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Empirical evidence, theories, and the Amundi experience

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About the author

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He started his professional career in 2001, as an Equity Risk Manager at Nextra Investment Management (Intesa Group), Milan. Still at Nextra SGR he has served as a quantitative Fund Manager from 2004 to 2005, when he has been appointed Head of the Quantitative Research team at the Italian branch of Credit Agricole Asset Management SGR.

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Executive Summary

Financial theory assumes that higher risk is compensated on average by higher returns. However, the outperformance of low volatility stocks during the last 50 years has been among the most puzzling anomalies in equity markets. At the same time, low risk investing has recently gained a remarkable interest, due to its documented performance coupled with the unprecedented volatility experienced during the last global financial crisis.

In our work we show how researchers have been documenting such anomaly since the early nineties: Fama and French (1992) show a rather negative relationship between risk and returns, and Baker and Haugen (1991) find significant reduction in volatility with no reduction in returns, for US minimum variance portfolios. We find that most of the relevant empirical studies focus on total or systematic volatility; some of them state that the low risk anomaly holds regardless of which of the two measures is used for stock selection. Only few exceptions instead (Ang *et al*, 2006) rather refer to idiosyncratic volatility.

After reviewing empirical evidence, we discuss how literature has so far explained such an anomaly. Among others, theories referring to leverage constraints are well represented: the idea behind is that investors willing to leverage and facing leverage constraints buy high beta stocks, thus lowering subsequent returns. In some other works where the mispricing belongs rather to behavioral bias, leverage constraints still prevent the anomaly from being arbitraged away. Other theories refer to delegated portfolio, benchmarking, and fund managers' utility function. We also investigate explanations related to distribution of equity returns: skewness and convexity in particular. The intuition here is that returns of high beta stocks are usually characterized by positive skewness and by a convex relationship with market returns. These features inflate the price paid for those stocks, thus lowering subsequent returns.

In the last section of our research, we annex a formal description of two Amundi investment processes (belonging to NextGen Equity Strategies) that successfully exploit this low risk anomaly: the Global Minimum Variance and the Global Smart Beta (risk parity).

1. Low Risk Equity Investing, empirical evidence of success

The first axiom of investing is that higher risk is compensated -on average- by higher returns.

In his Portfolio Selection (1952), Markowitz defines portfolio risk as the volatility (or variance) of its returns and states that, while taking investment decisions, investors maximize expected returns for a given amount of portfolio risk: this leads to a positively sloped and concave efficient frontier.

In Sharpe's CAPM (1964) risk is defined as portfolio beta relative to the Market Portfolio. The idea behind the CAPM is that any asset's average return may be expressed as the sum of the risk-free rate and a risk premium, the latter being proportional to the asset's covariance with the Market Portfolio (its beta).

According to the theory, both the Minimum Variance portfolio and the Market Portfolio belong to the efficient frontier; the latter exhibiting a higher expected return than the former.

As mentioned, our goal is not to take a position in the everlasting dispute in favor or against the CAPM. However it is worth mentioning some preliminary tests that provided some comforting results during the early seventies, as well as many subsequent studies proving the outperformance of low volatility stocks versus high volatility stocks, and its persistence during recent decades.

Black, Jensen and Scholes (1972), and Fama and MacBeth (1973) form portfolios on the basis of in sample estimations of beta, and then look at realized Beta and stock returns, finding that higher beta is positively correlated to higher returns. Yet, the slope of the linear relationship is flatter than anticipated by CAPM. For instance, in Fama and MacBeth, the hypothesis that beta is uncorrelated with returns is significantly rejected only in the full sample period (1935-1968) while it is not rejected in any of the 10-year sub-periods.

Despite some statistical weakness, these contributions held for a long time as a general confirmation of the traditional positive relationship between risk and return.

In fact, more recent evidence shows that well-constructed low risk portfolios contradict this axiom, as they deliver higher returns than riskier portfolios.

In 1992, Fama and French showed that value and size factors generate large returns in the US markets and, quite surprisingly, that the relationship between beta and returns is rather negative, when corrected for size effect.

Ang *et al* (2006) find strong evidence of negative relationship between stocks' returns and their sensitivity to changes in aggregate volatility (daily and monthly changes in VIX index and daily and monthly returns of an equity portfolio, replicating the VIX itself). They explain such evidence by the premium that investors pay (thus reducing subsequent returns) for stocks that are positively correlated with volatility jumps, thus providing a hedge in case of market drops (as typically market drops take place together with volatility jumps). They also find a strong negative relationship between idiosyncratic risk (residual component in a Fama French environment) and future returns.

Intuitively, as the aggregate volatility (VIX factor) is significant indeed, but missing in the Fama French model, companies with greater sensitivity to the VIX factor should exhibit higher idiosyncratic risk in a Fama French framework.

Surprisingly, after controlling for aggregate volatility, the negative relationship between returns and idiosyncratic risks is little changed, and completely unexplained.

As a consequence, the authors associate the low-risk high-return anomaly to idiosyncratic risk, rather than systematic and total risk.

Blitz and Vliet (2007) focus on systematic and total volatility. They state that the low-risk anomaly holds regardless of which of the two measures is used for stock selection: low variance stocks exhibit low beta, while high variance stocks exhibit high beta.

The author's controls for factors such as value, momentum, and size, are both via linear regressions with those factors' returns as explanatory variables (with little if any variance explained), and via cross basket analysis: returns of the volatility factors are investigated among stocks with similar momentum, size or value characteristics. The significance of the low risk effect is little reduced, and only within baskets of stocks with similar market capitalization.

They also argue (probably underestimating the cost in terms of loss of flexibility and risk of incurring an undesired exposure) that there is no need to rely on sophisticated risk models and that an equal weighted portfolio of stocks with low historical volatility is all investors need to achieve superior returns.

Baker, Bradley and Wurgler (2011), agree that there is no major change in addressing the superior risk adjusted performance of low risk stocks whether we rank our universe (CRSP) by total volatility or by beta.

We have replicated these transparent tests on the MSCI World constituents from January 2003 to June 2012 (excluding stocks with less than two years of presence in the index), and we confirm that grouping stocks for (ex-post) beta rather than (ex-post) volatility does not have a big impact: in both cases average risk-adjusted returns decrease with risk measure, with baskets built according to beta showing somehow better regularity and monotony (Annex 1, tables 1 and 2, chart 1).

Baker and Haugen (2012) implement deciles analysis for each of the 21 developed and 12 emerging countries of their sample: they group stocks from 1990 to 2011 according to their historical volatility and they find that lowest risk stocks exhibit higher returns and better Sharpe ratio, compared to high risk stocks, in any observed country. They also point out that only rarely do the highest volatility deciles outperform lowest deciles, in a rolling window of three years.

In their intentionally simple approach, they use an equally weighted basket and 24 months historical volatility as a measure of risk, apparently supporting argument by Blitz and Vliet (2007) for no need for a sophisticated risk model. Actually no control is done for valuation, size, or any other factor that could help to explain the anomaly.

