Crisis-Robust Bond Portfolios

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Abstract

This paper defines a "crisis-robust portfolio" that satisfies the minimal crisis-to-quiet time volatility ratio. This type of portfolio is less demanding for the investor than a regime-wise asset allocation. Although general, the concept of a crisis-robust portfolio is especially pertinent when applied to the bond market, which offers a flight-to-quality trade-off during crises (all volatilities increase but most correlations decrease). Using three categories of bonds (sovereign, investment grade corporate and high yield corporate) in the U.S. and Eurozone for the period 1998-2007, we demonstrate the composition of crisis-robust portfolios and discuss the stabilizing role played by low-quality bonds during crises.

A financial crisis is typically associated with a rise in the volatility of most assets. Moreover, if the crisis is "contagious", things become even worse for investors because correlations among asset returns also increase and diversification becomes less efficient than during quiet periods. As most identified financial crises have been positively tested for contagion (Loretan and English (2000), Hartmann *et al*. (2001), Bekaert *et al*. (2005)), this problem warrants close attention.

To reduce investors' excessive exposure to crisis effects (Chow *et al*. (1999)), this paper builds "crisis-robust" portfolios, i.e. those exhibiting the least change in volatility during crises. Such portfolios enable investors to minimize as much as possible the perverse effects of volatility caused by a crisis. Holding a crisis-robust portfolio is an alternative to a regime-switching asset allocation (Ang and Bekaert (2002, 2004)), which implies recognizing early signs of a crisis and switching regimes appropriately[1]. In this respect, a crisis-robust portfolio is less demanding; and therefore safer, for the investor.

As far as possible, crisis-robust portfolios are supposed to eliminate the need to rebalance asset allocations according to the climate in the financial markets. Choosing a crisis-robust portfolio not only reduces transaction costs; it means that investors do not have to worry constantly about changing their allocations. Furthermore, it is difficult to adapt portfolios to the changing volatility/correlation picture in a timely manner because most crises occur suddenly.

In practice, crisis-robust portfolios can help asset managers respect a risk threshold fixed by their clients or publicly announced along with a fund's objectives. Indeed, the risk thresholds produced by our model ensure that crisis periods interfere as little as possible with a portfolio's composition and risk. This is an innovation, since funds guaranteeing a maximal risk level typically evaluate that risk using time intervals that are long enough to be dominated by non-crisis subperiods.

Although general, the concept of crisis-robust portfolio is especially appealing when applied to markets that exhibit the mixed effects of crises on volatilities and correlations. Typically, these markets are characterized by a so-called flight-to-quality (FTQ) effect (i.e. all volatilities increase but the correlations between the safest and the riskiest assets decrease). Empirical work has detected this effect between bonds and stocks and among also bonds (Bekaert *et al*. (2005), Berben and Jansen (2005)), while stocks seem to be subject only to contagion (correlations rise).

Thus, contrary to the stock market, the bond market offers an FTQ trade-off during crises that can be exploited to keep portfolios as crisis-insensitive as possible, the optimum being the crisis-robust portfolio.

One might think that crisis-robust bond portfolios are composed solely of the highest quality securities, i.e. developed countries' sovereign bonds and highly rated corporate bonds, because the volatility of investment grade bonds is low, even during periods of turmoil. However, FTQ effect undermines this argument. The fact that investors shift from risky to safe securities during crises (Beber *et al*. (2006)) causes a decline in the correlations between risky and safe assets during such periods (Stivers and Sun (2002), Baur and Lucey (2006)). This is good news for diversification benefits.

Regarding bond portfolios, crises tend to increase volatilities and decrease correlations between highly rated and speculative bonds. Therefore, thanks to more effective diversification, a bond portfolio including some high yield bonds–but not too many, for volatility reasons–may withstand crises better than a 100%- safe portfolio. The empirical part of this paper is devoted to assessing the extent of this phenomenon, that is, to measuring the fraction of risky assets devoted to optimally "robustifying" the portfolio against financial turmoil.

Regarding the consequences of crises, therefore, bondholders are better placed than stockholders, who are subject to harmful contagion. Indeed, bondholders benefit from an FTQ-linked risk trade-off. Crisis-conscious bond portfolio managers should therefore take this into account. The crisis-robust approach is one way to fully exploit this trade-off in order to keep portfolios as crisis-insensitive as possible.

In the empirical part of this paper, we have identified seven financial crises between 1998 and 2007 on the basis of the literature. The full dataset is composed of weekly returns of government (GVT) and corporate bond indices in the U.S. and in the Eurozone. Corporate bonds are split into investment grade (IG) and high yield (HY) categories. We therefore deal with six different assets. First, we show that the FTQ has a key effect in the bond class. Indeed, the two correlation matrices (U.S., Eurozone) show that diversification benefits rise during troubled times in four cases out of six. Accordingly, in these cases, the crisis-robust portfolio includes the lowest-grade asset, confirming the impact of the volatility trade-off.

The paper is organized as follows. Section I defines crisis-robust portfolios in a general framework and furnishes their analytical expression in a simple two-asset setting. Section II presents the original bond-market dataset, explains how crisis periods have been delineated, and documents the presence of FTQ. Section III applies the crisis-robust concept to this dataset. Section IV concludes.

1. Crisis-robust portfolios

Because financial markets can be unpredictable, both in crisis periods and in quiet periods, the *crisis-robust portfolio* is defined as the one minimizing the volatility ratio between the two types of period.

Consider the following model. There are two possible regimes: the crisis regime C and the quiet regime Q, but the probabilities of these regimes are unknown. A crisis is therefore seen as a totally unpredictable event that affects the markets.

The market includes n risky assets. The stochastic return vector $R = (R_1, \ldots, R_n)$ is taken randomly from either the crisis-period multivariate distribution with covariance matrix Σ_C or the quiet time distribution with covariance matrix Σ_Q. Crises happen exogenously[2] but investors are "crisis-conscious" (Kole *et al.* (2006)), i.e. they know that crises are possible. However, they have no prior information on the probability of occurrence. A rationale for the impossibility of inferring the probability of a next-day crisis could stem from the fact that crises are highly erratic, so the probability is unknown and, in most cases, unknowable.

