (4.854) | | | | (5.707) | |
| $\Delta J^-_t$ | | | -0.049 | | | | -0.035 | |
| | | | (-4.312) | | | | (-5.245) | |
| $\Delta J^+_{t,r^-_t}$ | | | | 0.097 | | | | 0.060 |
| | | | | (3.738) | | | | (5.526) |
| $\Delta J^-_{t,r^-_t}$ | | | | -0.049 | | | | -0.034 |
| | | | | (-4.006) | | | | (-5.845) |
| $\Delta J^+_{t,r^+_t}$ | | | | 0.177 | | | | 0.095 |
| | | | | (11.224) | | | | (2.822) |
| $\Delta J^-_{t,r^+_t}$ | | | | -0.053 | | | | -0.063 |
| | | | | (-0.503) | | | | (-1.610) |
| Adj.$R^2$ | -0.30 | 6.23 | 7.09 | 7.37 | 0.16 | 8.31 | 8.97 | 9.27 |
| | Six-month horizon | | | | Twelve-month horizon | | | |
| $\Pi^C_{t,t+1}$ | 0.023 | 0.029 | 0.023 | 0.022 | 0.015 | 0.017 | 0.014 | 0.013 |
| | (1.432) | (1.709) | (1.335) | (1.282) | (1.061) | (1.128) | (0.950) | (0.884) |
| $\Delta J_t$ | | 0.023 | | | | 0.008 | | |
| | | (5.717) | | | | (2.474) | | |
| $\Delta J^+_t$ | | | 0.044 | | | | 0.019 | |
| | | | (4.665) | | | | (2.721) | |
| $\Delta J^-_t$ | | | -0.015 | | | | -0.004 | |
| | | | (-3.745) | | | | (-1.413) | |
| $\Delta J^+_{t,r^-_t}$ | | | | 0.045 | | | | 0.024 |
| | | | | (4.489) | | | | (3.868) |
| $\Delta J^-_{t,r^-_t}$ | | | | -0.016 | | | | -0.005 |
| | | | | (-4.155) | | | | (-1.719) |
| $\Delta J^+_{t,r^+_t}$ | | | | 0.036 | | | | -0.012 |
| | | | | (1.258) | | | | (-0.437) |
| $\Delta J^-_{t,r^+_t}$ | | | | -0.007 | | | | 0.017 |
| | | | | (-0.370) | | | | (1.272) |
| Adj.$R^2$ | 1.28 | 5.43 | 6.49 | 6.53 | 1.01 | 1.91 | 2.45 | 3.26 |

Notes: This table reports parameter estimates from the predictive return regressions in (25) with VRP, sentiment indicators and their interaction with the state of the market as regressors. Newey-West t-statistics are in brackets and adjusted $R^2$s are expressed in percentages. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

of the regression including only $\Pi^C_{t,t+1}$ and $\Delta J_t$. Similarly, the one year horizon $R^2$s are respectively 3.26 and 1.91. Thus it turns out that predictability indeed comes mostly from the way agents adapt their expectations when exposed to extreme variance realizations as detected by the third regime in our model.

With respect to the state of the market, as expected, we find that fear in conjunction with negative market performances impacts the future market performance over longer horizons. In fact, the coefficients associated with $\Delta J^+_{t,r^-_t}$ (Fear&Bear) are significant at all horizons exhibiting the slowly decaying behavior discussed above. Contrary, the effect of fear associated to bullish markets dies out much faster as shown by the insignificant coefficients starting from the six month horizon. Similar conclusion can be drawn for the case of optimistic variance expectations where $\Delta J^-_{t,r^-_t}$ (Optimism&Bear) contributes to a large extent to explain future aggregate market returns at all horizons, while optimism in conjunction with bullish markets contributes only marginally.

This result supports the hypothesis of an asymmetric response of the excess aggregate market returns to the sign of the shock in the aggregate price. It is worth noting that both $\Delta J^+_{t,r^-_t}$ and $\Delta J^+_{t,r^+_t}$ correlate positively with the future aggregate market excess return, meaning that agents price any source of market instability when perceived to be long lasting. The marginal impact of the Fear&Bull indicator is larger in the short term while that of the Fear&Bear indicator dominates on longer horizons.

The natural question to ask is what justifies the sharp difference in return predictability discussed in Section 2 of the different models typically used to compute the variance expectation needed to estimate the VRP. Using our sentiment indicators, we can pin down exactly which component in the VRP based on the random walk and on autoregressive type models effectively explains future return variation. To be precise, since the random walk implied VRP is completely exposed to jumps, both in the realized variance and its risk-neutral expectations, our results suggest that the degree of predictability of $\Pi^{RW}_{t,t+1}$ has to be dominated by that of $\Delta J_t^+$ and $\Delta J_t^-$. To test this hypothesis, we regress the future excess returns on the difference between $\Pi_{t,t+1}^{RW}$ and the sum of the fear and optimism indicators. We find that the resulting $R^2$s drop at all horizons, e.g. from 12.7 to 3.2 at the four-month horizon. Similarly, we argue that return predictability associated with $\Pi_{t,t+1}^{AR(1)}$, which is instead based on smoother variance expectations, is rather exposed to jumps in the risk-neutral expectations, thus being dominated by $\Delta J_t^+$. Subtracting the fear indicator ($\Delta J_t^+$) from AR(1) implied $\Pi_{t,t+1}^{AR(1)}$, the return predictability in fact aligns to the same levels observed for the smooth component $\Pi_{t,t+1}^{C}$.

## 5.3 Predictability with other economic predictors

We now compare the forecasting power of our four sentiment indicators with several macroeconomic variables that have been shown to have predictive power when it comes to return forecasting. This allows us to examine whether our sentiment indicators' forecasting power is driven by omitted economic variables related to business cycle fundamentals or changes in investor risk aversion. To test this we augment the predictive regression to take the following form

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+j} = a_0(h) + \sum_{j=1}^{m} a_j(h) SI_{j,t} + \sum_{i=1}^{k} b_i(h) X_{i,t} + u_{t+h,t}. \tag{26}$$

The set of regressors, $X_{i,t}$, we consider are:

- *Basic financial variables*: Dividend-payout ratio (DPO): difference between the log of dividends and log of earnings on the S&P 500 index; Price-earnings ratio (PE): difference between the log of earnings and log of prices on the S&P 500 index, where earnings are measured using a one-year moving sum; Dividend yield (DY): difference between the log of dividends and log of lagged prices on the S&P 500 index. All variables are downloaded from Robert Shiller's website.

- *Yield related variables*: Term spread (TMS): difference between the long-term yield and Treasury bill rate (10-Year Treasury Constant Maturity Minus 3-Month Treasury Constant Maturity); Default yield spread (DFY): difference between Moody's Seasoned BAA- and AAA-rated corporate bond yields; Default return spread (DFR): difference between long-term corporate bond and long-term government bond returns (Moody's Seasoned AAA Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity). All variables are downloaded from the St. Louis Federal Reserve website.
- *Traditional risk factors*: Small minus big (SMB) and high minus low (HML) are constructed using value-weighted portfolios of NYSE, AMEX and NASDAQ stocks formed on size and book-to-market. SMB is the average return on the small portfolios minus the average return on the big portfolios, HML is the average return on the value portfolios minus the average return on the growth portfolios. MOM is the Carhart (1997) momentum factor. All variables are downloaded from Kenneth R. French's website.
- *Macro-economic variables*: Consumption-wealth ratio (CAY): residual of regressing consumption on asset wealth and labor income from Lettau and Ludvigson (2001) and downloaded from Lettau's website. Re-spanned on monthly frequency as in Bollerslev, Tauchen, and Zhou (2009); Inflation (CPI): calculated from the consumer price index for all urban consumers as used in Giglio, Kelly, and Pruitt (2016); Output gap (OG): deviation of the logarithm of total industrial production from a trend that includes both a linear component and a quadratic component, see Cooper and Priestley (2009). We also include a cubic trend to capture better the recent economic downturn. The variables CPI and total industrial production are downloaded from the St. Louis Federal Reserve website.

- *Implied volatility*: The squared CBOE implied volatility index (VIX) downloaded from Datastream.

The left panels of Tables 9 and 10 considers the predictive regressions with each single economic variable, for respectively the one/three month and six/twelve month horizons. We report the slope coefficients of the regressions along with the adjusted $R^2$s. A coefficient with a star indicates that the coefficient is significant at the 10% level. As a reference, we also include the regression results using the smooth component of the VRP (i.e. $\Pi^C$) extracted from Table 7. Out of the 14 economic predictors, only DY exhibit significant predictive power for the market aggregate future return at the 10% or better significance levels for all the considered horizons. The $R^2$s increase from 0.49% to 12.26% when the horizon increases from one to twelve months. The variable OG has significant predictive power from the three month horizon onwards with $R^2$s from 3.06% to 17.17% and finally MOM is significant from the sixth month horizon with largest $R^2$ of 1.33% at the twelve month horizon.

The right panels of Tables 9 and 10 reports multivariate regressions to test for incremental predictive power of the sentiment indicators and their interaction with the state of the market, $SI_{j,t}$. This allows investigating whether the forecasting power of $SI_{j,t}$ documented in the previous section remains significant after controlling for economic predictors. The results show that the estimates of the slope coefficients associated with the sentiment indicators are indeed in line, in terms of sign and size, with those reported in Table 8. More precisely, the coefficient $a_1$ associated with Fear&Bear remains statistically significant at all horizons when the regression is augmented by the economic predictors. The coefficients of the indicators Optimism&Bear and Fear&Bull, i.e. $a_2$ and $a_3$, are significant at the one and three month horizons but less so at the six and twelve month horizons. The coefficient $a_4$ associated with Optimism&Bull is significant several times at the twelve month horizon.

With respect to the magnitude of the adjusted $R^2$s of these augmented regressions, they are substantially larger than those based on the economic predictors alone. For instance, at the one month horizon, the $R^2$s increase from about zero percent on average to at least 7%. Regressions combining the sentiment indicators with economic predictors found individually significant seems to increase predictability in an additive way. For example at the six month horizon, DY alone has an $R^2$ of 4.83% compared to an $R^2$ of 11.26% when the sentiment indicators are included in the model. This confirms that the latter indicators bring in genuinely different explanatory power. In sum, these results demonstrate that the sentiment indicators contain considerable complementary forecasting information beyond what is contained in the usual economic predictors.

# 6 International evidence

To study the properties of the VRP and its return predictability beyond the S&P 500 index, we now estimate our model on eight other market indices. Apart from the Dow Jones industrial average (DJIA), we consider the following foreign market indices: the STOXX Europe 50 index covering 18 European countries, the CAC 40 from France, the DAX index from Germany, the AEX index from the Netherlands, the FTSE 100 from the United Kingdom, the SMI from Switzerland and the NIKKEI 225 from Japan. We first describe the data and discuss the parameter estimates and the resulting VRP and sentiment indicators obtained with our model. We next report results for the predictive regressions with the smooth component of the VRP and the sentiment indicators and compare this to the existing evidence.