In 1991, Baker and Haugen first investigate Minimum Variance portfolios in the US equity market, pointing out a 30% reduction in portfolio volatility, compared to both a common US index and randomly selected portfolios, with no reduction in average returns.

Clarke, de Silva and Thorley (2006) build Minimum Variance portfolios on the largest 1000 US stocks over the period 1968 – 2005. They estimate a covariance matrix with Bayesian methods for shrinkage -in order to avoid error maximization problems- as well as principal component analysis.

They first detail some well-known portfolio characteristics like typical concentration (75 to 250 stocks with 3% cap on a single company), turnover (143% with monthly rebalancing), positive exposure to size and value factors, and zero mean but rather volatile exposure to momentum factor. They confirm Baker and Haugen's evidence (1991) of 30% reduction in volatility with substantially no reduction in average returns. Results are little changed after constraining fundamental factors' exposures and reducing turnover to 56% through quarterly rebalancing.

In their more recent (2011) "Minimum Variance Portfolio Composition", Clarke, de Silva and Thorley update previous (2006) statistics with basically no change. However their work is worth mentioning mostly because, while proposing an interesting analytical solution of the Minimum Variance portfolio composition, they point out that systematic risk dominates in the construction of such a portfolio: in their simplified single factor model, stocks with beta higher than a threshold are strictly excluded from the portfolio, while high idiosyncratic risk "only" contributes to lowering the stock's weight in the portfolio.

Carvalho, Lu, and Moulin (2011) discuss typical low beta and small cap exposure of Minimum Variance portfolios. Furthermore in their view, changes in the correlation matrix generate higher than justified turnover, thus they suggest constructing portfolios by applying stable weighting schemes based on stocks' beta, rather than optimizing.

In our opinion, a weighting scheme may be a reasonable solution, even though optimization remains our optimum for Minimum Variance portfolio construction, as an optimization package enables turnover to be reduced, while controlling many other constraints.

Thomas and Shapiro (State Street Global Advisors, 2007) show their encouraging results on a Minimum Variance portfolio built on the Russell 3000, using a standard optimization package (BARRA). Their contribution is relevant to us because -as we will discuss deeper in section 3- they recognize the advantage of using such an optimization package, in order to avoid the typical drawbacks of a Minimum Variance investment (excessive concentration in a few low risk sectors and stocks, lack of control for involuntary factor exposure), while enhancing their performance by tilting the portfolio toward some long term successful alpha strategy, like the historical dividend yield.

2. Why low risk stocks outperform

Most of the recent literature addressing the low risk anomaly has also offered some theoretical framework.

Explanations account for borrowing constraints and other market frictions, behavioral hypothesis, equilibrium models addressing the utility function of fund managers (rather than of investors) in delegated portfolio management, or taking into account higher moments (and their relative premium) than the mean and the variance of returns.

Among the others, theories referring to leverage constraints are probably the most represented.

2.1. Leverage Constraints

Blake identifies borrowing restriction as one of the possible sources of low risk stocks outperformance in his "Beta and Returns" (1995). Investors are supposed to pass through such a limitation, investing massively in high beta stocks thus lowering subsequent returns.

Frazzini and Pedersen (2011) develop a model of asset equilibrium populated by two categories of investors: investors with no leverage constraints but with margin requirements, and investors for which leverage is forbidden. They argue that while the latter overweight high beta assets, causing those assets to offer lower subsequent returns, the former buy low beta stocks and leverage their holdings by short selling the high beta stocks. Intuitively both the low beta and the high beta stocks have their demand in the market, thus it is not immediately clear why only high beta stocks should be overpriced, thus delivering poorer returns than the CAPM would predict. However the analytical solution of the required rate of return in equilibrium shows that the alpha of each security monotonically decreases with its beta, thus reducing the slope of asset returns relative to beta itself. This slope is flatter as the tightness of the funding constraint increases.

The authors build a "betting against beta" factor by going long on low beta stocks and shorting a smaller amount of high beta stocks, thus obtaining a beta neutral factor. They provide empirical evidence for alpha decreasing with security beta, and for the "betting against beta" factor exhibiting positive returns in equity, treasury, credit, and currency markets.

Carvalho, Lu, and Moulin (2011) agree that leverage constraint has a major impact on the preference towards high beta stocks. However, they add that, as they simply seek high returns, investors prefer riskier stocks, and create a demand imbalance.

Blitz and Vliet (2007) state that as long as leverage is limited and leveraging a low beta portfolio is needed to match the volatility of the market (in order to obtain a volatility neutral strategy), the anomaly is not fully arbitraged away.

However, among the possible explanations of the anomaly, they take into account a behavioral theory by Shefrin and Statman (2000), where investors allocate their wealth according to two layers: a low risk layer designed to avoid poverty, and a high aspiration layer, for which they are much less risk averse. In this case, investors will overpay for (often few and badly diversified) risky stocks, which are perceived to be similar to lottery tickets.

The authors finally mention another well represented family of theories that will be further discussed in the next section, which relates the low risk anomaly to the utility function of the fund manager: as the largest inflows go toward outperforming fund managers and to well performing asset classes, these portfolio managers may seek to maximize outperformance in the upward markets, thus systematically preferring high beta stocks.

2.2. Delegated portfolio management, benchmarking, and fund managers' utility function

Cornell and Roll (2005) recognize that the considerable market share of investment being nowadays delegated to professional fund managers, impose asset pricing models to incorporate the objective function of the agents, together with the traditional utility function of the final investors. The objective function of the fund manager is not the investor's wealth maximization, but the maximization of active return versus the benchmark of the delegated mandate.

Without a direct implication on performance of low risk versus high risk stocks, they show how a pricing model based on delegated investments (where fund managers' objective function dominates the investors' utility function), in equilibrium implies some cross sectional relationship between stocks' alpha and their beta relative to the benchmark. These relationships violate CAPM.

Baker, Bradley and Wurgler (2009 and 2011), make a step forward focusing explicitly on the low risk anomaly.

The anomaly itself is indeed a consequence of irrational behavior of non-professional and professional investors, who (1) favor lottery-type investments, (2) consider few cases of success of high volatile stocks as representative of all the other high volatility stocks, (3) are overconfident, trade more actively when optimistic than when pessimistic, and are likely to disagree on future stocks returns, especially on high volatility stocks. However, like Cornell and Roll (2005), the authors recognize the prominent role of delegated and benchmarked investment, as a limit to arbitrage: behavioral bias result in high absolute risk stocks delivering similar returns than low absolute risk stocks, but the latter still generate a higher tracking error relative to the mandate benchmark, and consequently fund managers are prevented from arbitraging the anomaly away (as they have low incentive to seek similar returns for a higher tracking error).

Backer and Haugen (2012), state that, as the risk return relationship is rather inverted, fund managers would have a concrete incentive in buying low risk stocks.

