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Working Paper | Cross asset Investment Strategy

WP-88-2019

Pricing Individual Stock Options using both Stock and Market Index Information

Research & Macro Strategy

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Abstract

When it comes to individual stock option pricing, most applications consider a univariate framework. From a theoretical point of view this is unsatisfactory as we know that the expected return of any asset is closely related to the exposure to the market risk factors. To address this, we model the evolution of the individual stock returns together with the market index returns in a flexible bivariate model in line with theory. We assess the model performance by pricing a large set of individual stock options on 26 major US stocks over a long time period including the global financial crisis.

Keywords: American option pricing, Economic loss, Forecasting, Multivariate GARCH

JEL classification: C10, C32, C51 C52, C53, G10

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1 Introduction

The majority of the work on empirical option pricing has been to the case of European style index options, in particular to options written on the S&P 500 Index.[1] Much less literature has focussed on pricing individual stock options. The reason for the focus on European style options is that they are easier to price since one does not have to consider and estimate the optimal early exercise strategy which is required to price the American style stock options. However, data from the Chicago Board of Options Exchange shows that in March 2018 the combined volume of the 500 most traded stock options exceeded that of the S&P 500 Index options by 26%. Thus, by considering only European style index options this large and important source of data is neglected. Moreover, individual stock options likely contain important information that allows us to learn about more about, e.g., investor preferences than can be deduced from index options alone.

Most, if not all, of the existing literature on individual stock option pricing uses univariate models, i.e. models in which the stock dynamics is considered in isolation, see

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We would like to thank participants at the 4th and 5th ESSEC empirical finance workshop, the 1st International Conference on Econometrics and Statistics (EcoSta 2017), the 2018 HEC/McGill Spring Finance Workshop, and at seminars at Amundi, Queen's University, University of Waterloo, and Wilfrid Laurier University for useful comments. Jeroen Rombouts acknowledges the support of the French Agence Nationale de la Recherche (ANR), under grant ANR-17-CE26-0001 (project BREAKRISK). Lars Stentoft acknowledges financial support from CREATES, funded by the Danish National Research Foundation (DNRF78). Francesco Violante acknowledges financial support from the Danish Council for Independent Research (1329-00011A) and CREATES, funded by the Danish National Research Foundation (DNRF78).
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[1]For some early contributions see for example Bollerslev and Mikkelsen (1996), Bollerslev and Mikkelsen (1999), Heston and Nandi (2000), Christoffersen and Jacobs (2004), and Hsieh and Ritchken (2005). In addition to the mentioned applications to the S&P 500 Index, GARCH models were found to perform well for European style options on the German DAX index by Härdle and Hafner (2000), on the Hang Seng Index by Duan and Zhang (2001), and on the FTSE 100 Index by Lehar, Scheicher, and Schittenkopf (2002).

Stentoft (2015) for a recent example. However, asset pricing models like, e.g., the Capital Asset Pricing Model imply that these dynamics and the expected returns depend on the asset's exposure to market risk factors. To fill this gap in the literature and to analyze the importance of allowing for interactions with and exposure to the market risk factors, we consider a bivariate discrete time model for the asset returns with time varying conditional volatility specified as a multivariate generalized autoregressive conditional heteroskedasticity (GARCH) process. Given our joint model for individual stock and the market index returns, we use the results of Rombouts and Stentoft (2011) to derive the risk neutral dynamics and we price American options using a simulation based approach that allows incorporating the early exercise feature.

Multivariate GARCH models have been previously used to test asset pricing models. One of the first papers to do so is Bollerslev, Engle, and Wooldridge (1988) who use GARCH models for the excess returns on bills, bonds, and stocks. See also Turtle, Buse, and Korkie (1994) and Gonzalez-Rivera (1996) who formulate bivariate models with size portfolios or individual stock returns and the market portfolio. More recent uses of multivariate GARCH models focus on portfolio risk management applications, see e.g. Francq and Zakoian (2018). Compared to this early literature, our paper considers not only the statistical support, in terms of significantly estimated coefficients for the risk parameters, but it also assesses the economic value of considering a bivariate framework by comparing out of sample option pricing errors.

To be specific, we ask the relevant question: does it pay, in the sense of yielding smaller pricing errors, to consider a bivariate model linking the market and the individual stock historical returns in terms of pricing the individual stock options? To assess this, we use an economically relevant metric for option pricing errors, a metric we believe is much more relevant for this type of comparisons, instead a purely statistical metric like the in sample fit. Our results show that the losses from using a univariate formulation amounts to 18% on average and may be as large as 80% for certain individual stocks. These results are robust, not only across option characteristics, such as moneyness and maturity, but also through time, in general, and in crisis periods, in particular, lending strong support to our proposed model.

We compare our flexible model to alternative specifications that uses the index returns as the one risk factor that should be priced. These results confirm that allowing for flexible dynamic conditional correlation is key to superior option pricing performance whereas the particular specification of risk premia is of second order only. For example, when considering a one factor model formulated in the spirit of e.g. Begin, Dorion, and Gauthier (2017) or Elkamhi and Ornthanalai (2010), the losses continue to be substantial and amounts to more than 10% on average. We also compare our model's performance to what would be obtained with a so-called affine specification. Affine dynamics are essential for obtaining closed form solutions for European options but in terms of accuracy of index option pricing, Christoffersen, Dorion, Jacobs, and Wang (2010) find that non-affine univariate GARCH models are dominated by affine specifications. Using a specification with affine conditional covariance dynamics, we confirm and generalize this finding to the multivariate setting. In particular, we document that using an affine specification instead of the more flexible and traditional non-affine specification results in large losses, that on average amount to 40% even when allowing for flexible conditional dependence.

In conclusion, our model shows excellent performance when it comes to option pricing. A potential drawback of our proposed approach, however, is that risk parameters are estimated from historical returns alone and that, therefore, potentially important information available from option data is not used. Previous literature, which has included option data, has used affine specifications of factor models with very restricted conditional dependence to allow obtaining semi-closed form solutions for European style option prices. Incorporating option data in a calibration exercise is very difficult in our model because option prices are unavailable in closed form and instead one has to use computationally expensive simulation based techniques. However, individual stock options are of American style and for these no closed form solutions are currently available rendering calibration to option prices difficult and, considering the large improvements our proposed methodology has over alternative methods that use affine formulations of simple one factor models and lead to large losses, we therefore argue that this "drawback" may not be that important after all because of our model's superior performance.

The rest of the paper is organized as follows: In Section 2 we present our bivariate model, we explain how to obtain the risk neutral dynamics, and we discuss the different choices of risk specifications. In Section 3 we describe the data and we present some overall results on model estimation and on the resulting aggregate pricing errors. In Section 4 we contrast the performance of our proposed methodology with various alternative models to assess the economic value of using a theoretically consistent model when pricing individual stock options. Section 5 conducts a series of robustness checks. Finally, Section 6 offers some conclusions and outlines future avenues for research.

2 A dynamic bivariate model for asset returns

We consider the following bivariate model

$$r_t = r\mathbf{i} + \Delta(\Sigma_t, \Lambda) - \frac{1}{2}(\Sigma_t \odot I)\mathbf{i} + \Sigma_t^{1/2}\eta_t, \quad (1)$$

where $r_t = (r_{1,t}, r_{2,t})'$, with asset 1 indicating the market index return and asset 2 indicating the individual stock returns, $\mathbf{i} = (1,1)'$, Σ_t is the conditional covariance matrix (with elements denoted by $\sigma_{ij,t}$), and η_t is a standard bivariate Gaussian innovation term. In (1), $\Delta(\Sigma_t, \Lambda)$ specifies the risk premia linking risk related parameters in Λ (with typical element λ_{ij}) to the conditional covariance Σ_t. We specify Σ_t according to the so-called BEKK-model (named after Baba-Engle-Kraft-Kroner) defined in Engle and Kroner (1995), though any other specification could have been used. Defining $N \times N$ matrices A_{ik} and B_{ik} and an upper triangular matrix C_0 the general version of the BEKK-model is given by

$$\Sigma_t = C_0^{\intercal} C_0 + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ik}\varepsilon_{t-i}\varepsilon_{t-i}^{\intercal}A_{ik}^{\intercal} + \sum_{k=1}^{K}\sum_{i=1}^{p} B_{ik}\Sigma_{t-i}B_{ik}^{\intercal}. \quad (2)$$