## 6.1 Data and parameter estimates

The monthly data for the panel of international indices spans the period from January, 2000, to September, 2015, totaling 189 observations. Implied variance time series are

Table 9: Predictive return regressions with economic predictors

| One-month horizon | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{r}_t^{(h)}=\hat{a}_0+\hat{b}_1 X_{1,t}$ | | $\hat{r}_t^{(h)}=\hat{a}_0+\sum_{j=1}^{4}\hat{a}_j SI_{j,t}+\hat{b}_1 X_{1,t}$ | | | | | | |
| $X_{1,t}$ | $\hat{b}_1$ | Adj.$R^2$ | $\hat{a}_1$ | $\hat{a}_2$ | $\hat{a}_3$ | $\hat{a}_4$ | $\hat{b}_1$ | Adj.$R^2$ |
| $\Pi^C$ | 6e-3 | -0.30 | 0.10* | -0.05* | 0.18* | -0.05 | 0.01 | 7.37 |
| DPO | -3e-3 | -0.33 | 0.10* | -0.05* | 0.18* | -0.05 | 0.06 | 7.30 |
| PE | -0.73 | 0.09 | 0.10* | -0.05* | 0.19* | -0.03 | -0.93 | 7.96 |
| DY | 1.31* | 0.49 | 0.10* | -0.05* | 0.19* | -0.05 | 1.69* | 8.64 |
| TMS | -0.08 | -0.27 | 0.10* | -0.05* | 0.18* | -0.05 | 0.02 | 7.30 |
| DFR | -0.42 | -0.10 | 0.10* | -0.04* | 0.19* | -0.03 | -0.44 | 7.52 |
| DFY | 0.69 | 0.12 | 0.10* | -0.04* | 0.18* | -0.03 | 0.31 | 7.37 |
| SMB | 0.06 | -0.12 | 0.10* | -0.05* | 0.18* | -0.05 | 0.05 | 7.48 |
| HML | -0.09 | 0.15 | 0.10* | -0.05* | 0.17* | -0.05 | -0.09 | 7.76 |
| MOM | -0.05 | -0.05 | 0.10* | -0.05* | 0.18* | -0.05 | -0.04 | 7.48 |
| LTR | -0.10 | 0.01 | 0.10* | -0.05* | 0.18* | -0.05 | -0.07 | 7.49 |
| CAY | -10.65 | -0.12 | 0.10* | -0.05* | 0.18* | -0.04 | -12.95 | 7.60 |
| CPI | 33.90 | -0.25 | 0.10* | -0.05* | 0.18* | -0.05 | -8.49 | 7.30 |
| OG | -8.99 | 0.55 | 0.10* | -0.05* | 0.18* | -0.05 | -8.77* | 8.13 |
| VIX | 0.00 | -0.32 | 0.11* | -0.04* | 0.19* | -0.04 | -0.01 | 7.37 |

| Three-month horizon | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\hat{r}_t^{(h)}=\hat{a}_0+\hat{b}_1 X_{1,t}$ | | $\hat{r}_t^{(h)}=\hat{a}_0+\sum_{j=1}^{4}\hat{a}_j SI_{j,t}+\hat{b}_1 X_{1,t}$ | | | | | |
| $X_{1,t}$ | $\hat{b}_1$ | Adj.$R^2$ | $\hat{a}_1$ | $\hat{a}_2$ | $\hat{a}_3$ | $\hat{a}_4$ | $\hat{b}_1$ | Adj.$R^2$ |
| $\Pi^C$ | 0.02 | 0.16 | 0.06* | -0.03* | 0.10* | -0.06 | 0.02 | 9.27 |
| DPO | 0.23 | -0.18 | 0.07* | -0.03* | 0.10* | -0.05 | 0.30 | 8.67 |
| PE | -0.47 | 0.16 | 0.07* | -0.03* | 0.11* | -0.04 | -0.57 | 9.12 |
| DY | 1.39* | 2.26 | 0.07* | -0.03* | 0.11* | -0.05 | 1.64* | 12.00 |
| TMS | -0.04 | -0.29 | 0.07* | -0.03* | 0.10* | -0.05 | 0.02 | 8.43 |
| DFR | -0.35 | 0.11 | 0.07* | -0.03* | 0.11* | -0.04 | -0.35 | 8.81 |
| DFY | 0.40 | 0.10 | 0.07* | -0.03* | 0.10* | -0.04 | 0.11 | 8.44 |
| SMB | -0.06 | 0.24 | 0.06* | -0.03* | 0.11* | -0.05 | -0.06 | 9.03 |
| HML | -0.06 | 0.29 | 0.07* | -0.03* | 0.09* | -0.05* | -0.07 | 9.10 |
| MOM | -0.03* | 0.96 | 0.07* | -0.03* | 0.09* | -0.06* | -0.06 | 9.58 |
| LTR | -0.15* | 1.90 | 0.06* | -0.03* | 0.10* | -0.05 | -0.14* | 10.28 |
| CAY | -0.39 | -0.22 | 0.07* | -0.03* | 0.10* | -0.05 | -1.46 | 8.43 |
| CPI | 45.80 | 0.05 | 0.07* | -0.03* | 0.10* | -0.04 | 17.62 | 8.47 |
| OG | -11.37* | 3.06 | 0.06* | -0.03* | 0.10* | -0.05 | -11.32* | 12.29 |
| VIX | 0.00 | -0.24 | 0.06* | -0.03* | 0.09* | -0.05 | 0.00 | 8.50 |

Notes: This table reports parameter estimates from the predictive return regressions in (26) with economic predictors and sentiment indicators as regressors. Newey-West t-statistics are in brackets and adjusted $R^2$ are expressed in percentages. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

Table 10: Predictive return regressions with economic predictors

| Six-month horizon | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\hat{r}_t^{(h)}=\hat{a}_0+\hat{b}_1X_{1,t}$ | | $\hat{r}_t^{(h)}=\hat{a}_0+\sum_{j=1}^{4}\hat{a}_jSI_{j,t}+\hat{b}_1X_{1,t}$ | | | | | |
| $X_{1,t}$ | $\hat{b}_1$ | Adj.$R^2$ | $\hat{a}_1$ | $\hat{a}_2$ | $\hat{a}_3$ | $\hat{a}_4$ | $\hat{b}_1$ | Adj.$R^2$ |
| $\Pi^C$ | 0.02 | 1.28 | 0.05* | -0.02* | 0.04 | -0.01 | 0.02 | 6.53 |
| DPO | 0.37 | 0.38 | 0.05* | -0.01* | 0.04 | -0.00 | 0.35 | 5.80 |
| PE | -0.36 | 0.19 | 0.05* | -0.01* | 0.05* | 0.02 | -0.49 | 6.11 |
| DY | 1.46* | 4.83 | 0.05* | -0.01* | 0.05* | -0.00 | 1.60* | 11.26 |
| TMS | 0.03 | -0.30 | 0.05* | -0.01* | 0.04 | 0.01 | 0.07 | 5.35 |
| DFR | -0.22 | -0.03 | 0.05* | -0.01* | 0.05* | 0.02 | -0.33 | 5.79 |
| DFY | -0.02 | -0.33 | 0.05* | -0.01 | 0.04 | 0.01 | -0.03 | 5.19 |
| SMB | -0.05 | 0.61 | 0.05* | -0.01* | 0.05 | 0.01 | -0.05 | 5.97 |
| HML | -0.08* | 1.27 | 0.05* | -0.01* | 0.03 | -0.00 | -0.08* | 6.79 |
| MOM | -0.04* | 0.92 | 0.05* | -0.01* | 0.04 | -0.00 | -0.04* | 6.32 |
| LTR | -0.09 | 1.02 | 0.05* | -0.01* | 0.04 | 0.01 | -0.08 | 6.23 |
| CAY | 7.18 | 1.15 | 0.05* | -0.01* | 0.04 | 0.01 | 5.81 | 5.47 |
| CPI | -45.38 | 0.34 | 0.05* | -0.01* | 0.04 | -0.00 | -56.58 | 6.12 |
| OG | -12.96* | 8.80 | 0.05* | -0.01* | 0.04 | -0.00 | -12.70* | 13.93 |
| VIX | 0.00 | 1.06 | 0.04* | -0.02* | 0.03 | -0.01 | 0.01 | 5.93 |
| **Twelve-month horizon** | | | | | | | | |
| | $\hat{r}_t^{(h)}=\hat{a}_0+\hat{b}_1X_{1,t}$ | | $\hat{r}_t^{(h)}=\hat{a}_0+\sum_{j=1}^{4}\hat{a}_jSI_{j,t}+\hat{b}_1X_{1,t}$ | | | | | |
| $X_{1,t}$ | $\hat{b}_1$ | Adj.$R^2$ | $\hat{a}_1$ | $\hat{a}_2$ | $\hat{a}_3$ | $\hat{a}_4$ | $\hat{b}_1$ | Adj.$R^2$ |
| $\Pi^C$ | 0.01 | 1.01 | 0.02* | -0.01* | 0.01 | -0.02 | 0.01 | 3.26 |
| DPO | 0.35 | 0.91 | 0.03* | -0.00 | -0.01 | 0.02 | 0.32 | 3.38 |
| PE | -0.51 | 1.58 | 0.03* | -0.00 | 0.00 | 0.03 | -0.60 | 5.00 |
| DY | 1.66* | 12.26 | 0.03* | -0.00 | 0.01 | 0.02 | 1.70* | 15.47 |
| TMS | 0.17 | 1.75 | 0.03* | -0.00 | -0.01 | 0.03* | 0.18 | 4.82 |
| DFR | -0.21 | 0.19 | 0.03* | -0.00 | 0.00 | 0.03* | -0.31 | 3.46 |
| DFY | -0.24 | 0.17 | 0.03* | -0.00 | -0.01 | 0.02 | -0.17 | 2.63 |
| SMB | -0.04 | 0.36 | 0.03* | -0.00 | -0.01 | 0.03* | -0.03 | 2.90 |
| HML | -0.05 | 0.71 | 0.03* | -0.00 | -0.02 | 0.02* | -0.05 | 3.52 |
| MOM | -0.04* | 1.33 | 0.03* | -0.00 | -0.01 | 0.02* | -0.04* | 4.05 |
| LTR | -0.05 | 0.38 | 0.03* | -0.00 | -0.01 | 0.02* | -0.04 | 2.98 |
| CAY | 16.23 | 3.44 | 0.02* | -0.00 | -0.01 | 0.02* | 15.60 | 5.86 |
| CPI | -61.15 | 1.82 | 0.03* | -0.01 | -0.01 | 0.01 | -64.68 | 4.57 |
| OG | -13.19* | 17.17 | 0.02* | -0.00 | -0.01 | 0.02* | -12.97* | 19.28 |
| VIX | 0.00 | 0.82 | 0.02* | -0.01 | -0.02 | 0.02 | 0.00 | 2.89 |

Notes: This table reports parameter estimates from the predictive return regressions in (26) with economic predictors and sentiment indicators as regressors. Newey-West t-statistics are in brackets and adjusted $R^2$ are expressed in percentages. The sample used is monthly data from January, 1990, to September, 2015, totalling 309 observations.

estimated by the CBOE volatility index applied to each index options dataset. Implied variances and interest rates used to compute excess returns are extracted from Datastream. Realized variance data, available from Oxford-Man Institute, are based on 5 minute returns along with the squared close-to-open overnight return.