Their explanation for fund managers not actually exploiting the anomaly is that they seek to maximize the probability of receiving a bonus, that is usually paid when performances are positive in absolute terms, and higher than a threshold (benchmark return plus the management fees, or an absolute discretionary threshold).

Portfolio managers exchange higher expected returns of low beta stocks, for a higher probability of beating their benchmarks (or a given target), provided by high volatility stocks. In other words they exchange a higher mean of returns distribution (low volatility stocks) for a higher expected value in the right-end tail of the distribution (high volatility stocks).

The authors also mention some additional incentives to hold volatile stocks, this time related to the delegated portfolio construction process: in order to impress colleagues, analysts are often willing to recommend stocks “in the news”, or stocks whose news flow is quite intense and that tend to exhibit higher than average volatility. Finally fund managers may find it easier to justify holdings or turnover of newsworthy stocks.

Supporting their intuition, the authors show that –for 1000 US stocks grouped in 10 homogeneous classes of market capitalization, from 2000 to 2011– companies with higher institutional ownership exhibit higher volatility than stocks with low institutional ownership. Finally, they find that analysts’ coverage (number of recommendations) is positively correlated with volatility.

2.3. Return distribution of stocks' return: may convexity and skewness have an impact?

Chunhachinda *et al* (1997) find that the returns of the world's 14 major stock markets are not normally distributed. Optimal portfolio compositions are computed (as allocations of 14 international stock indexes) incorporating investors' preferences for skewness. The empirical findings suggest that the incorporation of skewness into the investor's investment process causes a major change in the construction of the optimal portfolio. The evidence also suggests that investors exchange expected return for positive skewness.

Similar findings are provided by Prakasha *et al*. (2003).

Kraus and Litzenberger (1976) find that, while having aversion to variance, investors exhibit a preference for positive skewness. As a consequence, if the capital asset pricing model is extended to include systematic skewness, this latter is associated with a positive price (instead of a discount, as it is the case for variance), and the zero intercept for the security market line is not rejected.

The intuition behind these three contributions is that, as skewness has a positive price, it should be higher among high beta or high risk stocks, so that these latter are priced at a premium compared to a CAPM equilibrium, and finally deliver lower average returns.

We tested this hypothesis in the last decade on the constituents of the MSCI World Index in the period from January 2003 to June 2012. We regressed stocks' weekly returns (excluding only those stocks with less than two years of available data) on the index returns; we then formed equally populated baskets according their betas. We then computed differences and T-statistics of the average skewness, for any pair of baskets. The signs and their significance prove some interesting monotony: higher skewness for higher beta stocks.

Results are summarized in tables 3 and 4 in Annex 2: skewness generally increases with beta, sometimes significantly, especially for baskets whose differences in beta are high enough.

We repeated the exercise using total ex-post volatility as a measure for sorting and grouping stocks (tables 5 and 6). Results are even more significant, as skewness increases almost monotonically with volatility.

We recognize our analysis is completely in sample and misses some predictive power; however, it is transparent and easily replicable. The hypothesis of skewness increasing with beta and volatility is confirmed, and this suggests that the premium that investors pay for a positive skewness may explain lower subsequent risk-adjusted returns for high beta and high volatility stocks.

Cowan and Wilderman (2011) find that high beta stocks exhibit a positive convexity relative to broad market index returns, while low beta stocks have negative convexity.

They explain that high beta stocks provide a call option payoff. In the case of positive market returns, high beta stocks deliver market returns, multiplied by a factor roughly equal to their beta (whatever the market return is, exactly like a leveraged position). Conversely, in case of negative market returns, losses are limited to 100% of invested capital, at worst.

The difference with leveraged investments is straightforward as the latter generate payoffs exactly equal to the returns of the unleveraged positions times the leverage, with theoretically no limit to downside.

The authors argue that investors exchange future returns for having this call-type convex payoff: they pay an additional premium for high beta stocks just like they paid a premium to buy a call option.

In our view this explanation may be convincing only for very extreme market returns, that is quite rarely (the convex profile takes place in the form of a stop loss, that is activated in case of market returns of, let's say, -33% in the case of a stock beta higher than 3, or -50% in case of beta higher than 2).

However, we apply the methodology described for skew in the previous paragraph, to test if we find increasing convexity, for increasing beta. For each MSCI World constituent (from January 2003 to June 2012, excluding companies with less than 100 weekly returns) we run two OLS regressions: the first with the series of MSCI World returns (in USD) as the only explanatory variable; and a second one with the series of MSCI World squared returns as an additional explanatory variable:

$$R_i = \beta_i R_{Msci} + u_i \qquad (1)$$

and

$$R_i = \gamma_i R_{Msci} + \lambda_i R_{Msci}^2 + v_i \qquad (2)$$

where a positive estimate of λ implies positive convexity of stocks' returns, relative to market returns.

We group stocks according to their estimate of β from OLS regression 1, and we compute the average and the standard deviation of the estimate λ for any basket. We then test the significance of differences in average basket convexities.

Results are summarized in Annex 3 and seem to support Cowan and Wilderman's intuition. Table 7 reports average convexity and standard deviation across any basket; while table 8 reports differences in average convexity for any pair of baskets.

The higher the distance of betas, the larger and more significant the difference in convexity: high beta stocks exhibit a higher and thus more profitable convexity than low beta stocks. Differences are often significant and this may explain some premium paid by investors for high beta stocks.

Table 9 and 10 report results for the same experiment, when we rank and group stocks according to total ex-post volatility. Results are even more significant as convexity increases almost monotonically with volatility.

3. Amundi NextGen Equities

We have lived for several years in very challenging markets: international equity indexes have exhibited high realized volatility and quite disappointing returns, during the last decade. Equity investors have been faced with a major and unfavorable change in traditional risk return payoffs.

In the last few years, Amundi has strongly invested in order to meet investors' needs in such a challenging market context, developing a range of innovative solutions aiming at Sharpe ratio improvement. Their risk-return profile differs as well as their behavior in up and down markets. Over the last decade, these strategies have all succeeded in enhancing risk return trade-off (Sharpe ratio has been systematically superior to that of relevant equity index as the MSCI World). They all belong to the absolute risk category: away from the notion of tracking error or information ratio, they focus on Sharpe ratio or risk-adjusted return, and volatility metrics. They are based either on the use of instruments providing favorable asymmetry (options and other derivatives), or on portfolio construction techniques as maximum diversification, minimum variance, and risk parity ("Smart Beta" -or simply "Smart"- from now on).

The latter two are particularly relevant to this document as both exploit the low risk anomaly discussed previously.

On an ex-post basis, the volatility of the Global Minimum Variance and Global Smart portfolios is significantly lower than the market index, with a 10 to 20% reduction for Smart, and up to a 35% reduction for Minimum Variance (Annex 5.1 and 5.2). However, most of the relevant literature strictly identifies low risk strategies with the selection of low risk stocks and -among several measures of risk- systematic risk and beta are by far the most significant. For this reason it is worth investigating whether Minimum Variance and Smart processes limit portfolio volatility mainly by selecting low risk stocks, or rather by enhanced diversifications.

