

***Parameter Learning, Sequential Model Selection,  
and Bond Return Predictability***

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## Abstract

The paper finds both statistically and economically significant out-of-sample evidence of bond return predictability for a Bayesian investor who learns about parameters, hidden states, and predictive models over time. We find that the factor extracted from a large panel of macroeconomic variables contains rich information on future excess bond returns and that introducing stochastic volatility can improve predictive performance. Interestingly, economic evidence is much more pronounced when we do not impose any investment weight constraints, and there seems to be a squeeze of intermediaries capital and a scarcity of arbitrage capital when investors require extreme long positions. We also document that model combinations work well in predicting excess bond returns..

**Keywords:** Bayesian Learning, Bond Return Predictability, Parameter Uncertainty, Liquidity, Model Combinations.

**JEL classification:** C11, G11, G12, G17

## 1. Introduction

The expectation hypothesis of the term structure of interest rates asserts that the expected one-period return on bonds is equal to the one-period interest rate plus a risk premium, which is constant over time. A large number of empirical studies have explored whether risk premia on treasury bonds are indeed constant. A standard way to test the expectations hypothesis is to run predictability regressions of excess bond returns on some predetermined predictors. Empirical investigations have uncovered some evidence of bond return predictability. Fama and Bliss (1987) and Campbell and Shiller (1991) find that excess bond returns are predictable by forward spreads or yield spreads. Cochrane and Piazzesi (2005) find that information contained in the entire term structure of interest rates can capture more than 30% of the variation of excess bond returns over the period from January 1964 to December 2003. Recently, empirical studies by Ludvigson and Ng (2009), Cooper and Priestly (2009), Huang and Shi (2014), and Joslin, Priebisch, and Singleton (2014) show that macroeconomic variables contain rich information on future excess bond returns beyond information only in yield curve.

However, most evidence found in the above studies is in-sample. Investors in markets may be more concerned about whether there exists out-of-sample evidence of bond return predictability and whether such out-of-sample statistical predictability can translate into their economic gains. Thornton and Valente (2012) find that information contained in forward rates can not generate systematic economic value to an investor who has mean-variance preferences. Sarno, Schneider, and Wagner (2016) find that under affine term structure model framework the evident statistical predictability of bond risk premia rarely turns into investors' economic gain. However, in a recent study, Gargano, Pettenuzzo, and Timmermann (2017) use non-overlapping excess bond returns and Bayesian Markov Chain Monte Carlo (MCMC) methods and find that statistically significant out-of-sample predictability can translate into economic value for a real-time investor.

In this paper, we revisit this seemingly contentious issue. We consider a Bayesian

investor who faces the same learning problems as confronted by the econometrician. Except the expectations hypothesis that takes the historical mean as the optimal forecast, she has access to additional predictive models that may feature stochastic volatility. She takes parameters, state variables, and predictive models as unknowns and updates her beliefs using Bayes rule sequentially at each time when new information becomes available. Our Bayesian investor computes the predictive return distribution at each time based on what she has learned and maximizes her expected utility by taking into account all relevant uncertainties. We implement Bayesian learning on predictive models by following the marginalized resample-move approach proposed by Fulop and Li (2013). One of key implications from Bayesian learning is that it generates persistent and long-term changes to the investor's beliefs. Our treatment here is similar to that of Johannes, Korteweg, and Polson (2014) who investigate effects of sequential learning on stock return predictability.

We take forward spreads from Fama and Bliss (1987), the forward factor proposed by Cochrane and Piazzesi (2005), and the macro factor proposed by Ludvigson and Ng (2009) as three predictors and refer to them as FB, CP, and LN. The predictive models are built by using any non-empty subsets of these three predictors. Furthermore, we consider both constant and stochastic volatility cases. Putting together, there are in total 14 individual predictive models to be considered. We also consider four model combination schemes based on both statistical and economic evidence.

We construct monthly bond excess returns on US zero-coupon bonds with maturity 2-, 3-, 4-, and 5-year using the updated dataset of Gurkaynak, Sack, and Wright (2007). Most studies in bond return predictability focus on predictive regressions for annual excess bond returns at monthly forecasting frequency. Bauer and Hamilton (2017) argue that the bond returns with overlapping holding-period may induce strong serial correlations in the error terms and may raise additional econometric problems when predictors are persistent. Therefore, similar to Gargano, Pettenuzzo, and Timmermann (2017), we consider one-month holding period and construct non-overlapping monthly excess bond

returns. Our data range from January, 1962 to December, 2011, in total 600 months.

We first explore statistical evidence of out-of-sample bond return predictability. Based on out-of-sample R-square,  $R_{OS}^2$ , in the sense of Campbell and Thompson (2008), we find statistically significant out-of-sample predictability and notice that introduction of stochastic volatility boosts predictive performance of the models to a large extent. We also find that the models that take LN as a predictor almost always perform better than the other models, no matter whether stochastic volatility is present or not. This result indicates that the macro factor extracted from a large panel of macroeconomic variables contains rich information on future excess bond returns.

We then investigate whether statistical evidence of bond return predictability eventually translates into investor's economic gains. Our investor is Bayesian. At each time, she maximizes her expected power utility over the one-period value of a portfolio consisting of a risk-free asset and a treasury bond by accounting for all sources of uncertainty. Given these portfolios, we compute certainty equivalence return (CER) for each model to measure its economic performance. Our results based on CER's are consistent with those based on  $R_{OS}^2$ , indicating that statistical predictability does translate into economic gains to our Bayesian investor.

The previous literature has predominantly adopted a weight constraint in testing the economic evidence. The usual lower and upper weight bounds are -1 and 2, respectively, allowing the possibility of shorting and borrowing. Our initial tests on economic gains also use these constraints. However, while such bounds seem natural for equity markets, government bonds are much less risky, resulting for instance in much lower margins in repo transactions backed by these securities. Hence, sophisticated fixed income investors may be able to achieve much more aggressive short and especially long positions than implied by these bounds. Therefore, we redo the asset allocation exercises without setting any weight constraints. Interestingly, we find much more pronounced economic evidence, associated with more aggressive long or short positions. We dig a bit deeper and try to

understand whether the periods when our investment strategy implies extreme weights have something special about them. In particular, we use two market pressure indices introduced by He, Kelly, and Manela (2017) and Hu, Pan, and Wang (2013) to test if there pressure from intermediary sector’s own balance-sheet or lack of arbitrage capital when the investors require aggressive weights. Results show that indeed this is the case for large positive weights. Hence exactly when market conditions would suggest extreme portfolio positions, such positions may not be that easy to implement.

Finally, we turn to investigate model combinations. In general, such methods perform remarkably well both statistically and economically. All the four schemes proposed generate positive and statistically significant  $R_{OS}^2$  and positive certainty equivalent returns for our Bayesian investor. Furthermore, we document that a simple CER-based model combination works quite well in forecasting long-term excess bond returns.

The above results are robust to investors with different risk aversion, to the choice of different out-of-sample periods, and to the investor’s initial beliefs.

Our work makes two main contributions to the literature. First, we provide a generic econometric framework that allows real-time Bayesian learning about bond return predictability. Such a Bayesian learning simultaneously takes into account belief-updating, parameter uncertainty, and model risk. Thornton and Valente (2012) and Sarno, Schneider, and Wagner (2016) follow classical approaches and therefore ignore parameter and model uncertainties. Gargano, Pettenuzzo, and Timmermann (2017) employ Bayesian MCMC methods and do allow for parameter and model uncertainties. However, to investigate out-of-sample predictability, MCMC needs to be repeatedly run at each time, leading to a large computational cost. Our Bayesian learning approach is tailor-made for sequential inference and is naturally parallel, allowing the use of parallel hardware. Furthermore, in our Bayesian learning method, the only model-dependent requirement is a filtering mechanism for the model in question that provides at least an unbiased likelihood estimate, hence it may be customised to different forecasting models more easily

compared to MCMC methods.

Second, our results suggest that statistical predictability of excess bond returns can translate into economic gains for a Bayesian investor who learns parameters, hidden states, and models over time with respect to information accumulation. In particular, we find that without any weight constraints the economic gains are much more evident but there is also more market pressure when investors require extreme long positions. We suspect that placing weight constraints could be a potential reason why some previous papers do not find the link between statistical and economic evidence.

The remainder of the paper is organized as follows. Section 2 presents the predictive models to be considered and introduces Bayesian learning approach. Section 3 discusses how to statistically and economically evaluate predictive performance of each model. Section 4 provides data and main empirical results. Section 5 implements several robustness checks. Finally, section 6 concludes the paper.

## 2. Bayesian Learning and Bond Return Predictability

### 2.1. Predictive Models

In line with the existing literature, we define the log-yield of an  $n$ -year bond as

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}, \quad (1)$$

where  $p_t^{(n)} = \ln P_t^{(n)}$ , and  $P_t^{(n)}$  is the nominal price of an  $n$ -year zero-coupon bond at time  $t$ . A forward rate is defined as

$$f_t^{(n-m,n)} \equiv p_t^{(n-m)} - p_t^{(n)}, \quad (2)$$

and the excess return of an  $n$ -year bond is computed as the difference between the holding period return from buying an  $n$ -year bond at time  $t$  and selling it  $m$ -period later and the



yield on a  $m$ -period T-bill rate at time  $t$ ,

$$rx_{t+m}^{(n)} = p_{t+m}^{(n-m)} - p_t^{(n)} - m \cdot y_t^{(m)}, \quad (3)$$

where  $m$  is the holding period and  $y_t^{(m)}$  is the annualized T-bill rate. In this paper, we assume  $m$  is one-month, and  $n$  can be 2, 3, 4, or 5 years.

The standard approach to investigate bond return predictability usually takes a model of the form

$$rx_{t+1}^{(n)} = \alpha + \beta X_t + \epsilon_{t+1}, \quad (4)$$

where  $X_t$  is a set of the pre-determined predictors,  $\epsilon_t \sim N(0, \sigma_{rx}^2)$  is a mean-zero constant variance error term, and the coefficients  $\alpha$ ,  $\beta$ , and  $\sigma_{rx}$  are unknown fixed parameters. Bear in mind that  $rx_{t+1}^{(n)}$  represents the excess return after one month, as our forecasting frequency is monthly.

However, there is considerable evidence that suggests that bond return volatility is time-varying (Gray, 1996; Bekaert, Hodrick, and Marshall, 1997; Bekaert and Hodrick, 2001). Therefore, except the standard model (4), we also introduce the stochastic volatility model, which takes the form of

$$rx_{t+1}^{(n)} = \alpha + \beta X_t + e^{h_{t+1}} \epsilon_{t+1}, \quad (5)$$

where  $\epsilon_t \sim N(0, 1)$  is a standard normal noise, and  $h_{t+1}$  is the log-volatility at time  $t+1$ , which is assumed to follow

$$h_{t+1} = \mu + \phi h_t + v_{t+1}, \quad (6)$$

where  $h_t$  is stationary and mean-reverting when  $|\phi| < 1$ , and  $v_t \sim N(0, \sigma_h^2)$ . For simplicity, we assume independence between  $\epsilon_t$  and  $v_t$ .

Empirical studies have found that forward rates or forward spreads have ability to forecast bond returns. Fama and Bliss (1987) find that the forward-spot spread has

predictive power for excess bond returns and that its forecasting power increases as the forecasting horizon becomes long. Cochrane and Piazzesi (2005) show that the whole term structure of forward rates can capture more than 30% of the variation of excess bond returns over the period from January 1964 to December 2003. Joslin, Priebsch, and Singleton (2014) provide evidence that macroeconomic variables contain rich information on yields. Recently, Ludvigson and Ng (2009) extract macro factors from a large set of 132 macroeconomic variables and show that these factors have predictive power for excess bond returns.

Therefore, in this paper, we consider three predictors from Fama and Bliss (1987), Cochrane and Piazzesi (2005), and Ludvigson and Ng (2009) and refer to them as FB, CP, and LN, respectively. Specifically, FB is the forward spread,

$$FB_t^{(n,m)} = f_t^{(n-m,n)} - m \cdot y_t^{(m)}. \quad (7)$$

We construct the CP factor following Cochrane and Piazzesi (2005). At each time  $t$ , average excess bond return across maturities is regressed on the one-year bond yield and the full term structure of forward rates,

$$\bar{r}x_{t+1} = \gamma_0 + \gamma \mathbf{f}_t + e_{t+1}, \quad (8)$$

where  $\bar{r}x_{t+1} = \frac{1}{4} \sum_{n=2}^5 rx_{t+1}^{(n)}$  and  $\mathbf{f}_t = [f_t^{(1-1/12,1)}, f_t^{(2-1/12,2)}, f_t^{(3-1/12,3)}, f_t^{(4-1/12,4)}, f_t^{(5-1/12,5)}]$ .