Let $\Phi = (\phi_1, \ldots, \phi_n)$, $\sum_{i=1}^{n} \phi_i = 1$, be the composition of portfolio P made up of n assets, where ϕ_i denotes the proportion of asset i in P. The variance of this portfolio depends on the regime:

$$\sigma^2(P) = \begin{cases} \sigma_C^2(P) = \Phi' \Sigma_C \Phi & \text{during crises} \\ \sigma_Q^2(P) = \Phi' \Sigma_Q \Phi & \text{during quiet times} \end{cases} \tag{2.1}$$

The crisis-robust portfolio $P*$ is defined as the one minimizing the variance ratio between the two regimes:

$$\frac{\sigma_C^2(P*)}{\sigma_Q^2(P*)} = \min_{P \in \left\{ \substack{realisable \\ portfolios} \right\}} \frac{\sigma_C^2(P)}{\sigma_Q^2(P)} \tag{2.2}$$

Its composition, denoted by $\Phi^* = \left(\phi_1^*, \ldots, \phi_n^*\right)$, with $\sum_{i=1}^{n} \phi_i^* = 1$, is thus such that:

$$\Phi^* = \arg\min \frac{\Phi'\Sigma_C\Phi}{\Phi'\Sigma_Q\Phi} \qquad (2.3)$$

With a crisis-robust portfolio, the investor minimizes the need to rebalance the asset allocation. Thus, a crisis-robust portfolio reduces transaction costs, all other things being equal. However, the benefits associated with crisis-robustness are likely related to the style of the portfolio manager. For active investors who react even to minor signals, looking for a crisis-robust portfolio makes little sense since they adapt to any changes they perceive on the market. Conversely, passive investors with a diversified portfolio should be keenly interested in finding a way to avoid–or at least weaken as much as possible–the impact of a financial crisis on their wealth.

The optimization in (2.2)-(2.3) may be performed on any set of financial assets provided that the two-regime setting in (2.1) is assumed. Numerical software may then be used to find the appropriate asset allocation. However, to determine the robust portfolio analytically in a simple case, we will assume from now on that the market is composed of two assets $(i=1,2)$ having volatilities $\sigma_{iQ}(i=1,2)$ during quiet times, and $\sigma_{iC}(i=1,2)$ during crises. The correlation coefficient between the returns of the two assets is ρ_Q during quiet times, and ρ_C during crises. We also introduce parameter γ representing the ratio of these correlations:

$$\frac{\rho_C}{\rho_Q} = \gamma \qquad (2.4)$$

The position of γ with respect to unity is left unconstrained. It may either be larger than one in case of contagion or smaller than one when FTQ arises. Therefore, the chosen parameterization will make it easier to conduct further discussions involving FTQ-specific issues.

To obtain tractable analytical results we assume that the crisis has the same relative impact on both assets' volatilities, that is, their crisis-to-quiet time volatility ratios are equal. We consequently introduce parameter β for this common ratio. This parameter takes values larger than unity since crises are associated with a rise in volatility:

$$\frac{\sigma_{1C}}{\sigma_{1Q}} = \frac{\sigma_{2C}}{\sigma_{2Q}} = \beta > 1 \tag{2.5}$$

Short positions are excluded. The two-regime variance (2.1) of portfolio P including a proportion $\phi \in [0,1]$ of asset 1 and, consequently, proportion $1-\phi \in [0,1]$ of asset 2, may be expressed as follows:

$$\sigma_Q^2(P) = \phi^2 \sigma_{1Q}^2 + (1-\phi)^2 \sigma_{2Q}^2 + 2\phi(1-\phi)\sigma_{1Q}\sigma_{2Q}\rho_Q$$
$$\sigma_C^2(P) = \phi^2 \sigma_{1C}^2 + (1-\phi)^2 \sigma_{2C}^2 + 2\phi(1-\phi)\sigma_{1C}\sigma_{2C}\rho_C \tag{2.6}$$

The first order condition for minimizing the ratio $\dfrac{\sigma_C^2(P)}{\sigma_Q^2(P)}$ leads to:

$$(\gamma-1)\left[4\phi(1-\phi)\left(\phi\sigma_{1Q}^2 - (1-\phi)\sigma_{2Q}^2\right) - 2(1-2\phi)\left(\phi^2\sigma_{1Q}^2 + (1-\phi)^2\sigma_{2Q}^2\right)\right] = 0 \tag{2.7}$$

If $\gamma \neq 1$, meaning that the correlation changes during crises, then the condition simplifies to:

$$\phi^2 \sigma_{1Q}^2 - (1-\phi)^2 \sigma_{2Q}^2 = 0 \tag{2.8}$$

or equivalently:

$$\frac{\phi}{1-\phi} = \frac{\sigma_{2Q}}{\sigma_{1Q}} \tag{2.9}$$

so that the unique portfolio allocation fulfilling the first-order condition associated with the crisis-robust criterion is given by[3] $\Phi^* = \left(\phi_1^*, \phi_2^*\right)$ with:

$$
\begin{cases}
\phi_1^* = \dfrac{\sigma_{2Q}}{\sigma_{1Q} + \sigma_{2Q}} \\[3mm]
\phi_2^* = 1 - \phi_1^* = \dfrac{\sigma_{1Q}}{\sigma_{1Q} + \sigma_{2Q}}
\end{cases}
\tag{2.10}
$$

However, as illustrated by Figures 1 and 2 for several point values of the parameters in question, this critical point may well correspond to a maximal value of the volatility ratio as well as to a minimal value depending on the sign of coefficient δ defined as:

$$
\delta = \rho_Q (\gamma - 1) = \rho_Q \left(\frac{\rho_C}{\rho_Q} - 1 \right) = \rho_C - \rho_Q
\tag{2.11}
$$

The sign of δ indicates whether the two assets are subject to contagion $(\delta > 0)$ or to FTQ $(\delta < 0)$. In the first case, the critical point corresponds to a maximal value and is therefore useless. Conversely, when the crises are associated with a decorrelation of the assets, then the critical point provides the optimal composition for a crisis-robust portfolio. Proposition 1 summarizes this result.