Table 11 provides descriptive statistics. In terms of persistence, as observed before, the implied variance ($IV_{t,t+1}$) is consistently more persistent than the realized variance for all indices. The unconditional VRP ranges between 7.153 for CAC and 27.209 for NIKKEI, with an exception of DAX with an abnormal unconditional VRP of -0.386. The latter value is due to an unusual period in the early 2000's where DAX realized variance was systematically above the implied variance for several months, possibly due to imprecise estimation of DAX volatility index during its early stages of introduction. The DAX results below should therefore be interpreted with caution.

Table 12 presents parameter estimates of our model for the eight indices. Generally speaking, we find similar results to the S&P 500 index case. In terms of the dynamics of $\Sigma_{t-1,t}$, we find values ranging from 0.754 to 0.914, with SMI and AEX being the least and the most persistent, respectively. As expected, the idiosyncratic stationary component of $\Pi^C_{t,t+1}$ is generally very persistent with mean reversion coefficients between 0.830 and 0.962 for SMI and DJIA respectively. In terms of regime duration, the first regime lasts on average six months, the second regime on average 1.5 months, and the third regime about three months with small variability across indices and the model filters out the jump component well as can be seen from the distribution of $J^P_{t-1,1}$ and $E^Q_t[J^Q_{t,t+1}]$ exhibiting a large location and variance compared to those of the continuous state variables. Finally, the link between $\Sigma_{t-1,t}$ and $\Pi^C_{t,t+1}$, represented by the parameter $d_{S_t}$, shows evidence of time variation, though it disappears for CAC and DAX in periods of high volatility and jumps.

Figures 7 to 10 display the smooth component of the VRP from our model, $\Pi^C_{t,t+1}$,

Table 11: Properties of the risk-neutral, physical variances and unconditional VRP

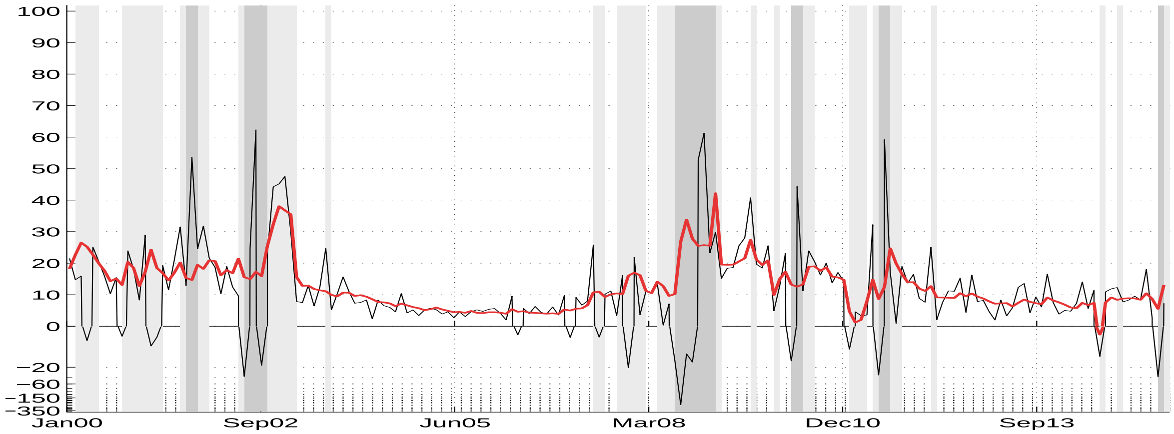
| Index | Variable | Mean | Std. deviation | Minimum | Maximum | AR(1) |
|---|---|---|---|---|---|---|
| DJIA | $IV_{t,t+1}$ | 36.669 | 34.493 | 8.367 | 259.75 | 0.816 |
| | $RV_{t-1,t}$ | 26.053 | 44.802 | 2.941 | 493.43 | 0.606 |
| | $\mathrm{E}(\Pi_{t,t+1})$ | 10.616 | | | | |
| STOXX | $IV_{t,t+1}$ | 59.021 | 50.482 | 12.772 | 313.55 | 0.787 |
| | $RV_{t-1,t}$ | 40.125 | 54.406 | 5.675 | 547.98 | 0.499 |
| | $\mathrm{E}(\Pi_{t,t+1})$ | 19.016 | | | | |
| CAC | $IV_{t,t+1}$ | 51.743 | 43.695 | 11.609 | 301.01 | 0.767 |
| | $RV_{t-1,t}$ | 44.851 | 57.917 | 5.325 | 569.98 | 0.525 |
| | $\mathrm{E}(\Pi_{t,t+1})$ | 7.153 | | | | |
| DAX | $IV_{t,t+1}$ | 47.420 | 43.402 | 10.528 | 279.95 | 0.776 |
| | $RV_{t-1,t}$ | 48.175 | 64.786 | 6.303 | 582.79 | 0.563 |
| | $\mathrm{E}(\Pi_{t,t+1})$ | -0.386 | | | | |
| AEX | $IV_{t,t+1}$ | 53.181 | 53.763 | 2.774 | 314.47 | 0.834 |
| | $RV_{t-1,t}$ | 40.584 | 63.169 | 5.549 | 642.27 | 0.539 |
| | $\mathrm{E}(\Pi_{t,t+1})$ | 12.781 | | | | |
| FTSE | $IV_{t,t+1}$ | 39.736 | 35.278 | 8.445 | 244.35 | 0.807 |
| | $RV_{t-1,t}$ | 27.814 | 42.711 | 3.816 | 457.3 | 0.549 |
| | $\mathrm{E}(\Pi_{t,t+1})$ | 12.007 | | | | |
| SMI | $IV_{t,t+1}$ | 33.524 | 34.237 | 7.038 | 284.90 | 0.781 |
| | $RV_{t-1,t}$ | 28.944 | 45.530 | 3.219 | 464.73 | 0.503 |
| | $\mathrm{E}(\Pi_{t,t+1})$ | 4.647 | | | | |
| NIKKEI | $IV_{t,t+1}$ | 62.172 | 62.877 | 15.436 | 696.93 | 0.625 |
| | $RV_{t-1,t}$ | 35.023 | 30.242 | 5.680 | 303.93 | 0.520 |
| | $\mathrm{E}(\Pi_{t,t+1})$ | 27.209 | | | | |

Notes: This table reports descriptive statistics for the implied variance, $IV_{t,t+1}$, and the realized variance, $RV_{t-1,t}$, for the eight market indices. AR(1) is the sample autocorrelation of order one and $\mathrm{E}(\Pi_{t,t+1})$ is the difference between the unconditional means of $IV_{t,t+1}$ and $RV_{t-1,t}$, respectively. The sample used is monthly data from January, 2000, to September, 2015, totalling 189 observations.

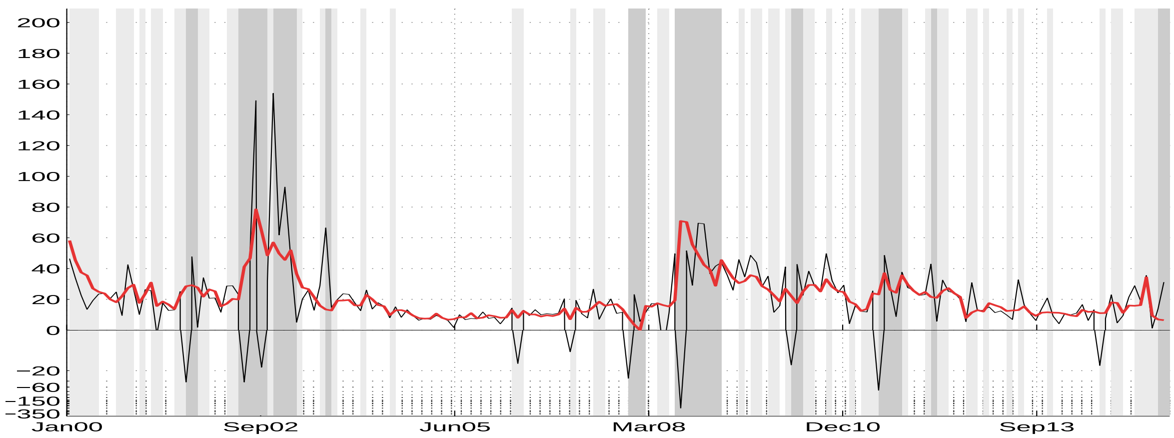
Table 12: Parameter estimates for our model - international evidence