Then the CP factor is computed as

$$CP_t = \hat{\gamma}_0 + \hat{\gamma} \mathbf{f}_t. \quad (9)$$

Finally, the LN factor is extracted from a large set of macroeconomic variables using principal component analysis. It is a linear combination of the estimated principal com-

ponents,  $\hat{\mathbf{F}}_t = [\hat{F}_{1,t}, \hat{F}_{1,t}^3, \hat{F}_{3,t}, \hat{F}_{4,t}, \hat{F}_{8,t}]$ ,

$$LN_t = \hat{\gamma}_0 + \hat{\gamma}\hat{\mathbf{F}}_t, \quad (10)$$

where  $\hat{\gamma}_0$  and  $\hat{\gamma}$  are estimated in the following regression

$$\overline{r}x_{t+1} = \gamma_0 + \gamma\hat{\mathbf{F}}_t + e_{t+1}. \quad (11)$$

We can see that both CP and LN are in-sample predictors. So in the out-of-sample forecasting exercise, CP and LN are reconstructed at each time  $t$  using the information available only up to time  $t$  in order to avoid any hindsight problems. We take  $X_t$  in Equations (4) and (5) as any non-empty subset of  $\{FB_t, CP_t, LN_t\}$ . There are in total 7 such non-empty subsets, that is,  $\{FB_t\}$ ,  $\{CP_t\}$ ,  $\{LN_t\}$ ,  $\{FB_t, CP_t\}$ ,  $\{FB_t, LN_t\}$ ,  $\{CP_t, LN_t\}$ , and  $\{FB_t, CP_t, LN_t\}$ , suggesting that there are in total 14 models, 7 constant volatility models and 7 stochastic volatility models. We name each model using the name(s) of its predictor(s) followed by the abbreviation of constant volatility (CV) or stochastic volatility (SV). For example, a model that takes CP and LN as its predictors and assumes stochastic volatility has a name of CPLN-SV. In model (4), when  $\beta = 0$ , no predictor is used, and the optimal forecast of excess bond returns is simply the historical mean. This case is in fact the expectations hypothesis, which will be taken as a benchmark for comparison with the above 14 predictive models.

## 2.2. Bayesian Learning and Belief Updating

We assume a Bayesian investor who faces the same belief updating problem as the econometrician (Hansen, 2007). She simultaneously learns about parameters, latent states, and models sequentially over time when new information arrives.

For a given predictive model  $\mathcal{M}_i$ , there is a set of unknown static parameters,  $\Theta$ , and/or a vector of the hidden state,  $h_t$ , when stochastic volatility is introduced. The ob-

servations include a time series of excess bond returns and predictors,  $y_{1:t} = \{rx_{1:t}^{(n)}, X_{1:t}^t\}$ . To account for the fact that the whole time series of CP and LN are changing at each time point in the out-of-sample period, we use  $X_{1:t_1}^{t_2}$  (where  $t_2 \geq t_1$ ) to denote the predictor time series from time 1 to  $t_1$ , with the information set at  $t_2$ . It is important to note that for CP and LN in the out-of-sample period,  $X_{1:t_1}^{t_1}$  and  $X_{1:t_1}^{t_2}$  may be totally different due to different regression results. In our learning algorithm, we take this into consideration and update the whole time series of CP and LN at each out-of-sample time point. So at each time  $t$ , Bayesian learning consists of forming the joint posterior distribution of the static parameters and the hidden state based on information available only up to time  $t$ ,

$$p(h_t, \Theta | y_{1:t}, \mathcal{M}_i) = p(h_t | \Theta, y_{1:t}, \mathcal{M}_i) p(\Theta | y_{1:t}, \mathcal{M}_i), \quad (12)$$

where  $p(h_t | y_{1:t}, \Theta, \mathcal{M}_i)$  solves the state filtering problem, and  $p(\Theta | y_{1:t}, \mathcal{M}_i)$  addresses the parameter inference issue. Updating of investor's beliefs therefore corresponds to updating this posterior distribution.

For the linear predictive model (4), Bayesian learning is straightforward using the particle-based algorithm proposed by Chopin (2002). However, when stochastic volatility is introduced, the model becomes non-linear and state-dependent. Therefore, for the purpose of state filtering and likelihood estimation, we rely on a particle filter whose detailed algorithm is given in Appendix A. We note that the decomposition (12) suggests a hierarchical framework for model inference and learning. At each time  $t$ , for a given set of model parameters proposed from some proposal, we can run a particle filter, which delivers the empirical distribution of the hidden states,  $p(h_t | \Theta, y_{1:t}, \mathcal{M}_i)$ , and the estimate of the likelihood,  $p(rx_{1:t}^{(n)} | \Theta, \mathcal{M}_i)$ , that can be used for parameter learning,  $p(\Theta | y_{1:t}, \mathcal{M}_i) \propto p(y_{1:t} | \Theta, \mathcal{M}_i) p(\Theta | \mathcal{M}_i)$ . To achieve this aim, we rely on the marginalized resample-move approach developed by Fulop and Li (2013). The key point here is that the likelihood estimate from a particle filter is unbiased (Del Moral 2004). Furthermore, in contrast to traditional Bayesian methods, our Bayesian learning approach can be easily parallelized,

making it computationally fast and convenient to use in practice.

The above Bayesian learning approach provides as natural outputs the predictive distribution of excess bond returns

$$p(rx_{t+1}^{(n)}|y_{1:t}, \mathcal{M}_i) = \int p(rx_{t+1}^{(n)}|h_t, \Theta, y_{1:t}, \mathcal{M}_i)p(h_t|\Theta, y_{1:t}, \mathcal{M}_i)p(\Theta|y_{1:t}, \mathcal{M}_i)dh_t d\Theta, \quad (13)$$

and an estimate of the marginal likelihood,

$$p(rx_{1:t}^{(n)}|\mathcal{M}_i) = \prod_{s=1}^{t-1} p(rx_{s+1}^{(n)}|y_{1:s}, \mathcal{M}_i). \quad (14)$$

Both Equations (13) and (14) account for all uncertainties related to parameters and state. Equation (14) summarizes model fit over time (model learning) and can be used to construct a sequential Bayes factor for sequential model comparison. For any two models  $\mathcal{M}_i$  and  $\mathcal{M}_j$ , the Bayes factor at time  $t$  has the following recursive formula

$$\mathcal{BF}_t \equiv \frac{p(rx_{1:t}^{(n)}|\mathcal{M}_i)}{p(rx_{1:t}^{(n)}|\mathcal{M}_j)} = \frac{p(rx_t^{(n)}|y_{1:t-1}, \mathcal{M}_i)}{p(rx_t^{(n)}|y_{1:t-1}, \mathcal{M}_j)} \mathcal{BF}_{t-1}, \quad (15)$$

which is completely out-of-sample, and can be used for sequential comparison of both nested and non-nested models.

Bayesian learning and belief updating generate persistent and long-term shocks to the agent beliefs. To see this, define  $\theta_t = E[\theta|y_{1:t}]$  as the posterior mean of a parameter  $\theta$  obtained using information up to time  $t$ . The iterated expectation indicates

$$E[\theta_{t+1}|y_{1:t}] = E[E[\theta|y_{1:t+1}]|y_{1:t}] = E[\theta|y_{1:t}] = \theta_t. \quad (16)$$

Therefore,  $\theta_t$  is a martingale, indicating that shocks to the agent beliefs on this parameter are not only persistent but also permanent. Thus, in Bayesian learning, the agent gradually updates her beliefs that the value of a parameter is higher or lower than that previously thought and/or that a model fits the data better than the other.

The Bayesian learning process is initialized by an agent’s initial beliefs or the prior distributions. We move the fixed parameters in one block using a Gaussian mixture proposal. Given that in our marginalized approach the likelihood estimate is a complicated nonlinear function of the fixed parameters, conjugate priors are not available. For parameters that have supports of real line, we assume normal distributions for the priors. However, if a parameter under consideration has a finite support, we take a truncated normal as its prior, and if a parameter under consideration needs to be positive, we take a lognormal as its prior. The hyper-parameters of the prior distributions are calibrated using a training sample, that is, an initial dataset is used to provide information on the location and scale of the parameters. The training-sample approach is a common way to generate the objective prior distributions (O’Hagan 1994). We find that the model parameters are not sensitive to the selection of the priors. Therefore, we give flat priors to the model parameters. Set one in Table 1 provides details of the selection of functional forms and hyper-parameters for the priors.

### **2.3. Model Combinations**

Model combination is an important tool to handle model uncertainty. Timmermann (2006) argues that model combination can be viewed as a diversification strategy that improves predictive performance in the same manner that asset diversification improves portfolio performance. Rapach, Strauss, and Zhou (2010) and Dangle and Halling (2012) show that model combinations can generate better forecasts than the individual models in forecasting stock returns. In this section, we introduce four model combination schemes for forecasting bond excess returns.

#### *2.3.1. Sequential Best Model*

Sequential best model (SBM) selects the model with the largest marginal likelihood at each time  $t$ , i.e., it gives a weight of one to the model that has the largest marginal

likelihood and a weight of zero to other models,

$$p_{SBM}(rx_{t+1}^{(n)}|y_{1:t}) = \max \left\{ p(rx_{t+1}^{(n)}|y_{1:t}, \mathcal{M}_i) \right\}_{i=1}^N, \quad (17)$$

where  $N$  is the number of models considered. The best model may change over time, suggesting that a different model may be used for forecasting at each time.

### 2.3.2. Equal-weighted Model Average

Equal-weighted model average (EMA) assumes equal weight on each model, that is,

$$p_{EMA}(rx_{t+1}^{(n)}|y_{1:t}) = \sum_{i=1}^N w_{i,t} \times p(rx_{t+1}^{(n)}|y_{1:t}, \mathcal{M}_i), \quad (18)$$

where  $w_{i,t} = 1/N$  for all  $i$  and all  $t$ . One advantage of this simple scheme is that the combining weights do not need to be estimated.

### 2.3.3. Bayesian Model Average

It could be beneficial to determine combining weights according to model performance. Bayesian model averaging (BMA) provides a coherent mechanism for this purpose (Hoeting et al., 1999). It is a model combination approach based on the marginal likelihood of each model,

$$p_{BMA}(rx_{t+1}^{(n)}|y_{1:t}) = \sum_{i=1}^N w_{i,t} \times p(rx_{t+1}^{(n)}|y_{1:t}, \mathcal{M}_i), \quad (19)$$

where  $w_{i,t} = p(\mathcal{M}_i|y_{1:t})$ , and  $p(\mathcal{M}_i|y_{1:t})$  is the posterior probability of model  $i$ ,

$$p(\mathcal{M}_i|y_{1:t}) = \frac{p(y_{1:t}|\mathcal{M}_i)p(\mathcal{M}_i)}{\sum_{j=1}^N p(y_{1:t}|\mathcal{M}_j)p(\mathcal{M}_j)}, \quad (20)$$

in which  $p(y_{1:t}|\mathcal{M}_i)$  denotes the marginal likelihood of model  $i$ , and  $p(\mathcal{M}_i)$  is the prior probability of model  $i$ . In implementation, we assume equal prior probability for each model.

### 2.3.4. CER-based Model Average

The above model combination schemes basically use statistical evidence to construct combining weights,  $w_{i,t}$ . However, investors are more concerned about whether the statistical evidence of predictability could translate into real economic gains. Therefore, it is tempting to construct combining weights according to models' economic performance. We will see in the next section that our investor is Bayesian and tries to maximize her expected utility using the predictive distribution of excess bond returns. Models' economic performance is then evaluated using the certainty equivalence returns (CER). Therefore, we propose a simple utility-based model average scheme (UMA) that constructs combining weights using CER's at each time. Specifically,

$$p_{UMA}(rx_{t+1}^{(n)}|y_{1:t}) = \sum_{i=1}^N w_{i,t} \times p(rx_{t+1}^{(n)}|y_{1:t}, \mathcal{M}_j), \quad (21)$$

where  $w_{i,t} = p(M_i|y_{1:t})$  and is given by

$$p(M_i|y_{1:t}) = \frac{CER_{i,t}}{\sum_{j=1}^N CER_{j,t}}, \quad (22)$$

in which  $CER_{i,t}$  is the certainty equivalent return computed using Equation (32) for the period from the beginning date of out-of-sample to the current time  $t$ . In empirical implementation, we find that this simple scheme works quite well.

## 3. Assessing Out-of-Sample Performance

### 3.1. Statistical Evaluation

Given the predictive distribution of excess bond returns, we can compute the posterior mean to obtain the point forecast at each time  $t$  for each model or model combination. Denote this point forecast as  $\widehat{rx}_{t+1}^{(n)}$  and define the sum of squared forecast errors (SSE)



from initial time of the out-of-sample period to time  $t$  as

$$\widehat{SSE}(t) = \sum_{s=1}^t (rx_{s+1}^{(n)} - \widehat{rx}_{s+1}^{(n)})^2. \quad (23)$$

Furthermore, the expectations hypothesis states that the optimal forecast of excess bond returns is the historical mean, that is,  $\overline{rx}_{t+1}^{(n)} = \frac{1}{t} \sum_{j=1}^t rx_j^{(n)}$ . Then the SSE for expectation hypothesis model is given by

$$\overline{SSE}(t) = \sum_{s=1}^t (rx_{s+1}^{(n)} - \overline{rx}_{s+1}^{(n)})^2. \quad (24)$$

A natural measure of the model predictive performance is the out-of-sample R-square,  $R_{OS}^2$ , proposed by Campbell and Thompson (2008). The  $R_{OS}^2$  statistic is computed as

$$R_{OS}^2 = 1 - \frac{\widehat{SSE}(T)}{\overline{SSE}(T)}, \quad (25)$$

where  $T$  denotes the end of the out-of-sample period. The  $R_{OS}^2$  is analogous to the standard in-sample  $R^2$  and measures the proportional reduction in prediction errors of the forecast from the predictive model relative to the historical mean. It is clear that when  $R_{OS}^2$  is positive, the predictive model statistically outperforms the expectations hypothesis. We can further test whether this outperformance is statistically significant using the statistic developed by Clark and West (2007). The Clark-West statistic adjusts the well-known Diebold and Mariano (1995) and West (1996) statistic and generates asymptotically valid inference when comparing nested model forecasts. Clark and West (2007) show that this statistic performs very well in terms of power and size properties.