For practical purposes, we will assume that $K = q = p = 1$ and we set $N = 2$ in this bivariate setup. In this case the Full BEKK bivariate model in (2) contains 11 parameters and implies the following dynamic model for typical elements of Σ_t:

$$\sigma_{11,t} = c_{11} + a_{11}^2\varepsilon_{1,t-1}^2 + 2a_{11}a_{12}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{12}^2\varepsilon_{2,t-1}^2$$

$$+b_{11}^2\sigma_{11,t-1} + 2b_{11}b_{12}\sigma_{21,t-1} + b_{12}^2\sigma_{22,t-1}, \quad (3)$$

$$\sigma_{21,t} = c_{21} + a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{21}a_{12} + a_{11}a_{22})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2$$
$$+b_{11}b_{12}\sigma_{11,t-1} + (b_{21}b_{12} + b_{11}b_{22})\sigma_{21,t-1} + b_{21}b_{22}\sigma_{22,t-1}, \quad (4)$$

$$\sigma_{22,t} = c_{22} + a_{21}^2\varepsilon_{1,t-1}^2 + 2a_{21}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2$$
$$+b_{21}^2\sigma_{11,t-1} + 2b_{21}b_{22}\sigma_{21,t-1} + b_{22}^2\sigma_{22,t-1}, \quad (5)$$

where $\varepsilon_t = \Sigma_t^{1/2}\eta_t$.

The Full BEKK model above allows for feedback effects from both assets, i.e. when $\alpha_{12} \neq 0$ the innovation to the second asset contributes to updating assets one's conditional variance. In the setup we have outlined above with the market as the first asset, this may not make much sense. For this reason we pick as our benchmark model a restricted version of the Full BEKK model in which A and B are lower diagonal. To be specific, in this model market innovations feed back into the stock volatility, but not the other way around. Also note that this assumption has implications for updating the covariance which now depends on the product of the innovations and the squared stock innovation but not the squared market innovation ε_1. We call this model the Triangular BEKK model. To assess the importance of allowing for flexible conditional covariance dynamics, we will contrast the performance of this model with one in which both off diagonal elements of A and B and where we force the covariance to be zero which results in a model with independent dynamics for the index and for the stock, that is a model that corresponds to what one would get if the stock returns were modelled univariately.

2.1 Risk neutral dynamics

In order to price options we need to derive the risk neutral dynamics implied by the system above. In the bivariate model we propose, markets are incomplete and hence there is no unique way to derive the equivalent martingale measure (EMM) needed for option pricing. In this paper, we follow the approach of Rombouts and Stentoft (2011) in which a candidate

EMM is specified from the following Radon–Nikodym derivative

$$\left.\frac{dQ}{dP}\right| \mathcal{F}_t \quad = \quad \exp\left(-\sum_{i=1}^{t}\left(\nu_i'\varepsilon_i + \Psi_i\left(\nu_i\right)\right)\right), \tag{6}$$

where ν_i is a N-dimensional vector sequence and $\Psi_t(\cdot)$ denotes the conditional cumulant generating function. This specification generalizes the results from Christoffersen, Elkamhi, Feunou, and Jacobs (2010) from a univariate case to the multivariate setting.

Rombouts and Stentoft (2011) show that the probability measure defined above is an EMM if and only if

$$0 \quad = \quad \Psi_t\left(\nu_t - e_j\right) - \Psi_t\left(\nu_t\right) - \Psi_t\left(-e_j\right) + \Delta(\Sigma_t, \Lambda), \tag{7}$$

for all $j = 1, ..., N$, where e_j is an $N-$dimensional vector of zeros except for position j where it is 1. Moreover, in this case it is straightforward to shown that under the risk neutral measure the conditional cumulant generating function of ε_t is given by

$$\Psi_t^Q(u) \quad = \quad \Psi_t\left(\nu_t + u\right) - \Psi_t\left(\nu_t\right). \tag{8}$$

When ε_t is bivariate Gaussian distributed, the conditional cumulant generating function is particularly simple and given by

$$\Psi_t(u) \quad = \quad \frac{1}{2}u'\Sigma_t u. \tag{9}$$

This very simple form means that option pricing is simple even in the case with conditional heteroskedasticity. In particular, it is straightforward to show that the distribution under Q is obtained from

$$\Psi_t^Q(u) \quad = \quad u'\Delta(\Sigma_t, \Lambda) + \frac{1}{2}u'\Sigma_t u. \tag{10}$$

Thus, it follows that the risk neutral dynamics remain Gaussian although with a shifted mean. The shift in the mean is exactly what is required to compensate investors for the risk associated with investing in the underlying risky assets. Moreover, in this case we can solve the EMM restriction in (7) explicitly to obtain

$$\nu_t \quad = \quad \Sigma_t^{-1}\Delta(\Sigma_t, \Lambda). \tag{11}$$

This means that there is a one-to-one correspondence between the ν-parameter, which is implicitly linked to investor preferences, and the elements of Λ, the λ-parameters, which we can estimated straightforwardly from historical returns.

The tight link between P and Q dynamics and the ability to link λ with ν is an important reason for maintaining the Gaussian assumption. Another important reason is that maintaining this assumption ensures that our results do not critically depend on the exact method used for risk-neutralization since all methods will yield similar results.[2] An important potential future extension of this paper is to generalize the innovation distribution to better fit the extant empirical evidence documenting that returns are in fact highly non-normal with excess kurtosis and potentially significant skewness.

2.2 Specifications of risk premia

The above section shows that the formulation of $\Delta(\Sigma_t, \Lambda)$ is key to specifying risk premia and to driving a wedge between the dynamics under P and Q and is, naturally, expected to be important for option pricing. In this paper we will consider several different specifications linking the risk related parameters in Λ denoted by λ_{ij} to the elements of the conditional variance in Σ_t denoted by $\sigma_{ij,t}$. The most general specification we consider is given by

$$\Delta(\Sigma_t, \Lambda) = \begin{bmatrix} \sigma_{11,t}\ \lambda_{11} \\ f(\sigma_{11,t}, \sigma_{21,t}, \sigma_{22,t})\ \lambda_{21} + \sigma_{22,t}\ \lambda_{22} \end{bmatrix}, \tag{12}$$

where $f(\sigma_{11,t}, \sigma_{12,t}, \sigma_{22,t})$ is a function of, potentially, all the elements in the conditional covariance matrix at time t. For example, we could simply set this function to be the covariance, i.e. $f(\,) = \sigma_{12,t}$, which we will refer to as our "benchmark" specification. In addition to this, we will consider specifications in which we set $f(\,)$ equal to the conditional correlation (that is $\rho_t = \sigma_{12,t}/(\sqrt{\sigma_{11,t}}\sqrt{\sigma_{22,t}})$, the conditional beta (that is $\beta_t = \sigma_{12,t}/\sigma_{11,t}$), and the conditional index variance $\sigma_{11,t}$.

[2]For example, Simonato and Stentoft (2015) document that very different empirical pricing performance can be obtained when using different standard methods for obtaining the risk neutral dynamics in univariate GARCH models with non-Normal innovations.

We will also consider the case when $\lambda_{21} = 0$ such that only (own) variance risk is priced in the model. In this case the risk premia are given by

$$\Delta(\Sigma_t, \Lambda) = \begin{bmatrix} \sigma_{11,t}\,\lambda_{11} \\ \sigma_{22,t}\,\lambda_{22} \end{bmatrix}. \tag{13}$$

When this specification for the risk premia is combined with a model with zero correlations it essentially corresponds to using a univariate model for option pricing purposes and we refer to it as the "Univariate" or "Independent" specification.

To assess the importance of allowing for flexible specifications of the risk premia we will contrast the performance of the above models to two alternatives which we discuss in detail in the Appendix. The first of these is a model formulated in the spirit of the one factor models. In this framework innovations are assumed independent but return correlation is introduced by assuming that market excess returns drives risk premia. This setup is straightforward to accommodate in our framework by allowing the function $f(\,)$ to depend on the market innovation also and setting $f(\,) = \lambda_{11}\sigma_{11,t} - \frac{1}{2}\lambda_{21}\sigma_{11,t} + \varepsilon_{1,t}$. The second alternative is a model formulated in the spirit of Duan and Wei (2005) in which the market is used as the driver of the stochastic discount factor directly. This setup is also straightforward to accommodate in our framework and simply involves a particular restriction on $\Delta(\Sigma_t, \Lambda)$ in which the only estimated risk parameter comes from the index equation.