Proposition 1: If the market is made up of two assets $(i = 1, 2)$ such that $\dfrac{\sigma_{1C}}{\sigma_{1Q}} = \dfrac{\sigma_{2C}}{\sigma_{2Q}} > 1$ and

$\rho_C < \rho_Q$, then the ratio $\dfrac{\sigma_C^2 (P)}{\sigma_Q^2 (P)}$ reaches an interior minimum for $P = P^*$

composed of $\phi_i^* = 1 - \dfrac{\sigma_{iQ}}{\sigma_{1Q} + \sigma_{2Q}}$ of asset i $(i = 1, 2)$.

[Insert Figure 1 near here.]

[Insert Figure 2 near here.]

Portfolio P^* is thus the crisis-robust portfolio. Its existence is ensured thanks to the downward shift in the correlation, which compensates, at least to some extent, for the rise in the volatilities of the two asset returns. In the case of contagion, such a compensation effect is

impossible and the minimal ratio is obtained for corner solutions, i.e. portfolios made up of a single asset.

What is the efficiency of the crisis-robustness criterion? In other words, how close to one is the optimal variance ratio $\dfrac{\sigma_C^2(P^*)}{\sigma_Q^2(P^*)}$? Replacing ϕ_1^* and ϕ_2^* by their expressions and computing both variances yields:

$$\frac{\sigma_C^2(P^*)}{\sigma_Q^2(P^*)} = \frac{1+\rho_Q}{(1+\gamma\,\rho_Q)\beta^2} = \frac{1+\rho_Q}{(1+\rho_C)\beta^2} \tag{2.12}$$

The next proposition follows from this equation.

Proposition 2: If the market is made up of two assets $(i=1,2)$ such that $\dfrac{\sigma_{1C}}{\sigma_{1Q}} = \dfrac{\sigma_{2C}}{\sigma_{2Q}} > 1$ and

$\rho_C < \rho_Q$, then the minimal ratio $\dfrac{\sigma_C^2(P^*)}{\sigma_Q^2(P^*)}$ is equal to 1 if and only if

$\dfrac{1+\rho_Q}{1+\rho_C} = \beta^2$.

When FTQ is present, the correlation spread between crisis and quiet period, $\delta = \rho_C - \rho_Q$, is negative and creates a volatility trade-off. Proposition 2 exhibits the threshold to be reached by this correlation spread in order to allow for full compensation, making the volatility of the crisis-robust portfolio totally insensitive to crises. Indeed, we have:

$$\frac{1+\rho_Q}{1+\rho_C} = \beta^2 \Leftrightarrow \delta = \overline{\delta} \text{ where } \overline{\delta} = \left(\frac{1}{\beta^2}-1\right)(1+\rho_Q) \tag{2.13}$$

By definition, $\beta > 1$ and $-1 \leq \rho_Q \leq 1$, then $\overline{\delta}$ is always negative. However, in practice, the correlation gap is generally insufficient ($0 > \delta > \overline{\delta}$) to lead to the existence of a portfolio that is totally insensitive to crises[4]. Consequently, crisis-robust portfolios should be seen as second-best choices.

On Figure 1, all chosen values of δ are such that $\delta > \bar{\delta} = -0.975$. Therefore, none of the represented curves reaches the level of one for the volatility ratio corresponding to a fully crisis-insensitive portfolio. Note that the result is not influenced by the quiet period volatilities of the two assets but only by the crisis-to-quiet period ratio β.

Conversely, on Figure 2, for $\delta = \bar{\delta} = -0.9$, the curve presents a minimum value with a unit ratio, while for $\delta = -1 < \bar{\delta}$ the volatility ratio can be lower than one. This is an unrealistic situation to be avoided, making the volatility during crisis periods smaller than in quiet times.

2. Data, descriptive statistics and correlation matrices

Government and Corporate Bonds

The period under study stretches from July 1998 to May 2007. The dataset is composed of weekly returns of the sovereign and corporate bond indices of two geographic zones: U.S. and Eurozone. Corporate bonds are split into IG and HY.

GVT bonds are represented by the 10-year benchmark indices supplied by Datastream[5]. These indices, which include coupon returns, are usually based on a single bellwether, namely the last bond issued by the country's Treasury in a given maturity. Factors such as liquidity, issue size and coupons are also taken into account when choosing the index component(s). For corporate bonds, two categories of index were used: IG, with ratings between AAA and BBB⁻, and HY, rated from BB⁺ to CCC. These indices exclude convertible bonds and include coupon returns. Data are sourced from Merrill Lynch (i.e., bids quoted by traders at the ML desk) at the market close[6].

Figures 3 presents GVT bond prices, Figure 4 draws all corporate bond prices. HY bond indices decreased sharply between 2000 and 2002 in the aftermath of the stock market collapse. Indeed, HY default rates reached historical peaks at that time. Starting from 1.6% in 1998, they reached 5.1% in 2000 and finally 12.8% in 2002 (an historical high on the period

1971-2006). The HY market also recovered very rapidly after 2002. In 2003, the default rate declined to 4.7%, causing a complete change in the HY performance trend at that date. The particular episode of 2000-2002 explains why globally, for the period 1998-2006 under review, the HY bond index underperforms IG bond returns, showing a negative risk premium to less risky assets.

[Insert Figure 3 near here.]

[Insert Figure 4 near here.]

Crises studied

Our definition of "crisis" is broad. It encompasses five types of event: currencies, sovereign debt, bond or equity crash, corporate bankruptcies (Enron, WorldCom), and crises of confidence (9/11). We have deliberately omitted crises of a purely banking nature and economic crises such as recessions or oil shocks. The real difficulty lies in establishing precise timeframes for the crises we have selected.

Table I shows the start and end dates of the crises studied in this article. They have been chosen carefully on the basis of previous research (see Appendix). Admittedly, while the onset of a crisis is usually easy to identify, the end date is much harder to pinpoint. This awkward problem is highlighted by the Asian crisis studied by several authors.

[Insert Table I near here.]

Descriptive statistics, correlations and volatilities

Table II provides descriptive statistics on returns (in local currencies) for all assets in the database, for the whole period (July 1998 - August 2006). All returns have been tested for stationarity (but the results are not reported here). The annualized mean return of GVT bonds is around 4.8%. IG corporate bonds exhibit somewhat larger returns with a risk premium between 108 and 164 bp while their volatility does not show higher values.

