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Illiquidity and Investment Decisions: A Survey

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Illiquidity and Investment Decisions: A Survey

Abstract

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Investing in illiquid assets has become increasingly popular among individual and institutional investors. Illiquid assets provide higher returns and interesting diversification alternatives for asset managers. To the contrary of traditional assets that investors are familiar with, the alternative investments require deep knowledge of their intrinsic properties. This knowledge is necessary to value the underlying and determine their share in portfolios. As the last financial crisis showed, misgauging the implications of illiquidity can have severe consequences. The current article surveys the existing literature that enhances our understanding of illiquid assets, their attractiveness, their valuation, and most importantly, their optimal allocation in investment portfolios.

Keywords: illiquid assets, alternative investments, illiquidity premium, asset allocation

JEL classification: G11; G12.

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1 Introduction

In the past decades, illiquid assets have emerged as investment vehicles and are gaining momentum in portfolio allocation. As a result, an increasing amount of wealth is allocated to venture capital, private equity, initial public offerings, commercial real estate, and hedge funds. These assets are usually priced at a discount due to their illiquidity. The high volatility of financial markets combined with the low interest rates have led to a growing interest in private markets among investors seeking attractive return profiles. The 2017 McKinsey Global Wealth & Asset Management Practice reports that private markets account in total for 7.5% of global assets under management at the end of the year 2016. Further, this figure is set to grow by 5% per year over a medium term. This highlights the necessity and the relevance of improving our understanding of the illiquidity discounts on these assets (Chen, Dyl, Jiang, and Juneja, 2015).

Asset pricing studies featuring illiquidity have also become increasingly popular in the empirical literature. Compared to traditional assets such as stocks, bonds and currencies, some inherent features of illiquid assets such as attractive risk-return tradeoffs and high diversification potential, make them particularly valuable in investment decisions. In general, traditional financial assets are held only for the purpose of generating returns while their imbedded utility is irrelevant for other considerations, although there are few exceptions such as shares held e.g. for voting rights. For alternative asset classes there are other considerable factors that drive investment decisions, for example inflation hedging and liability matching. Moreover, unlike many factor risks, illiquidity risk cannot be diversified away, i.e., one cannot offset an illiquidity exposure by going short an illiquid security. This is just a short illustration that shows the growing interest and motivation for studies on illiquid assets.

Going further, Swensen (2009, page 82) argues that markets are typically illiquid, "... since rewarding investments tend to reside in dark corners, not in the glare of floodlights". Investors thus face an important dilemma. In traditional markets there are few opportunities to enjoy excess returns. In contrast, non traditional markets which are illiquid by nature are more subject to market inefficiencies and provide opportunities to harvest excess returns. However, in such markets it is difficult to use standard investment decision processes which rely on the existence of statistical characteristics of assets (means, variances, correlations, and higher-order moments) that are not easily available for illiquid assets. Besides, minimal disclosure requirements and specialized investment mandates in illiquid assets provide investors with skills a unique opportunity to harness market inefficiencies.¹

The latest financial crisis highlighted the danger of inadequate allocations to illiquid assets and has urged major investors to carefully assess the trade-off between harnessing the potential of alternative assets and meeting their investment goals. The lack of appropri-

¹This is in stark contrast to standard asset pricing models derived under assumptions that are appropriate for traditional financial markets, i.e. with transparent, liquid, and low-friction transactions.

ate and sound investment process that includes illiquidity proves to be very harmful and damaging for high profile investors. This has been featured in major financial and investment outlets where even well-established investors such as CalPERS, Harvard, Yale, etc., experienced severe liquidity constraints.²

In light of the critical importance of alternative assets and the growing reliance on them, a coherent and methodological approach to handle the full scope of illiquidity within the existing modern portfolio framework (Capital Asset Pricing Model, Arbitrage Pricing Theory, and their derivatives, for example the Black-Litterman model) with an emphasis on practical implementation is a paramount. The basic premise being investigated is whether assets with different levels of liquidity will earn different returns. In other words, the goal is to assess whether illiquidity is a significant criterium to determine portfolio performance, i.e. is illiquidity a priced source of risk distinct from previously identified factor risks?

The aim of this survey is to review the literature on theoretically well-grounded and practical problems related to investment decisions involving illiquid assets. The article reviews different illiquidity measurement methods as well as frameworks that integrate illiquid assets into the investment process. The ultimate goal is to provide the reader with answers to practical questions relevant to investment decisions when illiquid securities belong to the asset menu. Such questions may for example relate to the major sources of asset illiquidity, the portfolio exposure to market-wide illiquidity factors, empirical challenges when dealing with illiquid asset data, or the weight on illiquid assets in a portfolio alongside traditional liquid assets.