3 Data and preliminary results

We consider 26 large US stocks for which full samples of options are available for the period from 2000 to 2015 and full samples of returns are available for the period from 1999 to 2015, both years included.[3] To decrease the computational burden involved with numerically evaluating option prices we only consider options traded on Wednesdays and

[3]We require an additional year of data for returns since, when pricing options, we need to filter out starting values of the conditional covariance matrix. These stocks are all part of the Dow Jones Industrial Avarage (DJIA) as of the most recent change to the index which happened after the close of trading on March 18, 2015.

to ensure that quoted option prices are for liquidly traded options we only consider out of the money options.[4] Table 1 shows the relevant stock tickers together with information about the option data and sample statistics for the asset returns for the period between 2000 and 2015.

The table shows that though the number of options available for the individual stocks does vary slightly, the split across moneyness categories is remarkably stable with most of the options, about 42%, being out of the money put options as expected. Across maturity most of the options fall in the long term category but the table shows that a significant amount, i.e. 31.6%, of the options in our sample have less than 2 months to maturity. For individual stocks, the results do differ much more across maturity than across moneyness. For example, for PFE almost one third of the options have long maturity and only around one eight of them have short maturity. The sample statistics for the return data shown in the right most columns in Table 1 confirm that all stocks share the typical empirical regularities found for financial asset returns with, in particular, significant excess kurtosis. Compared to what is typically observed for index returns, though, the skewness of individual stock returns is much smaller in absolute terms.

3.1 Estimation

All the specifications considered in this paper are straightforward to estimate with the maximum likelihood method. In particular, by construction, all the parameters needed for pricing can be identified from historical return data since we are implying the pricing parameters, i.e. the λ's. This is a clear advantage of the methodology and means that it could be used even in cases where options do not exist. On the other hand and as was previous mentioned, option data may have important information and incorporating this information is a valuable future area of research. This would, though, require one to develop faster pricing methodologies than the state of the art simulation based method used in this paper. Note that since we are considering individual stock options which are

[4] These restrictions are in line with most of the literature. In particular, we consider only options, put options as well as call options though, that are less than 20% out of the money.

Table 1: Data

	Panel A: Option data								Panel B: Return data			
Ticker	All	Put	ATM	Call	ST	MT	LT	Ticker	Mean	StDev	Skew	Kurt
AXP	22,266	40.9%	22.3%	36.8%	37.0%	18.4%	44.6%	AXP	0.042	2.373	0.398	9.848
BA	28,239	43.0%	19.7%	37.4%	39.4%	18.6%	42.1%	BA	0.058	1.936	-0.038	5.156
CAT	28,943	42.2%	19.3%	38.5%	37.4%	20.1%	42.5%	CAT	0.058	2.068	0.121	4.328
CSCO	12,261	38.7%	19.9%	41.4%	18.1%	23.9%	58.0%	CSCO	0.020	2.616	0.568	9.598
CVX	24,425	43.1%	21.3%	35.5%	40.0%	17.4%	42.6%	CVX	0.045	1.649	0.405	12.780
DD	18,021	41.8%	25.4%	32.9%	34.1%	17.4%	48.5%	DD	0.032	1.822	0.084	5.437
DIS	19,251	41.5%	22.9%	35.6%	37.7%	18.2%	44.2%	DIS	0.057	1.993	0.238	8.741
GE	13,023	42.6%	21.1%	36.3%	13.4%	22.4%	64.2%	GE	0.019	1.978	0.370	9.176
HD	21,614	41.8%	21.9%	36.2%	31.7%	19.2%	49.1%	HD	0.045	2.056	-0.379	13.875
IBM	29,799	41.4%	18.0%	40.6%	35.8%	19.0%	45.2%	IBM	0.027	1.694	0.201	8.108
INTC	14,912	39.8%	20.6%	39.6%	21.2%	23.8%	55.0%	INTC	0.035	2.457	-0.102	7.319
JNJ	15,596	43.2%	28.4%	28.3%	33.4%	17.4%	49.2%	JNJ	0.038	1.240	-0.216	14.005
JPM	27,461	41.3%	21.2%	37.5%	27.0%	24.4%	48.6%	JPM	0.052	2.632	0.840	13.835
KO	13,965	41.9%	25.3%	32.7%	16.5%	20.2%	63.3%	KO	0.029	1.367	0.287	8.921
MCD	21,059	43.7%	25.8%	30.5%	34.2%	18.1%	47.7%	MCD	0.048	1.519	0.029	6.017
MMM	18,523	42.3%	21.1%	36.6%	33.1%	18.2%	48.7%	MMM	0.053	1.517	0.202	5.212
MRK	17,190	41.6%	25.2%	33.2%	28.6%	19.5%	52.0%	MRK	0.026	1.791	-0.830	19.204
MSFT	19,183	40.9%	22.5%	36.5%	23.9%	23.9%	52.2%	MSFT	0.028	2.005	0.241	9.824
NKE	21,810	41.8%	21.0%	37.2%	41.8%	16.5%	41.8%	NKE	0.082	1.935	-0.078	10.144
PFE	12,528	42.6%	23.2%	34.2%	12.7%	21.7%	65.6%	PFE	0.027	1.636	-0.097	4.929
PG	16,212	41.3%	28.1%	30.6%	30.7%	18.2%	51.1%	PG	0.030	1.392	-2.948	70.261
UNH	25,092	42.1%	18.7%	39.2%	36.0%	18.9%	45.2%	UNH	0.096	2.095	0.980	26.560
UTX	21,149	41.5%	22.8%	35.7%	34.7%	18.0%	47.2%	UTX	0.050	1.754	-0.834	21.134
VZ	16,595	45.9%	26.5%	27.5%	18.7%	25.1%	56.2%	VZ	0.027	1.624	0.365	7.173
WMT	19,098	41.0%	24.7%	34.3%	27.7%	18.2%	54.1%	WMT	0.016	1.545	0.294	6.227
XOM	21,432	42.8%	24.7%	32.5%	38.0%	17.3%	44.7%	XOM	0.039	1.588	0.362	11.138
All	519,647	42.0%	22.4%	35.6%	31.6%	19.6%	48.8%	Average	0.042	1.857	0.018	12.652

This table shows details for the option data in the left hand side and sample statistics for the returns in the right hand side.

American style and hence allow for early exercise this is a non-standard task as no closed (or semi-closed) form pricing formulas exist.

Table 2 shows the likelihood values for the Triangular BEKK model with 6 different risk premia specifications. The table shows that likelihoods always increase when adding risk premia to the equation and this most importantly so when adding all three premia. In fact, the increase in likelihood is large, around 20 on average, and significant for all the models with three risk premia parameters. The likelihood also increases significantly at a 10% level for all of the index-stock combinations when only considering (own) volatility risk premia in the index and stock equation. The table shows that a model with covariance risk is the one that in most cases results in the largest likelihood value. This happens for 13 of the stocks. Thus, from the return series alone there is strong support of models that include risk premia and in terms of likelihood values the best model is a model with priced covariance risk in the stock equation.

Table 3 provides parameter estimates for this model and Table 4 provides the corresponding statistics from a t-test of their significance. The tables first of all show that, as expected, in general most of the estimated risk premia are positive. In particular, all the estimated parameters from the index equation are positive and in the stock equation the covariance risk is positive for all of the stocks and the own variance risk is positive for 22 of the stocks. For the 4 stocks with negative own variance risk premia neither of these are significant. In terms of statistical significance of the individual stock risk premia only 6 of the own variance risk premia are significantly positive.[5] However, when considering the covariance risk in addition to the own variance risk, 11 of the 26 individual stocks have significant risk premia. Table 4 also shows that most of the dynamic parameters, the c's, a's and b's, are precisely estimated and significantly different from zero. In particularly all the $a_{i,i}$'s and $b_{i,i}$'s, for $i = 1, 2$, are significant with the only exception being $a_{2,2}$ for UNH. The parameter estimates for the feedback effects, i.e. $a_{2,1}$ and $b_{2,1}$, are also significant in the majority of the cases, 20 and 16, respectively. In total, 199 of the 225 dynamic parameters are significant lending strong support to our choice of specification.

[5]It is well known that estimating these precisely from historical asset return data alone is difficult.