In Annex 4, we provide some empirical evidence on two back-tested Minimum Variance and Smart portfolios, during the period December 2003 – December 2011.

Table 11 shows that the ex-post beta of the Minimum Variance portfolio on full data sample is only 0.55 (significantly lower than 1 at a 1% confidence level). On an ex-ante basis with quarterly observations, portfolio beta ranges from 0.48 to 0.65. As for the Global Smart portfolio, table 11 shows that the ex-post beta relative to MSCI World is 0.85 (again significantly lower than 1 at 1% confidence level) and the ex-ante beta ranges from 0.7 to 0.87.

In table 12 we further investigate the systematic risk characteristics of the two portfolios and the benchmark in a multifactor framework. For each of them at every quarter of our back-test, we compute the weighted average common factor risks extracted from the BIM model by BARRA[1]. Weighted average common factor risk of the Minimum Variance portfolio is lower than that of Global Smart; both of them are lower than the MSCI World.

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[1] Weighted average common factor risk, at any time t of our sample, is computed as follows:

$$CF(t) = \Sigma_i w_i(t) CF_i(t)$$

where $CF_i(t)$ is the common factor risk of the ith stock at time t, and $w_i(t)$ is its weight at time t.

Charts 2 and 3 show average portfolio weight over stocks grouped by beta and common factor risk[2]. Minimum Variance and Smart portfolios overweight stocks with lower than average beta, compared to the MSCI World. The same result holds for average common factor risk. Aggregate weights decrease with increasing risk for Minimum Variance and Smart, while weights are distributed much more uniformly for the MSCI World, with slightly higher frequency on stocks with average risk.

This evidence supports the intuition by Carvalho, Lu, and Moulin (2011), who infer that the exposure to low beta and low systematic risk plays a major role in explaining the superior performance of Minimum Variance portfolios; we further extend these findings to Smart Beta as well.

In fact, such exposure of Smart portfolios to low risk stocks is somehow intuitive if we consider the sector allocation process (sector weights are inversely proportional to their marginal contribution to risk), and the stock weighting scheme (inversely proportional to their total volatility). The two of them are described in detail in section 3.2.

3.1. Amundi Global Minimum Variance: an enhanced process

As shown, the most straightforward way to exploit the low risk anomaly is certainly building a portfolio of stocks with the lowest possible risk.

In this section we describe our Minimum Variance approach on a Global Developed Equity universe. We claim several years of experience in Minimum Variance management, with two Europe portfolios (since 2007 and 2009 respectively), a very recent Global portfolio, and our paper portfolios on developed World, Japan, Emerging markets, Pacific ex Japan, and other customized universes. However in this document, we focus on the Global Minimum Variance as it is the most complete case for descriptive purposes.

Efficient frontier and the Minimum Variance portfolio

The efficient frontier represents the set of portfolios that earn the maximum rate of return for every given level of risk. We use an optimization process to build a portfolio sitting on the very edge of the efficient frontier. In building such a portfolio, expected returns are not needed as the only requirement is to minimize volatility, while being fully invested.

[2] Every quarter we build three equally populated baskets of stocks, according to their beta and common factor risk (high, median, and low risk). We then compute the aggregate weight of stocks in each group, for any portfolio. Finally we compute the historical average of aggregate weights.

Our simple objective function is thus:

$$\text{Min } (w^T V w)$$

$$\text{Such that } e^T w = 1$$

where w is the vector of the optimal portfolio weights, V is the variance-covariance matrix, and e^T is a vector of ones.

Although we recognize the advantage of such a process being transparent and intuitive, we are conscious of some typical drawbacks that may arise from Minimum Variance portfolios: as shown in Clarke, de Silva and Thorley (2006), Minimum Variance portfolios may be quite concentrated on a few low volatility stocks, may exhibit rather high turnover, positive exposure to value and small capitalization stocks (with some relevant implications on liquidity), and some volatile exposure to momentum factor.

Similarly, Thomas and Shapiro (2007) highlight the risk of the Minimum Variance portfolio being excessively concentrated on few low risk sectors, and the lack of control for involuntary factor exposure. They also express their preference for tilting portfolios toward some successful stock ranking criteria.

We agree that most of those are relevant issues in portfolio construction and we do believe that handling them through an optimization package is strictly needed in order to come out with reasonable and investable portfolios, as only few of them might be addressed correctly with some clever weighting schemes (like in the case of limiting turnover through an equally weighted basket of low beta stocks, as suggested by Carvalho, Lu, and Moulin).

That’s why we implement our enhanced portfolio construction process in Barra One, as described below.

Quality Stocks

We believe that fundamental equity selection can provide some valuable enhancement in the risk return profile of equity portfolios, at least in the long run. At the same time we don’t want to renounce an optimization process which is completely independent from expected returns. Expected returns are very noisy in forecast and thus responsible for well-known “error maximization” problems.

For this reason, we apply a qualitative filter to our investment universe, excluding the lowest quality stocks from the optimization. Basically, each quarter we rank the constituents of the MSCI World Developed Markets according to a Piotroski (2000) score and we exclude the two bottom quintiles.

Keeping 60% of constituents available for investments, the optimizer is left with a high degree of freedom and it tilts the optimal portfolio toward good quality stocks, without using explicit expected returns.

Table 11 in Annex 4 reports the balance sheet, income statement and corporate actions employed in the Piotroski score, and table 4 shows that -in the last decade- the top quality 33% of MSCI constituents (equally weighted) have outperformed the market index with lower volatility. At the same time, the median basket has performed in line with the market, and the bottom basket has underperformed with even higher volatility.

Turnover and liquidity

High turnover is a critical issue in many systematic investment strategies like Minimum Variance.

In our case, turnover in the investment universe is limited as the Piotroski score is based on balance sheet data that varies very little during one quarter. Furthermore we also rebalance our portfolio quarterly, as suggested by Baker and Haugen (1991).

Nevertheless, more than turnover itself, our concern is liquidity indeed: we aim to avoid small illiquid companies as we want to be able to liquidate our portfolio in a reasonable time lag, without incurring significant market impact costs.

To address this requirement, we limited the amount held in any stock to the following percentage:

$$UB_i = 25\% \frac{ADV_i}{NOT} D$$

where UB_i is the upper bound on the ith stock, D is the number of days that we accept to liquidate the fund, ADV_i is the average daily volume over the last quarter, and NOT is a notional amount of assets under management of USD 1 billion: quite conservative as it is still far above the current size of our fund.

Sector, country, and stock concentration

As mentioned above, Minimum Variance portfolios may tend to be poorly diversified across sectors, countries or single stocks. We have thus applied some constraints at these levels, without preventing the optimizer from choosing solutions that are far enough from a market index.