| State Var. | Parameter | DJIA | STOXX | CAC | DAX | AEX | FTSE | SMI | NIKKEI |
|---|---|---|---|---|---|---|---|---|---|
| | | Index | | | | | | | |
| $\Sigma_{t-1,t}$ | $b_0$ | 10.456 | 12.133 | 17.672 | 20.506 | 8.765 | 10.620 | 9.906 | 22.391 |
| | $b_1,\ b_2$ | 22.738 | 23.188 | 37.024 | 43.939 | 22.570 | 22.827 | 22.356 | 30.500 |
| | $\phi^{\Sigma}$ | 0.806 | 0.845 | 0.837 | 0.797 | 0.914 | 0.828 | 0.754 | 0.789 |
| | $\sigma^2_{\Sigma,0}$ | 6.656 | 5.461 | 12.307 | 27.404 | 12.265 | 3.301 | 3.505 | 15.183 |
| | $\sigma^2_{\Sigma,1},\ \sigma^2_{\Sigma,2}$ | 24.304 | 54.175 | 64.770 | 247.156 | 35.233 | 36.920 | 96.508 | 25.401 |
| $\Pi^C_{t,t+1}$ | $a_0$ | 4.859 | 8.678 | [2.916] | -7.876 | [2.843] | 7.564 | [1.501] | 12.517 |
| | $a_1,\ a_2$ | 7.048 | 11.258 | 9.232 | 5.926 | 8.659 | 9.813 | 6.429 | 20.696 |
| | $\phi^C$ | 0.962 | 0.862 | 0.838 | 0.932 | 0.904 | 0.950 | 0.830 | 0.909 |
| | $d_0$ | 0.122 | 0.652 | 0.400 | 0.203 | 0.534 | 0.930 | 0.474 | 0.480 |
| | $d_1$ | 0.917 | 0.829 | [0.189] | [-0.082] | 0.783 | [0.538] | 0.479 | 0.783 |
| | $d_2$ | 0.382 | 0.996 | [0.510] | [0.186] | 1.064 | 1.684 | 0.722 | 0.966 |
| | $\sigma^2_{C,0}$ | [0.569] | 0.684 | [0.008] | [1.492] | 0.786 | [0.019] | [1.048] | [0.236] |
| | $\sigma^2_{C,1},\ \sigma^2_{C,2}$ | 3.919 | 13.235 | 22.690 | 14.299 | 11.109 | 2.589 | 7.655 | 9.078 |
| $J^P_{t-1,1}$ | $j^P$ | 49.398 | 56.825 | 59.015 | 75.127 | 67.353 | 48.092 | 60.479 | 31.837 |
| | $\sigma^2_{J^P}$ | 728.49 | 1765.5 | 2031.8 | 3509.5 | 2256.9 | 1131.1 | 1461.5 | 580.84 |
| $E^Q_t[J^Q_{t,t+1}]$ | $j^Q$ | 44.971 | 56.982 | 47.217 | 44.687 | 59.403 | 32.617 | 37.340 | 42.055 |
| | $\sigma^2_{J^Q}$ | 306.57 | 513.57 | 623.61 | 306.50 | 1600.3 | 387.49 | 233.09 | 544.81 |
| $\varepsilon^{RV}_t$ | $\sigma^2_{RV,0}$ | 1.951 | 7.496 | 13.900 | 9.732 | [4.972] | 8.149 | 4.692 | 13.780 |
| | $\sigma^2_{RV,1},\ \sigma^2_{RV,2}$ | 142.49 | 87.216 | 114.24 | 21.102 | 144.08 | 92.130 | 59.119 | 55.206 |
| $\varepsilon^{SW}_t$ | $\sigma^2_{SW,0}$ | 11.314 | 5.449 | 14.218 | 11.554 | 7.320 | 12.163 | 4.842 | [0.904] |
| | $\sigma^2_{SW,1},\ \sigma^2_{SW,2}$ | 11.791 | 45.938 | 125.19 | 82.389 | 58.811 | 41.953 | 33.536 | 29.662 |
| $P$ | $p_{00}$ | 0.847 | 0.637 | 0.801 | 0.813 | 0.742 | 0.837 | 0.827 | 0.777 |
| | $p_{11}$ | 0.628 | 0.341 | 0.247 | 0.159 | 0.205 | 0.407 | 0.439 | 0.696 |
| | $p_{12}$ | 0.061 | 0.122 | 0.173 | 0.091 | 0.035 | 0.029 | 0.117 | 0.085 |
| | $p_{22}$ | 0.751 | 0.667 | 0.674 | 0.594 | 0.598 | 0.681 | 0.679 | 0.649 |

Notes: This table provides maximum likelihood estimates for our model. Parameters that are insignificant at a ten percent level are in brackets. The sample used is monthly data from January, 2000, to September, 2015, totalling 189 observations.



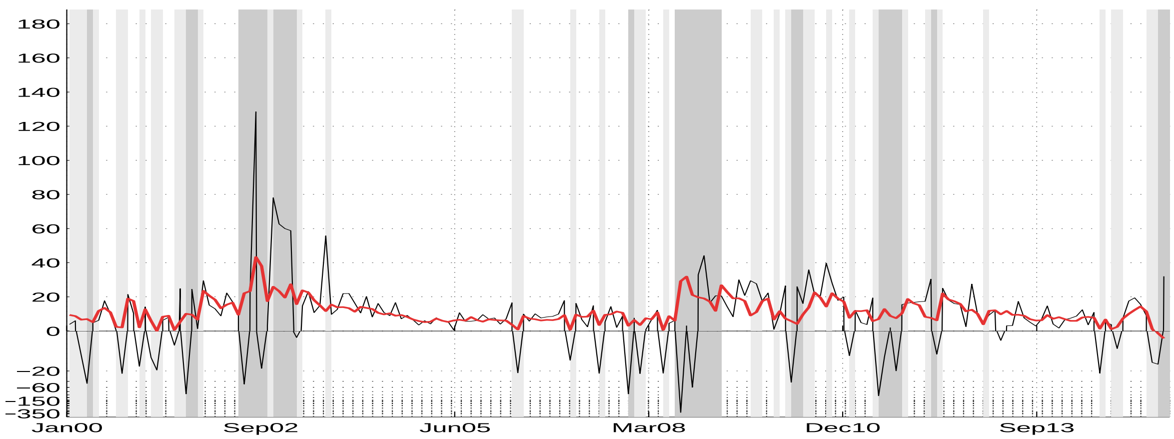

(a) DJIA



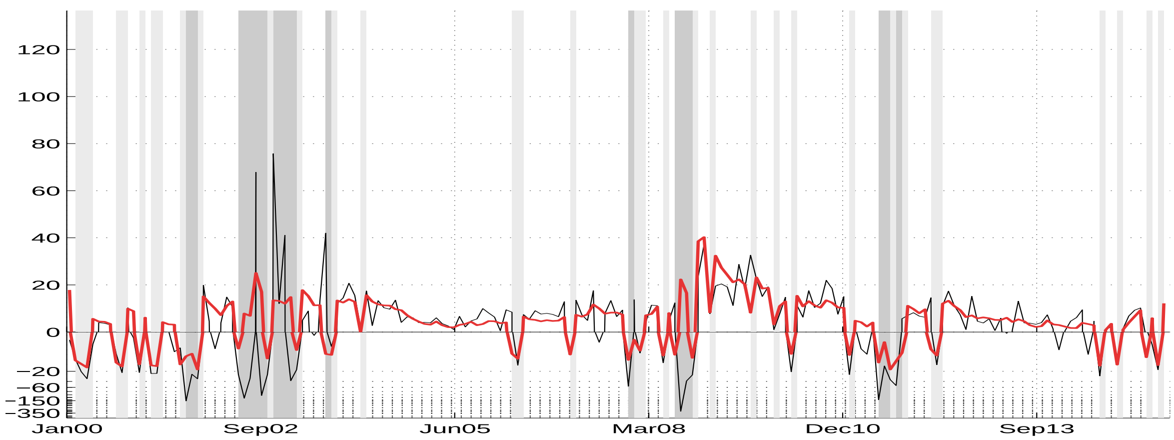

(b) STOXX

Figure 7: $\Pi^{C}_{t,t+1}$ (red) vs $\Pi^{RW}_{t,t+1}$ (black). The shaded areas identify the high vol-of-vol (light) and jump (dark) regimes

along with the VRP from a model that assumes random walk dynamics for the realized variance, $\Pi^{RW}_{t,t+1}$. We see that the positivity of $\Pi^{C}_{t,t+1}$ over the sample period is generally satisfied with only a few exceptions and these are numerically small. These occurrences may appear because the short sample (189 observations) available does not allow to precisely separate the signal from the noise. Furthermore, the peculiar behaviour of the $IV_{t,t+1}$ series for DAX, is inherited by $\Pi^{C}_{t,t+1}$ as can be seen from Figure 8(b) which show multiple positivity violations, though to a lesser extent than $\Pi^{RW}_{t,t+1}$.




(a) CAC




(b) DAX

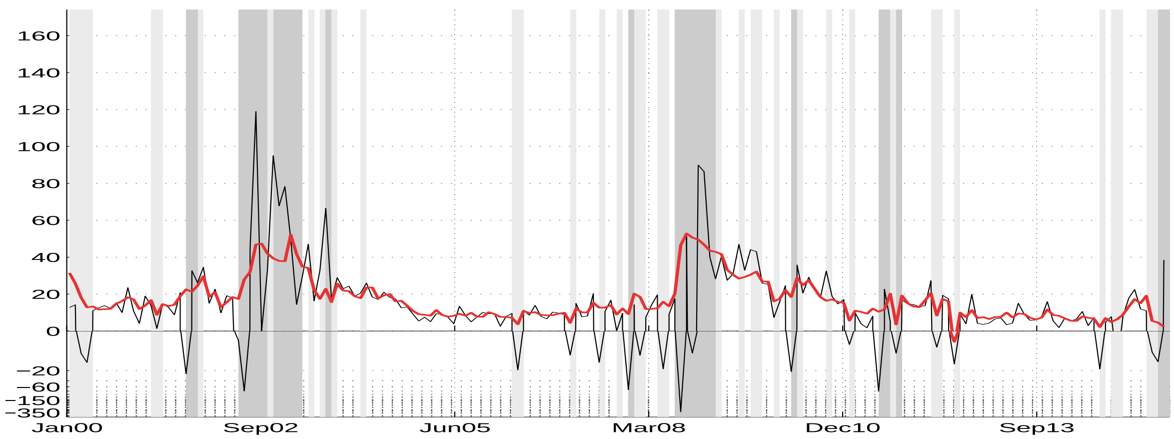
Figure 8: $\Pi^{C}_{t,t+1}$ (red) vs $\Pi^{RW}_{t,t+1}$ (black). The shaded areas identify the high vol-of-vol (light) and jump (dark) regimes

The regime capturing the extreme shocks on the market, i.e. the shaded areas in the figures, is generally common across markets and precisely identifies the well known market turmoil events since 2000. For all markets, we observe an increase in $\Pi^C_{t,t+1}$ coinciding with the peak of the global financial crisis (September, 2008), which instead triggers an unrealistic sharp fall in the VRP computed under the random walk dynamics. The same behaviour is observed with respect to the flash crash and the European Sovereign debt crisis. Table 13 presents descriptive statistics for the sentiment indicators, computed as in Section 4. Overall, jumps occur in about 10 percent of the observations. Note that NIKKEI is the only index where the implied variance lies above realized variance with only few sparse exceptions. This is reflected in $\Delta J^+_t$ and $\Delta J^-_t$, in that NIKKEI is the only case where fear reactions to extreme events have a higher mean compared to optimistic ones.