Surprisingly, the mean returns of HY bonds are lower than those of IG bonds, and in the case of Europe, even lower than the return on GVT bonds. The most probable explanation for this, as already mentioned, is the presence in the sample period of a huge breakdown (2000-2002) on the HY bond market, leading to a sharp increase in the default rate.

[Insert Table II near here.]

Tables III and IV give the descriptive statistics for crises and quiet periods, respectively. Logically, as testified by their average returns, corporate bonds suffer much more from crises than do GVT assets. In fact, HY bond returns exhibit negative mean returns during crises.

As crises are the exception rather than the rule, the quiet period statistics generally resemble the whole period data. Nevertheless, the anomaly regarding the risk premia over the whole period (i.e. riskier bonds having smaller mean returns) disappears in the quiet-period dataset (Table IV), testifying to the impact of crises on this stylized fact.

[Insert Table III near here.]

[Insert Table IV near here.]

Tables V and VI provide the correlation matrices for, respectively, the whole period, the crises, and the quiet period. In all cases, sovereign and IG bonds exhibit positive correlations. The same is true within the HY bond class. However, correlations between HY bonds and the other categories take values between 0.61% and 29.97% during quiet periods. Most of the time they drop sharply during crises, reaching values comprised between -6.15% and 34.59%. The FTQ effect during crises is clearly observed among bonds.

[Insert Table V near here.]

[Insert Table VI near here.]

Finally, as the theory has shown (see Propositions 1 and 2 in Section 2), correlation spreads and volatility ratios are key elements in determining crisis-robust portfolios. This information

is provided in Tables VII to IX, respectively, thus helping to measure the effects of crises on volatilities and diversification benefits.

[Insert Table VII near here.]

[Insert Table VIII near here.]

[Insert Table IX near here.]

Tables VII and VIII show that FTQ is observed in four out of six cases, meaning that crises lead to lower correlations. For instance, on the U.S. market during turmoil, the correlation between sovereign and IG corporate bonds declined more than 8%, from 93.27% to 86.59%. Likewise, the decline in correlation between sovereign and HY bonds reaches 11.14% while intra-corporate bonds (IG and HY) show some contagion (+4.62%). This could mean that, during crises, U.S. bondholders consider public debt as the only really safe asset. Be that as it may, Tables VII and VIII testify to the simultaneous occurrence of contagion and FTQ in the bond market, thus confirming the relevance of our two-correlation-regime approach.

Table IX shows that volatility ratios in our dataset all lie between 1.46 (for GVT bonds in Europe) and 2.26 (for HY bonds in the U.S.). Not surprisingly, the ratios take the largest values in the HY bond category. The two-asset theoretical model in Section 2 assumes a constant ratio. While this hypothesis is clearly not met for every pair of assets, it seems still realistic for U.S. and E.U. sovereign and IG bonds at least. Crisis-robust portfolios can however be determined empirically in all cases. The drawback of not respecting the constant volatility ratio hypothesis is that neither the existence nor the uniqueness of a crisis-robust portfolio is guaranteed by the theory.

3. Determining crisis-robust bond portfolios

For each geographical zone, we determine the crisis-robust portfolio made up of two or three assets. In the two-asset case, only one parameter is required for indicating the portfolio

composition. Consequently, it is possible to represent the portfolio volatilities in both regimes, as well as their ratios, as functions of the proportion of an asset in the portfolio.

Take for example the case of U.S. GVT and HY bonds. Figure 5 shows that both volatility curves are U-shaped and thus reach a minimal risk value for a certain proportion of HY bonds in the portfolio (69.91% during calm periods, 50.14% during crisis periods). The volatility ratio is trickier[7] but still reaches a minimal value at an interior point of the 0%-100% interval. Precisely, the crisis-robust portfolio has a volatility ratio of 1.55 and includes 12.68% of HY bonds. Consequently, HY bonds are indeed present in the crisis-robust portfolio. However, this portfolio is not fully crisis-insensitive since the minimal ratio remains above the unit value.

[Insert Figure 5 near here.]

More generally, Table X offers the results for the various possibilities of two- or three-asset U.S. and E.U. portfolios. In each case, we determine the crisis-robust bond portfolio from the standpoint of a domestic investor.

Table X indicates (left column) the assets allowed in the portfolio, then the presence or absence of the FTQ effect. The latter piece of information is derived from Tables VII or VIII depending on the geographical zone in question. In accordance with the theory, the two cases of no-FTQ effect correspond to corner solutions: 100% IG bonds in IG-HY U.S. portfolios, 100% GVT bonds in GVT-IG E.U. portfolios.

[Insert Table X near here.]

When the FTQ effect is present, the U.S. and E.U. portfolios differ from one another by the respective proportions of the riskiest asset required to reach crisis-robustness. While European investors find their optimum with small proportions (2.8%, 4.9%), U.S. investors need much larger proportions (12.7%, or even 59.3%). Moreover, the optimal ratio is more advantageous

for Europeans, as it is (slightly) closer to unity, the best ratio being 1.45 for the E.U. and 1.52 for the U.S.

The difference between the US and Euro portfolio composition is probably attributable to the 2000-2002 "anomaly" mentioned earlier. In fact, in the U.S. during quiet times, IG bond volatility appears lower than sovereign bond volatility (0.77 < 0.94, see Table IV). According to Proposition 1, the share of asset 1 volatility in the sum gives the proportion of asset 2 in the crisis-robust portfolio. Even if the results in Table X do not *stricto sensu* obey the theory (because the volatility ratio is not constant), Proposition 1 is still a good proxy for the composition of the crisis-robust portfolio. The same "anomaly" occurs in Europe, too, but only in the GVT-IG case, where the FTQ effect does not appear. Conversely, the HY European bonds exhibit greater volatility than do the two other classes. As these cases correspond to the occurrence of the FTQ effect, only relatively small amounts of HY bonds are to be included in crisis-robust portfolios.

Note also that, in both the U.S. and the Eurozone, putting the three assets together does not make the optimal portfolio more crisis-robust. Since the theory is restricted to the two-asset case, we cannot infer the generality of this observation. But, on the basis of this stylized fact, we conjecture that only a few securities exhibiting FTQ effects would be required to (partially) hedge a portfolio against financial turmoil.