Despite being far from exhaustive given the huge amount of work covered by the extant literature, the present survey tries to provide an interesting mix of both academic, industry and policy-related studies analyzing illiquidity in asset pricing and portfolio choice, with a focus on the most relevant aspects for applied research and practical implementation of new investment strategies.

2 Importance of liquidity in investments

Investors benefit from liquidity in their investment decisions, and depending on their use of these benefits, the components of the portfolio can be classified either as assets or as liabilities. According to Kinlaw, Kritzman, and Turkington (2013), liquidity has some appealing features in the portfolio choice context, such as being able to

- rebalance a portfolio;
- meet capital calls;
- reallocate part of the portfolio to newly discovered opportunities;

²See for example Tett (2010), Hechinger (2009), Pensions&Investments (2009), and Bloomberg (2010).

- exit from unproductive investments; and
- respond to shifts in risk tolerance.

We discuss these features in more detail.

2.1 Rebalancing

If market conditions change considerably to render initial asset allocation less efficient, investors can improve the portfolio risk-return tradeoff by tilting initial allocation towards the optimal one to account for new market conditions. The latest financial crisis was characterized by a sudden and severe impairment of liquidity across a range of asset markets. As a result, banks were often unable to promptly exit or hedge certain risky illiquid positions without materially affecting market prices. Moreover, large swings in illiquidity premia, defined as the additional compensation required by investors to hold illiquid instruments, led to substantial mark-to-market losses on fair-valued instruments as liquidity conditions deteriorated rapidly. Over time, rebalancing is necessary to offset the effects of different returns on various asset classes. Because transactions are not cost-free, investment managers need a strategy to determine when rebalancing should be undertaken. Sun, Fan, Chen, Schouwenaars, and Albot (2006) used dynamic programming to develop a rebalancing approach that weighs trading costs against the expected costs of not acting. Higher liquidity risk bears an implicit cost as it decreases the opportunity to rebalance the portfolio weights to comply with the target weights. Buchner (2016) argues that there is an incentive to rebalance when correlations with liquid securities are low. Terhaar, Staub, and Singer (2003) show that the performance of illiquid assets fairly compensates for the lack of flexibility to rebalance.

2.2 Capital calls

The tradeoff between performance, risk and liquidity is a key element in managing investments subject to inflows and outflows of cash. Analysing open-end funds, Edelen (1999) estimates the direct liquidity costs from forced transactions to about 1.5–2% annually. Chordia (1996) concludes that open-end fund managers must hold a cash position larger than they would otherwise need in order to deal with the potential redemptions.³ Some investors (pension funds, endowments, etc.) may immediately need cash to meet unanticipated expenses/revenues. The financial crisis of 2007-2009 highlighted the importance of liquidity to many investors. That is, widespread liquidity shortage forced investors to meet outflows and margin calls where prices dropped as liquidity melted. University endowment funds⁴ for example were forced to sell publicly traded securities at substantially depressed values in order

³According to the Wall Street Journal issue of December 8, 2007, more than \$32 billion of assets managed by Putnam were redeemed in a single month.

⁴Including Harvard, Yale, Norwegian Sovereign Fund and CalPERS

to meet funding commitments to private investments. Hedge funds engaged in fire sales of publicly traded securities to meet margin calls from lenders and redemption demands from clients, and financial institutions sold assets at substantial discounts. Such spending needs must come from cash or public security holdings, regardless of prevailing market conditions and valuation discrepancies. Investors could guard against these capital calls by maintaining a reserve of highly liquid securities (Siegel, 2008), but this protection would require them to sacrifice the expected return premium of less liquid investments. Thus the necessity of keeping a reasonable share of liquid assets to face outflows without burdening a prohibitive liquidity cost is salient.