Moreover, to trace the predictive performance of each model over time, Goyal and Welch (2008) recommend to use the cumulative difference of squared forecast errors between the expectation hypothesis and a predictive model,

$$cumSSE(t) = \overline{SSE}(t) - \widehat{SSE}(t). \quad (26)$$

A positive and increasing *cumSSE* curve indicates that the predictive model always outperforms the historical mean.

### 3.2. Economic Value and Certainty Equivalent Returns

In evaluating the economic predictability, we consider a real-time investor who construct a portfolio consisting of a risk-free zero-coupon bond and a risky bond with maturity  $n$  and maximizes her expected utility over the next period portfolio value,  $W_{t+1}$ ,

$$\max_{\omega} \mathbf{E}[U(W_{t+1})|y_{1:t}, \mathcal{M}_i], \quad (27)$$

where  $U(\cdot)$  represents the investor's utility function and the portfolio value evolves according to

$$W_{t+1} = (1 - \omega_t^{(n)}) \exp(r_t^f) + \omega_t^{(n)} \exp(r_t^f + rx_{t+1}^{(n)}), \quad (28)$$

where  $r_t^f$  is the risk-free rate, and  $\omega_t^{(n)}$  is the portfolio weight on the risky bond with maturity  $n$ .

We assume that our investor has a power utility with the relative risk aversion controlled by  $\gamma$ ,

$$U(W_{t+1}) \equiv U(\omega_t^{(n)}, rx_{t+1}^{(n)}) = \frac{W_{t+1}^{1-\gamma}}{1-\gamma}. \quad (29)$$

The expected utility can be computed for each model as follows,

$$\mathbf{E}[U(W_{t+1})|y_{1:t}, \mathcal{M}_i] = \int U(\omega_t^{(n)}, rx_{t+1}^{(n)}) p(rx_{t+1}^{(n)}|y_{1:t}, \mathcal{M}_i) dx_{t+1}^{(n)}, \quad (30)$$

where the predictive distribution of excess bond returns,  $p(rx_{t+1}^{(n)}|y_{1:t}, \mathcal{M}_i)$ , is given by Equation (13).

Our investor is Bayesian. When computing expected utility in Equation (30), she takes into account all relevant uncertainties. At each time, the investor choose the portfolio weight to maximize her expected utility. In our Bayesian learning, we have  $M$  particles

for each variable at each time. Then the optimal weight can be obtained by

$$\hat{w}_t^{(n)} = \arg \max \frac{1}{M} \sum_{j=1}^M \left\{ \frac{\left[ (1 - \omega_t^{(n)}) \exp(r_t^f) + \omega_t^{(n)} \exp(r_t^f + r x_{t+1}^{(n),j}) \right]^{1-\gamma}}{1 - \gamma} \right\}. \quad (31)$$

The above portfolio weight in Equation (31) is then used to compute the investor's realized utility at each time  $t$ . We denote the realized utility from a predictive model as  $\hat{U}_t$  and denote the realized utility from the EH benchmark as  $\bar{U}_t$ . Then the certainty equivalence return (CER) for each predictive model is defined as a value that equates the average realized utility from the model to that from the expectations hypothesis over the forecasting period. Following Pettenuzzo, Timmermann, and Valkanov (2014), we have

$$CER = \left( \frac{\sum_{t=1}^T \hat{U}_t}{\sum_{t=1}^T \bar{U}_t} \right)^{\frac{1}{1-\gamma}} - 1. \quad (32)$$

In addition, similar to *cumSSE*, we construct the cumulative sum of CER over time,

$$cumCER(t) = \sum_{s=1}^t \log(1 + CER_s). \quad (33)$$

where  $CER_t$  captures the real-time CER of a model and is given by  $CER_t = \left( \hat{U}_t / \bar{U}_t \right)^{\frac{1}{1-\gamma}} - 1$ . A positive and increasing *cumCER* curve suggests that the model always economically performs better than the historical mean.

## 4. Empirical Results

### 4.1. Data

We construct monthly yields on US zero-coupon bonds with maturity 2-, 3-, 4-, and 5-year using the updated dataset of Gurkaynak, Sack, and Wright (2007).<sup>1</sup> Most studies in bond return predictability focus on predictive regressions for annual excess bond returns in

<sup>1</sup> The data is from: <https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html>

monthly forecasting frequency, that is,  $m$  is one year in Equation (3). Bauer and Hamilton (2017) argue that the overlapping bond returns induce strong serial correlations in the error terms in predictive regressions, and may raise additional econometric problems when predictors are persistent. Therefore, similar to Gargano, Pettenuzzo, and Timmermann (2017), we consider one-month holding period and construct non-overlapping monthly excess bond returns. This implies that  $m$  is equal to one month in Equations (2) and (3).

Our sample spans from January, 1962 to December, 2011. In total, there are 600 months. Table 2 presents summary statistics for full-sample excess bond returns and predictors. Panel A shows that both mean and standard deviation of the annualized monthly excess returns increase with respect to maturity. For example, the mean excess return is about 1.41% with a standard deviation of 2.97% for the 2-year bond, whereas it increases to 2.19% with a standard deviation of 6.23% for the 5-year bond. Furthermore, we notice that both skewness and kurtosis decreases with respect to maturity. For example, the skewness and kurtosis for the 2-year excess bond returns are 0.50 and 14.9, respectively, whereas they are only 0.01 and 6.58, respectively, for the 5-year returns. This suggests that the short-maturity excess bond returns are more right-skewed and more leptokurtic than the long-maturity ones. Both short- and long-maturity excess bond returns are very weakly autocorrelated, as the first-order autocorrelations range from 0.12 (5-year) to 0.17 (2-year). Figure 1 plots the time series of excess returns for 2-, 3-, 4-, and 5-year bonds. We can see that all excess returns are quite volatile during the period of 1980-1983, whereas during the period of the recent global financial crisis, return volatility is by no means comparable.

Panel B presents summary statistics for the three predictors: FB, CP, and LN. We find that (1) the FB factors are left-skewed and their kurtosis's are close to 3, whereas both CP and LN are right-skewed and their kurtosis's are larger than 3, 5.3 and 5.6, respectively; (2) the FB factors are much more persistent than both CP and LN, and LN is the least persistent with the first-order autocorrelation being about 0.43. Panel C

reports correlations between predictors. The FB factors are highly correlated with each other, with the correlation ranging from 0.88 between 2- and 5-year FB's to 0.99 between 4- and 5-year FB's. The CP factor has relatively small correlations with the FB factors, ranging from 0.46 with 2-year FB to 0.50 with 5-year FB. LN is almost not correlated with FB and CP. Its correlation with FB varies from -0.09 to 0.02, and its correlation with CP is only about 0.2.

It needs to emphasize that when we implement Bayesian learning, both CP and LN are reconstructed at each time  $t$  using the information available only up to time  $t$  in order to avoid any hindsight problems.

#### 4.2. Parameter Learning and Sequential Model Comparison

Different from batch estimation methods, our Bayesian learning approach provides us with the whole picture of how parameter posteriors evolve over time with respect to accumulation of information for each. In this section, we focus on a stochastic volatility model and a constant volatility model, both of which take the LN factor as their only predictor (LN-SV and LN-CV). Figure 2 presents the sequential learning of the fixed parameters for LN-SV, and Figure 3 presents the sequential learning of the fixed parameters for LN-CV, on 3-year excess bond returns. For each parameter, the posterior mean (solid line) and the 5th and 95th percentile credible interval (dashed lines) are reported.

There are a number of notable features from these two figures. First, the investor's beliefs on parameters are quite uncertain in the beginning as the 5th and 95th percentile credible intervals are very large for all parameters. Then, as information accumulates, the credible intervals become narrower over time, and parameter uncertainty diminishes.

Second, the speed of learning is different across parameters. For the expected return parameters,  $\alpha$  and  $\beta$ , learning is faster for  $\alpha$  than for  $\beta$  in both LN-SV and LN-CV. It takes only several years for  $\alpha$  to reach small credible intervals, whereas it takes more than 10 years for  $\beta$  to have relatively small credible intervals. This evidence is particularly

striking in LN-SV. For the parameters governing volatility,  $\mu$ ,  $\phi$ , and  $\sigma_h$ , the learning speed for  $\sigma_h$  is much slower than the others. Its posterior mean is slowly going up in the beginning, and then is slowly going down after around 1970. Moreover, it takes very long time for its 5th and 95th percentile credible interval to get sufficiently narrowed.

Third, the last panel of Figure 2 presents the sequential estimates of volatility. Consistent with the investor's beliefs on parameters, her belief on volatility is quite uncertain in the beginning, whereas after a short period, she becomes quite certain on volatility dynamics, mirrored by very narrow 5th and 95th percentile credible intervals.

Fourth, the learning process of  $\sigma_{rx}$  in LN-CV reveals evidence of misspecification of the constant volatility model, as its learned value slowly drifts up and reaches its highest value around 1982 when bond returns are very volatile, and then it keeps going down up to the end of the sample.

Moreover, thanks to its recursive nature, our Bayesian learning approach produces the sequential marginal likelihood at each time  $t$  for each model  $i$  as shown in Equation (14). We can then construct the sequential Bayes factors and use them for real-time model analysis and comparison. The last panel of Figure 3 presents the sequential log Bayes factors between LN-SV and LN-CV. It gives us a richer picture of model performance over time. First, no matter which maturity is considered, when market information is scarce and the variation of excess bond returns is very small (see Figure 1) in the beginning of the sample, LN-SV performs nearly the same as LN-CV. Second, as the market information accumulates over time, the data begin to strongly favour the stochastic volatility model. Third, most of the up-moves in Bayes factors happen during market turmoils. This phenomenon is particularly striking around 1982 when all four time series of excess bond returns have high volatility and indicates that the investors mainly update their beliefs on model specifications during market turmoils. Fourth, before 1982, the stochastic volatility model performs particularly better than the constant volatility model on the 5-year excess bond returns, whereas after 1982, its performance becomes much stronger on the 2-year

excess bond returns.

### 4.3. Predictive Performance of Individual Models

We use the second half of the sample as the out-of-sample evaluation period, that is, the out-of-sample period is from January 1987 to December 2011, in total, 300 months.

#### 4.3.1. Statistical Evidence

At each time  $t$ , our Bayesian learning approach provides us with the full predictive density for each model,  $p(rx_{t+1}^{(n)}|y_{1:t}, \mathcal{M}_i)$ , based on which we take its posterior mean as the point forecast to construct  $R_{OS}^2$  and  $cumSSE$  for evaluating its predictive performance. Table 3 presents  $R_{OS}^2$ 's for all the 14 models considered. Panel A is for the constant volatility models, and Panel B for the stochastic volatility models. We have the following notable findings. First, in the 2-year excess bond returns, there is significant statistical evidence of predictability in LN-CV and FBLN-CV. For example, LN-CV generates an  $R_{OS}^2$  of 2.42%, and FBLN-CV generates an  $R_{OS}^2$  of 1.07%, Both are highly statistically significant. Second, when moving to 3-, 4-, and 5-year excess bond returns, we find that all constant volatility models that take LN as a predictor generate statistically significantly positive  $R_{OS}^2$ , and in general, predictability of these models improves with respect to maturity. For example, FBCPLN-CV has an  $R_{OS}^2$  of 0.35% in the 3-year excess bond returns, whereas its  $R_{OS}^2$  increases to 1.52% in the 4-year excess bond returns and further increases to 1.93% in the 5-year excess bond returns.

Third, when stochastic volatility is introduced, the predictability of most of the models dramatically improves, especially for those models that take LN as a predictor. For example, in 3-year excess bond returns,  $R_{OS}^2$  is about 2.64% in FBLN-CV, whereas it becomes 4.58% in FBLN-SV. This suggests that stochastic volatility is an important feature of bond return dynamics. Fourth, among constant volatility models, LN-CV performs the best no matter which maturity is concerned, and among stochastic volatility

models, LN-SV outperforms the other models in 2- and 3-year excess bond returns and FBLN-SV stands out in 4- and 5-year excess bond returns. This result suggests that the macro factor extracted from a large panel of macroeconomic variables contains rich information on the yield curve.

We can also check how each model performs over time in forecasting excess bond returns by taking look at *cumSSE* recommended by Goyal and Welch (2008). Figure 4 presents *cumSSE*'s for four models: CP-CV, CP-SV, LN-CV, and LN-SV. According to  $R_{OS}^2$ 's in Table 3, the models that take CP as the predictor performs almost the worst, whereas the models that take LN as the predictor performs almost the best. We see from Figure 4 that *cumSSE*'s for LN-CV and LN-SV are always positive and increasing over time no matter which maturity is concerned, whereas *cumSSE*'s for CP-CV and CP-SV are almost negative for all the four maturities. We also see that LN-CV performs better than LN-SV for predicting 2- and 3-year excess bond returns over time before the recent financial crisis in 2008, whereas LN-CV outperforms LN-SV for predicting 4- and 5-year excess bond returns almost all over time, even after the 2008 financial crisis.