Table 2: Likelihood values for Triangular BEKK model with different risk premia

	Likelihood values						Best model			
Ticker	NoRisk	IndOwn	CBeta	Corre	Covar	Index	CBeta	Corre	Covar	Index
AXP	24857.9	24869.4	24869.6	24869.6	24869.5	24869.5		*		
BA	24348.7	24359.0	24359.2	24359.3	24359.5	24359.6				*
CAT	24301.0	24313.7	24314.0	24313.8	24313.7	24313.8	*			
CSCO	23551.9	23561.0	23561.8	23561.2	23563.0	23563.2				*
CVX	25442.7	25456.2	25456.3	25456.2	25457.0	25456.6			*	
DD	25142.8	25153.4	25153.4	25153.4	25153.6	25153.4			*	
DIS	24642.1	24652.0	24652.5	24652.6	24652.8	24652.9				*
GE	25385.3	25395.7	25395.8	25395.7	25397.1	25396.8			*	
HD	24518.5	24531.3	24531.3	24531.6	24535.3	24535.3				*
IBM	25214.3	25227.0	25229.1	25229.5	25227.3	25227.0		*		
INTC	23853.6	23864.6	23865.8	23864.8	23865.9	23866.2				*
JNJ	26288.8	26303.5	26303.9	26304.2	26303.8	26303.7		*		
JPM	24735.5	24750.7	24750.9	24750.9	24750.7	24750.7	*			
KO	25750.2	25759.2	25759.6	25761.2	25762.4	25761.7			*	
MCD	25073.5	25082.8	25083.6	25084.7	25088.5	25087.4			*	
MMM	24276.7	24288.6	24289.1	24288.9	24289.3	24289.1			*	
MRK	24120.4	24126.6	24127.6	24126.9	24131.3	24130.4			*	
MSFT	24474.0	24485.3	24486.3	24486.0	24485.6	24486.0	*			
NKE	24146.4	24162.1	24163.3	24163.7	24163.0	24162.5		*		
PFE	24981.7	24987.9	24986.4	24988.1	24993.7	24992.2			*	
PG	25783.3	25792.5	25792.7	25792.8	25795.4	25793.6			*	
UNH	23738.9	23757.9	23758.0	23758.0	23759.3	23758.2			*	
UTX	25407.1	25419.0	25419.3	25419.2	25419.3	25419.6				*
VZ	25211.3	25220.1	25220.9	25220.3	25227.3	25224.9			*	
WMT	25279.0	25284.1	25288.7	25285.7	25290.4	25290.3			*	
XOM	25694.6	25706.0	25706.4	25706.4	25706.8	25706.5			*	
Average	24854.63	24865.77	24866.36	24866.34	24867.75	24867.36	3	4	13	6

This table shows the likelihood values for the Triangular BEKK multivariate GARCH model with various risk premia. In the right part of the table a "*" indicates which specification of risk premia leads to the highest likelihood.

Table 3: Parameter estimates for Triangular BEKK model with priced covariance risk

	Parameter estimates											
Ticker	$\lambda_{1,1}$	$\lambda_{2,1}$	$\lambda_{2,2}$	$c_{1,1}$	$c_{2,1}$	$c_{2,2}$	$a_{1,1}$	$a_{2,1}$	$a_{2,2}$	$b_{1,1}$	$b_{2,1}$	$b_{2,2}$
AXP	5.144	1.239	2.388	0.136	0.147	0.066	0.293	0.207	0.173	0.949	-0.056	0.983
BA	5.096	2.640	2.207	0.136	0.155	-0.074	0.305	0.213	0.122	0.945	-0.057	0.990
CAT	5.215	0.531	3.272	0.132	0.168	0.106	0.302	0.202	0.128	0.947	-0.051	0.989
CSCO	5.180	3.772	1.414	0.121	0.124	0.109	0.292	0.200	0.101	0.950	-0.040	0.992
CVX	5.626	3.764	2.401	0.131	0.138	0.097	0.291	0.082	0.221	0.950	-0.028	0.971
DD	5.263	1.719	2.476	0.140	0.158	0.074	0.312	0.220	0.128	0.942	-0.060	0.989
DIS	5.097	3.302	1.793	0.125	0.164	0.196	0.276	0.060	0.272	0.955	-0.015	0.953
GE	5.720	5.331	0.602	0.146	0.070	0.094	0.319	0.116	0.180	0.940	-0.012	0.975
HD	5.532	7.169	1.001	0.140	0.138	0.068	0.314	0.174	0.149	0.942	-0.048	0.988
IBM	5.001	1.713	3.502	0.121	0.064	0.287	0.277	-0.082	0.363	0.955	0.049	0.907
INTC	5.442	3.602	1.623	0.143	0.116	0.148	0.314	0.123	0.174	0.941	-0.025	0.980
JNJ	5.405	1.800	4.696	0.123	0.096	0.096	0.272	0.039	0.280	0.957	-0.013	0.955
JPM	5.378	0.044	2.717	0.138	0.113	0.068	0.304	0.165	0.190	0.946	-0.035	0.979
KO	5.524	6.978	0.555	0.137	0.104	0.071	0.303	0.101	0.180	0.946	-0.031	0.980
MCD	5.287	7.787	1.024	0.144	0.078	0.060	0.320	0.077	0.147	0.940	-0.019	0.988
MMM	5.466	3.299	3.004	0.138	0.140	0.064	0.309	0.178	0.122	0.944	-0.048	0.990
MRK	5.091	7.540	-0.123	0.131	0.118	0.130	0.299	0.177	0.096	0.947	-0.043	0.991
MSFT	5.382	1.960	2.388	0.132	0.108	0.110	0.303	0.153	0.137	0.946	-0.033	0.987
NKE	5.347	3.544	3.198	0.137	0.128	0.115	0.312	0.174	0.119	0.943	-0.044	0.990
PFE	5.529	8.370	-0.513	0.141	0.090	0.108	0.293	0.021	0.211	0.948	-0.004	0.973
PG	5.451	5.432	1.682	0.134	0.084	0.074	0.300	0.079	0.168	0.947	-0.022	0.983
UNH	5.418	5.245	2.736	0.128	0.167	0.134	0.287	0.163	0.163	0.952	-0.051	0.981
UTX	5.142	1.881	3.146	0.134	0.141	0.042	0.306	0.201	0.111	0.945	-0.050	0.993
VZ	5.257	10.762	-0.905	0.140	0.114	0.077	0.301	0.095	0.198	0.946	-0.031	0.978
WMT	4.958	9.310	-1.272	0.134	0.066	0.081	0.302	0.078	0.153	0.947	-0.017	0.985
XOM	5.298	3.803	2.039	0.131	0.140	0.088	0.299	0.088	0.224	0.948	-0.030	0.971
mean	5.3173	4.3284	1.8097	0.1343	0.1203	0.0958	0.3002	0.1271	0.1734	0.9469	-0.0314	0.9785
pos	26	26	22	26	26	25	26	25	26	26	1	26
neg	0	0	4	0	0	1	0	1	0	0	25	0

This table shows the parameter estimates for the Triangular BEKK multivariate GARCH model with covariance risk priced in the mean equation for the stock.