As pointed out by Holmstrom and Tirole (2001), liquidity is defined in terms of firms' financial constraints. Firms avoid assets whose return is low when constraints are severe. However, when assets are subject to a lot of constraints in terms of illiquidity, the expected return is high in equilibrium.⁵ From Amihud (2002) and Acharya and Pedersen (2005), illiquidity takes the form of exogenous time-varying transaction costs. An increase in the costs of trading an asset raises the expected return that investors require to hold it and lowers its price. With yet another motivation to analyze the impact of illiquidity, Garleanu and Pedersen (2011) introduce margin constraints in an infinite horizon setting with multiple assets. They show that assets with higher margin requirements earn higher expected returns and are more sensitive to changes in the wealth of the margin-constrained agents. When market prices fall, market participants need funding to finance their positions which become credit constrained (e.g., due to margin calls). This forces them to liquidate their assets, which in turn makes the market less liquid and makes liquidity more valuable. As liquidity can prove very harmful to investors, taking a somewhat different approach, Golts and Kritzman (2010) introduce an options valuation framework where investors can purchase liquidity options to meet unscheduled capital calls.

2.3 Asset restrictions

Restrictions on assets have consequences in at least two dimensions. First, they don't allow investors to cease new opportunities, and second, they don't allow investors to exit their position when it no longer appears interesting.

2.3.1 New opportunities

New discoveries in market opportunities and/or strategies present yet another motivation for holding liquid assets, since only liquid assets allow to cease these opportunities. Illiquid assets face various restrictions. The most common is a restriction on redemptions on some assets that limits investor liquidity. Everything else equal, such an asset should promise the

⁵In the literature there are numerous approaches to model illiquidity in the context of financial constraints for a firm.

investor a positive illiquidity premium. Lo, Petrov, and Wierzbicki (2003) add liquidity as an additional constraint within the Markowitz’s mean-variance framework. Ang, Papanikolaou, and Westerfield (2014) investigate the impact of trading restrictions on investment decisions and found that risk aversion is positively related to, and leads to significant distortions in illiquid shares in the portfolio. Beachkofski, Gallardo, Kuo, Lim, Liu, and Tovikkai (2011) assess portfolio performance under trading restrictions and conclude that restrictions can considerably impair investment opportunities and associated utility gains. Aragon (2007) documents a significant returns spread of about 3–7% between hedge funds with and without lockup periods, suggesting that restrictions enable some funds to harvest illiquidity rents.

2.3.2 Exit from unproductive investments

Investments that were initially held to meet some investment goals can be withdrawn when failed to achieve a pre-specified performance hurdle rate.⁶ To do so, the current positions need to be liquid enough. This necessity was illustrated in the recent financial crisis. Many shareholders have suffered great losses in down markets and, at the same time, faced with liquidity restrictions which forbid selling shares (Kahl, Liu, and Longstaff, 2003). This reduces the managers’ willingness to hold illiquid assets, and raises the illiquidity premium. In this context, withdrawals become more likely when performance falls below a certain threshold. For example, in an effort to reduce complexity and costs in its investment program, the California Public Employees’ Retirement System (CalPERS) recently decided to spread its entire \$4 billion investment in twenty-four hedge funds and six hedge funds-of-funds.⁷ Intuitively, investors are willing to pay more for assets that allow them to exit positions at a reasonable cost during pervasive market declines or liquidity dry-ups. Derman (2007) introduces a valuation framework where investors may swap capital invested in underperforming funds to the outperforming ones in the absence of lockup period and trade restrictions.

2.4 Shifting risk tolerance

Investors can change their risk aversion so that initial asset allocation departs significantly from risk appetite. It is generally accepted and empirically verified that investors’ effective risk-aversion increases with volatility. Thus, in times of high volatility, illiquid assets comove with the market more strongly than other assets, and this is also the case when managers are the most risk-averse. Ang, Papanikolaou, and Westerfield (2014) show that fluctuations

⁶The relevant example is from the Norwegian Sovereign Fund that reconsiders its allocations in real estate, as featured in Reuters (<http://www.reuters.com/article/us-norway-swf-realestate-idUSKBN16L0U3>) and Wall Street Journal (<https://www.wsj.com/articles/norway-fund-bulks-up-on-real-estate-1424795145>).

⁷Ted Eliopoulos, CalPERS Interim Chief Investment Officer, stated, “hedge funds are certainly a viable strategy for some, but at the end of the day, when judged against their complexity, cost, and the lack of ability to scale at CalPERS’ size, the ARS program is no longer warranted.” (Source: <http://www.calpers.ca.gov/index.jsp?bc=/about/newsroom/news/eliminate-hedge-fund.xml>).

in the share of illiquid assets in the portfolio go hand-in-hand with endogenous time-varying risk aversion.

To sum up, liquidity provides a necessary tool to investors in order to be flexible and quickly react to changing market environment. However, due to many reasons already mentioned such as attractive risk-return tradeoffs, high diversification potential, inflation hedging, liability matching, and a large range of exploitable market inefficiencies, investors increasingly pay more attention to illiquid assets.