On countries and sectors we accept deviations from the market index of 500 to 1000 bp, while for single stocks we apply a general upper bound (GUB), thus modifying the actual upper bound as follows:

$$\overline{UB}_i = \min\left[GUB; 25\% \frac{ADV_i}{NOT} D\right]$$

Management of asymmetries in factor returns

Furthermore, we are conscious that Minimum Variance portfolios may be exposed to fundamental factors as size, value or momentum.

We observe that much of our size exposure is corrected away by the liquidity constraints. As for other factor exposures, we have decided not to manage them systematically as –again– we don't want to excessively constrain the optimization process.

On the other hand, we regularly monitor the behavior of all the risk factors of the BARRA model (size, value, growth, momentum, leverage…). The goal of this monitoring is to detect bubbles or suspicious asymmetries like excessive positive skewness in recent performance (Sornette, 2003; Morel, Malongo, and Lambinet, 2013): in the case of significant alerts, we punctually hedge the risk of an exploding bubble, imposing a neutral exposure to the suspected factor.

As for performance (Annex 5.1, table 15), results are very satisfactory indeed: from the beginning of 2003 to the third quarter of 2012, Minimum Variance portfolios outperformed the standard index by a minimum of 3.6% in Pacific (All Countries) ex Japan, to a maximum of 5% in the World developed Markets, while volatilities and draw-downs were reduced by one fourth to one third. As a consequence, risk adjusted returns are twice as high as for the standard index[3].

Minimum Variance portfolios belong to the absolute return investment category, as it is confirmed by ex-post tracking errors ranging from 9 to 10% roughly.

Real money performance of our two Europe portfolios (from end 2007 and mid 2009 respectively) confirms the results of our back test with both of them outperforming the standard index in absolute and risk-adjusted terms[4].

3.2. Amundi Global "Smart Beta": A systematic risk parity approach

In this section we discuss our risk parity approach on global developed markets. We recall that we currently manage several risk parity portfolios (Euro Area, Europe, World), and we are investigating the behavior of such strategies in many others areas (World ex Japan, Pacific ex Japan, Japan, Emerging Markets, Emerging Markets with Sharia filter). Our approach is consistent across the regions with the Developed World being probably the best example for descriptive purposes.

Risk parity means that each asset (asset class, equity sector, single stock) has an equal contribution to the total risk of the portfolio.

As Maillard, Roncalli, and Teiletche (2009) have pointed out, full risk parity cannot be obtained in a closed formula unless some unrealistic hypotheses (such as equal correlation among all the assets in the investment universe) are made, and may not be achieved either through optimization, if the number of assets involved is somehow relevant, and correlations are very heterogeneous.

As for the number of assets involved, we typically deal with 1,500 to 2,000 constituents of the MSCI World. For this reason we have decided to split our portfolio construction process into two steps: the region-sector allocation, and the stock weighting in each regional sector basket.

Optimization (for instance, the minimization of the cross-section standard deviation of assets' contribution to risk) does not guarantee a full risk parity solution. Furthermore, it can be

[3] As for the Emerging Markets portfolio (from December 2005), in order to be eligible to minimum variance optimization, stocks must be held on the Amundi emerging market flagship fund, or must be top-ranked (33%) according to a Piotroski score.

[4] The former of the two European portfolios has been designed in order not to exceed the ex-ante volatility of 10%. That's the main source of performance difference between them.

time-consuming, and should rely on a robust numerical algorithm. For these reasons we have investigated alternative solutions among some reasonable systematic weighting procedures.

While defining the region and sector allocation of our risk parity portfolio, we distinguish three regions: North America, Europe, and Pacific. Then, within each geographical region, we operate on the 10 regional sectors according to GICS definition (Level 1) as homogeneous groups of stocks.

In the allocation of each economic sector within one region, as well as of each region within the global portfolio, we reject the very popular method of weighting baskets by the inverse of their volatility: this procedure ignores correlations that should be taken into account explicitly instead, as they may be highly heterogeneous across sectors and regions.

One way to account for them is to use the measure of marginal contribution to total risk.

If W is the vector of weights of portfolio P, and σ is the volatility of portfolio P, MC_i is the marginal contribution to risk of each asset, and is equal to:

$$MC_i = \frac{\partial \sigma}{\partial W_i}$$

In order to come out with full risk parity, the following relation must hold:

$$MC_i W_i = MC_j W_j \text{ for any } i \neq j$$

In other words the risk contribution should be the same for any basket:

$$RC = MC \otimes W = Ke$$

where RC is the vector of risk contributions, k is a constant, MC is the vector of marginal contributions, e is a vector of ones, and $\otimes$ is the element by element product operator. Marginal contributions are function of volatilities and correlations of any basket with the rest of the portfolio, with correlations depending on portfolio composition itself.

In our equation, weights are the unknowns and should lead to a constant vector when multiplied by MC, which depends on weights themselves: the problem is clearly recursive, and the solution is endogenous.

Intuitively, we should set the target weight of each basket as proportional to the inverse of its marginal contribution. Starting from a discretionary non-optimal initial portfolio composition W^{INIT}:

$$W^{TGT} \sim \frac{1}{MC^{INIT}}$$

Unfortunately, moving from W^{INT} to W^{TGT}:

$$MC^{TGT} \neq MC^{INIT}$$

Marginal contributions change as a consequence of weights' change, and as the change in marginal contributions is different for any asset, the risk budget is no longer equal across all the assets, and the weights calculated accordingly no longer guarantee risk parity. As a consequence, W^{TGT} may be somewhat far from optimal.

It is clear that the choice of the starting point where we compute marginal contributions is crucial. For instance, a standard market index, where sectors and regions are weighted by market cap, is not a good configuration for estimating marginal contributions, as they may be exacerbated by index characteristics (very low weight in some extremely volatile sector like IT in the Euro zone, may lead to an artificially low contribution).

In order to come out with a satisfactory solution in a reasonable time, we have chosen to observe marginal contribution in the most neutral portfolio composition: the equally weighted composition.

Equal weights as starting point have the advantage of not being far from the (still unknown) optimal solution: in this way the marginal contributions that we use for target weight calculation are a very good proxy for the marginal contribution that we will observe after weight calculation, thus ensuring a truly well balanced risk contribution.

In order to check for the accuracy of our solution, we have computed percentage contribution (PC_i), for any basket, at any date of our back test.

$$PC_i = \frac{W_i MC_i}{\sigma}$$

As for the second step, that is the methodology at a stock level, we weight stocks inversely proportionally to their total volatility. We have already evoked that, with equal correlations, inverse of total volatility is a weighting scheme that guarantees risk parity across stocks, while we observe that for similar correlation this solution is a very efficient proxy. Within the same region and sector, correlations among stocks are very similar and most of the dispersion in stock's returns is explained by their difference in beta and by idiosyncratic risk.

Also, we prefer using total risk as the relevant metric because it is intuitive and easy to estimate, even without resorting to a complex risk model: while it is easy to challenge or validate an existing risk model in re-estimating marginal contributions on 10 sectors and 3 regions (or even on 30 region-sector baskets together), it is not such an easy task to do the same with the roughly 2,000 constituents of an index.