4.   Conclusion

Several past events have pointed to the necessity for investors to take into account the possibility that financial crises may occur. That said, each crisis has its own pattern: some start slowly, others abruptly; some are short, others last longer; some hit specific assets, others are general. Therefore, waiting for a crisis to erupt in order to rebalance a portfolio is hazardous. So this paper offers the concept of a crisis-robust portfolio, that is, a portfolio

exhibiting the lowest volatility ratio between turmoil and quiet periods. Our work highlights the somewhat paradoxical result that introducing risky assets into a portfolio can "hedge" against a rise in volatility during crises. This result is established analytically as well as empirically. In a two-asset framework, the presence of the riskiest asset in the crisis-robust portfolio is solely linked to the occurrence of a flight-to-quality (FTQ) effect , namely an asset decorrelation during crises.

The empirical application provided here is rather simple, aimed at illustrating the fact that crisis-robust portfolios can include a substantial fraction of risky assets when FTQ is observed instead of contagion. In this respect, the bond market offers a natural arena for examples. Other cases of FTQ have also been identified, in particular in the bond-stock correlation (Hartmann *et al.* (2001), Baur and Lucey (2006)). Therefore, a natural extension of this work would be to test crisis robustness in the case of larger portfolios that also include stocks. It could be of particular interest for European portfolios where sector diversification decreased due to European integration (Meon and Weill (2004)). Indeed, contagion and its unpleasant consequences for portfolio management have been now extensively described. But surprisingly, little attention has been paid to the benefits that can be obtained by overweighting assets whose correlations decrease during crises. Actually, acknowledging the FTQ effect helps reduce the perceived "drama" surrounding financial crises. More empirics are obviously needed to gauge the size of FTQ on global financial markets.

Crisis-robustness is, however, a debatable criterion for portfolio management, for several reasons. First, in this framework, the investor does not choose the level of risk as such. As safer assets tend to be less influenced by crises than highly risky assets, a crisis-robust portfolio will be low-risk overall, but not necessarily the one with the minimal risk over the whole period or under a given regime. In this respect, the investor's classic level of risk aversion is replaced by an aversion to higher volatility during crises.

Second, crisis-robustness concerns volatility only. The fact that returns decrease during crises is not taken into account. Further research could investigate the distorsions of the risk-return relationship during crises and check whether the volatility gains associated with crisis-robust portfolios are adversely affected by return costs. The intuition is that the result would depend on the level of investors' crisis-awareness. In markets where portfolios are generally composed in a single regime setting, crisis-robust portfolios offer free hedging, whereas in two-regime markets, recognized by investors and integrated in their optimization along the lines of our model, asset pricing would incorporate the pricing of the hedge.

Third, crisis-robustness is formulated here in relative terms, as a ratio between the volatilities in the two regimes. Other formulations involving, for example, the frequency of a crisis or some kind of weighted average volatility level, could lead to better criteria. A ratio has been chosen here for simplicity but our main goal was to advocate looking for a crisis-conscious investment rather than having to scrutinize the market each morning and decide whether or not to change the portfolio's composition.

Going further on this issue would require a more general theoretical framework for investors' fears and attitudes regarding crises. In this respect, behavioral finance (Malkiel *et al.* (2005), Bourachnikova (2007)) has shown that people tend to overestimate the probability of detrimental but rare events. By not introducing the probability of a crisis occurrence into our model, we provide a theoretical approach compatible with any view (rational or not) about the possibility of a crisis. Moreover, constraints could be added to the proposed optimization, such as a maximal level of volatility in either of the given regime, or even in both.

Finally, the model in this paper is static since no composition update is considered. This is of course a major limitation, because extensive evidence of highly unstable correlations can be found in econometric papers (Engle (2002), Goetzmann et al. (2005)). Further research could

therefore consider a dynamic counterpart, including for instance a two-regime random correlation coefficient.

Acknowledgements

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Notes

[1] "Because the regime is not observable, the estimation involves inferring from the data which regime prevails at each point in time" (Ang and Beckaert, 2004, p.89).

[2] In this respect, the applications of the model are widely open as no crisis specification is needed. Indeed, different portfolio management styles may be sensitive to different crisis types.

[3] Other solutions may exist if the no-short selling condition is waived.

[4] However, in theory, it can happen that two interior portfolios are crises-insensitive because the correlation gap is so huge that it overcompensate the variance growth. In such a situation, as a matter of fact, the variance-ratio minimization is inappropriate due to the fact that the denominator may exceed the numerator.

[5] For Eurozone, the German bond index.

[6] The indices have minor differences. For investment grade indices, we selected a maturity of 7 to 10 years. However, for HY indices, maturity was not proposed as a selection parameter, so there are small differences in duration. Furthermore, the geographical indices for both IG and HY securities have median ratings that may vary slightly. Moreover, the number of index components varies from one country to another.

[7] If the assumptions made in the theoretical part were fully satisfied, this curve would also exhibit a U-shape. This is not the case since the GVT bond volatility ratio is 1.56 while the HY one is much larger (2.26). Nevertheless, a numerical optimisation can always be performed.
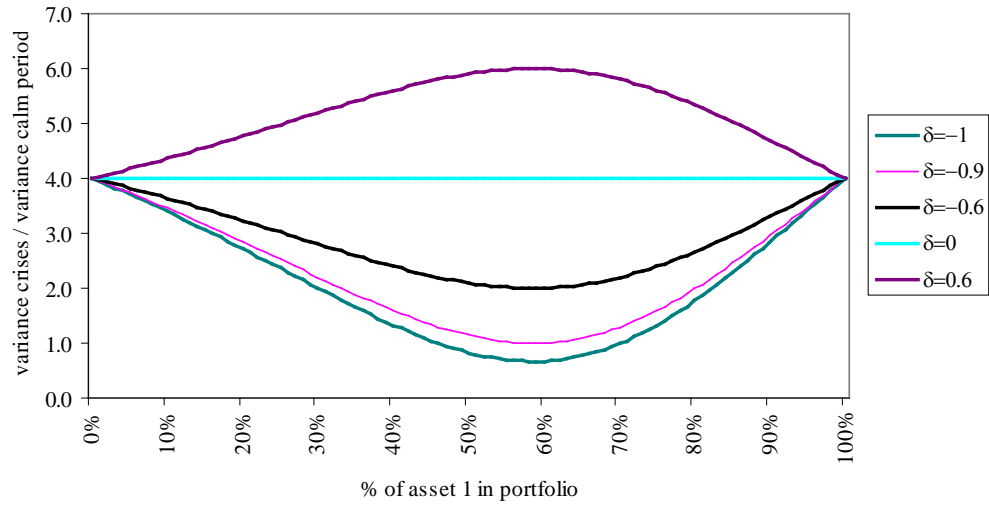
Figure 1