#### 4.3.2. *Economic Evidence*

The statistical evidence of predictability does not necessarily translate into economic gains for investors. Our investor is Bayesian, who takes into account all relevant uncertainty when maximizing her expected utility in Equation (27). In empirical implementation, we restrict the portfolio weight,  $\omega_t^{(n)}$ , between -1 and 2 as in Thornton and Valente (2012) and Sarno, Schneider, and Wagner (2016) to prevent extreme investments (Goyal and Welch, 2008; Ferreira and Santa-Clara, 2011). In fact, the above restrictions on portfolio weight allow the investor to take full short sales. We assume that the coefficient of investor's relative risk aversion,  $\gamma$ , is equal to 5, and compute the corresponding certainty equivalent return (CER) for each model using formula (32). To test if the annualized CER values are statistically greater than zero, we use a Diebold-Mariano test similar to that used in



Gargano, Pettenuzzo, and Timmermann (2017)<sup>2</sup>

Table 3 presents the annualized CER values for all the 14 models considered. CER's deliver almost the same implications as  $R_{OS}^2$ 's do. We do find economic gains from bond return predictability. No matter whether constant volatility models or stochastic volatility models are considered, the economic gains increase with respect to maturities. In general, introduction of stochastic volatility improves economic gains to a large extent. However, we find that among all models, LN-CV generates the largest CER's in 2-, 4-, and 5-year excess bond returns, which are about 0.63%, 2.57%, and 2.83%, respectively, whereas LN-SV produces the largest economic gain in 3-year excess bond returns, which is about 1.91%.

Figure 5 presents the cumulative sum of CER's over time for the four models: CP-CV, CP-SV, LN-CV, and LN-SV. We see that LN-CV and LN-SV perform much better than CP-CV and CP-SV over time. The cumCER's of LN-CV and LN-SV have increasing tendency over time, though this tendency in 2-year excess bond returns is not as strong as that in the other maturities. However, the cumCER of CP-CV is always negative and almost decreasing over time in all four time series of excess bond returns. In spite of evidence that the cumCER of CP-SV slightly increases over time, it is too weak.

#### 4.4. Unbounded Weights and Economic Value

The above economic evidence is examined by limiting the weight on risky bond between -1 and 2. However, investors in fixed income markets may be able to borrow or short with more extreme positions, facilitated by Repo agreements for instance. In light of this, we allow the investor to take her investment decision without any weight bounds. In the out-of-sample period from 1987 to 2012, we still assume a risk aversion of 5 and report the new CER values in Table 4.

For risky bonds with maturities of 2, 3, 4, and 5 years, the maximum weights are

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<sup>2</sup>We run the regression,  $U_{i,t+1} - U_{EH,t+1} = \alpha + \epsilon_{t+1}$ , where  $U_{i,t}$  and  $U_{EH,t}$  are the realized utilities at time  $t + 1$  for model  $i$  and  $EH$ , and test if  $\alpha$  equals zero.

about 44, 29, 31, and 16, respectively; the minimum weights are about -23, -13, -9 and -6, respectively; the average weights are about 4.91, 2.74, 1.96 and 1.54, respectively, decreasing with maturity; and the associated average annualized CERs are about 5.37%, 4.02%, 3.22% and 2.49%, respectively. If we compare Table 4 with Table 3, almost all CER values increase dramatically. Some of the high CER's are not significant because the associated realized utilities have high standard deviations. Without imposing any weight constraints, the economic evidence is much more pronounced, especially for the short-maturity risky bonds. The investment on the 2-year bond is now the most profitable across all maturities. We recall that bounding the weights is a common practice in almost all previous papers, so we suspect that this could be one reason why some studies do not identify the link between statistical evidence and economic value.

This seems a new result in the literature on fixed income predictability and seems to contrast with the general finding on equities where constraints on the portfolio weights tend to improve out-of-sample performance (see, for example, Pettenuzzo, Timmermann, and Valkanov, 2014). The reason may be that government bonds are much less risky compared to equity and even small fluctuations in the conditional expected returns suggest wildly varying portfolio weights. Of course, such highly levered positions may be sensitive to the funding liquidity conditions in the market, i.e., when the investor would like to take up an extreme position, finding financing may get harder.

One reason could relate to financial intermediaries, who play crucial roles in many asset markets. The intermediary sector's net worth (or equivalently, its equity capital ratio) is a key determinant of its marginal value of wealth and is an important indicator for any markets, as documented in He, Kelly, and Manela (2017, HKM henceforth). Borrowing their analogy, we argue that when the investment decision is extreme, primary dealers are unwilling to facilitate the trade because they face much more pressure from their own balance sheet. We select the three best performing models from Table 3 and Table 4 and use quantile regressions to see the potential relations between market pressure

and portfolio weights at different weight quantiles. We regress the unbounded portfolio weights of long-term bonds on the market pressure index  $1/\text{HKM}$  (the leverage in the intermediary balance-sheet) from 1987 to 2012.<sup>3</sup> Results are reported in Table 5. We find that all coefficients are significant at 1% level. Coefficients generally increase from low quantile to high quantile. The results on the high quantiles (extreme long positions) are in line with our conjecture. When investors seek high leveraged positions, primary dealers are under more pressure from their own balance sheet and thus they may be reluctant to provide more liquidity. However, the same mechanics do not seem to show for extreme short positions (low quantiles of weight distributions), where changes in funding liquidity are only weakly related to the weights.

Another reason could be the shortage of arbitrage capital, as argued by Hu, Pan, and Wang (2013, HPW henceforth). They state that when there is no sufficient arbitrage forces in the market, assets' prices could deviate significantly away from the fundamental values. Thus, price deviations can be seen as a proxy of market illiquidity. When liquidity dries up, it becomes difficult to implement extreme investment actions. Therefore, we use the illiquidity measure from HPW and perform a similar quantile regression as for HKM measure. We regress the unbounded portfolio weights for risky bonds on the HPW measure.<sup>4</sup> Results are reported in Table 6. Apart from a few exceptions in stochastic volatility models, results in the high quantiles show that when investors require extreme long positions from the market, there is a tightening of liquidity due to the lack of arbitrage capital.

#### 4.5. Model Combinations and Predictive Performance

We now move on to take a look at statistical evidence and economic gains of bond return predictability in model combinations. From here on, we still bound the weight at -1 and 2 in asset allocation. Table 7 presents both  $R_{OS}^2$ 's and CER's resulting from the four

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<sup>3</sup>The data is from Asaf Manela's website: <http://apps.olin.wustl.edu/faculty/manela/data.html>

<sup>4</sup>The data is from Jun Pan's website: <http://www.mit.edu/~junpan/>

model combination schemes introduced previously. We have the following main findings. First, all the four combination schemes generate highly statistically significant  $R_{OS}^2$ 's and positive CER's, no matter which maturity is concerned. Statistically significant  $R_{OS}^2$  and positive CER's indicate that there exists evidence of bond return predictability which also has economic value. Second, having compared these model combination schemes, we find that in 2-year excess bond returns, SBM performs the best according to  $R_{OS}^2$ , whereas BMA performs the best according to CER. However, in 3-, 4-, and 5-year excess bond returns, UMA outperforms the other three schemes according to both  $R_{OS}^2$  and CER. For example, UMA generates an  $R_{OS}^2$  of 4.20% and a CER of 1.50% in 3-year excess bond returns, an  $R_{OS}^2$  of 4.44% and a CER of 2.32% in 4-year excess bond returns, and an  $R_{OS}^2$  of 4.60% and a CER of 2.87% in 5-year excess bond returns.

Figure 6 presents the cumulative sum of squared errors, and Figure 7 presents the cumulative sum of CER's, for these four model combination schemes. These two figures give us a full picture how these four combinations perform over time. The *cumSSE*'s for both SBM and UMA are always positive and have stronger increasing tendency than the other two. Similar pattern can also be found in Figure 7.

## 5. Robustness Checks

### 5.1. Risk Aversion and Predictability

We assume in our main analysis in Section 4 that the coefficient of investor's relative risk aversion is equal to 5. To see how sensitive our results are to this coefficient, we also consider another two scenarios: lower risk aversion,  $\gamma = 3$ , and higher risk aversion,  $\gamma = 10$ . In this section, we constrain all investment weights between -1 and 2.

Table 8 presents CER's resulted from using these two parameters of risk aversion. In general, we see that when the investor's risk aversion becomes lower, the economic gains become smaller, whereas when risk aversion becomes higher, the economic gains become larger. For example, when  $\gamma = 3$ , the CER generated from LN-SV is reduced from 0.47%

to 0.36; and when  $\gamma = 10$ , the CER generated by the same model is much higher, about 1.49%. Also, as we lower the value of risk aversion, CER decreases on average. This is mainly due to the fact that with lower  $\gamma$ , the weights on the risky asset reach more often the upper bound, for both the EH benchmark and other predictive models, thus making it more difficult to differentiate between these models.

However, our main results remain unchanged. First, no matter whether  $\gamma = 3$  or  $\gamma = 10$ , the models that take LN as a predictor always perform much better. For example, among constant volatility models, LN-CV generates the highest CER's in most cases except that FBLN-CV generates the highest CER when  $\gamma = 10$  in 5-year excess bond returns; and among stochastic volatility models, the model with LN as the predictor again performs the best in most cases. Second, when stochastic volatility is introduced, the economic gains generally become much stronger. Third, model combinations improve economic gains in general, and among the four combination schemes, UMA performs the best in most cases, and SBM is runner-up.

## 5.2. Different Out-of-Sample Periods

In Section 4, we take the second half of the sample as the out-of-sample period. In this section, we explore whether our results are sensitive to the choice of out-of-sample periods. There is substantial evidence that the Federal Reserve changes its policy rule during the early 1980s. We therefore choose an out-of-sample period that starts in January 1982 and ends in December 2011. Gargano, Pettenuzzo, and Timmermann (2017) set the out-of-sample period ranging from January 1990 to December 2011. We then take this period as another out-of-sample period for robustness check.

Table 9 presents  $R_{OS}^2$ 's and CER's resulted from all the models and model combinations for the out-of-sample period ranging from January 1982 to December 2011. We find that in general with comparison to results in Table 3, the overall predictive performance of the individual models and model combinations improve a lot in this period accord-

ing to both  $R_{OS}^2$  and CER. For example, in 2-year excess bond returns, the FBLN-CV model generates an  $R_{OS}^2$  of 5.96% and a CER of 0.73%, whereas these two values are only 1.07% and 0.13%, respectively, in Table 3. However, our main results still hold. The models with LN as a predictor perform better than other models; introduction of stochastic volatility dramatically improves predictive performance of the models; and all model combinations generate positive and highly significant  $R_{OS}^2$ 's, which finally translate into positive economic gains.

Table 10 presents statistical and economic evidence of predictability resulted from all the models and model combinations for the out-of-sample period ranging from January 1990 to December 2011. The results are very similar to what we have seen in Table 3 and Table 9. We therefore conclude that our main results are robust to the choice of out-of-sample period.

### 5.3. Sensitivity to Priors

Our Bayesian learning is initialized by the investor's priors on model parameters. We then test whether our results are robust to alternative priors. We use the second set of priors in Table 1, which assumes a normal distribution for a parameter that has support of real line, and assumes a truncated normal distribution for a parameter that has finite support. The hyper-parameters are chosen such that the priors are not informative. We assume that the coefficient of investor's relative risk aversion is equal to 5, and take the second half of the sample as the out-of-sample period.

We obtain exactly the same results as those in Table 3 and Table 7. Therefore, we conclude that our results are not sensitive to the choice of priors at all.

## 6. Concluding Remarks

The paper finds statistically and economically significant evidence of out-of-sample bond return predictability for a Bayesian investor who learns about parameters, hidden states,

and models at each time when new information becomes available. We compare the performance of the expectation hypothesis model and predictive models. We take forward spreads of Fama and Bliss (1987), the forward factor of Cochrane and Piazzesi (2005), and the macro factor of Ludvigson and Ng (2009) as predictors and build in total 14 individual models and 4 model combination schemes.

Most studies in bond return predictability focus on predictive regressions for annual excess bond returns in monthly data. Such overlapping returns induce strong serial correlations in the error terms and may raise additional econometric problems when predictors are persistent (Bauer and Hamilton, 2017). Therefore, similar to Gargano, Pettenuzzo, and Timmermann (2017), we consider one-month holding period and construct non-overlapping monthly excess bond returns. Our data range from January, 1962 to December, 2011, in total 600 months. Both out-of-sample  $R^2$  and certainty equivalent returns suggest that predictability improves with respect to bond maturities and that introducing stochastic volatility in a model can enhance its predictive performance to a large extent. Interestingly, economic evidence is much more pronounced when we do not impose any weight constraints. Therefore, bounding investment weights may be a potential reason why some previous papers do not identify significant economic evidence. We use quantile regressions to show that the pressure from intermediary sector's balance-sheet and shortage of arbitrage capital could be the forces that may constrain investors in implementing such positions. Furthermore, we find that the factor extracted from a large panel of macroeconomic variables contains rich information on future excess bond returns. We also document that a simple utility-based combination scheme works well in forecasting long-term bond returns.

## Appendix

### A. Particle Filter Algorithm

Our state-space model takes the form:

$$rx_{t+1} = \alpha + \beta X_t + \exp(h_{t+1})\epsilon_{rx,t+1}, \quad (34)$$

$$h_{t+1} = \mu + \phi h_t + \sigma_h \epsilon_{h,t+1}, \quad (35)$$

where  $rx_t$  and  $X_t$  are observations of excess returns and predictors. The first equation is the observation equation and second is the state transition equation.