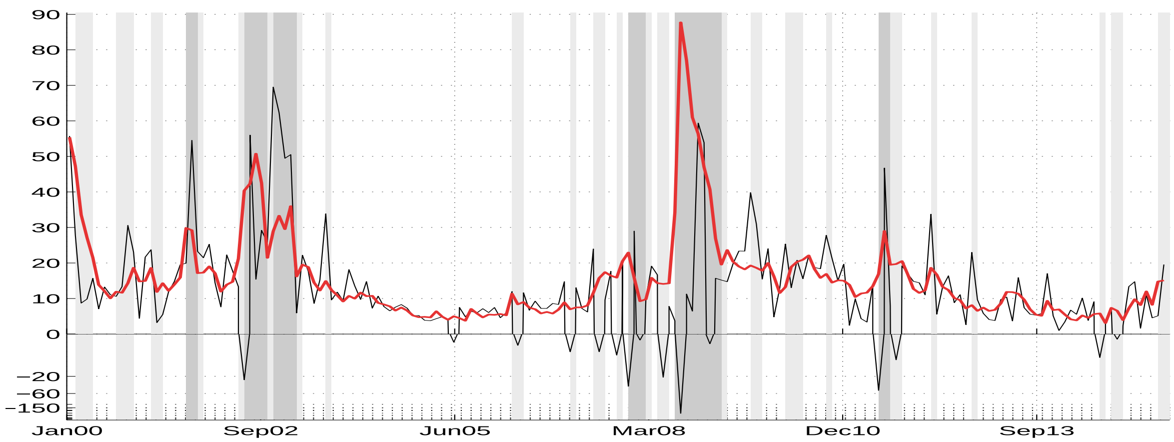
## 6.2 Predictive return regressions

Tables 14 to 17 display the adjusted $R^2$s of predictive return regressions with the VRP, sentiment indicators and their interaction with the state of the market as regressors. For the sake of comparison, we also report the results for the standard approaches described in Section 2. Since realized measures of variance robust to extreme events are available from the Oxford-Man Institute starting from January 2000, we also include an ARX type model denoted by BH. The latter model, inspired by Bekaert and Hoerova (2014), is based on the non-parametric decomposition of the realized variance into its jump and continuous part.

With respect to the smooth part of the VRP, $\Pi^C_{t,t+1}$ generally has no predictive power on future aggregate market excess returns for any horizons. An exception is SMI, showing sizeable $R^2$s up to the five month horizon with a maximum of 4.45 at the three month horizon. When we add to the predictive regression our sentiment indicators, the $R^2$s globally increase in the same fashion as observed in the case of S&P 500 index. This result

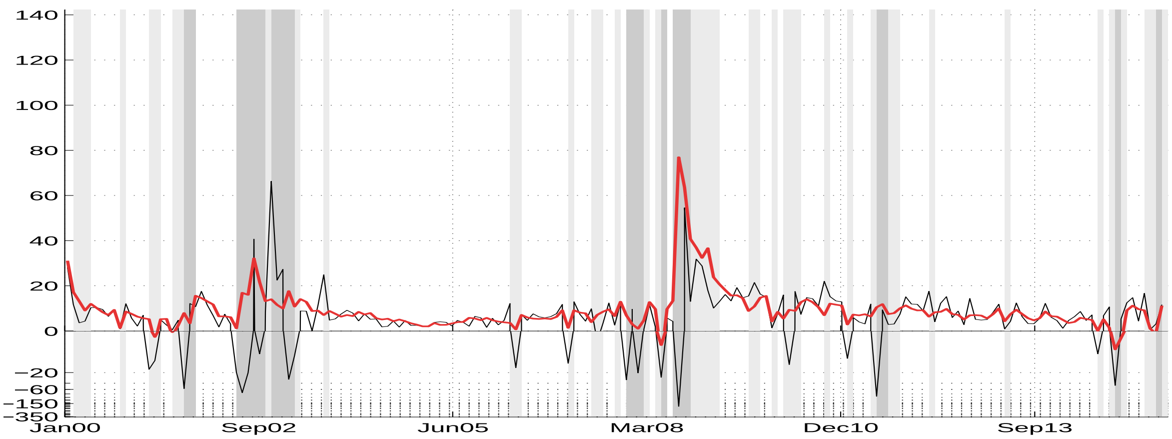



(a) AEX

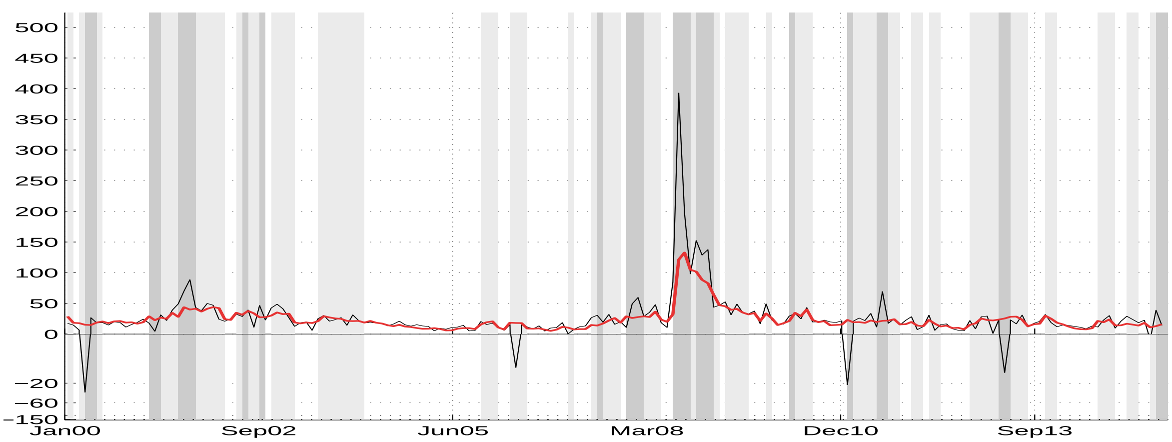



(b) FTSE

Figure 9: $\Pi^{C}_{t,t+1}$ (red) vs $\Pi^{RW}_{t,t+1}$ (black). The shaded areas identify the high vol-of-vol (light) and jump (dark) regimes




(a) SMI




(b) NIKKEI

Figure 10: $\Pi^{C}_{t,t+1}$ (red) vs $\Pi^{RW}_{t,t+1}$ (black). The shaded areas identify the high vol-of-vol (light) and jump (dark) regimes

Table 13: Descriptive statistics for international attitude and sentiment indicators

| | | Sentiment | | Interaction with state of the market | | | |
|---|---|---|---|---|---|---|---|
| Index | | $\Delta J_t^+$ | $\Delta J_t^-$ | $\Delta J_{t,r_t^-}^+$ | $\Delta J_{t,r_t^-}^-$ | $\Delta J_{t,r_t^+}^+$ | $\Delta J_{t,r_t^+}^-$ |
| DJIA | Mean | 31.881 | 53.998 | 35.417 | 56.889 | 21.275 | 30.865 |
| | N.obs | 8 | 9 | 6 | 8 | 2 | 1 |
| STOXX | Mean | 27.236 | 73.111 | 31.337 | 89.544 | 10.832 | 34.769 |
| | N.obs | 20 | 10 | 16 | 7 | 4 | 3 |
| CAC | Mean | 27.268 | 77.107 | 28.714 | 92.566 | 21.486 | 25.575 |
| | N.obs | 15 | 13 | 12 | 10 | 3 | 3 |
| DAX | Mean | 43.615 | 79.245 | 43.615 | 89.622 | – | 22.167 |
| | N.obs | 6 | 13 | 6 | 11 | 0 | 2 |
| AEX | Mean | 32.049 | 67.450 | 38.351 | 79.982 | 13.144 | 25.677 |
| | N.obs | 12 | 13 | 9 | 10 | 3 | 3 |
| FTSE | Mean | 20.781 | 55.413 | 19.917 | 68.099 | 22.797 | 26.870 |
| | N.obs | 10 | 13 | 7 | 9 | 3 | 4 |
| SMI | Mean | 17.076 | 55.676 | 21.038 | 67.194 | 5.192 | 29.760 |
| | N.obs | 8 | 13 | 6 | 9 | 2 | 4 |
| NIKKEI | Mean | 50.083 | 22.957 | 47.066 | 24.368 | 64.165 | 20.606 |
| | N.obs | 17 | 8 | 14 | 5 | 3 | 3 |

Notes: The statistics are computed considering only the non-zero values for the variables. The sample used is monthly data from January, 2000, to September, 2015, totalling 189 observations.

clearly indicates that the information contained in the jump component of the VRP bears most of, if not all, the predictive power on future aggregate market returns. The only exception seems again to be SMI where adding the sentiment indicators to the predictive regression only marginally increases the predictive power at all horizons and though often significantly so.

Isolating the asymmetric effect of $\Delta J_t^+$ and $\Delta J_t^-$ with respect to the state of the market, allows to further increase the $R^2$s particularly at the short and long horizons. The most striking example is DJIA, where the interaction of the sentiment indicators with the sign of the market performance explain up to 13.8% of the future return variation at the four month horizon. Overall the results for DJIA are very similar to our findings for the S&P 500 index in the previous section. The STOXX and FTSE indices show the same sharp increase in return predictability though to a lower extent. For instance, in the case of FTSE, the four indicators contribute to a staggering increase in return predictability from 0.33 to 6.81 at the four month horizon.

Figure 11 provides an overall view of the $R^2$ patterns of the regressions with the interaction of the sentiment indicators with the sign of the market performance, i.e. the $R^2$s in the 4th row of Tables 14 to 17. We clearly see that for DJIA, STOXX, CAC, DAX and AEX, the inverse U-shape is positioned within the one and six month horizons before flattening out at their initial levels. For SMI and FTSE, the $R^2$s trajectory decays much slower. Contrary to what is normally observed, the NIKKEI reveals an inverse U-shape $R^2$ pattern with peaks at horizon nine.