Variance in crisis / variance in quiet period
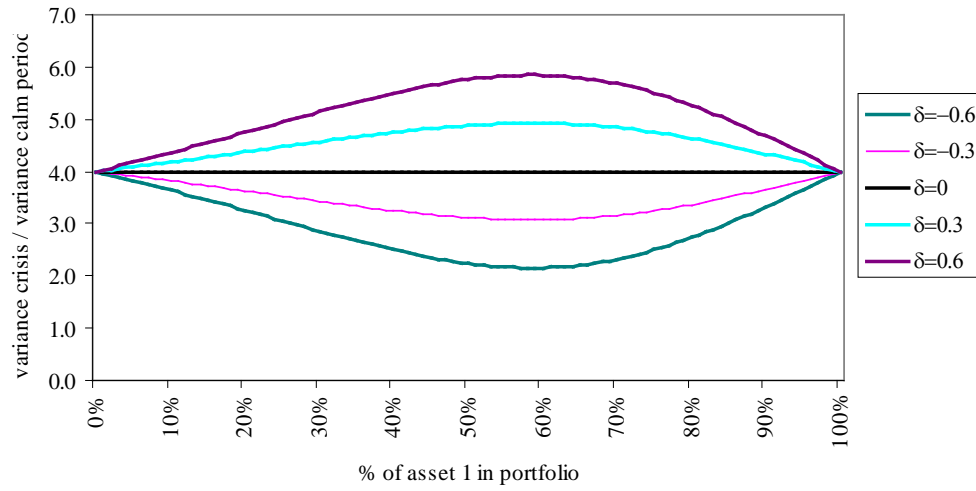
The variance ratio is calculated as a function of $\delta = \rho_Q(\gamma - 1) = \rho_C - \rho_Q$ for $\sigma_{1Q}^2 = 2 \times 10^{-4}, \sigma_{2Q}^2 = 4 \times 10^{-4}, \beta = 2, \rho_Q = 0.3.$

Figure 2

Variance in crisis / variance in quiet period

The variance ratio is calculated as a function of $\delta = \rho_Q(\gamma - 1) = \rho_C - \rho_Q$ for $\sigma_{1Q}^2 = 2 \times 10^{-4}, \sigma_{2Q}^2 = 4 \times 10^{-4}, \beta = 2, \rho_Q = 0.2.$

Figure 3

Government bond prices

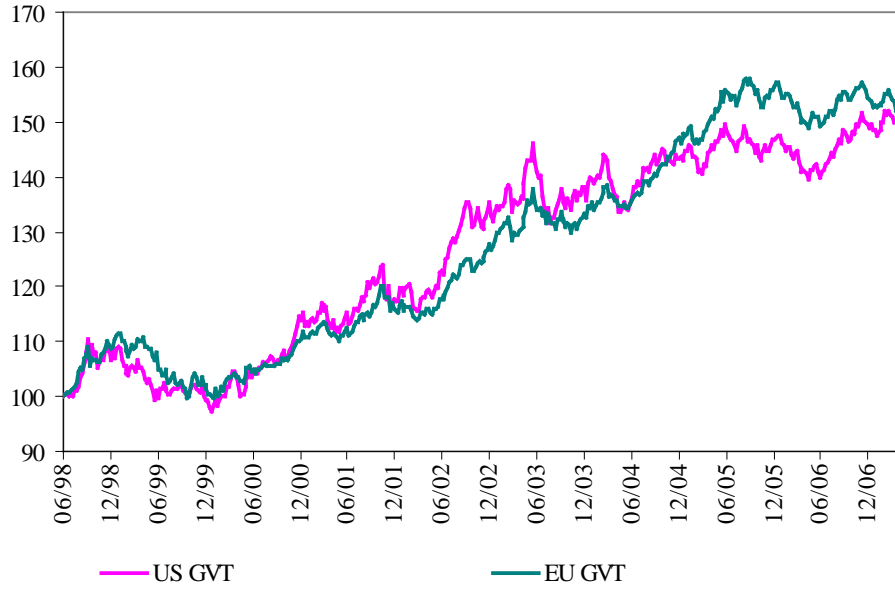**Cumulative weekly returns in local currency, July 1998 –May 2007**

Figure 4

Corporate bond prices

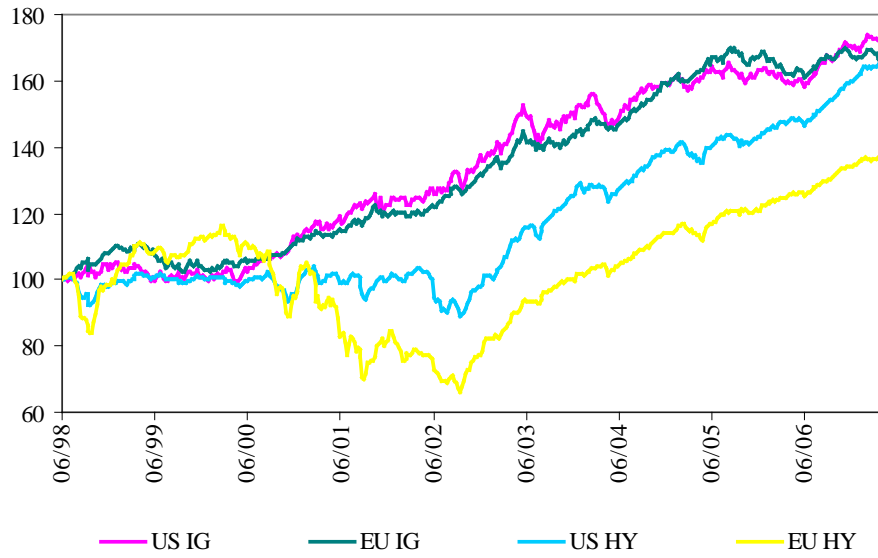**Cumulative weekly returns in local currency, July 1998 –May 2007**