Denote the parameter set  $\theta = \{\alpha, \beta, \mu, \phi, \sigma_h\}$ . We employ a bootstrap particle filter as follows:

**Step 1:** We initialize the filter and set state particles to be:  $h^{(i)} = \mu/(1 - \phi), i = 1, \dots, M$  and give each particle a weight  $1/M$ . Note that there is a lag between the excess return and the predictor.

For time  $t=2, \dots, T$ :

**Step 2:** We run the particle filter on the new observation. Run one step forward to sample new states using the transition equation to get  $\psi(h_t^{(i)}|h_{t-1}^{(i)}, \theta)$ . Compute the particle weight by:  $w_t^{(i)} \sim \text{Normal}(rx_t - \alpha - \beta X_{t-1}, \exp(h_t^{(i)}))$ . Normalize the weight by  $\pi_t^{(i)} = w_t^{(i)} / \sum_{k=1}^M w_t^k$ .

**Step 3:** Stratified resample step: first draw i.i.d. uniform random numbers  $U_t^{(i)}, i = 1, \dots, M$ , then draw new particle indices by inverting the cumulative distribution function (CDF) of the multinomial characterized by  $\pi_t^{(i)}$  at the stratified uniforms  $\frac{i+U_t^{(i)}}{M}$ . Update and draw state particles population according to the new indices and the new sample is equally weighted now. The marginal likelihood can be computed as:  $p(rx_t|rx_{1:t}, X_{1:t}, \theta) = \frac{1}{M} \sum_{k=1}^M p(rx_t|rx_{1:t-1}, X_{1:t}, h_t^k)$ .



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Table 1: **The Prior Distributions**

|               | Set One                                       | Set Two                                       |
|---------------|---|---|
| $\alpha$      | $N(0, 10)$                                    | $N(0, 10)$                                    |
| $\beta$       | $N(0, 10)$                                    | $N(0, 10)$                                    |
| $\sigma_{rx}$ | $\log(\sigma_{rx}) \sim N(-2, 5)$             | Truncated Normal: $N(0, 10), \sigma_{rx} > 0$ |
| $\mu$         | $N(0, 5)$                                     | $N(0, 5)$                                     |
| $\phi$        | Truncated Normal: $N(0, 5), \phi \in (-1, 1)$ | Truncated Normal: $N(0, 5), \phi \in (-1, 1)$ |
| $\sigma_h$    | $\log(\sigma_h) \sim N(-2, 5)$                | Truncated Normal: $N(0, 15), \sigma_h > 0$    |

The table shows two sets of prior distributions we consider. The linear model is given in equation (4). Parameters for linear models are:  $\alpha, \beta, \sigma_{rx}$ . The SV model is given in equation (5) and (6). Parameters for SV models are:  $\alpha, \beta, \mu, \phi, \sigma_h$ .

Table 2: **Summary Statistics**

| Panel A: Excess Bond Returns |        |        |        |        |  |  |
|------------------------------|--------|--------|--------|--------|--|--|
|                              | 2-Year | 3-Year | 4-Year | 5-Year |  |  |
| Mean                         | 1.415  | 1.732  | 1.987  | 2.194  |  |  |
| St.dev                       | 2.971  | 4.156  | 5.217  | 6.225  |  |  |
| Skew                         | 0.500  | 0.208  | 0.057  | 0.015  |  |  |
| Kurt                         | 14.86  | 10.65  | 7.900  | 6.580  |  |  |
| AC(1)                        | 0.169  | 0.153  | 0.138  | 0.124  |  |  |

| Panel B: Predictors |        |        |        |        |       |       |
|---------------------|--------|--------|--------|--------|-------|-------|
|                     | FB     |        |        |        | CP    | LN    |
|                     | 2-Year | 3-Year | 4-Year | 5-Year |       |       |
| Mean                | 0.108  | 0.129  | 0.145  | 0.158  | 0.153 | 0.153 |
| St.dev              | 0.100  | 0.116  | 0.129  | 0.138  | 0.213 | 0.308 |
| Skew                | -0.072 | -0.238 | -0.223 | -0.143 | 0.785 | 0.841 |
| Kurt                | 3.756  | 3.378  | 3.041  | 2.755  | 5.288 | 5.613 |
| AC(1)               | 0.878  | 0.899  | 0.913  | 0.923  | 0.673 | 0.428 |

| Panel C: Correlations |       |       |       |       |       |        |
|-----------------------|-------|-------|-------|-------|-------|--------|
|                       | FB2   | FB3   | FB4   | FB5   | CP    | LN     |
| FB2                   | 1.000 | 0.973 | 0.926 | 0.879 | 0.460 | -0.093 |
| FB3                   |       | 1.000 | 0.987 | 0.961 | 0.472 | -0.055 |
| FB4                   |       |       | 1.000 | 0.993 | 0.490 | -0.016 |
| FB5                   |       |       |       | 1.000 | 0.500 | 0.017  |
| CP                    |       |       |       |       | 1.000 | 0.187  |
| LN                    |       |       |       |       |       | 1.000  |

This table presents the summary statistics of bond excess returns and full-sample predictors. Panel A shows the mean, standard deviation, skewness, kurtosis and first-order autocorrelation of annualized monthly excess returns (in percentage). Panel B shows the mean, standard deviation, skewness, kurtosis and first-order autocorrelation of predictors. Panel C shows the correlation matrix of predictors. Full-sample data is from Jan, 1962 to Dec, 2011.

Table 3: **Out-of-Sample Predictability**

| Panel A: Constant Volatility Models   |            |       |            |         |            |         |            |         |
|---------------------------------------|------------|-------|------------|---------|------------|---------|------------|---------|
|                                       | 2-Year     |       | 3-Year     |         | 4-Year     |         | 5-Year     |         |
|                                       | $R_{OS}^2$ | CER   | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| FB                                    | 0.02       | -0.95 | 1.14       | -0.36   | 1.51**     | 0.28    | 1.66**     | 0.54    |
| CP                                    | -4.62      | -1.06 | -2.53      | -0.96   | -1.27      | -0.90   | -0.64      | -0.79   |
| LN                                    | 2.42***    | 0.63  | 3.73***    | 1.58**  | 4.97***    | 2.57*** | 4.79***    | 2.83*** |
| FBCP                                  | -4.25      | -0.17 | -2.39      | -0.81   | -0.56      | -0.09   | -0.17*     | 0.07    |
| FBLN                                  | 1.07***    | 0.13  | 2.64***    | 0.86*   | 3.15***    | 1.68*   | 3.31***    | 2.11*   |
| CPLN                                  | -2.09***   | -0.08 | 0.92***    | 0.62*   | 2.36**     | 1.59*   | 3.11**     | 2.08*   |
| FBCPLN                                | -1.71***   | -0.03 | 0.35***    | 0.28    | 1.52***    | 0.81    | 1.93***    | 1.21    |
| Panel B: Stochastic Volatility Models |            |       |            |         |            |         |            |         |
|                                       | 2-Year     |       | 3-Year     |         | 4-Year     |         | 5-Year     |         |
|                                       | $R_{OS}^2$ | CER   | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| FB                                    | -1.33      | -1.00 | 0.21*      | -0.67   | 1.25**     | 0.20    | 1.43**     | 0.49    |
| CP                                    | -0.81      | 0.07  | -0.39      | 0.11    | 0.41       | 0.35    | 0.86       | 0.68    |
| LN                                    | 5.43***    | 0.47  | 4.64***    | 1.91*** | 4.21***    | 2.49*** | 3.71**     | 2.66*** |
| FBCP                                  | -0.63      | -0.77 | 0.41*      | -0.37   | 1.26**     | 0.48    | 1.85**     | 0.81    |
| FBLN                                  | 5.03***    | 0.36  | 4.58***    | 1.16*   | 4.53***    | 1.90**  | 4.18***    | 2.11*   |
| CPLN                                  | 3.40***    | 0.46  | 3.34***    | 1.24*   | 4.04***    | 2.08**  | 4.11**     | 2.73*** |
| FBCPLN                                | 4.21***    | 0.35  | 3.69***    | 0.96    | 4.03***    | 1.54*   | 3.97***    | 2.16*   |

This table presents the out-of-sample R-2 and annualized CERs for linear and stochastic volatility forecasting models. Linear or Stochastic models can use different combinations of predictors: one predictor (FB, CP, LN), two predictors (FBCP, FBLN, CPLN), or three predictors (FBCPLN). The  $OOS - R^2$  is given in equation (25). The statistical significance measure is from Clark and West (2007). \* means significance at 10% level. \*\* means significance at 5% level. \*\*\* means significance at 1% level. The CER is given in equation (32). Risk aversion is 5. The out-of-sample period is from Jan, 1987 to Dec, 2011.

Table 4: **Out-of-Sample CERs: unbounded weight case**

|                                       | 2-year   | 3-year  | 4-year  | 5-year |
|---------------------------------------|----------|---------|---------|--------|
| Panel A: Constant Volatility Models   |          |         |         |        |
| FB                                    | 0.79     | 1.51    | 1.93    | 2.06   |
| CP                                    | -0.45    | -0.43   | -0.26   | -0.21  |
| LN                                    | 7.45***  | 6.25*** | 5.87*** | 4.72** |
| FBCP                                  | 0.08     | 0.11    | 0.87    | 0.72   |
| FBLN                                  | 8.30***  | 7.52*** | 6.52**  | 5.13   |
| CPLN                                  | 6.00***  | 4.98**  | 4.03*   | 3.12   |
| FBCPLN                                | 7.17***  | 5.96**  | 4.97    | 3.61   |
| Panel B: Stochastic Volatility Models |          |         |         |        |
| FB                                    | 0.02     | 0.07    | -0.06   | 0.42   |
| CP                                    | 1.96     | 0.96    | 0.98    | 1.06   |
| LN                                    | 12.27*** | 8.17*** | 5.63**  | 4.51** |
| FBCP                                  | 1.25     | 0.51    | 0.42    | 0.65   |
| FBLN                                  | 10.70*** | 7.69*** | 5.03*   | 2.88   |
| CPLN                                  | 9.95***  | 6.62**  | 4.83*   | 3.10   |
| FBCPLN                                | 9.67***  | 6.37*   | 4.32    | 3.13   |
| Average                               | 5.37     | 4.02    | 3.22    | 2.49   |

The table presents out-of-sample annualized CER values for model combinations in the out-of-sample period from Jan, 1987 to Dec, 2011. No bound is imposed on the weights of risky and riskless assets. Risk-aversion is 5.

Table 5: Quantile Regression - HKM (2017)

|                                       | Quantile | 0.10    | 0.25    | 0.50    | 0.75    | 0.90    |
|---------------------------------------|----------|---------|---------|---------|---------|---------|
| Panel A: Constant Volatility Models   |          |         |         |         |         |         |
| 2Y                                    | LN       | 0.20*** | 0.25*** | 0.40*** | 0.50*** | 0.51*** |
|                                       | FBLN     | 0.41*** | 0.44*** | 0.54*** | 0.61*** | 0.59*** |
|                                       | CPLN     | 0.44*** | 0.43*** | 0.55*** | 0.63*** | 0.68*** |
| 3Y                                    | LN       | 0.12*** | 0.18*** | 0.26*** | 0.33*** | 0.41*** |
|                                       | FBLN     | 0.29*** | 0.28*** | 0.39*** | 0.47*** | 0.47*** |
|                                       | CPLN     | 0.30*** | 0.28*** | 0.37*** | 0.43*** | 0.47*** |
| 4Y                                    | LN       | 0.08*** | 0.12*** | 0.18*** | 0.26*** | 0.32*** |
|                                       | FBLN     | 0.23*** | 0.22*** | 0.33*** | 0.38*** | 0.37*** |
|                                       | CPLN     | 0.21*** | 0.23*** | 0.28*** | 0.33*** | 0.36*** |
| 5Y                                    | LN       | 0.07*** | 0.09*** | 0.16*** | 0.19*** | 0.27*** |
|                                       | FBLN     | 0.20*** | 0.20*** | 0.29*** | 0.34*** | 0.34*** |
|                                       | CPLN     | 0.18*** | 0.18*** | 0.22*** | 0.27*** | 0.33*** |
| Panel B: Stochastic Volatility Models |          |         |         |         |         |         |
| 2Y                                    | LN       | 0.30*** | 0.27*** | 0.35*** | 0.53*** | 0.73*** |
|                                       | FBLN     | 0.70*** | 0.51*** | 0.57*** | 0.63*** | 0.84*** |
|                                       | CPLN     | 0.50*** | 0.43*** | 0.53*** | 0.54*** | 0.80*** |
| 3Y                                    | LN       | 0.15*** | 0.15*** | 0.23*** | 0.35*** | 0.30*** |
|                                       | FBLN     | 0.41*** | 0.29*** | 0.29*** | 0.49*** | 0.54*** |
|                                       | CPLN     | 0.27*** | 0.25*** | 0.29*** | 0.33*** | 0.33*** |
| 4Y                                    | LN       | 0.09*** | 0.09*** | 0.14*** | 0.21*** | 0.24*** |
|                                       | FBLN     | 0.26*** | 0.19*** | 0.24*** | 0.31*** | 0.33*** |
|                                       | CPLN     | 0.15*** | 0.17*** | 0.21*** | 0.28*** | 0.27*** |
| 5Y                                    | LN       | 0.05*** | 0.05*** | 0.08*** | 0.15*** | 0.16*** |
|                                       | FBLN     | 0.19*** | 0.13*** | 0.19*** | 0.25*** | 0.30*** |
|                                       | CPLN     | 0.09*** | 0.08*** | 0.13*** | 0.19*** | 0.15*** |

This table presents results from quantile regression of weight on market pressure measure (1/HKM) for the 3 best performing models. \*\*\* means significance at 1% level. \*\* means significance at 5% level. \* means significance at 10% level. Out-of-sample period is from Jan, 1987 to Dec, 2011.