In terms of marginal contribution, Fear&Bear explains a large part of the return predictability at all horizons for the DJIA, SMI and to a limited extent AEX and FTSE. The $R^2$s of DJIA are particularly high at the long horizons while the reverse is true for SMI. The Optimism&Bear indicator yields similar return predictability. The combinations Fear&Bull and Optimism&Bull contribute to explaining the return variation only to a lim-

Table 14: Adjusted $R^2$ from predictive return regressions

**DJIA**

| Regressors | | Horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^i$ | $SI_{j,t}$ | 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
| $\Pi^C_{t,t+1}$ | | -0.56 | -0.56 | -0.49 | -0.22 | 0.26 | 0.17 | 0.13 | 0.62 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t$ | 4.16 | 5.39 | 8.63 | 12.76 | 8.26 | 4.81 | 1.13 | 0.85 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t^+$ $\Delta J_t^-$ | 6.06 | 8.06 | 10.36 | 13.66 | 9.76 | 7.52 | 5.31 | 5.00 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^-}$ $\Delta J^-_{t,r_t^-}$ $\Delta J^+_{t,r_t^+}$ $\Delta J^-_{t,r_t^+}$ | 7.20 | 8.18 | 10.44 | 13.80 | 9.87 | 7.68 | 7.03 | 7.18 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t^+$ | 3.91 | 5.46 | 5.20 | 4.94 | 5.21 | 5.71 | 5.29 | 4.88 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t^-$ | 1.90 | 2.43 | 5.19 | 9.13 | 5.25 | 2.29 | 0.19 | 0.67 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^-}$ | 2.27 | 4.04 | 4.46 | 4.07 | 4.17 | 5.53 | 6.83 | 6.60 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r_t^-}$ | 2.18 | 2.28 | 4.94 | 8.71 | 5.02 | 2.27 | 0.20 | 0.64 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^+}$ | 1.37 | 0.83 | 0.19 | 0.59 | 1.25 | 0.47 | 0.21 | 0.83 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r_t^+}$ | -0.10 | -0.35 | -0.18 | 0.30 | 0.56 | 0.18 | 0.22 | 0.89 |
| $\Pi^{RW}_{t,t+1}$ | | 3.30 | 5.24 | 10.63 | 14.06 | 10.31 | 6.84 | 1.41 | 0.53 |
| $\Pi^{AR(1)}_{t,t+1}$ | | 0.92 | 1.97 | 3.42 | 5.18 | 5.58 | 5.55 | 3.98 | 3.88 |
| $\Pi^{ARX2}_{t,t+1}$ | | 0.40 | 1.17 | 1.96 | 3.22 | 3.99 | 4.44 | 3.87 | 4.04 |
| $\Pi^{BH}_{t,t+1}$ | | 0.30 | 1.14 | 1.89 | 3.12 | 4.21 | 4.83 | 4.15 | 4.37 |

**STOXX**

| Regressors | | Horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^i$ | $SI_{j,t}$ | 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
| $\Pi^C_{t,t+1}$ | | -0.56 | -0.51 | -0.35 | -0.39 | -0.51 | -0.57 | -0.22 | -0.02 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t$ | 0.03 | 0.70 | 1.20 | 2.26 | 1.37 | -0.21 | 0.05 | 0.50 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t^+$ $\Delta J_t^-$ | 0.70 | 0.70 | 1.21 | 2.29 | 1.37 | -0.01 | 0.09 | 0.68 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^-}$ $\Delta J^-_{t,r_t^-}$ $\Delta J^+_{t,r_t^+}$ $\Delta J^-_{t,r_t^+}$ | 1.53 | 1.90 | 2.09 | 3.10 | 2.71 | 0.29 | 0.24 | 1.28 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t^+$ | -0.49 | -0.13 | 0.05 | 0.73 | 0.18 | -0.07 | 0.00 | 0.55 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t^-$ | 0.49 | 0.51 | 1.03 | 1.64 | 1.00 | -0.43 | -0.07 | 0.23 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^-}$ | -0.39 | -0.31 | -0.12 | 0.42 | -0.10 | -0.20 | 0.05 | 0.74 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r_t^-}$ | 0.54 | 0.17 | 0.67 | 1.23 | 0.59 | -0.47 | -0.06 | 0.25 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^+}$ | 0.22 | 0.42 | 0.39 | 0.50 | 0.74 | -0.19 | -0.13 | 0.39 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r_t^+}$ | -0.55 | 0.19 | 0.21 | 0.09 | 0.15 | -0.50 | -0.21 | -0.01 |
| $\Pi^{RW}_{t,t+1}$ | | -0.15 | 0.68 | 0.78 | 1.68 | 1.31 | -0.08 | -0.22 | 0.23 |
| $\Pi^{AR(1)}_{t,t+1}$ | | -0.51 | -0.55 | -0.56 | -0.46 | -0.41 | -0.48 | -0.35 | 0.06 |
| $\Pi^{ARX2}_{t,t+1}$ | | -0.10 | -0.20 | 0.02 | -0.25 | -0.43 | -0.57 | -0.53 | -0.41 |
| $\Pi^{BH}_{t,t+1}$ | | -0.01 | -0.06 | 0.25 | -0.09 | -0.30 | -0.52 | -0.57 | -0.51 |

Notes: This table reports the adjusted $R^2$s in percentage from the predictive return regressions in (25) with VRP, sentiment indicators and their interaction with the state of the market as regressors. The sample used is monthly data from January, 2000, to March, 2015, totalling 189 observations.

Table 15: Adjusted $R^2$ from predictive return regressions

**CAC**

| Regressors $\Pi^i$ | $SI_{j,t}$ | Horizon 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^C_{t,t+1}$ | | -0.51 | -0.14 | 0.16 | 0.06 | -0.42 | -0.57 | -0.44 | -0.13 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t$ | 1.82 | 3.52 | 4.59 | 4.36 | 2.96 | 0.78 | 1.10 | 1.91 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ $\Delta J^-_t$ | 1.83 | 3.76 | 4.76 | 4.78 | 3.15 | 1.30 | 1.64 | 2.25 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^-_t}$ $\Delta J^-_{t,r^-_t}$ $\Delta J^+_{t,r^+_t}$ $\Delta J^-_{t,r^+_t}$ | 2.44 | 4.53 | 5.72 | 6.21 | 4.39 | 1.60 | 1.74 | 2.34 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ | 0.03 | 1.38 | 1.71 | 2.18 | 0.95 | 0.72 | 0.96 | 1.14 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_t$ | 1.53 | 2.74 | 3.78 | 3.29 | 2.25 | 0.25 | 0.52 | 1.31 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^-_t}$ | -0.31 | 0.72 | 0.91 | 1.10 | 0.19 | 0.33 | 0.65 | 1.04 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r^-_t}$ | 1.57 | 2.41 | 3.49 | 2.92 | 1.93 | 0.12 | 0.53 | 1.34 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^+_t}$ | 0.32 | 0.88 | 1.65 | 2.05 | 1.14 | -0.14 | -0.19 | -0.08 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r^+_t}$ | -0.50 | 0.47 | 0.54 | 0.76 | 0.22 | -0.21 | -0.44 | -0.12 |
| $\Pi^{RW}_{t,t+1}$ | | 1.31 | 3.59 | 4.60 | 4.33 | 4.13 | 1.92 | 2.03 | 2.83 |
| $\Pi^{AR(1)}_{t,t+1}$ | | -0.53 | -0.53 | -0.54 | -0.53 | -0.46 | -0.52 | -0.24 | 0.29 |
| $\Pi^{ARX2}_{t,t+1}$ | | 0.04 | -0.08 | 0.11 | -0.03 | -0.27 | -0.39 | -0.58 | -0.49 |
| $\Pi^{BH}_{t,t+1}$ | | -0.16 | -0.32 | -0.20 | -0.27 | -0.44 | -0.51 | -0.57 | -0.38 |

**DAX**

| Regressors $\Pi^i$ | $SI_{j,t}$ | Horizon 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^C_{t,t+1}$ | | -0.33 | 0.11 | -0.21 | -0.45 | -0.11 | 0.38 | 0.34 | 1.60 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t$ | 0.87 | 1.54 | 1.44 | 2.33 | 1.75 | 0.73 | 0.70 | 2.62 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ $\Delta J^-_t$ | 1.72 | 2.49 | 3.04 | 3.10 | 3.00 | 1.43 | 0.80 | 2.67 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^-_t}$ $\Delta J^-_{t,r^-_t}$ $\Delta J^+_{t,r^+_t}$ $\Delta J^-_{t,r^+_t}$ | 1.96 | 2.65 | 3.34 | 3.46 | 3.32 | 1.44 | 0.98 | 3.27 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ | -0.05 | 0.41 | 0.42 | -0.35 | 0.29 | 0.74 | 0.35 | 1.61 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_t$ | 1.39 | 2.13 | 2.32 | 2.96 | 2.53 | 1.03 | 0.79 | 2.67 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^-_t}$ | -0.05 | 0.41 | 0.42 | -0.35 | 0.29 | 0.74 | 0.35 | 1.61 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r^-_t}$ | 1.24 | 2.23 | 2.48 | 3.16 | 2.70 | 1.04 | 0.84 | 2.82 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^+_t}$ | -0.33 | 0.11 | -0.21 | -0.45 | -0.11 | 0.38 | 0.34 | 1.60 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r^+_t}$ | 0.05 | 0.18 | -0.06 | -0.26 | 0.06 | 0.38 | 0.47 | 2.06 |
| $\Pi^{RW}_{t,t+1}$ | | 1.08 | 2.61 | 2.67 | 3.43 | 2.94 | 0.82 | 0.57 | 1.96 |
| $\Pi^{AR(1)}_{t,t+1}$ | | -0.56 | -0.50 | -0.56 | -0.49 | -0.50 | -0.56 | -0.58 | -0.24 |
| $\Pi^{ARX2}_{t,t+1}$ | | 0.08 | 0.02 | 0.69 | 0.18 | 0.06 | 0.09 | -0.26 | -0.53 |
| $\Pi^{BH}_{t,t+1}$ | | 0.03 | -0.08 | 0.53 | 0.05 | -0.05 | 0.02 | -0.31 | -0.56 |

Notes: This table reports the adjusted $R^2$s in percentage from the predictive return regressions in (25) with VRP, sentiment indicators and their interaction with the state of the market as regressors. The sample used is monthly data from January, 2000, to March, 2015, totalling 189 observations.

Table 16: Adjusted $R^2$ from predictive return regressions

**AEX**

| Regressors $\Pi^i$ | $SI_{j,t}$ | Horizon 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^C_{t,t+1}$ | | 0.14 | 0.16 | 0.72 | 0.33 | -0.24 | -0.53 | -0.26 | -0.45 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t$ | 1.03 | 2.29 | 2.47 | 2.49 | 2.21 | 1.01 | 0.86 | 0.58 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ $\Delta J^-_t$ | 1.04 | 2.53 | 2.88 | 2.74 | 2.22 | 1.05 | 0.92 | 0.67 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^-}$ $\Delta J^-_{t,r_t^-}$ $\Delta J^+_{t,r_t^+}$ $\Delta J^-_{t,r_t^+}$ | 2.97 | 2.99 | 3.60 | 3.18 | 3.00 | 1.22 | 1.39 | 0.81 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ | 0.67 | 1.93 | 2.52 | 2.38 | 1.12 | 0.75 | 0.54 | 0.42 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_t$ | 1.03 | 1.95 | 2.11 | 2.22 | 2.15 | 0.90 | 0.73 | 0.44 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^-}$ | 0.76 | 1.70 | 2.21 | 2.28 | 1.07 | 0.77 | 0.68 | 0.49 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r_t^-}$ | 1.19 | 1.95 | 2.08 | 2.09 | 1.91 | 0.83 | 0.74 | 0.43 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^+}$ | 1.47 | 1.74 | 2.54 | 1.85 | 1.07 | 0.53 | 0.60 | 0.18 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r_t^+}$ | 1.56 | 1.14 | 1.61 | 2.25 | 1.80 | 0.72 | 0.27 | 0.11 |
| $\Pi^{RW}_{t,t+1}$ | | 0.63 | 1.14 | 1.58 | 1.70 | 0.94 | 0.52 | 0.26 | 0.10 |
| $\Pi^{AR(1)}_{t,t+1}$ | | 0.34 | -0.21 | 0.00 | 0.25 | 0.06 | 0.23 | -0.04 | -0.28 |
| $\Pi^{ARX2}_{t,t+1}$ | | 1.09 | 0.96 | 1.42 | 1.73 | 1.90 | 1.65 | 1.16 | 0.66 |
| $\Pi^{BH}_{t,t+1}$ | | -0.50 | -0.54 | -0.56 | -0.36 | -0.10 | -0.13 | 0.89 | 1.07 |