Figure 5

Volatility during the quiet period and during a crisis, and volatility ratio:
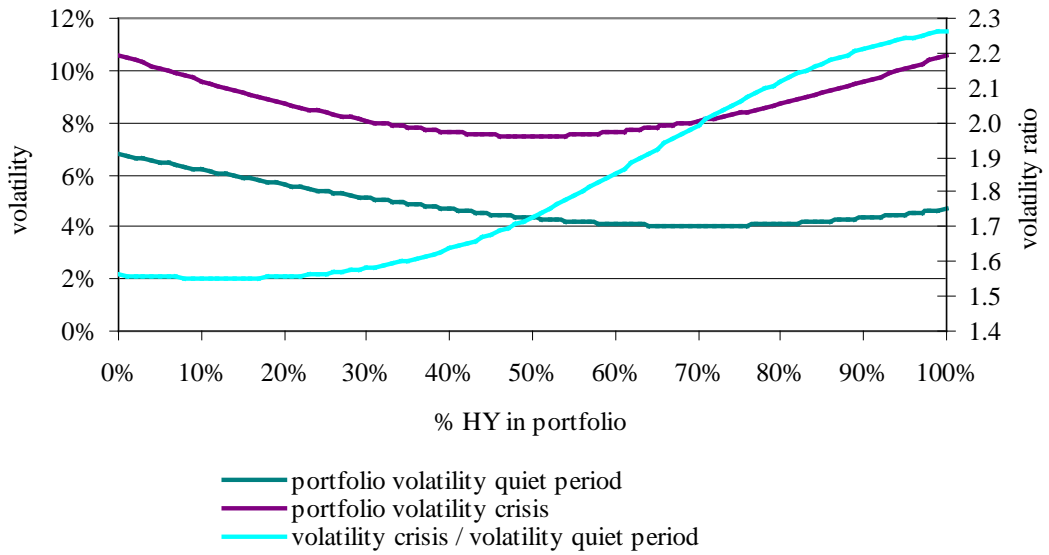
U.S. portfolios (GVT and HY bonds)

Table I

Crises used in this study

	Start date	*End date*	*Type of crisis*
Russia and LTCM 1998	08/17/1998	10/15/1998	Sovereign debt and corporate bankruptcy
Brazil 1999	01/13/1999	01/31/1999	Currency
e-crash 2000	03/28/2000	04/14/2000	Market crash
Argentina 2001	10/01/2001	12/23/2001	Sovereign debt
9/11	09/11/2001	09/28/2001	Confidence
Enron 2001	11/28/2001	12/31/2001	Corporate bankruptcy
WorldCom 2002	06/25/2002	07/31/2002	Corporate bankruptcy

Table II

Descriptive statistics for returns: All bonds, whole period

Weekly returns in local currency, July 1998 –May 2007

	U.S.	E.U.*
GVT bond indices		
Mean	0.09%	0.09%
Annualized mean	4.78%	4.91%
Median	0.10%	0.15%
Min	-4.25%	-3.17%
Max	2.45%	2.00%
Standard deviation	1.00%	0.73%
Skewness	-0.53	-0.53
Kurtosis	3.91	4.11
IG corporate bond indices		
Mean	0.12%	0.11%
Annualized mean	6.42%	5.99%
Median	0.14%	0.16%
Min	-3.27%	-3.17%
Max	2.15%	1.68%
Standard deviation	0.81%	0.58%
Skewness	-0.46	-0.65
Kurtosis	3.85	5.01
HY corporate bond indices		
Mean	0.11%	0.08%
Annualized mean	5.97%	3.70%
Median	0.19%	0.15%
Min	-5.47%	-8.40%
Max	2.68%	5.54%
Standard deviation	0.76%	1.40%
Skewness	-1.34	-1.02
Kurtosis	11.22	9.95

* Germany for GVT bond indices.

Table III

Descriptive statistics for returns: All bonds, Crisis period

Weekly returns in local currency, July 1998 –May 2007

	U.S.	E.U.*
GVT bond indices		
Mean	0.40%	0.27%
Annualized mean	17.47%	10.68%
Median	0.65%	0.37%
Min	-4.25%	-3.17%
Max	2.31%	2.00%
Standard deviation	1.50%	1.06%
Skewness	-1.38	-1.04
Kurtosis	4.90	4.83
IG corporate bond indices		
Mean	0.10%	0.12%
Annualized mean	3.58%	6.10%
Median	0.43%	0.19%
Min	-3.27%	-3.17%
Max	2.07%	1.68%
Standard deviation	1.23%	0.86%
Skewness	-1.03	-1.57
Kurtosis	3.85	7.29
HY corporate bond indices		
Mean	-0.46%	-0.47%
Annualized mean	-20.64%	-25.70%
Median	0.01%	-0.34%
Min	-5.47%	-8.40%
Max	1.25%	4.26%
Standard deviation	1.48%	2.61%
Skewness	-1.43	-0.95
Kurtosis	4.97	4.65

Germany for GVT bond indices.

Table IV

Descriptive statistics for returns: All bonds, quiet period

Weekly returns in local currency, July 1998 –May 2007

	U.S.	*E.U.**
GVT bond indices		
Mean	0.07%	0.08%
Annualized mean	3.47%	4.23%
Median	0.09%	0.13%
Min	-3.05%	-2.97%
Max	2.45%	1.84%
Standard deviation	0.94%	0.70%
Skewness	-0.35	-0.42
Kurtosis	3.23	3.55
IG corporate bond indices		
Mean	0.13%	0.11%
Annualized mean	6.66%	5.86%
Median	0.13%	0.16%
Min	-2.01%	-1.59%
Max	2.15%	1.66%
Standard deviation	0.77%	0.56%
Skewness	-0.23	-0.35
Kurtosis	3.14	3.28
HY corporate bond indices		
Mean	0.16%	0.13%
Annualized mean	8.81%	6.61%
Median	0.20%	0.16%
Min	-2.21%	-6.02%
Max	2.68%	5.54%
Standard deviation	0.65%	1.24%
Skewness	0.00	-0.47
Kurtosis	5.33	8.44