In a focus on the Euro zone, Chart 4 of Annex 5.2 reports the highest and lowest percentage contribution for any sectors, during the 10 years of our sample, and Chart 5 reports full sample means.

If we implement sector allocation after re-building the sector with stocks' weights inversely proportional to their volatility (we refer to this procedure as "bottom up" as step 2 is performed before step 1), percentage risk contribution ranges from 9.9% to 10.1%, leading to almost perfect risk parity. Performing step 1 before step 2 ("top down"), risk parity is slightly less accurate, as marginal contributions are estimated on sector baskets where stocks are weighted for free float adjusted market cap, thus diverging from the actual baskets (where stocks are finally weighted for the inverse of volatility). Both solutions are definitely satisfactory, if we consider that, within the MSCI Index, risk contributions range from 3% to 25%.

As for performance (Annex 5.2, table 16), data are again extremely good: from the beginning of 2003 to third quarter of 2012, "Smart" portfolios outperformed MSCI indexes by roughly 5% annually in World Developed markets, World Emerging markets, EMU area, and Pacific All Countries ex Japan. As for Japan, outperformance is 3.5% when the "Smart" portfolio is built on MSCI index constituents, and it rises to 5% if it is built on TOPIX index constituents[5].

Volatilities are reduced by 10 to 25% and draw-downs by 20 to 25%.

[5] With MSCI Japan constituents, we perform risk parity on the 10 GICS level 1 sectors. With TOPIX constituents, we involve 17 sectors, according to TSE classification.

Real time data in Europe and in the EMU area (from mid-2010) are really encouraging indeed, as both portfolios outperform the standard index in absolute and risk-adjusted terms, with volatilities and draw-downs reduced similarly to the back-tests.

The global portfolio (available since January 2012 only) so far exhibits the same risk-adjusted return as the MSCI index.

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ANNEX 1

(Ex-post risk and returns in World Developed markets)[6]

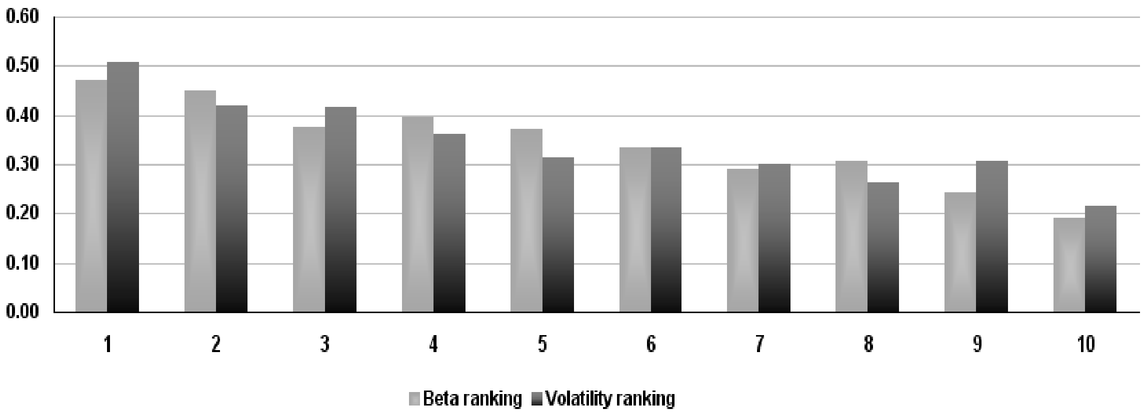
Table 1: Average returns, volatility and risk adjusted returns of baskets ranked for beta (decile 1 lowest beta, 10 highest)

Deciles	1	2	3	4	5	6	7	8	9	10
Average Annual	2.64%	2.87%	2.74%	3.28%	3.44%	3.43%	3.28%	3.88%	3.47%	3.38%
St. Deviation	5.63%	6.37%	7.33%	8.31%	9.31%	10.26%	11.36%	12.60%	14.35%	17.68%
Risk Adj. returns	0.469	0.450	0.374	0.395	0.370	0.334	0.289	0.308	0.242	0.191

Table 2: Average returns, volatility and risk adjusted returns of baskets ranked for volatility (decile 1 lowest volatility, 10 highest)

Deciles	1	2	3	4	5	6	7	8	9	10
Average Annual	2.79%	2.83%	3.18%	3.15%	3.03%	3.45%	3.41%	3.22%	4.02%	3.46%
St. Deviation	5.47%	6.78%	7.66%	8.71%	9.61%	10.37%	11.40%	12.22%	13.10%	16.05%
Risk Adj. returns	0.509	0.418	0.415	0.361	0.315	0.333	0.299	0.264	0.307	0.216

Chart 1: Risk adjusted returns (01-2003; 06-2012)



[6] Source for all figures: Amundi, Factset

ANNEX 2

(Skewness in World Developed markets) [7]

Table 3: Average skewness of baskets ranked for beta (decile 1 is the lowest beta, 10 is the highest)

Deciles	1	2	3	4	5	6	7	8	9	10
Average	0.083	0.006	0.100	0.130	0.080	0.140	0.165	0.357	0.402	1.111
St. Deviation	0.804	0.882	0.859	0.704	0.856	1.178	1.157	1.366	1.515	2.172

Table 4: Difference in average skewness for baskets ranked for beta (basket in row minus basket in column)

Deciles	1	2	3	4	5	6	7	8	9	10
1										
2	-0.077									
3	0.018	0.095								
4	0.047	0.124*	0.029							
5	0.003	0.074	0.021	0.050						
6	0.057	0.134	0.039	0.010	0.060					
7	0.082	0.159*	0.065	0.035	0.085	0.025				
8	0.275***	0.351***	0.257***	0.228**	0.278***	0.218**	0.192*			
9	0.319***	0.396***	0.301***	0.272***	0.322***	0.262**	0.237**	0.044		
10	1.029***	1.106***	1.011***	0.982***	1.032***	0.972***	0.946***	0.754***	0.710***	

Table 5: Average skewness for baskets ranked for volatility (decile 1 is the lowest volatility, 10 is the highest)

Deciles	1	2	3	4	5	6	7	8	9	10
Average	-0.175	-0.059	-0.048	-0.021	0.060	0.056	0.153	0.161	0.559	1.643
St. Deviation	0.544	0.600	0.507	0.613	0.633	0.601	0.769	0.538	1.136	2.298

Table 6: Difference in average skewness for baskets ranked for volatility (basket in row minus basket in column)

Deciles	1	2	3	4	5	6	7	8	9	10
1										
2	0.116**									
3	0.128***	0.011								
4	0.154***	0.038	0.027							
5	0.235***	0.119**	0.108**	0.081						
6	0.231***	0.115**	0.104**	0.077	-0.004					
7	0.328***	0.212***	0.201***	0.174***	0.093	0.097*				
8	0.337***	0.220***	0.209***	0.182***	0.101**	0.105**	0.008			
9	0.734***	0.618***	0.606***	0.579***	0.499***	0.502***	0.405***	0.397***		
10	1.818***	1.702***	1.690***	1.663***	1.583***	1.587***	1.489***	1.481***	0.534***	