Table 4: Test statistics for Triangular BEKK model with priced covariance risk

	T-stats											
Ticker	$\lambda_{1,1}$	$\lambda_{2,1}$	$\lambda_{2,2}$	$c_{1,1}$	$c_{2,1}$	$c_{2,2}$	$a_{1,1}$	$a_{2,1}$	$a_{2,2}$	$b_{1,1}$	$b_{2,1}$	$b_{2,2}$
AXP	3.880	1.311	3.224	7.782	4.077	2.054	14.053	3.892	5.106	134.295	-3.263	142.522
BA	0.795	0.516	0.765	3.247	3.061	-2.387	8.219	4.721	6.562	58.097	-2.785	306.885
CAT	0.593	0.065	1.104	5.569	3.139	0.832	4.148	1.811	2.901	44.293	-1.570	83.282
CSCO	2.050	0.881	0.830	7.514	5.112	4.732	15.662	6.757	10.030	161.696	-4.977	626.789
CVX	0.707	0.192	0.181	4.179	3.472	3.521	7.365	1.143	6.980	67.226	-1.367	119.676
DD	1.617	0.568	1.979	7.691	5.644	4.476	14.674	5.720	10.261	125.233	-4.405	427.340
DIS	1.157	0.426	1.112	7.811	4.893	3.343	9.125	0.453	2.707	115.518	-0.428	34.511
GE	1.404	0.773	0.437	4.839	2.421	2.419	9.284	3.213	3.537	67.823	-0.846	74.442
HD	4.097	2.995	1.876	8.597	6.788	2.876	16.786	5.860	8.596	147.231	-4.903	334.071
IBM	4.822	0.389	1.317	5.103	0.569	3.587	10.256	-0.739	3.326	105.120	1.298	19.841
INTC	1.341	1.545	0.566	5.821	3.678	4.272	12.027	2.696	4.988	90.185	-1.884	130.559
JNJ	4.371	0.859	3.301	8.093	4.607	2.404	12.213	0.921	3.279	141.374	-0.960	34.399
JPM	4.875	1.384	6.385	8.913	4.264	4.330	15.209	3.397	11.430	146.462	-2.101	259.868
KO	2.413	1.758	1.105	6.054	4.893	4.257	12.032	4.032	6.765	102.005	-3.651	171.415
MCD	5.075	4.201	2.552	7.606	4.520	3.719	15.267	3.721	8.914	121.018	-2.669	364.337
MMM	1.562	1.292	0.892	4.867	4.000	2.087	13.642	7.540	4.438	101.346	-5.150	180.670
MRK	2.537	2.556	-0.587	7.813	2.163	1.007	15.838	5.260	2.323	150.167	-2.886	79.066
MSFT	3.712	0.924	1.953	8.099	2.126	4.195	15.734	2.403	8.113	151.317	-1.214	296.131
NKE	2.195	0.792	2.121	7.814	5.196	4.125	16.426	4.032	6.656	143.443	-3.338	353.828
PFE	1.261	2.883	-0.186	3.484	1.862	2.405	4.035	0.150	3.045	38.147	-0.101	54.428
PG	4.319	2.075	1.417	8.544	4.486	4.864	14.372	3.125	6.166	136.488	-3.269	283.476
UNH	0.104	0.070	0.443	0.785	0.418	0.357	3.212	2.465	0.975	19.951	-0.667	16.177
UTX	3.755	0.728	2.163	8.061	7.555	1.872	16.418	7.262	8.199	151.903	-6.126	418.131
VZ	2.638	2.367	-1.303	8.649	5.256	3.624	14.434	4.358	5.644	134.076	-4.745	122.177
WMT	2.751	1.980	-0.540	5.838	3.505	6.306	11.098	4.145	14.440	94.175	-2.477	538.283
XOM	0.535	0.397	1.088	4.199	3.822	2.377	15.600	2.407	5.869	112.007	-4.112	86.623
Significant	15	6	6	25	23	22	26	20	25	26	16	26
Pos	15	6	6	25	23	21	26	20	25	26	0	26
Neg	0	0	0	0	0	1	0	0	0	0	16	0

This table shows the test statistics for significance of the parameters for the Triangular BEKK multivariate GARCH model with covariance risk priced in the mean equation for the stock.

3.2 Simulated option prices

Given a set of parameter estimates, the risk-neutral distribution derived above is easily simulated from, and since option values are essentially conditional discounted expectations, option pricing is possible in all the models outlined above. Option pricing using simulation for European style derivatives dates back to, at least, Boyle (1977). To accommodate the early exercise feature in individual stock options we implement the Least Squares Monte Carlo (LSMC) pricing method initially suggested by Longstaff and Schwartz (2001). The method has since then been used in several empirical papers, see e.g. Stentoft (2005) for the first empirical application of a (univarite) GARCH model to price individual stock options and for a detailed description of how the method can be implemented.[6]

The LSMC method of Longstaff and Schwartz (2001) approximates the value of holding the American style option at a given point in time along a specific simulated path by the predicted value from a cross-sectional regression using all the in-the-money paths. The method proceeds as follows: First, given the full sample of random paths, the pricing step is initiated at the maturity date of the option. At this time, it is possible to decide along each path if the option should be exercised because the future value trivially equals zero. Hence, the pathwise cash flows may be easily determined at maturity. Second, working backwards through time, a cross-sectional regression is performed at the previous point in time where early exercise is to be considered. In the regression, the discounted future cash flows are regressed on transformations of the current state variables, i.e. the asset prices and volatility levels. The fitted values from this regression are then used as estimates of the pathwise conditional expected values of holding the option for one more period. The decision of whether to exercise or not along each path can now be made by comparing the estimated conditional expected value of holding the option with the value of immediate exercise. Third, once the decision has been recorded for each path, we can move back through time to the previous early exercise point and perform a new cross-sectional regression with the appropriate pathwise cash flows based on the previously

[6] See also Stentoft (2008) and Stentoft (2015) for additional results on the use of simulation methods to price American style individual stock options in GARCH models with non-Gaussian innovations.

Table 5: RIVRMSE for various risk premia specifications

		RIVRMSE				Best model			
Ticker	Count	CBeta	Correl	Covar	Index	Cbeta	Correl	Covar	Index
AXP	22,266	0.2137	0.2137	0.2134	0.2136			*	
BA	28,239	0.1538	0.1537	0.1538	0.1537		*		
CAT	28,943	0.1872	0.1874	0.1876	0.1878	*			
CSCO	12,261	0.1951	0.1949	0.1949	0.1950			*	
CVX	24,425	0.1755	0.1755	0.1754	0.1755			*	
DD	18,021	0.1875	0.1874	0.1875	0.1875		*		
DIS	19,251	0.2661	0.2658	0.2665	0.2657				*
GE	13,023	0.2537	0.2534	0.2540	0.2530				*
HD	21,614	0.1686	0.1679	0.1669	0.1666				*
IBM	29,799	0.2974	0.2948	0.3007	0.3006		*		
INTC	14,912	0.1937	0.1941	0.1944	0.1943	*			
JNJ	15,596	0.2019	0.2017	0.2013	0.2014			*	
JPM	27,461	0.2712	0.2713	0.2718	0.2717	*			
KO	13,965	0.1425	0.1421	0.1420	0.1416				*
MCD	21,059	0.1445	0.1445	0.1441	0.1442			*	
MMM	18,523	0.1513	0.1513	0.1507	0.1508			*	
MRK	17,190	0.2281	0.2282	0.2276	0.2262				*
MSFT	19,183	0.2203	0.2203	0.2202	0.2205			*	
NKE	21,810	0.1950	0.1950	0.1950	0.1949				*
PFE	12,528	0.1688	0.1707	0.1698	0.1691	*			
PG	16,212	0.1843	0.1840	0.1829	0.1831			*	
UNH	25,092	0.1682	0.1683	0.1682	0.1686			*	
UTX	21,149	0.1525	0.1525	0.1522	0.1522				*
VZ	16,595	0.1601	0.1599	0.1593	0.1570				*
WMT	19,098	0.1661	0.1658	0.1665	0.1657				*
XOM	21,432	0.1856	0.1856	0.1848	0.1861			*	
All	519,647	0.2002	0.1999	0.2003	0.2002	4	3	10	9

This table shows the relative implied volatility root mean squared error for option pricing models Triangular BEKK specifications and various formulations for the risk premium. Left hand columns contain the actual RIVRMSE and right hand columns contains a "*" for the best performing model among the four specifications.

determined optimal choices. Fourth and finally, with the early exercise strategies along each path, an estimate of the American option value can be obtained as a simple average of the discounted pathwise cash flows.

In our application we use $M = 100,000$ paths in the simulation and we use the complete set of polynomials of order less than or equal to two in the stock price and the stock volatility in the cross-sectional regressions. Table 5 contains the pricing errors expressed in terms of relative implied volatility root mean squared errors, RIVRMSEs, for the four different risk premia specifications with a Triangular BEKK specification for the conditional covariance.[7] Implied volatilities, IVs, are often used as a reasonable and informative non-linear transformation of option prices that allows comparing options with different maturity and moneyness on an equal footing. The first thing to notice from the table is that there is generally very little difference between the RIVRMSEs for different specifications of the risk premium. This holds for all individual stocks, in general, and on average, in particular. The Covar specification is the one that most often has the lowest RIVRMSE for the individual stocks but is the specification with the, marginally, largest average RIVRMSE. Thus, our first conclusion is that the particular specification used for the risk premium is of second order and in the following we therefore consider the Covar specification, for the simple reason that this model is the one that most often has the highest likelihood value.[8]

4 Evaluating option pricing models

To evaluate our proposed option pricing model's performance we now compare its RIVRMSE to what would be obtained with several alternative specifications. The most obvious alternative is the univariate models. Specifically we consider the relative losses specified by

$$Loss \quad = \quad \frac{RIVRMSE(alternative)}{RIVRMSE(multivariate)} - 1. \tag{14}$$

Positive relative losses directly measures the cost that would be incurred if instead of using the multivariate model one was to use the standard univariate framework, or any of the other alternatives considered. Table 6 shows the relative implied root mean squared errors, RIVRMSE, for various alternative models along with their losses, calculated from

[7] We imply the volatility by inverting the Black-Sholes-Merton formula.
[8] Since it is the worst performing benchmark model our results should be robust to this choice.