3 The rise of illiquid assets in investments

In this section, we review the theory of illiquidity premium and its asset-pricing implications.

3.1 Defining illiquidity

To underscore the difficulties inherent in defining and measuring illiquidity, Amihud (2002) points out that “liquidity can be regarded as empirical proxies that measure different aspects of illiquidity. It is doubtful that there is one single measure of liquidity.”⁸ Inspired by the definition of high-quality liquid asset (HQLA) in Basel III, assets are considered HQLA if they can be easily and immediately converted into cash at little or no cost. The liquidity of an asset under Basel Committee (2013) can be related to its fundamental and market characteristics as follows:

- *Fundamental characteristics*: low risk, ease and certainty of valuation (high degree of agreement on valuation is likely with standardization, homogeneity, and simplicity of products), low correlation with risky assets, and listing on a developed and recognized exchange. In that sense, Eisfeldt (2004) finds that liquidity and productivity are closely related both across industries and economies.
- *Market-related characteristics*: active, sizeable market (low bid-ask spreads, high trading volumes, large and diverse number of [committed] market participants), low volatility (of price and volume during stressed periods), and flight to quality (assets typically sought in times of systemic crisis). As pointed out by Constantinides (1986) and Lo, Mamaysky, and Wang (2004), the quantity limitations/volumes⁹ can generate an illiquidity premium. This intuition is further proven by Longstaff (2001) and Kahl, Liu,

⁸Lancaster (1966) provides a stylized microeconomic foundation of asset features where the intrinsic characteristics of what goods or assets contain are more relevant than assets themselves. Ang, Goetzmann, and Schaefer (2009, 2011) describe an asset class as a collection of fundamental factors or beta sources (e.g., credit risk, term risk, currency, liquidity, value/growth, small/large, momentum and volatility) and suggest that asset classes should be viewed as vehicles for underlying factor exposures that drive risk and return.

⁹The way the size of the trade impacts the liquidity can be stylized along the lines of Glosten (1989) who finds that the trade size is an important dimension of the liquidity factor.

and Longstaff (2003) who show that, c.p., illiquidity premia are higher for more volatile assets and for more risk-averse agents.

Keeping in mind the fundamental and market-related characteristics to determine the illiquidity of an asset, we can now have a closer look at the main sources of illiquidity.

3.2 Sources of illiquidity

There are three major sources of illiquidity:

- *Transaction costs*: Usually they affect more short-term investments than long-term investments and have greater influence on trading frequency.
- *Market organization*: The structure of markets (trading exchange, peer to peer, etc) is one fundamental element of liquidity.
- *Diversity in valuations*: Difference in opinions about the real/fair value of an asset.

We discuss these measures in more detail.

3.2.1 Transactions costs

Transaction cost related to illiquidity comes from the fact that less liquid financial assets have higher execution costs to investors, and therefore investors are not willing to pay as much as they would for pure liquid assets which are easy to sell quickly and at relatively fair price levels. This refers to limited liquidity (Amihud and Mendelson, 2012). Lower liquidity increases uncertainty of investors who therefore demand a higher discount to compensate the lack of marketability (Chen, Dyl, Jiang, and Juneja, 2015). This illiquidity cost is based on three main components: direct trading costs, price-impact costs and opportunity costs.

Direct transaction costs Classical securities are partially marketable and can always be traded at posted prices but with transactions costs. As pointed out by Constantinides (1986), if two otherwise identical assets with the same characteristics except that the first asset is subject to a proportional transaction cost but the second is not, are held in equilibrium, the expected return on the first asset must exceed that of the liquid counterpart by some illiquidity premium. According to Constantinides (1986) and subsequent literature (Jang, Keun Koo, Liu, and Loewenstein, 2007, Liu and Loewenstein, 2002 and Dai, Li, Liu, and Wang, 2016), the illiquidity premium represents the maximum expected return an investor is willing to exchange for zero transaction cost.¹⁰ The direct transaction cost of an asset is

¹⁰Li, Mooradian, and Zhang (2007) document a strong positive relationship between equity returns and the aggregate commission rate for NYSE trading, which itself exhibits a strong correlation with illiquidity

closely related to its holding period and the frequency of trading. Direct trading costs include brokerage commissions, exchange fees and taxes that materialize due to the transaction. Higher transaction costs lead to low returns on short-term investments and result in irregular trading. The constraint to trade at will can reduce the demand of an asset and generates significant illiquidity discount; thus the necessity of accepting either a longer duration of sale or a lower price. This is consistent with the findings of Jang, Keun Koo, Liu, and Loewenstein (2007) where a greater illiquidity premium mainly comes from the higher transaction cost payment caused by lower trading frequency.