**FTSE**

| Regressors $\Pi^i$ | $SI_{j,t}$ | Horizon 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^C_{t,t+1}$ | | 0.14 | 0.16 | 0.72 | 0.33 | -0.24 | -0.53 | -0.26 | -0.45 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t$ | 0.53 | 1.34 | 4.18 | 4.66 | 2.89 | 1.94 | 1.03 | 0.44 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ $\Delta J^-_t$ | 0.98 | 2.23 | 5.36 | 6.49 | 5.53 | 5.01 | 1.97 | 1.55 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^-}$ $\Delta J^-_{t,r_t^-}$ $\Delta J^+_{t,r_t^+}$ $\Delta J^-_{t,r_t^+}$ | 3.61 | 6.00 | 6.71 | 6.81 | 6.46 | 5.44 | 2.57 | 2.12 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ | 0.23 | 1.97 | 4.09 | 5.05 | 4.91 | 4.70 | 1.67 | 1.45 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_t$ | 0.72 | 0.87 | 3.16 | 3.28 | 1.58 | 0.75 | 0.52 | 0.01 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^-}$ | 0.93 | 0.18 | 1.51 | 2.70 | 1.88 | 2.15 | 1.84 | 1.74 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r_t^-}$ | 0.39 | 0.49 | 2.64 | 3.22 | 1.72 | 0.79 | 0.64 | 0.06 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r_t^+}$ | 0.76 | 4.76 | 4.23 | 2.57 | 2.94 | 1.87 | -0.18 | -0.41 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r_t^+}$ | 1.27 | 1.33 | 1.08 | 0.35 | -0.08 | -0.48 | -0.10 | -0.38 |
| $\Pi^{RW}_{t,t+1}$ | | -0.37 | 0.71 | 3.36 | 3.93 | 2.74 | 2.38 | 0.38 | 0.45 |
| $\Pi^{AR(1)}_{t,t+1}$ | | -0.47 | -0.50 | -0.22 | -0.07 | 0.05 | 0.41 | 0.17 | 0.01 |
| $\Pi^{ARX2}_{t,t+1}$ | | -0.21 | -0.46 | -0.44 | -0.49 | -0.57 | -0.49 | -0.30 | -0.44 |
| $\Pi^{BH}_{t,t+1}$ | | -0.35 | -0.23 | -0.34 | -0.12 | 0.02 | 0.37 | -0.27 | -0.53 |

Notes: This table reports the adjusted $R^2$s in percentage from the predictive return regressions in (25) with VRP, sentiment indicators and their interaction with the state of the market as regressors. The sample used is monthly data from January, 2000, to March, 2015, totalling 189 observations.

Table 17: Adjusted $R^2$ from predictive return regressions

**SMI**

| Regressors | | Horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^i$ | $SI_{j,t}$ | 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
| $\Pi^C_{t,t+1}$ | | 2.29 | 2.75 | 4.45 | 4.21 | 2.50 | 0.56 | -0.58 | -0.58 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t$ | 4.07 | 3.36 | 4.45 | 4.57 | 3.06 | 1.10 | -0.13 | -0.34 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ $\Delta J^-_t$ | 5.47 | 4.93 | 5.71 | 4.79 | 3.21 | 1.14 | 0.13 | -0.33 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^-_t}$ $\Delta J^-_{t,r^-_t}$ $\Delta J^+_{t,r^+_t}$ $\Delta J^-_{t,r^+_t}$ | 7.91 | 5.52 | 5.79 | 5.23 | 4.83 | 3.87 | 0.63 | -0.17 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ | 4.77 | 4.85 | 5.61 | 4.26 | 2.51 | 0.56 | -0.53 | -0.57 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_t$ | 3.32 | 2.95 | 4.48 | 4.70 | 3.17 | 1.14 | 0.02 | -0.33 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^-_t}$ | 4.77 | 5.06 | 5.67 | 4.28 | 2.54 | 0.56 | -0.56 | -0.56 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r^-_t}$ | 2.66 | 2.85 | 4.49 | 4.50 | 2.79 | 0.69 | -0.18 | -0.41 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^+_t}$ | 2.31 | 2.89 | 4.46 | 4.31 | 3.01 | 1.02 | -0.26 | -0.51 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r^+_t}$ | 5.31 | 3.09 | 4.45 | 4.65 | 3.85 | 3.19 | -0.21 | -0.42 |
| $\Pi^{RW}_{t,t+1}$ | | 0.65 | -0.48 | -0.29 | 0.73 | 0.83 | 0.52 | -0.24 | -0.36 |
| $\Pi^{AR(1)}_{t,t+1}$ | | 4.50 | 3.19 | 3.49 | 2.37 | 1.29 | 0.25 | -0.47 | -0.48 |
| $\Pi^{ARX2}_{t,t+1}$ | | 0.86 | 1.81 | 4.33 | 5.22 | 3.93 | 2.01 | -0.03 | -0.15 |
| $\Pi^{BH}_{t,t+1}$ | | -0.42 | -0.10 | 0.61 | 0.73 | 0.32 | -0.22 | -0.58 | -0.51 |

**NIKKEI**

| Regressors | | Horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\Pi^i$ | $SI_{j,t}$ | 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
| $\Pi^C_{t,t+1}$ | | -0.53 | -0.33 | 0.31 | 0.35 | -0.28 | -0.46 | 0.26 | 1.10 |
| $\Pi^C_{t,t+1}$ | $\Delta J_t$ | 0.90 | 2.93 | 0.89 | 0.40 | 0.10 | 0.23 | 4.03 | 2.62 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ $\Delta J^-_t$ | 1.29 | 3.01 | 1.06 | 0.41 | 0.21 | 0.36 | 4.40 | 2.69 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^-_t}$ $\Delta J^-_{t,r^-_t}$ $\Delta J^+_{t,r^+_t}$ $\Delta J^-_{t,r^+_t}$ | 2.12 | 3.06 | 1.43 | 0.74 | 0.35 | 1.65 | 5.42 | 3.05 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_t$ | 0.40 | 2.94 | 0.68 | 0.38 | -0.04 | 0.02 | 3.09 | 2.30 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_t$ | 0.41 | -0.24 | 0.72 | 0.38 | -0.02 | -0.10 | 1.68 | 1.53 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^-_t}$ | -0.14 | 2.47 | 0.78 | 0.35 | -0.10 | -0.29 | 2.03 | 1.98 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r^-_t}$ | 0.65 | -0.20 | 0.95 | 0.35 | 0.10 | 0.50 | 2.22 | 1.80 |
| $\Pi^C_{t,t+1}$ | $\Delta J^+_{t,r^+_t}$ | 0.11 | -0.32 | 0.40 | 0.59 | -0.27 | -0.03 | 0.77 | 1.17 |
| $\Pi^C_{t,t+1}$ | $\Delta J^-_{t,r^+_t}$ | -0.52 | -0.33 | 0.33 | 0.48 | -0.28 | -0.20 | 0.27 | 1.12 |
| $\Pi^{RW}_{t,t+1}$ | | -0.31 | -0.29 | -0.35 | -0.05 | -0.49 | -0.56 | -0.56 | -0.36 |
| $\Pi^{AR(1)}_{t,t+1}$ | | -0.32 | -0.33 | -0.30 | 0.04 | -0.44 | -0.57 | -0.58 | -0.10 |
| $\Pi^{ARX2}_{t,t+1}$ | | -0.33 | -0.36 | -0.28 | 0.06 | -0.42 | -0.57 | -0.56 | 0.05 |
| $\Pi^{BH}_{t,t+1}$ | | -0.17 | -0.26 | -0.34 | 0.07 | -0.44 | -0.57 | -0.58 | -0.10 |

Notes: This table reports the adjusted $R^2$s in percentage from the predictive return regressions in (25) with VRP, sentiment indicators and their interaction with the state of the market as regressors. The sample used is monthly data from January, 2000, to March, 2015, totalling 189 observations.

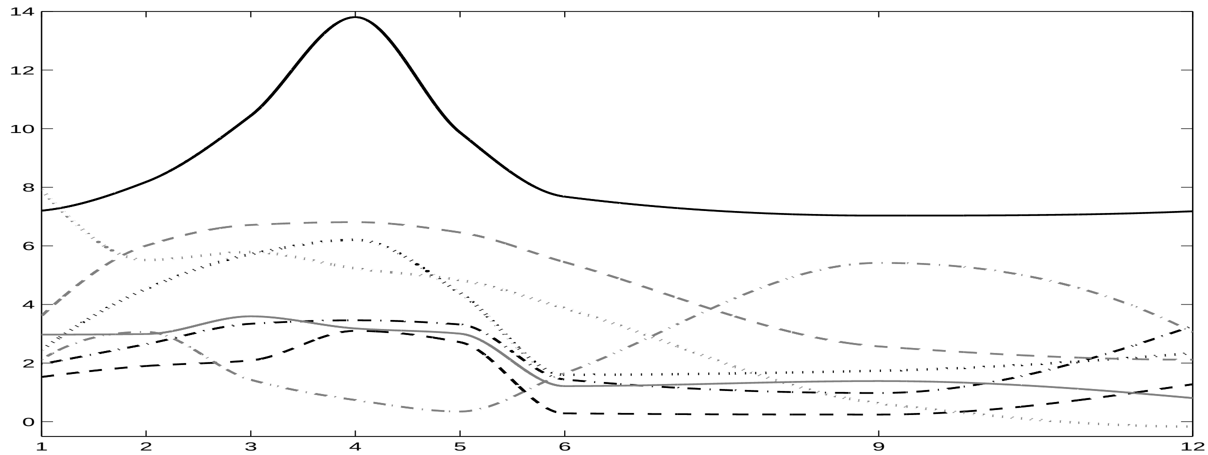



Figure 11: Adjusted $R^2$s from predictive return regressions with sentiment indicators for the eight international indices: DJIA (black), STOXX (black dashed), CAC (black dotted), DAX (black dot-dashed), AEX (grey solid), FTSE (grey dashed), SMI (grey dotted) and NIKKEI (grey dot-dashed)

ited extent, e.g. CAC has $R^2$s equal to 2.05 and 0.76 for Fear&Bull and Optimism&Bull, respectively, at the four month horizon. For STOXX, NIKKEI and DAX, the individual contribution of each indicator is difficult to assess. The heterogeneity of the return predictability shows that the attitude of agents towards risk as well as the pricing of different risk factors varies across markets. In the Euro area, the sentiment factors are priced only to a lower extent and in the short term. In the Japanese, Swiss and UK markets, the effect on prices is stronger and longer lasting. Last, for the US markets, asset prices are the most reactive to agents beliefs towards the future occurrence of extreme shocks.