Table 6: Quantile Regression - HPW (2013)

|                                       | Quantile | 0.10    | 0.25    | 0.50    | 0.75    | 0.90    |
|---------------------------------------|----------|---------|---------|---------|---------|---------|
| Panel A: Constant Volatility Models   |          |         |         |         |         |         |
| 2Y                                    | LN       | 0.22    | 0.27**  | 0.94*** | 1.74*** | 2.02*** |
|                                       | FBLN     | 0.73*** | 0.73*** | 1.49*** | 2.10*** | 2.09*** |
|                                       | CPLN     | 0.97*** | 1.12*** | 1.49*** | 2.21*** | 2.37*** |
| 3Y                                    | LN       | 0.05    | 0.22*** | 0.61*** | 1.18*** | 1.22*** |
|                                       | FBLN     | 0.55*** | 0.49*** | 1.15*** | 1.48*** | 1.24*** |
|                                       | CPLN     | 0.63*** | 0.68*** | 1.02*** | 1.45*** | 1.37*** |
| 4Y                                    | LN       | 0.07    | 0.14*** | 0.45*** | 0.94*** | 0.92*** |
|                                       | FBLN     | 0.45*** | 0.42*** | 0.91*** | 1.34*** | 1.42*** |
|                                       | CPLN     | 0.49*** | 0.51*** | 0.77*** | 1.14*** | 1.04*** |
| 5Y                                    | LN       | 0.05    | 0.09*   | 0.38*** | 0.74*** | 0.85*** |
|                                       | FBLN     | 0.39*** | 0.38*** | 0.75*** | 0.96*** | 0.78*** |
|                                       | CPLN     | 0.37*** | 0.40*** | 0.59*** | 0.89*** | 0.86*** |
| Panel B: Stochastic Volatility Models |          |         |         |         |         |         |
| 2Y                                    | LN       | 0.63*   | 0.19    | 0.35    | 0.60    | 0.06    |
|                                       | FBLN     | 1.39*** | 0.87*** | 0.83*** | 0.67**  | 0.63    |
|                                       | CPLN     | 1.07*** | 0.81*** | 0.99*** | 0.79**  | 0.27    |
| 3Y                                    | LN       | 0.34**  | 0.25*** | 0.28*** | 0.40*   | 0.45    |
|                                       | FBLN     | 0.79*** | 0.55*** | 0.46*** | 0.47*** | 0.22    |
|                                       | CPLN     | 0.56*** | 0.58*** | 0.57*** | 0.56*** | 0.22    |
| 4Y                                    | LN       | 0.24*** | 0.12**  | 0.30*** | 0.50*** | 0.30*   |
|                                       | FBLN     | 0.60*** | 0.33*** | 0.38*** | 0.38*** | 0.39**  |
|                                       | CPLN     | 0.47*** | 0.41*** | 0.43*** | 0.59*** | 0.43**  |
| 5Y                                    | LN       | 0.08    | 0.04    | 0.19*** | 0.32*** | 0.30**  |
|                                       | FBLN     | 0.41*** | 0.28*** | 0.31*** | 0.45*** | 0.44*** |
|                                       | CPLN     | 0.28*** | 0.24*** | 0.24*** | 0.39*** | 0.34*** |

This table presents results from quantile regression of weight on market pressure measure (HPW) for the 3 best performing models. \*\*\* means significance at 1% level. \*\* means significance at 5% level. \* means significance at 10% level. Out-of-sample period is from Jan, 1987 to Dec, 2011.

Table 7: Model Combinations and Predictability

|     | 2-Year     |      | 3-Year     |        | 4-Year     |         | 5-Year     |         |
|-----|------------|------|------------|--------|------------|---------|------------|---------|
|     | $R_{OS}^2$ | CER  | $R_{OS}^2$ | CER    | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| SBM | 5.74***    | 0.50 | 4.08***    | 1.16*  | 3.39***    | 1.77**  | 3.55***    | 2.84*** |
| EMA | 4.47***    | 0.07 | 4.07***    | 0.70   | 4.31***    | 1.49    | 4.24***    | 2.11*   |
| BMA | 5.24***    | 0.58 | 2.78***    | 0.15   | 3.06***    | 0.92    | 2.65***    | 0.93    |
| UMA | 4.00***    | 0.56 | 4.20***    | 1.50** | 4.44***    | 2.32*** | 4.60***    | 2.87*** |

The table presents out-of-sample R-squared and CER results for model combinations in the out-of-sample period. The  $OOS - R^2$  is given in equation (25). The statistical significance measure is from Clark and West (2007). \* means significance at 10% level. \*\* means significance at 5% level. \*\*\* means significance at 1% level. The CER is given in equation (32). Risk-aversion is 5. Out-of-sample period is from Jan, 1987 to Dec, 2011.

Table 8: **Out-of-Sample Predictability: Risk Aversions**

| Panel A: Constant Volatility Models   |              |               |              |               |              |               |              |               |
|---------------------------------------|--------------|---------------|--------------|---------------|--------------|---------------|--------------|---------------|
|                                       | 2-Year       |               | 3-Year       |               | 4-Year       |               | 5-Year       |               |
|                                       | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ |
| FB                                    | -1.11        | -0.03         | -1.14        | 0.58          | -0.50        | 0.93          | 0.07         | 1.03          |
| CP                                    | -1.05        | -0.50         | -1.66        | -0.31         | -1.52        | -0.22         | -1.72        | -0.22         |
| LN                                    | 0.49         | 1.34***       | 0.78         | 1.93***       | 1.95**       | 2.21***       | 2.66**       | 2.01**        |
| FBCP                                  | -1.12        | -0.28         | -1.55        | -0.09         | -0.90        | 0.31          | -0.15        | 0.30          |
| FBLN                                  | 0.12         | 1.05**        | -0.08        | 1.90***       | 0.57         | 2.20**        | 1.28         | 2.32**        |
| CPLN                                  | -0.13        | 0.84**        | -0.22        | 1.44**        | 0.48         | 1.57**        | 1.35         | 1.49          |
| FBCPLN                                | -0.11        | 0.69          | -0.50        | 1.34*         | -0.21        | 1.75*         | 0.33         | 1.79          |
| Panel B: Stochastic Volatility Models |              |               |              |               |              |               |              |               |
|                                       | 2-Year       |               | 3-Year       |               | 4-Year       |               | 5-Year       |               |
|                                       | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ |
| FB                                    | -1.19        | -0.03         | -1.68        | 0.25          | -0.66        | 0.50          | -0.08        | 0.44          |
| CP                                    | 0.02         | 0.71**        | -0.56        | 0.40          | -0.56        | 0.51          | 0.20         | 0.62          |
| LN                                    | 0.36         | 1.49***       | 1.10         | 2.34***       | 1.90*        | 2.33***       | 2.42**       | 2.02***       |
| FBCP                                  | -0.92        | 0.10          | -1.18        | 0.30          | -0.32        | 0.63          | 0.44         | 0.68          |
| FBLN                                  | 0.31         | 1.18***       | 0.45         | 1.79***       | 1.03         | 2.00**        | 1.36         | 1.71          |
| CPLN                                  | 0.45         | 1.27***       | 0.40         | 1.84***       | 1.15         | 2.18***       | 2.23**       | 2.12**        |
| FBCPLN                                | 0.28         | 1.18***       | 0.25         | 1.65**        | 0.73         | 1.89**        | 1.56         | 1.76          |
| Panel C: Model Combinations           |              |               |              |               |              |               |              |               |
|                                       | 2-Year       |               | 3-Year       |               | 4-Year       |               | 5-Year       |               |
|                                       | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ | $\gamma = 3$ | $\gamma = 10$ |
| SBM                                   | 0.33         | 1.47***       | 0.35         | 1.92***       | 0.86         | 1.68*         | 2.28**       | 1.80*         |
| EMA                                   | 0.15         | 0.94**        | -0.19        | 1.53***       | 0.46         | 1.74**        | 1.09         | 1.58*         |
| BMA                                   | 0.53         | 1.46***       | -0.73        | 1.02          | 0.08         | 1.07          | 0.51         | 0.67          |
| UMA                                   | 0.47         | 1.30***       | 0.74         | 2.10***       | 1.45*        | 2.17***       | 2.16**       | 1.92**        |

The table presents a robustness check of Certainty Equivalent Returns for linear models, stochastic volatility models and model combination schemes, relative to the EH benchmark, in the out-of-sample period. The CER is given in equation (32). Risk aversion coefficient  $\gamma$  is 3 or 10. Out-of-sample period is from Jan, 1987 to Dec, 2011.

Table 9: **Out-of-Sample Predictability: 1982-2011**

| Panel A: Constant Volatility Models   |            |        |            |         |            |         |            |         |
|---------------------------------------|------------|--------|------------|---------|------------|---------|------------|---------|
|                                       | 2-Year     |        | 3-Year     |         | 4-Year     |         | 5-Year     |         |
|                                       | $R_{OS}^2$ | CER    | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| FB                                    | 3.87***    | -0.10  | 3.36***    | 0.89    | 3.48***    | 1.79*   | 3.45***    | 2.16*   |
| CP                                    | -3.69      | -1.17  | -2.08      | -0.89   | -0.99      | -0.61   | -0.39      | -0.46   |
| LN                                    | 2.77***    | 0.65   | 3.48***    | 1.94**  | 4.29***    | 3.10*** | 3.98***    | 3.28*** |
| FBCP                                  | 1.17**     | -0.39  | 1.43**     | 0.24    | 2.27**     | 1.10    | 2.51***    | 1.39    |
| FBLN                                  | 5.96***    | 0.73   | 5.48***    | 1.87**  | 5.61***    | 3.18*** | 5.63***    | 3.86*** |
| CPLN                                  | -0.18***   | 0.25   | 1.39***    | 1.25    | 2.39***    | 2.31**  | 2.71***    | 2.63**  |
| FBCPLN                                | 3.84***    | 0.69   | 4.31***    | 1.66*   | 4.64***    | 2.55**  | 4.79***    | 3.13**  |
| Panel B: Stochastic Volatility Models |            |        |            |         |            |         |            |         |
|                                       | 2-Year     |        | 3-Year     |         | 4-Year     |         | 5-Year     |         |
|                                       | $R_{OS}^2$ | CER    | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| FB                                    | 3.16**     | -0.25  | 3.41***    | 0.34    | 3.73***    | 1.25    | 3.46***    | 1.47    |
| CP                                    | -1.27      | -0.14  | -0.30      | 0.20    | 0.28       | 0.51    | 0.69       | 0.85    |
| LN                                    | 5.05***    | 0.37   | 4.09***    | 1.71**  | 4.06***    | 2.46*** | 3.38***    | 2.56*** |
| FBCP                                  | 3.32***    | -0.22  | 3.60***    | 0.50    | 3.90***    | 1.44    | 4.12***    | 1.78    |
| FBLN                                  | 9.05***    | 1.06** | 7.30***    | 2.12*** | 6.79***    | 2.87*** | 5.98***    | 2.94*** |
| CPLN                                  | 4.05***    | 0.56   | 3.97***    | 1.62**  | 4.29***    | 2.32*** | 3.97***    | 2.84*** |
| FBCPLN                                | 8.52***    | 1.09** | 7.12***    | 2.11*** | 6.65***    | 2.70**  | 6.08***    | 3.04**  |
| Panel C: Model Combinations           |            |        |            |         |            |         |            |         |
|                                       | 2-Year     |        | 3-Year     |         | 4-Year     |         | 5-Year     |         |
|                                       | $R_{OS}^2$ | CER    | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| SBM                                   | 6.62***    | 0.99** | 3.93***    | 1.38*   | 3.88***    | 2.49*** | 4.76***    | 3.71*** |
| EMA                                   | 6.41***    | 0.70   | 5.47***    | 1.70**  | 5.19***    | 2.38*** | 4.90***    | 2.84*** |
| BMA                                   | 5.13***    | 0.54   | 4.85***    | 0.94    | 4.39***    | 1.70*   | 4.08***    | 1.70    |
| UMA                                   | 5.27***    | 0.63   | 5.03***    | 1.78**  | 4.90***    | 2.46*** | 4.88***    | 2.88*** |

This table presents the out-of-sample R-squared and CERs for linear models, stochastic volatility forecasting models and model combinations. The  $R^2$  is given in equation (25). The statistical significance measure is from Clark and West (2007). \* means significance at 10% level. \*\* means significance at 5% level. \*\*\* means significance at 1% level. The CER is given in equation (32). Risk aversion is 5. The out-of-sample period is from Jan, 1982 to Dec, 2011.