*** statistically significant at 1%

** statistically significant at 5%

* statistically significant at 10%

[7] Source for all figures: Amundi, Factset

ANNEX 3

(Convexity in World Developed markets)[8]

Table 7: Average convexity for baskets ranked for beta (decile 1 is the lowest beta, 10 is the highest)

Deciles	1	2	3	4	5	6	7	8	9	10
Average	-1.067	-0.610	-0.734	-0.165	-0.316	-0.227	0.215	0.237	0.385	1.983
St. Deviation	1.747	1.695	1.559	1.857	1.824	2.137	2.098	2.317	2.582	3.586

Table 8: Difference in average convexity for baskets ranked for beta (basket in row minus basket in column)

Deciles	1	2	3	4	5	6	7	8	9	10
1										
2	0.457***									
3	0.332**	-0.125								
4	0.902***	0.445***	0.570***							
5	0.750***	0.293**	0.418***	-0.152						
6	0.840***	0.383**	0.508***	-0.062	0.090					
7	1.281***	0.824***	0.949***	0.379**	0.531***	0.441**				
8	1.304***	0.847***	0.972***	0.402**	0.554***	0.464**	0.023			
9	1.452***	0.995***	1.119***	0.550***	0.701***	0.612***	0.170	0.148		
10	3.050***	2.593***	2.717***	2.147***	2.299***	2.210***	1.768***	1.745***	1.598	

Table 9: Average convexity for baskets ranked for volatility (decile 1 is the lowest Volatility, 10 is the highest)

Deciles	1	2	3	4	5	6	7	8	9	10
Average	-0.908	-0.429	-0.509	-0.127	-0.071	-0.062	0.343	0.410	0.450	0.917
St. Deviation	1.454	1.503	1.537	1.480	1.900	1.917	1.966	2.263	2.753	4.746

Table 10: Difference in average convexity for baskets ranked for volatility (basket in row minus basket in column)

Deciles	1	2	3	4	5	6	7	8	9	10
1										
2	0.479***									
3	0.399***	-0.080								
4	0.781***	0.302	0.382							
5	0.837***	0.357**	0.438**	0.055						
6	0.846***	0.367**	0.447**	0.065	0.010					
7	1.251***	0.772***	0.852***	0.470***	0.414	0.405*				
8	1.318***	0.839***	0.919***	0.537***	0.482**	0.472**	0.067			
9	1.358***	0.879***	0.959***	0.577***	0.522***	0.512***	0.107***	0.040***		
10	1.825***	1.346***	1.426***	1.044***	0.989***	0.979***	0.574***	0.507***	0.035***	

*** statistically significant at 1%

** statistically significant at 5%

* statistically significant at 10%

[8] Source for all figures: Amundi, Factset

ANNEX 4

(Amundi NextGen Equities and low risk exposure)[9]

Table 11: beta, data from 12-2003 to 12-2011

	Ex Ante - Quarterly				Ex Post
Portfolio	**Minimum**	**Median**	**Average**	**Maximum**	**Full Sample**
Global Minimum Variance	0.48	0.59	0.58	0.65	0.55
Global Smart	0.70	0.81	0.80	0.87	0.85
MSCI World	1.00	1.00	1.00	1.00	1.00

Table 12: weighted average common factor risk, data from 12-2003 to 12-2011

	Ex Ante - Quarterly			
Portfolio	**Minimum**	**Median**	**Average**	**Maximum**
Global Minimum Variance	0.12	0.17	0.16	0.22
Global Smart	0.16	0.20	0.21	0.28
MSCI World	0.16	0.21	0.22	0.33

Chart 2: BETA
weight distribution (average 12/2003 - 12/2011 quarterly data)

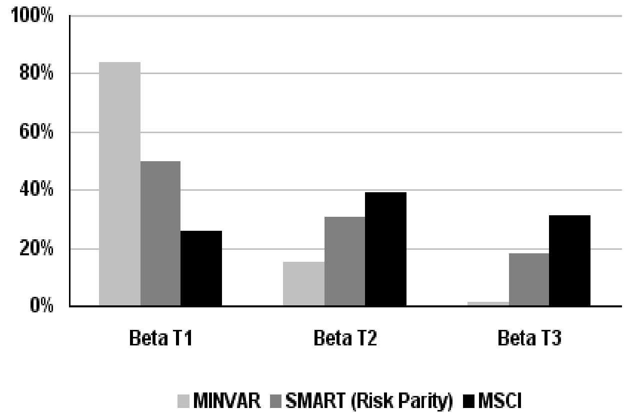
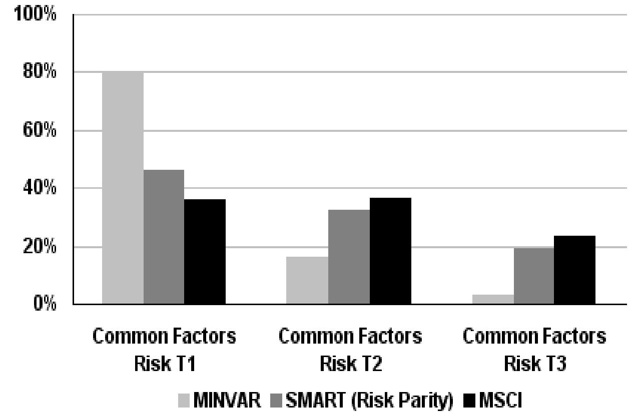


Chart 3: weighted average common factor risk
weight distribution (average 12/2003 - 12/2011 quarterly data)



[9] Source for all figures: Amundi, Factset

ANNEX 5

(Amundi NextGen Equities)[10]

Annex 5.1 – Minimum Variance

Table 13: Piotroski Score (Balance Sheet, Income Statement And Corporate Actions)

Profitability

Positive return on assets in the current year
Positive operating cash flow in the current year
Higher return on assets (ROA) in the current period compared to the ROA in the previous year
Cash flow from operations are greater than ROA

Leverage, Liquidity and Source of Funds

Lower ratio of long term debt to in the current period compared value in the previous year
Higher current ratio this year compared to the previous year
No new shares were issued in the last year

Operating Efficiency

A higher gross margin compared to the previous year
A higher asset turnover ratio compared to the previous year

Table 14: Piotroski Score (Value Added)

from 12/2000 to 02/2012	cumulative return	Av. Annual Return	Annual Volatility	Risk Adj. Return
Piotroski T1	201.3%	10.4%	17.3%	0.60
Piotroski T2	148.8%	8.5%	18.5%	0.46
Piotroski T3	116.7%	7.2%	21.2%	0.34
MSCI World Constituents EQW	149.3%	8.5%	18.8%	0.45