* Germany for GVT bond indices

Table V

Correlation matrix: U.S. bonds

Weekly returns in local currency, July 1998 –May 2007

Panel A : Whole period

	US GVT	US IG	US HY
US GVT		93.27%	6.32%
US IG			30.37%
US HY			

Panel B : Crisis period

	US GVT	US IG	US HY
US GVT		86.59%	-0.22%
US IG			34.59%
US HY			

Panel C : Quiet period

	US GVT	US IG	US HY
US GVT		95.18%	10.92%
US IG			29.97%
US HY			

Table VI

Correlation matrix: E.U. bonds

Weekly returns in local currency, July 1998 –May 2007

Panel A : Whole period

	EU GVT	EU IG	EU HY
EU GVT		93.51%	-1.35%
EU IG			8.63%
EU HY			

Panel B : Crisis period

	EU GVT	EU IG	EU HY
EU GVT		95.44%	-6.15%
EU IG			3.93%
EU HY			

Panel C : Quiet period

	EU GVT	EU IG	EU HY
EU GVT		93.28%	0.61%
EU IG			10.13%
EU HY			

Table VII

Correlation spread matrices*, U.S. bonds

Weekly returns in local currency, July 1998 –May 2007

	US GVT	US IG	US HY
US GVT		-8.59%	-11.14%
US IG			4.62%
US HY			

*Differences between correlations in crisis and correlations during quiet periods.
The cells in grey correspond to the presence of FTQ (correlation decrease)

Table VIII

Correlation spread matrices*, E.U. bonds

Weekly returns in local currency, July 1998 –May 2007

	EU GVT	EU IG	EU HY
EU GVT		2.16%	-6.77%
EU IG			-6.20%
EU HY			

*Differences between correlations in crieis and correlations during quiet periods.
The cells in grey correspond to the presence of FTQ (correlation decrease)*

Table IX

Volatility ratios*

Weekly returns in local currency, July 1998 –May 2007

US GVT	EU GVT	US IG	EU IG	US HY	EU HY
1.56	1.46	1.55	1.55	2.26	2.06

* crisis volatility over quiet-period volatility.

Table X

Composition of the crisis-robust portfolios

Assets allowed in portfolio	FTQ	GVT	IG	HY	Volatility ratio (crisis / quiet period.)
U.S Portfolios					
GVT IG	Yes	40.7%	59.3%	-	1.52
GVT HY	Yes	87.3%	-	12.7%	1.55
IG HY	No	-	100%	0%	1.55
GVT IG HY	Yes	40.7%	59.3%	0%	1.52
E.U. Portfolios					
GVT IG	No	100%	0%	-	1.46
GVT HY	Yes	95.1%	-	4.9%	1.45
IG HY	Yes	-	97.2%	2.8%	1.55
GVT IG HY	Yes	95.1%	0%	4.9%	1.45

Appendix: Crises selected for study

In this study, we examine five types of crisis: (1) currency crises , (2) sovereign debt crises, (3) crises triggered by an equity or bond crash, (4) corporate bankruptcies or loss of confidence (e.g. the collapse of Enron), and (5) crises of confidence arising from severe external events (e.g. 9/11).

Currency crisis

Brazil 1999

Dungey *et al.* (2006) say that the crisis began on January 13, 1999 with the devaluation of the real. It is hard to establish an end date because no landmark events occurred. However, the crisis is generally referred to as the "January 1999 Brazilian crisis". We have therefore taken the final date to be the end of January 1999.

Sovereign debt crises

Russia 1998

The Russian crisis began on August 17, 1998, when the country defaulted on its debt, and continued until September of that year, when another crisis was triggered by the collapse of the hedge fund LTCM. We have therefore considered these two crises jointly, setting the end date for both at the end of the LTCM crisis.

Argentina 2001

The crisis began on the November 1, 2001 when Argentina announced a debt restructuring plan. On December 5, the IMF refused to release funds to help the country, and the Argentine president was forced to resign on December 20. On December 23, 2001, the country announced that it was in default. For investors, the announcement marked the end of the crisis, and emerging spreads began to narrow (BIS, March 2002).

Crashes

e-crash 2000

Triggered by the crash in tech stocks, the equity meltdown began on March 28, 2000. We have dated the end of the crisis to April 14, 2000, when prices stopped falling. Thereafter, the market entered a period of stagnation.

Corporate bankruptcies and crises of confidence

LTCM 1998

The hedge fund Long Term Capital Management (LCTM) collapsed on September 23, 1998. Dungey *et al.* (2006) consider that the crisis ended when the U.S. Federal Reserve decided to cut interest rates in order to contain the fallout. The Fed's decision was taken unexpectedly between two FOMC meetings on October 15, 1998.

Enron 2001

The onset of the crisis can be dated to November 28, 2001, when Moody's Investor Services decided to downgrade Enron, taking it from IG to HY. Although it was Moody's decision that sparked the mood of wariness which spread to all financial markets, signs that Enron was in trouble had emerged much earlier. On October 16, 2001, the company lowered its earnings guidance (BIS, March 2002), and on November 8, it announced a retroactive adjustment to all its results since 1997. Enron filed for bankruptcy on December 2. It is extremely difficult to set a precise end date, and we consider that the crisis lasted throughout December.

WorldCom 2002

The crisis related to the bankruptcy of WorldCom began on June 25, 2002, when the company revealed accounting inaccuracies concealing losses of $3.8 billion in 2001 and 2002; it also announced 17,000 job cuts, equivalent to 20% of the workforce. WorldCom filed for bankruptcy on July 11, and its share fell 80% over the next four months. Once again it is very

hard to establish an end date because the loss of confidence was exacerbated by fears relating to terrorist attacks in May and June 2002 and to political tensions between India and Pakistan. According to the BIS (September 2002), the most significant crisis-related market movements occurred between July 10 and 23. We therefore consider that the crisis lasted until end-July 2002.

Other crisis of confidence

9/11

The terrorist attacks on the USA on September 11, 2001 sparked a crisis of confidence across markets worldwide. It is hard to say precisely when the crisis ended, but we have considered that it lasted for the whole of September.

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