Table 10: **Out-of-Sample Predictability: 1990-2011**

| Panel A: Constant Volatility Models   |            |       |            |         |            |         |            |         |
|---------------------------------------|------------|-------|------------|---------|------------|---------|------------|---------|
|                                       | 2-Year     |       | 3-Year     |         | 4-Year     |         | 5-Year     |         |
|                                       | $R_{OS}^2$ | CER   | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| FB                                    | 0.87*      | -0.79 | 2.23**     | 0.10    | 2.69**     | 0.91    | 2.93**     | 1.31    |
| CP                                    | -1.81      | -1.08 | -0.11      | -0.64   | 0.31       | -0.42   | 0.65       | -0.30   |
| LN                                    | 1.61***    | 0.33  | 3.78***    | 1.46**  | 5.01***    | 2.39*** | 5.08***    | 2.75*** |
| FBCP                                  | -0.70*     | -0.94 | 0.76*      | -0.27   | 2.04**     | 0.65    | 2.40**     | 1.07    |
| FBLN                                  | -0.04***   | 0.02  | 2.72***    | 0.97    | 3.48***    | 2.00**  | 4.00***    | 2.75**  |
| CPLN                                  | -1.94***   | -0.08 | 1.97***    | 0.95    | 3.43**     | 2.05**  | 3.90**     | 2.46**  |
| FBCPLN                                | -1.59***   | 0.01  | 1.55***    | 0.77    | 2.78***    | 1.58    | 3.42***    | 2.23    |
| Panel B: Stochastic Volatility Models |            |       |            |         |            |         |            |         |
|                                       | 2-Year     |       | 3-Year     |         | 4-Year     |         | 5-Year     |         |
|                                       | $R_{OS}^2$ | CER   | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| FB                                    | -1.14      | -0.76 | 1.02*      | -0.15   | 1.91**     | 0.90    | 2.31**     | 1.44    |
| CP                                    | 0.24       | 0.12  | 0.68       | 0.61    | 0.93       | 0.79    | 1.32       | 1.05    |
| LN                                    | 5.94***    | 0.32  | 5.24***    | 1.82**  | 4.68***    | 2.49*** | 4.04***    | 2.66*** |
| FBCP                                  | 0.31*      | -0.55 | 1.73**     | 0.18    | 2.54**     | 1.20    | 3.14**     | 1.74    |
| FBLN                                  | 5.81***    | 0.31  | 5.84***    | 1.72**  | 5.47***    | 2.57*** | 5.31***    | 3.05*** |
| CPLN                                  | 4.85***    | 0.54  | 4.76***    | 1.71*** | 5.09***    | 2.66*** | 5.18**     | 3.40*** |
| FBCPLN                                | 5.37***    | 0.39  | 5.45***    | 1.53**  | 5.51***    | 2.32**  | 5.39***    | 3.19*** |
| Panel C: Model Combinations           |            |       |            |         |            |         |            |         |
|                                       | 2-Year     |       | 3-Year     |         | 4-Year     |         | 5-Year     |         |
|                                       | $R_{OS}^2$ | CER   | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     | $R_{OS}^2$ | CER     |
| SBM                                   | 6.58***    | 0.51  | 4.89***    | 1.34**  | 4.00**     | 1.92**  | 4.04**     | 3.08*** |
| EMA                                   | 5.70***    | 0.10  | 5.34***    | 1.13**  | 5.24***    | 2.01**  | 5.28***    | 2.58*** |
| BMA                                   | 5.79***    | 0.55  | 4.19***    | 0.75    | 4.13**     | 1.73*   | 3.67**     | 1.75    |
| UMA                                   | 4.20***    | 0.27  | 4.88***    | 1.29**  | 5.37***    | 2.37*** | 5.51***    | 2.65**  |

This table presents the out-of-sample R-squared and CERs for linear models, stochastic volatility forecasting models and model combinations. The  $R^2$  is given in equation (25). The statistical significance measure is from Clark and West (2007). \* means significance at 10% level. \*\* means significance at 5% level. \*\*\* means significance at 1% level. The CER is given in (32). Risk-aversion is 5. The out-of-sample period is from Jan, 1990 to Dec, 2011.

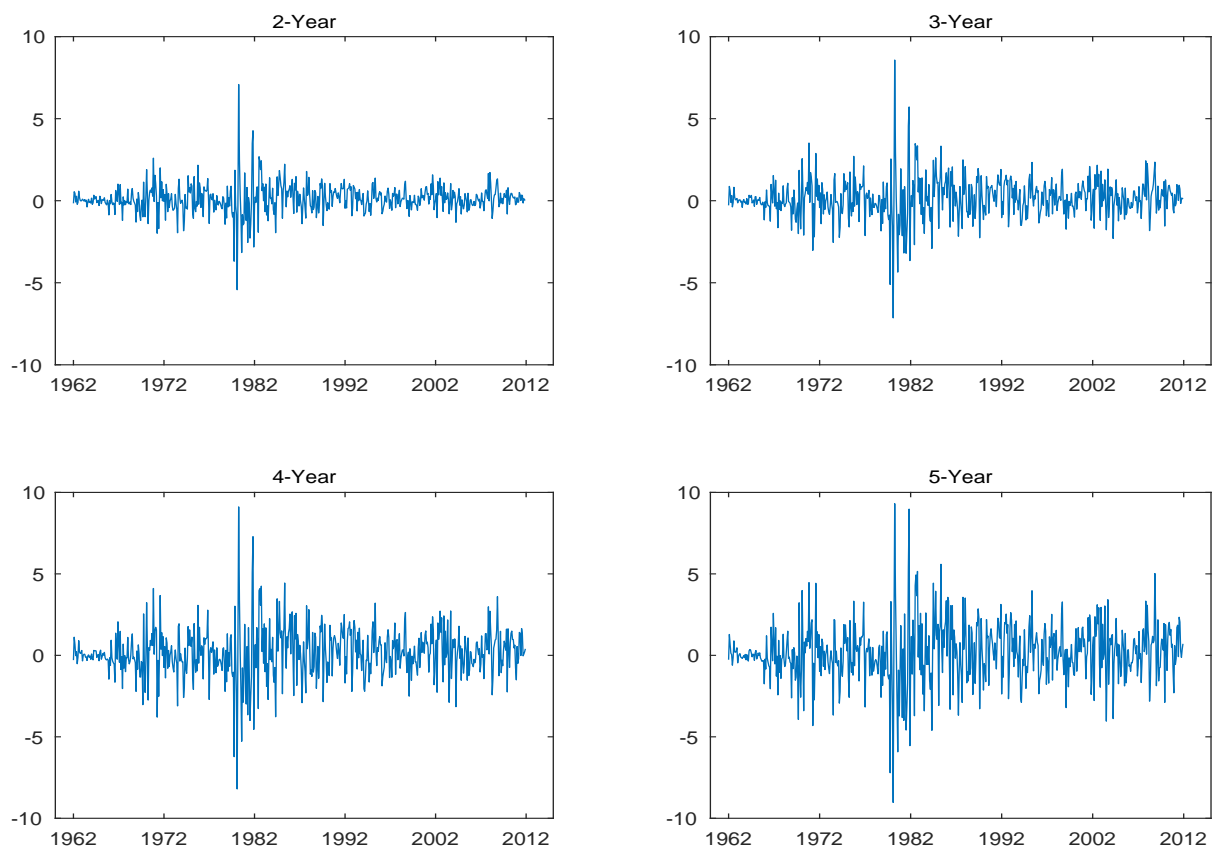
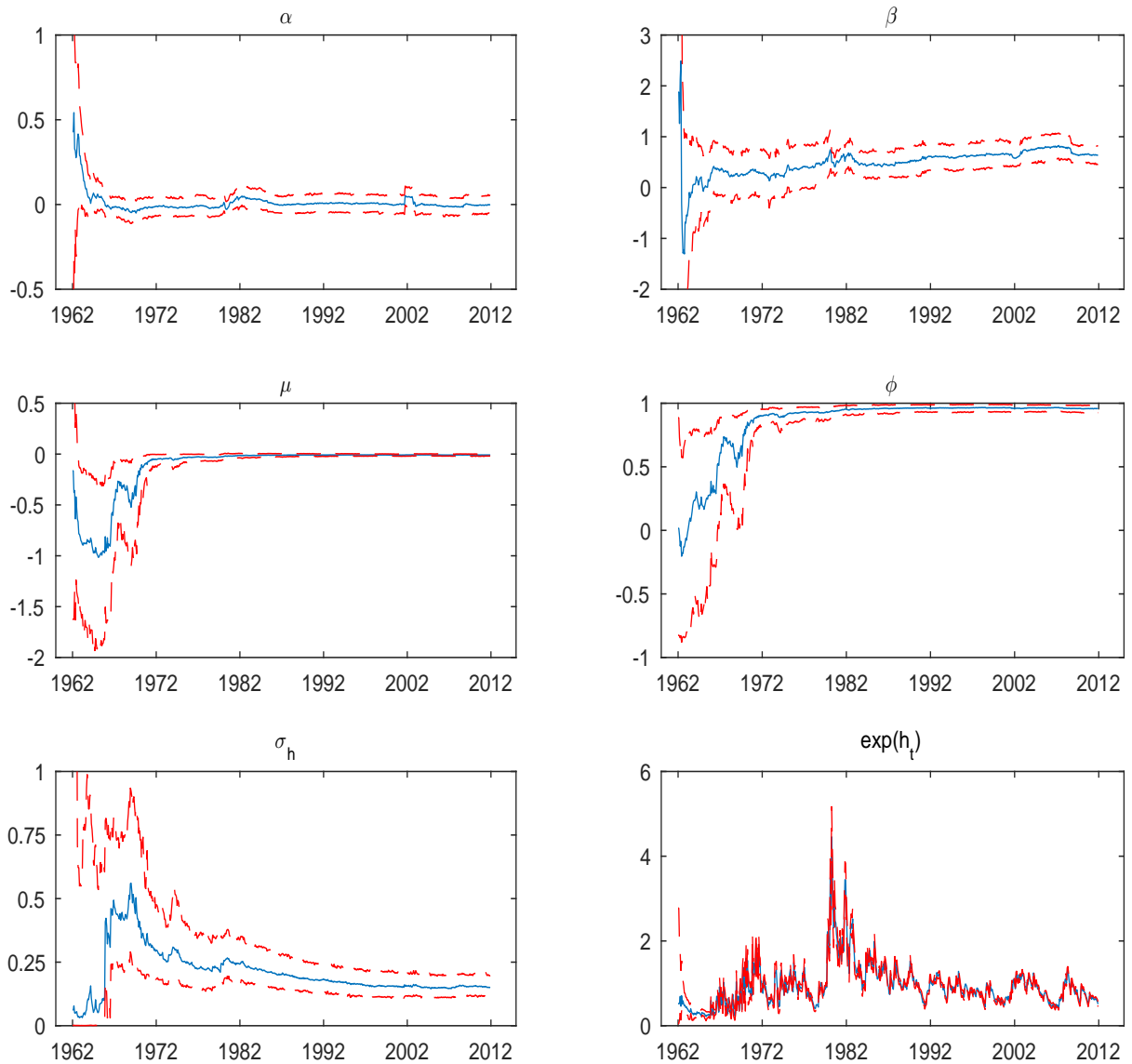


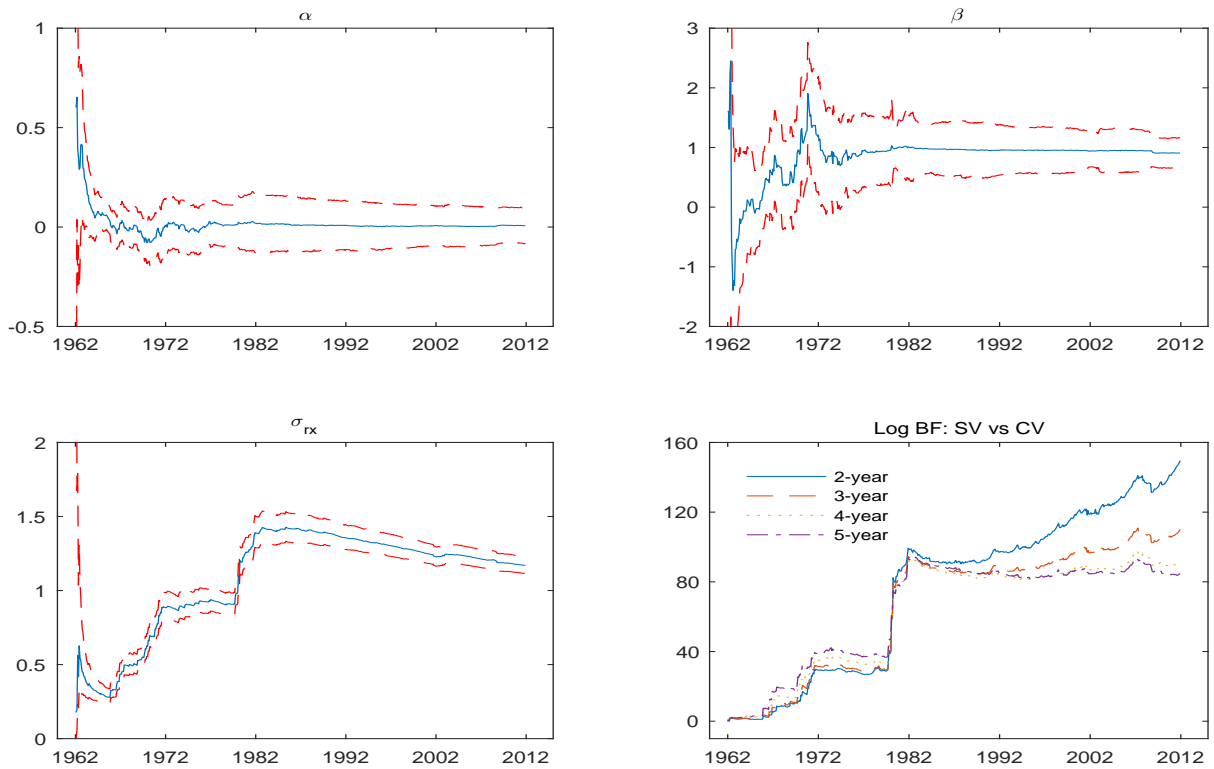
Figure 1: **The Time Series of Excess Bond Returns**

This figure plots the time series of 4 excess bond returns (in percentage), from Jan, 1962 to Dec, 2011.