Table 6: RIVRMSE for alternative option pricing models

Ticker	Indep	Loss	Factor	Loss	Index	Loss	Affine	Loss
AXP	0.2221	0.0408	0.2561	0.2001	0.2154	0.0092	0.3834	0.7964
BA	0.2162	0.4062	0.1679	0.0920	0.1544	0.0038	0.2422	0.5752
CAT	0.1957	0.0429	0.1998	0.0650	0.1881	0.0023	0.2388	0.2728
CSCO	0.2294	0.1767	0.4280	1.1956	0.1948	-0.0006	0.3036	0.5575
CVX	0.1962	0.1186	0.1998	0.1394	0.1765	0.0066	0.2258	0.2874
DD	0.2255	0.2024	0.2087	0.1128	0.1877	0.0010	0.2575	0.3731
DIS	0.2802	0.0513	0.3120	0.1706	0.2696	0.0117	0.2393	-0.1021
GE	0.2540	-0.0001	0.2679	0.0547	0.2542	0.0007	0.3431	0.3510
HD	0.2648	0.5868	0.1772	0.0619	0.1703	0.0205	0.2992	0.7932
IBM	0.2928	-0.0262	0.3503	0.1649	0.3077	0.0234	0.2576	-0.1432
INTC	0.2207	0.1354	0.2005	0.0316	0.1952	0.0041	0.2938	0.5114
JNJ	0.2198	0.0919	0.1847	-0.0824	0.2004	-0.0048	0.2647	0.3145
JPM	0.2735	0.0062	0.2375	-0.1260	0.2703	-0.0055	0.4309	0.5855
KO	0.1919	0.3511	0.1667	0.1734	0.1424	0.0024	0.2431	0.7112
MCD	0.1703	0.1821	0.1557	0.0805	0.1448	0.0052	0.2407	0.6707
MMM	0.2249	0.4917	0.1560	0.0346	0.1514	0.0045	0.2454	0.6277
MRK	0.3428	0.5061	0.2062	-0.0938	0.2280	0.0020	0.3619	0.5903
MSFT	0.2861	0.2995	0.2175	-0.0123	0.2217	0.0068	0.2789	0.2666
NKE	0.2297	0.1779	0.1946	-0.0023	0.1957	0.0036	0.2526	0.2953
PFE	0.1934	0.1393	0.2282	0.3443	0.1720	0.0130	0.2359	0.3893
PG	0.2104	0.1507	0.1807	-0.0117	0.1823	-0.0031	0.2429	0.3284
UNH	0.2357	0.4012	0.1810	0.0765	0.1676	-0.0036	0.2478	0.4733
UTX	0.2752	0.8076	0.1623	0.0664	0.1524	0.0012	0.2424	0.5927
VZ	0.1722	0.0813	0.1535	-0.0362	0.1606	0.0086	0.2663	0.6718
WMT	0.1758	0.0558	0.1740	0.0447	0.1661	-0.0023	0.2743	0.6472
XOM	0.2002	0.0836	0.1999	0.0817	0.1854	0.0036	0.2479	0.3416
All	0.2359	0.1774	0.2213	0.1045	0.2015	0.0059	0.2803	0.3994

This table shows the relative implied volatility root mean squared error for various alternative option pricing models. Column 2 shows results for the benchmark Triangular BEKK multivariate GARCH model with covariance risk premia. Columns 3 and 4 shows results and relative losses for the univariate restriction, columns 5 and 6 shows results and relative losses for a one factor model, and columns 7 and 8 shows results and relative losses for the Duan model (delete this though). Finally, columns 8 and 9 shows results and losses for the Affine specification.

(14), when compared to the benchmark Triangular BEKK multivariate GARCH model with covariance risk premia.

Column 3 in the table shows results for the restricted version of our model without conditional covariances equivalent to assuming that a univariate model is used and column

4 shows the relative losses one would incur when using this model instead of our proposed benchmark model. The table shows that the losses amount to 17.74% on average. For most, i.e. 24 out of 26, stocks, the univariate model performs worse and losses may be very large with the maximum loss amounting to 80.76% for UTX. The average loss in RIVRMSE from using the univariate specification is 0.037 larger than with the multivariate benchmark model and a simple T-test for equal performance of the two specifications is strongly rejected with a test statistic equal to 5.40. IBM is the only stock for which the univariate model actually performs slightly better. The parameter estimates in Table 3 shows that for IBM $a_{2,1}$ and $b_{2,1}$ are estimated with the opposite sign of all other models and this could potentially be the reason for the loss in performance.

The one factor model maintains the assumption of independent innovations but essentially introduces return correlation through the mean specification which involves a term that compensates for index excess return. The results shown in column 5 and 6, though, shows that using this model for option pricing results in losses of 10.45% on average and that individual losses could be as large as 119.56%. Losses are incurred for 21 of the 26 stocks, the average losses are 0.021 larger than with the benchmark model, and again the assumption of equal performance is rejected with a test statistic of 2.18. The three stocks for which the one factor model works the best are JPM, MRK, and JNJ. The estimation results, not shown here, show that all parameters are estimated significantly different from zero in the factor specification. However, given the pricing results statistical significance is clearly not an indication that economical improvements in terms of pricing precision will ensue. The specification proposed by Duan and Wei (2005), in which the market index return is used indirectly as the driver of the stochastic discount factor, allows for more flexible conditional covariance when used with the Triangular BEKK specification, performs much better than the one factor model and almost as well as the covariance risk specification benchmark we propose, yet still results in losses that are statistically significant with a test statistic of 2.74.

Finally, columns 7 and 8 in Table 6 show the results for an affine covariance specification of the Triangular BEKK multivariate GARCH model.[9] Surprisingly, this simple change of specification in the dynamics of the conditional covariance results in very poor relative pricing performance which is on average 39.94% worse. The average loss in RIVRMSE from using the affine specification is 0.082 larger than with our proposed multivariate model and a simple T-test for equal performance of the two specifications is massively rejected with a test statistic equal to 8.95. Across the individual stocks the affine model works better for only 2 stocks, DIS and IBM, but may result in losses that are as high as 79.64%.

5 Robustness checks

To examine the robustness of the above results we now compare the performance of the multivariate model to the independent or univariate special case across various interesting option characteristics. Again we use the losses from (14) for comparisons and Table 7 shows the results across option moneyness, option maturity, and through time.

Columns 2-4 in the table show the losses from imposing the univariate restriction across option moneyness. The columns labelled "Put" and "Call" corresponds to put and call options that are between 2% and 20% out of the money, respectively, and the column labelled "ATM" corresponds to out of the money put and call options that are less than 2% out of the money. The first thing to notice is that on average losses are incurred for all these categories and these losses increase with the strike price, i.e. losses are larger for out of the money call options than for out of the money put options. In fact, the losses are always positive for call options strongly demonstrating the improved performance of our model with flexible specifications of the conditional covariance. The losses are also primarily positive for options close to being at the money though the univariate model occasionally performs the best for the out of the money put options.

Columns 5-7 show the losses from imposing the univariate restriction across option maturity. The column labelled "ST" corresponds to options with maturity of less then 2 months, "MT" corresponds to options with maturity of more than 2 months but less than

[9]The affine model is implemented by using η_t instead of $\varepsilon_t = \Sigma_t^{1/2}\eta_t$ in the Triangular BEKK specification.