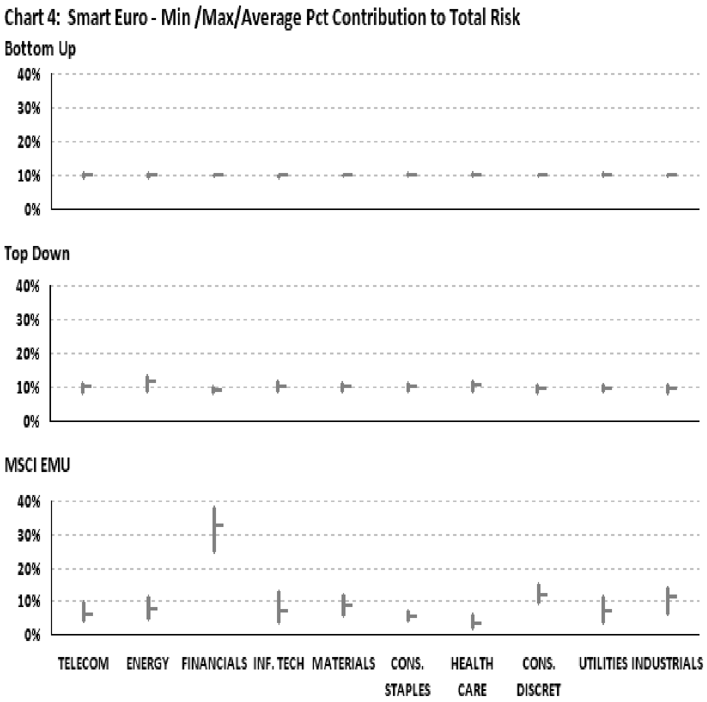
Table 15: Performance Summary - Minimum Variance

Paper / Real Money	Region / Index	Currency	cumulative return	Av. Annual Return	Annual Volatility	Max Draw-down	Risk Adj. Return	Tracking Error
Real Money Portfolios	Minimum Variance Europe (since 04/29/2009)	EUR	60.4%	15.3%	14.0%	-14.2%	1.09	8.0%
	MSCI Europe		47.8%	12.5%	19.4%	-24.3%	0.64	
	Minimum Variance Europe, limited volatility (since 12/07/2007)	EUR	9.2%	1.9%	10.3%	-20.6%	0.19	22.4%
	MSCI Europe		-16.7%	-3.8%	26.7%	-56.2%	-0.14	
Paper Portfolios	Mimimum Variance World (since 01/31/2003)	USD	220.6%	12.8%	10.1%	-31.5%	1.26	9.2%
	MSCI World		107.6%	7.8%	16.4%	-54.0%	0.48	
	Mimimum Variance Emerging (since 12/31/2005)	USD	154.3%	14.8%	22.5%	-48.0%	0.66	9.5%
	MSCI Emerging Markets		66.7%	7.9%	28.3%	-61.4%	0.28	
	Minimum Variance Japan (since 03/31/2003)	JPY	56.5%	4.8%	13.3%	-41.4%	0.36	9.2%
	MSCI Japan		11.2%	1.1%	18.4%	-60.7%	0.06	
	Min. Variance AC Pacific Ex Jap. (since 01/31/2003)	JPY	419.1%	18.4%	15.6%	-47.4%	1.18	9.7%
	MSCI AC Pacific Ex Japan		283.8%	14.8%	22.5%	-61.3%	0.66	

Amundi and Benchmarks' data as at the end of September 2012, gross performance with reinvested dividend, monthly frequency
Past performance is not a reliable indicator of future results, or guarantee of future results

[10] Source for all figures: Amundi, Factset

Annex 5.2 – Smart Beta (risk parity)[11]



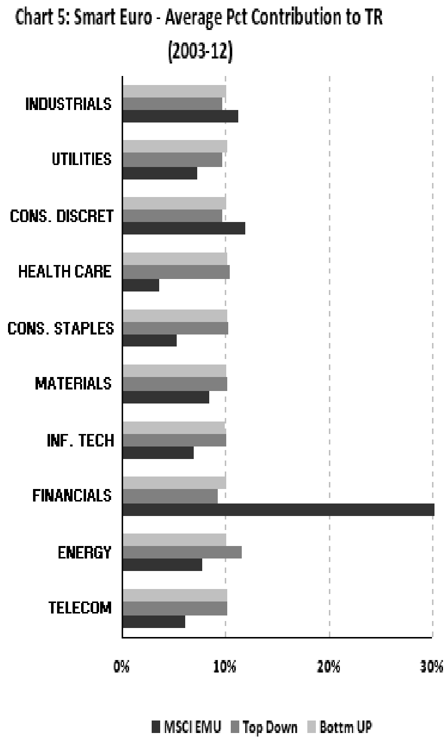


Table 16: Performance Summary - Smart Beta (Risk Parity)

Paper / Real Money	Region / Index	Currency	cumulative return	Av. Annual Return	Annual Volatility	Max Draw-down	Risk Adj. Return	Tracking Error
Real Money Portfolios	Smart Euro (since 06/24/2010)	EUR	9.6%	4.3%	19.4%	-24.6%	0.22	5.2%
	MSCI EMU		3.0%	1.4%	23.2%	-30.6%	0.06	
	Smart Europe (since 12/06/2010)	EUR	7.8%	4.3%	18.0%	-20.6%	0.24	3.2%
	MSCI Europe		4.8%	2.7%	19.5%	-24.3%	0.14	
	Global Smart (since 01/31/2012)	EUR	7.7%	12.1%	7.2%	-5.9%	1.67	4.3%
	MSCI World		9.5%	15.1%	9.0%	-7.7%	1.69	
Paper Portfolios Since 01/31/ 2003	Global Smart	EUR	165.7%	10.6%	11.8%	-40.6%	0.90	4.2%
	MSCI World		73.2%	5.8%	13.0%	-49.0%	0.45	
	Smart EMU	EUR	135.6%	9.3%	14.9%	-49.6%	0.62	4.7%
	MSCI EMU		59.6%	5.0%	17.7%	-56.2%	0.28	
	Smart Emerging	EUR	460.9%	19.5%	18.6%	-48.4%	1.05	5.1%
	MSCI Emerging Markets		266.2%	14.4%	20.5%	-56.3%	0.70	
	Smart Japan	JPY	39.5%	3.5%	14.1%	-42.7%	0.25	7.3%
	MSCI Japan		4.7%	0.5%	18.6%	-57.3%	0.03	
	Smart TOPIX	JPY	74.3%	5.9%	15.0%	-44.0%	0.40	7.3%
	TOPIX		5.6%	0.6%	18.2%	-56.2%	0.03	
	Smart AC Pacific Ex Japan	USD	430.5%	18.8%	20.3%	-53.2%	0.93	4.5%
	MSCI AC Pacific Ex Japan		264.6%	14.3%	22.5%	-61.6%	0.64	

Amundi and Benchmarks' data as at the end of September 2012, gross performance with reinvested dividend, monthly frequency
Past performance is not a reliable indicator of future results, or guarantee of future results

[11] Source for all figures: Amundi, Factset

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