We next turn our attention to the performance of the VRPs estimated using the standard approaches in the literature. In terms of size of the $R^2$s, we observe large differences in the level of DJIA excess return predictability compared to the other markets. In particular, the US markets show higher predictability when compared to the other markets considered here. The VRP based on random walk expectations shows higher explanatory power then the one based on autoregressive type models. This holds for all markets except for NIKKEI and SMI. In fact, for NIKKEI, none of the standard approaches yields excess return predictability at any horizon. For SMI, in turn, $\Pi^{RW}_{t,t+1}$ does not show any predictability, but VRP estimates based on autoregressive type models do. For example, $\Pi^{ARX2}_{t,t+1}$ explains 4.93% of the excess return variation at the one month horizon and exhibits sizeable though decreasing $R^2$s up to the five month horizon. These results are generally in line with Bollerslev, Marrone, Xu, and Zhou (2014), though their findings only extend to the case of $\Pi^{RW}_{t,t+1}$, rely on a shorter sample period ending in December 2011 and considers only a subset of the indices used in this paper. For example, in terms of $R^2$s' size and patterns, they find similar predictability for the CAC, DAX, FTSE and the NIKKEI. However, a difference is SMI for which instead they document the usual inverse U-shape $R^2$s pattern with substantial predictability at medium to long horizons.

Compared to the case of the standard approaches, the $R^2$s of the predictive return regressions including the sentiment indicators and their interaction with the state of the market as explanatory variables largely dominates over most if not all horizons those using the random walk based VRP for all indices. The results reported for DJIA, which shows stronger gains at the short and long horizons, displays very similar patterns and levels as observed for the S&P 500 index. For FTSE, the extra degree of flexibility provided by the sentiment indicators, produces gains in predictability from 3.36 to 6.71, and 2.38 to 5.44 at the three and six month horizon respectively. Another example is SMI with gains from -0.29 to 5.79, and 0.52 to 3.87 at the three and six month horizon respectively. Thus, the international evidence not only confirms our findings from the US but also shows that for several markets the sentiment indicators have significantly more predictive power than what is obtained with any of the existing methods for modeling the VRP.

# 7 Conclusion

This paper proposes a flexible approach for retrieving the variance risk premia which is essential for risk management, asset pricing and portfolio management. The method delivers more refined, precise and realistic estimates of the market price of risk and it is straightforward to implement using maximum likelihood. Moreover, our model allows identifying not only the smooth component associated with the price of risk in periods of normal market activity but also to infer the occurrence and size of extreme variance events or jumps and the risk neutral expectation of these. Based on this decomposition we show how to construct a number of sentiment indicators based on agents' expectations response to extreme shocks.

Our empirical application to the S&P 500 index documents the importance of allowing for interactions, discontinuities and occurrence of extreme events. In particular, our model specification is strongly supported by the data and all structural parameters are estimated with a high degree of precision. Moreover, the filtered variance predictions and the risk neutral variance expectations match adequately the level and the dynamics of their observable counterparts along the entire sample. Finally and most importantly, the resulting filtered variance risk premium satisfies the positivity constraint, shows a reasonable degree of smoothness and it appropriately reacts to changes in level and variability.

We address a puzzle in the existing literature related to return predictability by disentangling the predictability stemming from the part of the variance risk premium associated with normal sized price fluctuations from that associated with extreme tail events, i.e. tail risk. In particular, using our sentiment indicators we show that predictability comes almost entirely from the way agents adapt their expectations when exposed to extreme variance realizations and our results essentially show that future market performance is driven by fear and optimism with respect to the current state of the market. The documented predictive power is distinct from that of alternative economic predictors. Our results extend internationally to eight major markets for which we find that excess returns are to a large extent explained by extreme tail events and only marginally, if at all, by the premium associated with normal price fluctuations.

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# A Technical details

This appendix provides further details on how the model in Section 3 can be written in state space form and can be estimated using straightforward filtering techniques.

## A.1 State space formulation

To rewrite the model in state space form, we define for measurement and transition equations of the physical variance $\Sigma^*_{t-1,t} = \Sigma_{t-1,t} + j^P_{S_t}$ and $\mathcal{E}^{RV}_t = \varepsilon^{RV}_t + \sigma_{J^P,S_t} u^P_t$, with $\mathrm{Var}(\mathcal{E}^{RV}_t) = \sigma^2_{RV,S_t} + \sigma^2_{J^P,S_t} = \varsigma^2_{RV,S_t}$. Then after the following transformation we can rewrite (15) as

$$\begin{aligned}
\Sigma^*_{t-1,t} &= (b_{S_t} + j^P_{S_t}) + \phi^{\Sigma}(\Sigma^*_{t-2,t-1} - b_{S_{t-1}} - j^P_{S_{t-1}}) + \sigma_{\Sigma,S_t} v^{\Sigma}_t \\
&= k^{(1)}_{S_t} + \phi^{\Sigma}(\Sigma^*_{t-2,t-1} - k^{(1)}_{S_{t-1}}) + \sigma_{\Sigma,S_t} v^{\Sigma}_t \qquad (27) \\
RV_{t-1,t} &= \Sigma^*_{t-1,t} + \mathcal{E}^{RV}_t \qquad (28)
\end{aligned}$$

where $k^{(1)}_{S_t}$ is a regime switching drift parameter.

We also re-parameterize and reduce the measurement and transition equations of the risk neutral expectation of the variance to functions of the latent states $F$ and $\Sigma$. We can first compute $E_t^P[\Sigma_{t,t+1}]$ as

$$\begin{aligned} E_t^P[\Sigma_{t,t+1}] &= b_{S_t} + \phi^\Sigma(\Sigma_{t-1,t} - b_{S_t}) \\ &= b_{S_t} + \phi^\Sigma(\Sigma^*_{t-1,t} - b_{S_t} - j^P_{S_t}) \\ &= (b_{S_t}(1-\phi^\Sigma) - \phi^\Sigma j^P_{S_t}) + \phi^\Sigma \Sigma^*_{t-1,t}. \end{aligned} \tag{29}$$

Second, the term in (18) which links the variance to the VRP can be written as

$$\begin{aligned} d_{S_t}(\Sigma_{t-1,t} - b_{S_t}) &= d_{S_t}(\Sigma^*_{t-1,t} - b_{S_t} - j^P_{S_t}) \\ &= -d_{S_t}(b_{S_t} + j^P_{S_t}) + d_{S_t}\Sigma^*_{t-1,t}. \end{aligned} \tag{30}$$

By defining $F^*_{t,t+1} = F_{t,t+1} + j^Q_{S_t} + b_{S_t}(1-\phi^\Sigma - d_{S_t}) - j^P_{S_t}(\phi^\Sigma + d_{S_t}) = F_{t,t+1} + k^{(2)}_{S_t} - a_{S_t}$, then we can write the transition equation for the latent state $F^*_{t,t+1}$ as

$$\begin{aligned} F^*_{t,t+1} &= k^{(2)}_{S_t} + \phi^F(F_{t-1,t} - a_{S_{t-1}}) + \sigma_{F,S_t} v^F_t \\ &= k^{(2)}_{S_t} + \phi^F(F^*_{t-1,t} - k^{(2)}_{S_{t-1}}) + \sigma_{F,S_t} v^F_t \end{aligned} \tag{31}$$

where $k^{(2)}_{S_t}$ is a Markov-state dependent parameter defining the drift in the reduced form of the second latent state.

Following the same rationale for the physical variance, we define for the risk-neutral variance $\mathcal{E}^{SW}_t = \varepsilon^{SW}_t + \sigma_{J^Q,S_t} u^Q_t$, with $\text{Var}(\mathcal{E}^{SW}_t) = \sigma^2_{SW,S_t} + \sigma^2_{J^Q,S_t} = \varsigma^2_{SW,S_t}$. The reduced form for the measurement equation of $SW_{t,t+1}$ in (16) is given by

$$\begin{aligned} SW_{t,t+1} &= F^*_{t,t+1} + (d_{S_t} + \phi^\Sigma)\Sigma^*_{t-1,t} + \mathcal{E}^{SW}_t \\ &= F^*_{t,t+1} + h^{(12)}_{S_t}\Sigma^*_{t-1,t} + \mathcal{E}^{SW}_t. \end{aligned} \tag{32}$$

Putting the pieces together, the reduced form model in its state space form is thus

$$\begin{bmatrix} RV_{t-1,t} \\ SW_{t,t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ h^{(12)}_{S_t} & 1 \end{bmatrix} \begin{bmatrix} \Sigma^*_{t-1,t} \\ F^*_{t,t+1} \end{bmatrix} + \begin{bmatrix} \mathcal{E}^{RV}_t \\ \mathcal{E}^{SW}_t \end{bmatrix}, \tag{33}$$

with

$$\begin{bmatrix} \Sigma^*_{t-1,t} \\ F^*_{t,t+1} \end{bmatrix} = \begin{bmatrix} k^{(1)}_{S_t} \\ k^{(2)}_{S_t} \end{bmatrix} + \begin{bmatrix} \phi^{\Sigma} & 0 \\ 0 & \phi^{F} \end{bmatrix} \left( \begin{bmatrix} \Sigma^*_{t-2,t-1} \\ F^*_{t-1,t} \end{bmatrix} - \begin{bmatrix} k^{(1)}_{S_t} \\ k^{(2)}_{S_t} \end{bmatrix} \right) + \begin{bmatrix} \sigma_{\Sigma,S_t} & 0 \\ 0 & \sigma_{F,S_t} \end{bmatrix} \begin{bmatrix} v^{\Sigma}_t \\ v^{F}_t \end{bmatrix}. \quad (34)$$

## A.2 Estimation by maximum likelihood

The model in (33)-(34) is estimated using Kim (1994) filter which consists of a combination of extended versions of the Kalman filter and the Hamilton filter, adapted with suitable approximations. We use a non-linear optimization procedure to maximize the approximated log-likelihood function with respect to the parameters of interest, in a quasi maximum likelihood framework, see Kim and Nelson (1999) for details. Computations are done in Matlab 2016a and the code developed by the authors is available upon request.

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