Table 7: Relative performance of the multivariate model for various categories

	Moneyness			Maturity			Time		
Ticker	Put	ATM	Call	ST	MT	LT	Pre	Crisis	Post
AXP	-0.074	-0.015	0.158	0.030	0.155	0.018	0.095	0.286	-0.027
BA	0.257	0.294	0.601	0.278	0.452	0.579	0.605	0.389	0.381
CAT	-0.045	0.005	0.143	0.066	0.075	0.000	0.206	-0.030	0.021
CSCO	-0.003	0.132	0.397	0.109	0.180	0.198	0.187	0.031	0.187
CVX	0.011	0.079	0.215	0.099	0.136	0.131	0.085	0.054	0.137
DD	0.048	0.176	0.358	0.223	0.182	0.190	0.240	0.087	0.210
DIS	-0.058	0.023	0.125	0.034	0.044	0.070	0.110	0.067	0.042
GE	-0.161	0.001	0.211	0.060	0.044	-0.019	0.054	-0.021	-0.019
HD	0.406	0.389	0.849	0.308	0.551	0.901	0.808	0.440	0.552
IBM	-0.058	-0.059	0.006	-0.011	-0.036	-0.034	-0.016	0.061	-0.036
INTC	-0.034	0.122	0.311	0.124	0.152	0.134	0.265	-0.038	0.118
JNJ	0.031	0.047	0.198	0.046	0.111	0.122	0.145	0.092	0.070
JPM	-0.226	-0.068	0.186	-0.014	0.061	-0.002	0.007	0.196	-0.096
KO	0.233	0.280	0.549	0.196	0.333	0.415	0.415	0.361	0.316
MCD	0.100	0.141	0.370	0.135	0.192	0.230	0.227	0.135	0.180
MMM	0.278	0.474	0.723	0.379	0.480	0.615	0.475	0.336	0.525
MRK	0.479	0.462	0.587	0.445	0.556	0.533	0.291	0.544	0.542
MSFT	0.189	0.256	0.456	0.216	0.311	0.355	0.287	0.184	0.312
NKE	0.122	0.130	0.269	0.153	0.259	0.205	0.203	0.120	0.177
PFE	0.052	0.119	0.266	0.081	0.130	0.153	0.085	0.053	0.180
PG	0.083	0.070	0.303	0.121	0.128	0.182	0.211	0.088	0.119
UNH	0.279	0.337	0.538	0.256	0.399	0.576	0.319	0.391	0.438
UTX	0.443	0.552	1.228	0.439	0.589	1.240	0.825	0.749	0.808
VZ	0.015	0.018	0.258	0.064	0.076	0.090	0.242	0.107	0.041
WMT	0.062	0.017	0.098	0.095	0.082	0.012	0.104	0.168	0.028
XOM	-0.029	0.070	0.170	0.080	0.101	0.081	-0.034	0.110	0.102
All	0.058	0.137	0.300	0.142	0.190	0.201	0.211	0.179	0.168

This table shows the relative performance of the independent or univariate special case relative to the multivariate model as defined in (14) across option moneyness, option maturity and through time. Columns 2-4 show results for different option moneyness with columns labelled “Put” and “Call” corresponding to put and call options that are between 2% and 20% out of the money, respectively, and the column labelled “ATM” corresponding to out of the money put and call options that are less than 2% out of the money. Columns 5-7 show results for different option maturities, with columns labelled “ST” corresponding to options with maturity of less then 2 months, “MT” corresponding to options with maturity of more than 2 months but less than 4 months, and “LT” corresponding to options with maturity of more than 4 months. Columns 8-10 show results through time with the pre-crises period covering 2000-2006, the crisis period covering 2007-2009, and the post-crisis period covering 2010-2015, all years included.

4 months, and "LT" corresponds to options with maturity of more than 4 months. The first thing to notice is that on average losses are incurred for all these categories and these losses increase with maturity, i.e. losses are larger for long term options than for short term options. For the individual stocks, most of the losses are also positive, may be as large as 124.0%, and only rarely does the univariate model perform the best.

Finally, columns 8-10 in Table 7 show results through time with pre-crises covering the 2000-2006 period, crisis covering the 2007-2009 period, and post-crisis covering the 2010-2015 period, all years included. The first thing to notice is that the average relative performance of our proposed option pricing models is remarkably stable across these very different periods of time, and much more stable than across option characteristics such as moneyness or maturity. For the 26 individual stocks the multivariate specification is the best model for 24, 23 and 22 stocks, respectively, for the three subperiods. The worst relative performance is for JPM in the post crisis period, a stock that was clearly hit hard by the events of the Global Financial Crisis of 2007-2009 and for which "idiosyncratic" effects are likely much more important than dependency on the market factor during and after this volatile period.

6 Conclusion

In this paper we use a joint model for individual stock returns and the market index to price options written on the individual stock and we ask the following question: does it pay, in the sense of yielding smaller pricing errors, to consider a flexible bivariate model linking the market and the individual stock historical returns in terms of pricing the individual stock options? Our paper is motivated by the fact that most, if not all, the existing literature on individual stock option pricing uses univariate models, i.e. models in which the stock dynamics are considered in isolation whereas most, if not all, asset pricing models implies that these dynamics, in general, and the expected returns, in particular, should depend on the asset's exposure to market risk factors.

To fill this gab in the literature and to analyze the importance of allowing for interactions with and exposure to the market risk factors, we consider a bivariate discrete time model for the asset returns, we use the class of multivariate GARCH processes to allow for time varying conditional volatility, we use a generalization of the no-arbitrage approach to derive the risk neutral dynamics allowing us to estimate all relevant parameters from historical returns alone, and we price the options using a simulation based approach that allows us to incorporate the early exercise of individual stock options which are of American style.

Our model improves significantly on the univariate models which are typically used in the existing literature. In particular, our results show that the losses from using a univariate formulation amount to 18% on average and may be as large as 80% for certain individual stocks. These results are robust, not only across option characteristics, such as moneyness and maturity, but also through time, in general, and in crisis periods, in particular, lending strong support to our proposed model. Similar results are obtained when comparing to alternative models that uses the index returns as the one risk factor that should be priced and to specifications that use affine specifications for the conditional covariance.

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A Related literature

In this Appendix we describe in detail the approach used in two related papers. We change the notation used in Tzang, Wang, and Yu (2016) to specify conditional volatilities as σ^2 instead of h and we change the specification in Duan and Wei (2005) to having risk premia being specified in terms of variances instead of volatilities.

A.1 The wrong factor model of Tzang, Wang and Yo (2016)

Tzang, Wang, and Yu (2016) consider a restricted version of the bivariate framework we propose in this paper. In particular, instead of formulating a general model with flexible dynamics the model they use is essentially a one-factor model where risk premia are introduced in terms of "beta pricing" of the excess market return. Their one factor model uses a mean equation given by

$$r_{m,t} = \mu_{m,t} - \frac{1}{2}\sigma_{m,t}^2 + \sigma_{m,t} z_{m,t}, \tag{15}$$

for the index and

$$r_{s,t} = \mu_{s,t} - \frac{1}{2}\sigma_{s,t}^2 + \beta_s (r_{m,t} - r) + \sigma_{s,t} z_{s,t}, \tag{16}$$

for the individual stock where r is the risk free rate. In this setting, it is assumed that the two innovations $z_{m,t}$ and $z_{s,t}$ are independent Gaussian. However, due to the term involving the excess market return in the mean for the stock asset returns will be correlated. Assuming the local risk neutral valuation relationship, LRNVR, of Duan (1995) holds it is straightforward to derive the risk neutral dynamics or alternatively to sort out what the risk neutral innovations should be so as to make both processes Q-martingales. For the market index this is simple and involves the following restriction on the market index pricing parameter

$$\lambda_{m,t}^Q = \frac{\mu_{m,t} - r}{\sigma_{m,t}}, \tag{17}$$

with $z^Q_{m,t} = z_{m,t} + \lambda^Q_{m,t} = z_{m,t} + \lambda_{m,t}\sigma_{m,t}$ being standard normally distributed. This means that though the innovations in the mean equation have zero mean and unit variance the "innovation" going into the GARCH updating equation have non-zero mean, similarly to Duan (1995). For the stock it is (only) slightly more complicated. If we substitute from the mean equation of the market index and use the restriction from the pricing parameter we obtain the following:

$$\begin{aligned}
r_{s,t} &= \mu_{s,t} - \frac{1}{2}\sigma^2_{s,t} + \beta_s(r_{m,t} - r) + \sigma_{s,t}z_{s,t} \\
&= \mu_{s,t} - \frac{1}{2}\sigma^2_{s,t} + \beta_s(\mu_{m,t} - \frac{1}{2}\sigma^2_{m,t} + \sigma_{m,t}z_{m,t} - r) + \sigma_{s,t}z_{s,t} \\
&= \mu_{s,t} - \frac{1}{2}\sigma^2_{s,t} + \beta_s(\lambda_{m,t}\sigma^2_{m,t} - \frac{1}{2}\sigma^2_{m,t} + \sigma_{m,t}z_{m,t}) + \sigma_{s,t}z_{s,t} \\
&= \mu_{s,t} - \frac{1}{2}\sigma^2_{s,t} + \beta_s\lambda_{m,t}\sigma^2_{m,t} - \frac{1}{2}\beta_s\sigma^2_{m,t} + \frac{1}{2}\beta_s^2\sigma^2_{m,t} - \frac{1}{2}\beta_s^2\sigma^2_{m,t} + \beta_s\sigma_{m,t}z_{m,t} + \sigma_{s,t}z_{s,t} \\
&= \mu_{s,t} + \frac{1}{2}\beta_s(\beta_s - 1)\sigma^2_{m,t} - \frac{1}{2}\sigma^2_{s,t} + \sigma_{s,t}z_{s,t} + \beta_s\lambda_{m,t}\sigma^2_{m,t} - \frac{1}{2}\beta_s^2\sigma^2_{m,t} + \beta_s\sigma_{m,t}z_{m,t},
\end{aligned}$$

where the next to last step is needed to ensure that the expected value of the exponential of $-\frac{1}{2}\beta_s^2\sigma^2_{m,t} + \beta_s\sigma_{m,t}z_{m,t}$ equals zero. While the last two terms have expectation zero under the physical measure by construction the last three terms have expectation zero under the risk neutral measure and because of this the requirement that the individual mean be a Q-martingale implies the following (generalized) restriction on the individual stock pricing parameter

$$\begin{aligned}
\lambda^Q_{s,t} &= \frac{\mu_{s,t} - r + \frac{1}{2}\beta_s(\beta_s - 1)\sigma^2_{m,t}}{\sigma_{s,t}} \\
&= \frac{\mu_{s,t} - r}{\sigma_{s,t}} + \frac{\beta_s(\beta_s - 1)\sigma^2_{m,t}}{2\sigma_{s,t}}, \qquad (18)
\end{aligned}$$

with $z^Q_{s,t} = z_{s,t} + \lambda^Q_{s,t} = z_{s,t} + \lambda_{s,t}\sigma_{s,t}$ being standard normally distributed.[10] Formulated in terms of λ the estimating equations are

$$r_{m,t} = r + \lambda_{m,t}\sigma^2_{m,t} - \frac{1}{2}\sigma^2_{m,t} + \sigma_{m,t}z_{m,t}, \qquad (19)$$

for the index and

$$r_{s,t} = r + \lambda_{s,t}\sigma^2_{s,t} - \frac{1}{2}\beta_s(\beta_s - 1)\sigma^2_{m,t} - \frac{1}{2}\sigma^2_{s,t} + \beta_s(r_{m,t} - r) + \sigma_{s,t}z_{s,t}$$

[10]Note that the term $\sigma^2_{m,t}$ is mistakenly missing from equation (8) in Tzang, Wang, and Yu (2016).

$$= r + \lambda_{s,t}\sigma^2_{s,t} - \frac{1}{2}\sigma^2_{s,t} - \beta_s(\lambda_{m,t}\sigma^2_{m,t} - \frac{1}{2}\beta_s\sigma^2_{m,t} + \sigma_{m,t}z_{m,t}) + \sigma_{s,t}z_{s,t}, \quad (20)$$

for the individual stock. Though we could simplify the last equation slightly, keeping it in terms of the innovations z allows comparison with the risk neutral dynamics in a simple way. In particular, the simulation equations are given by

$$r_{m,t} = r - \frac{1}{2}\sigma^2_{m,t} + \sigma_{m,t}z^Q_{m,t}, \quad (21)$$

for the index and

$$r_{s,t} = r - \frac{1}{2}\sigma^2_{s,t} - \beta_s(-\frac{1}{2}\beta_s\sigma^2_{m,t} + \sigma_{m,t}z^Q_{m,t}) + \sigma_{s,t}z^Q_{s,t}, \quad (22)$$

for the individual stock. It is easily seen that this model can be implemented in our setup by simply setting $\lambda_{21} = \beta_s$ and estimating this parameter from an equivalent formulation of (20).

A.2 The market index model of Duan and Wei (2005)

The framework used in the current paper specifies a particular Radon-Nikodym derivative, and implicitly a price of risk, that can be directly estimated from historical data. An alternative to this approach is to obtain the risk neutral dynamics be specifying the stochastic discount factor directly in a general equilibrium framework. Duan and Wei (2005) is an example of the latter approach. In that paper, a standard mean equation like

$$r_{s,t} = r + \lambda_{s,t}\sigma^2_{s,t} - \frac{1}{2}\sigma^2_{s,t} + \sigma_{s,t}z_{s,t}, \quad (23)$$

is used for the stock returns but unlike most of the previous literature $\lambda_{s,t}$ is not estimated from data on the stock.[11] Instead it is characterized explicitly in terms of "systematic" risk by specifying a utility function depending on consumption and defining $Y_t = \kappa - \ln(U'(C_t)/U'(C_{t-1}))$ as the one period stochastic discount factor. It is then shown that in

[11] Note that Duan and Wei (2005) specifies risk premia as being specified in terms of $\lambda_{s,t}\sigma_{s,t}$ instead of $\lambda_{s,t}\sigma^2_{s,t}$ as we essentially do. The calculations goes through with either specification, though, since it is the product that is important for risk neutralization. This is a benefit of using maintaining the assumption of Gaussian innovations.

equilibrium

$$\lambda_{s,t} = \frac{q_t \delta_t}{\sigma_{s,t}}, \tag{24}$$

where $q_t = Corr^P(r_{s,t}, Y_t|\phi_{t-1})$ and $\delta_t^2 = Var^P(Y_t|\phi_{t-1})$, with ϕ_t denoting the information set at time t.

If the market index is assumed to be the only factor, Y_t it can be expressed as

$$Y_t = a + br_{I,t} + \epsilon_t, \tag{25}$$

where $r_{I,t} = \ln(I_t/I_{t-1})$ is the market index return. With this specification we have that

$$\delta_t = \sqrt{b^2\sigma_{I,t}^2 + c}, \tag{26}$$

where $\sigma_{I,t}^2 = Var^P(r_{I,t}|\phi_{t-1})$ and $c = Var^P(\epsilon_t)$ and it follows that

$$q_t = \frac{bCov^P(r_{s,t}, r_{I,t}|\phi_{t-1})}{\sigma_{s,t}\sqrt{b^2\sigma_{I,t}^2 + c}} = \frac{b\beta_t\sigma_{I,t}^2}{\sigma_{s,t}\sqrt{b^2\sigma_{I,t}^2 + c}}, \tag{27}$$

where $\beta_t = Cov^P(r_{s,t}, r_{I,t}|\phi_{t-1})/\sigma_{I,t}^2$. As a result we can write

$$\lambda_{s,t} = \frac{b\beta_t\sigma_{I,t}^2}{\sigma_{s,t}^2}, \tag{28}$$

and therefore we have that $\lambda_{s,t}\sigma_{s,t}^2 = b\beta_t\sigma_{I,t}^2$. Note that this depends on the parameter b, which is constant across different assets, and can be identified from the market portfolio since the market portfolio's own systematic risk equals one by definition. In particular, we have for the market index that

$$\begin{aligned} r_{I,t} &= r + \lambda_{I,t}\sigma_{I,t}^2 - \frac{1}{2}\sigma_{I,t}^2 + \sigma_{I,t}z_{I,t} \\ &= r + b1\sigma_{I,t}^2 - \frac{1}{2}\sigma_{I,t}^2 + \sigma_{I,t}z_{I,t} \end{aligned} \tag{29}$$

and the estimated b thus corresponds to our λ_{11}. Since there is no λ_{21} in this formulation it is seen to lead to a very restricted formulation of the risk premium compared to what we propose. Thus, in our setting this model is easily implemented and involves only estimating the parameter b from the market index equation and using this times the conditional covariance in the individual stock equation.

Cross asset

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