**Figure 2: Parameter Learning for LN-SV**

The figure shows time series parameter estimates of stochastic volatility model with LN predictor for 3-year bond excess returns. The estimation is based on full-sample information. The model form is given in equation (5) and (6). The last panel shows the stochastic volatility estimate. The two dashed lines are 5-th and 95-th percentiles of estimate distribution. The solid line is the mean estimate for each parameter. Full-sample is from Jan, 1962 to Dec, 2011.



**Figure 3: Parameter Learning for LN-CV and Log Bayes Factors**

The figure shows time series parameter estimates of constant volatility model with LN predictor for 3-year bond excess returns. The estimation is based on full-sample information. The linear model form is given in equation (4). The last panel shows the log Bayes factor of LN-SV and LN-CV models, for all 4 maturities. The two dashed lines are 5-th and 95-th percentiles of estimate distribution. The solid line is the mean estimate for each parameter. Full-sample is from Jan, 1962 to Dec, 2011.





Figure 4: **The  $cumSSE$ 's for Individual Models**

This figure shows  $cumSSE$ 's for CP-CV, CP-SV, LN-CV and LN-SV models.  $cumSSE$  is given in equation (26). Out-of-sample period is from Jan, 1987 to Dec, 2011.

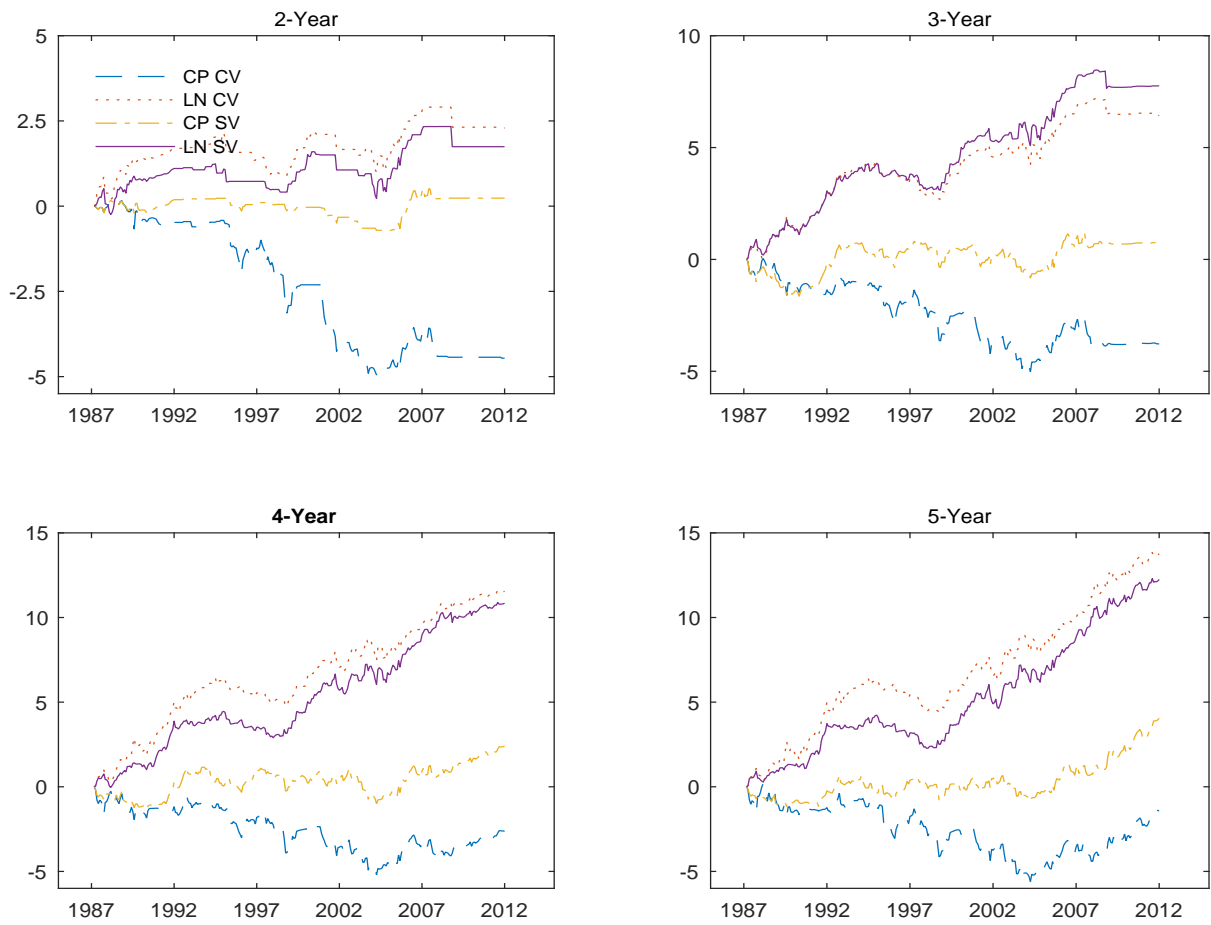


Figure 5: **The *cumCER*'s for Individual Models**

This figure shows *cumCER*'s for CP-CV, CP-SV, LN-CV and LN-SV models. *cumCER* is given in equation (33) and equation (?). Out-of-sample period is from Jan, 1987 to Dec, 2011.

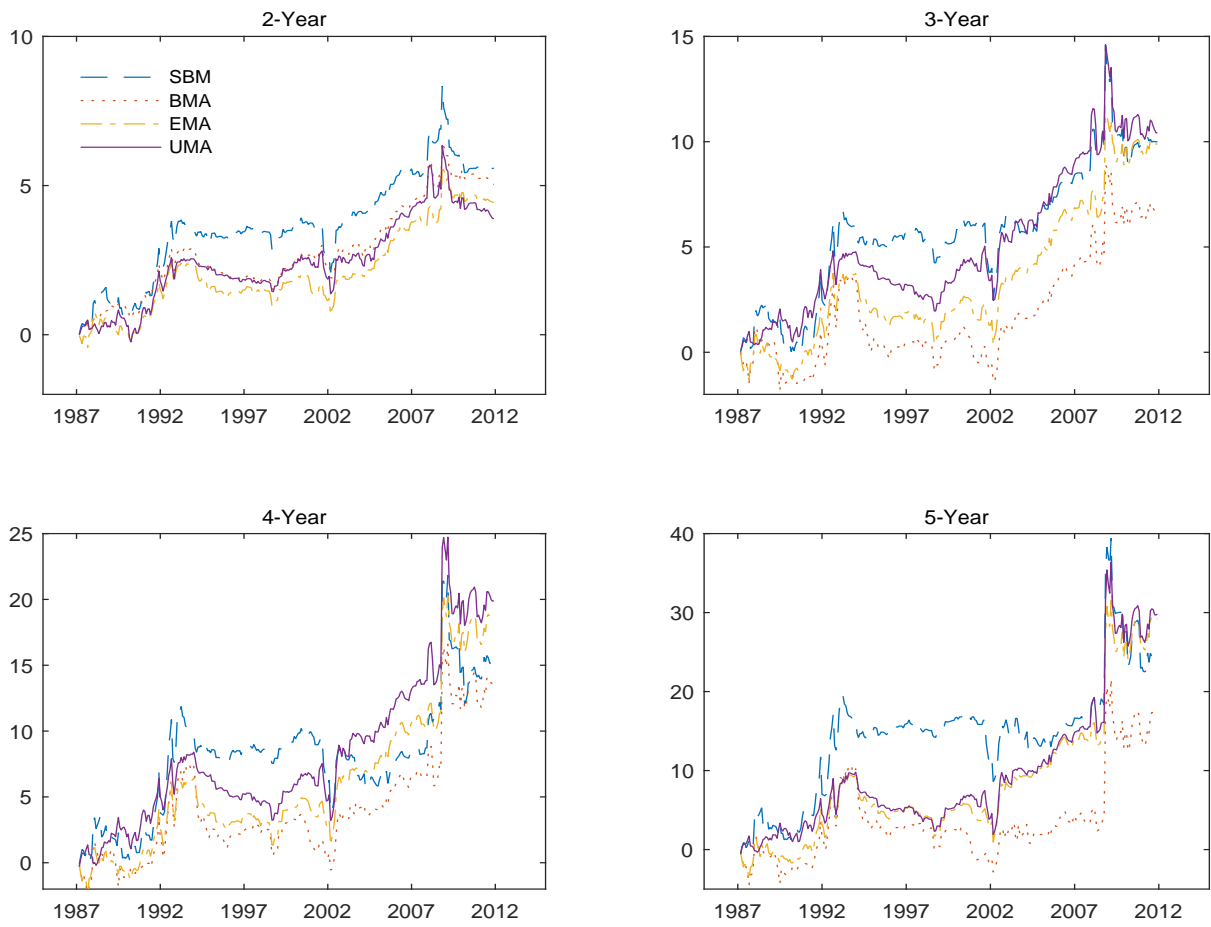


Figure 6: **The  $cumSSE$ 's for Model Combinations**

This figure shows  $cumSSE$ 's for four model combination schemes.  $cumSSE$  is given in equation (26). Out-of-sample period is from Jan, 1987 to Dec, 2011.

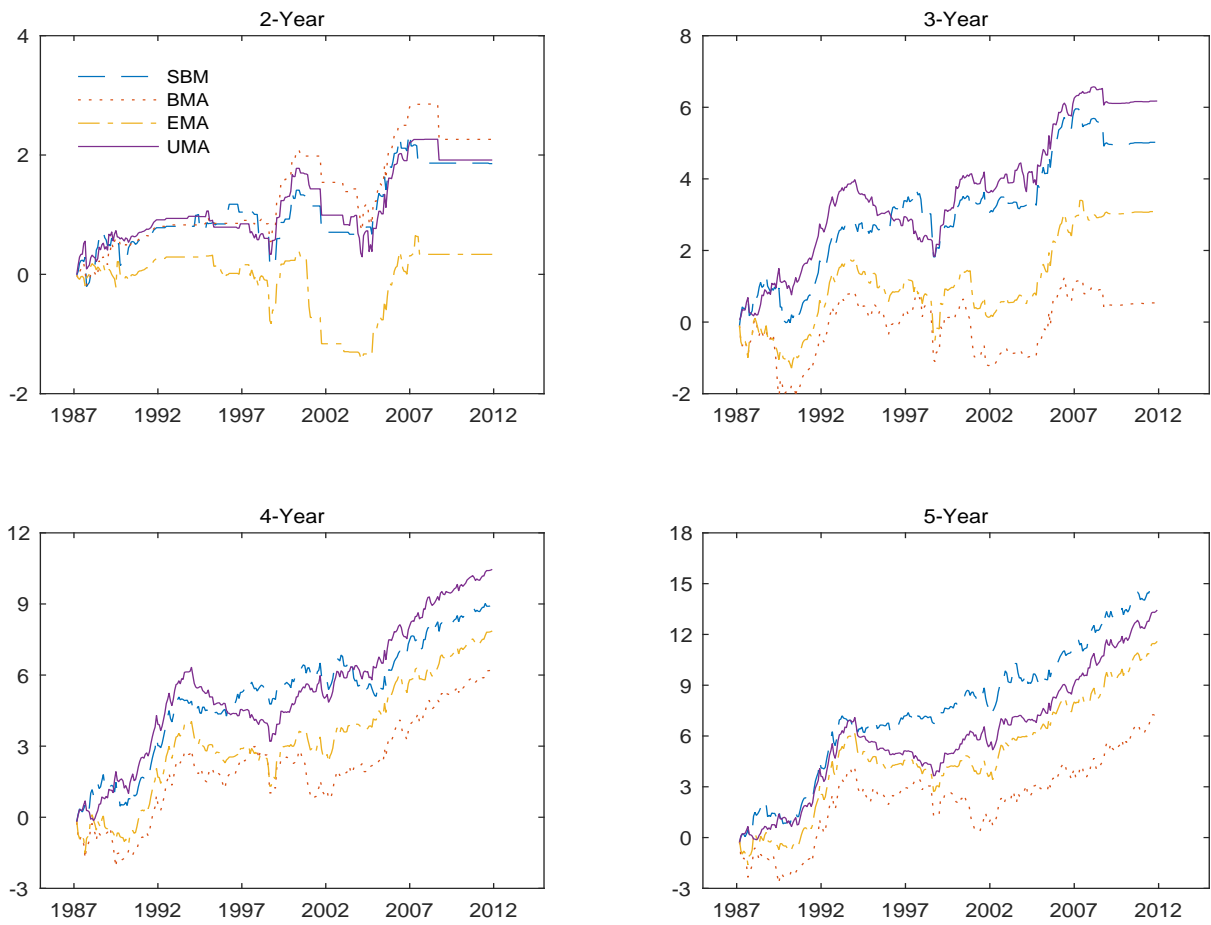


Figure 7: **The *cumCER*'s for Model Combinations**

This figure shows *cumCER*'s for four model combination schemes. *cumCER* is given in equation (33) and (?). Out-of-sample period is from Jan, 1987 to Dec, 2011.

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