Indirect transaction costs For an organized market, the indirect transaction cost is the price-impact cost,¹¹ or the bid-ask spread. It can be viewed as a “round-trip cost” (the cost of simultaneous purchase and sale of an asset). In other words it represents the amount of asset value that an investor forfeits in the investment process due to other factors beyond supply-demand. Demsetz (1968) defines this spread as transaction cost. Further, Miller (1965) mentions that both price and time dimensions of the liquidation process can be expressed in money units and interpreted as costs. Pastor and Stambaugh (2003), and Acharya and Pedersen (2005) show that assets whose own price-impact measure is high are more sensitive to the liquidity factor measured as changes in the aggregate price-impact measure. Thus, the notion of liquidity as an asset characteristic is related to that as a priced factor.¹²

Opportunity costs The fact that each investor has a different set of opportunities leads to heterogeneity in personal valuations of the achievable receipts at sale or expenditures at purchase. For instance, expected returns on alternative investments (private equity, real estate, hedge funds, etc.) vary across the business cycle. That is, capital calls are made when economic conditions are shrinking, which entails a higher opportunity cost than capital called when economic conditions are improving. Accordingly, there should be a return premium associated with providing liquidity to fund managers particularly in bad times, i.e. when the opportunity cost is high. In other words, funds that are likely to call capital under stressed market conditions should earn higher average returns. Dai, Li, Liu, and Wang (2016) develop a model in which illiquidity is driven by the opportunity cost of pricey rebalancing to harness the time-varying volatility. As a result, the illiquidity factor plays an economically meaningful role in investment decisions. It captures the idea that liquidity providers should

indicators such as bid-ask spreads, order imbalance, and small-trade ratio. Their results show that a one standard deviation change in the illiquidity factor corresponds to a 4.68 percentage point change in quarterly aggregate market returns. Their findings further support the hypothesis that market liquidity is an important risk factor that significantly affects expected returns in the cross-section.

¹¹The price-impact cost is the price effect when selling or buying an asset. The price-impact cost is a premium when buying an illiquid asset and a discount when selling an illiquid asset.

¹²It is worth noting that this concept is more relevant for dealer markets (where a specialized dealer or market-maker publicly quotes the price at which she is willing to trade).

be compensated for their role in the liquidity allocation mechanism in the economy.

3.2.2 Market organization

The form of market organization differs with respect to the technical procedures and the character of the search for a trading partner (Svensson and Werner, 1993). Searching for trading partner has positive value as it enables to potentially achieve higher sales proceeds. It can be viewed as a process of obtaining valuable information (Stigler, 1961); in which case search is costly. Duffie, Garleanu, and Pedersen (2005, 2007) analyze the impact of search and bargaining on asset prices and liquidity. The implications of their model reveal that search frictions, such as the lack of a trading partner at any given point in time, give rise to a liquidation cost, while intermediation leads to bid-ask spreads and novel dynamics. Using search-based models, Vayanos and Wang (2007) and Vayanos and Weill (2008) derive the liquidity premium from an endogenous concentration of traders in segmented markets. Complexity and weak comparability of assets result in the necessity for individual assessment of the true value of each instrument. Verifying the true characteristics of real assets is a time and money consuming process (for real estate, checking the true state of the property is necessary; buying private equity entails in-depth due diligence; and buying an art/collectible requires specialized expertise; etc.).

The market organization may lead to a significant distortion of asset characteristics (understated volatility, understated correlations, and overstated returns) that in turn may be associated with unrealistic higher shares of illiquid assets in portfolios. Consequently, forms of market organization with an efficient process of finding trading partners lead to higher trading activity and attract more trading actors.

It is widely and empirically recognized that less liquid assets exhibit a higher degree of positive serial correlation. The lack of exchange-based trading combined with the inherent heterogeneity of private asset markets means that, by definition, return data are based on irregular transactions and associated valuations rather than regular and accurate price data. As a result, return data are smoothed, leading to an artificial reduction in volatility and the alteration of correlation estimates.

Under the market structure paradigm, illiquidity risk also means that some asset markets are closed for certain time intervals of possibly random duration. Conversely, when investors are able to continuously trade as for liquid assets, Vayanos (1998) finds that transaction costs have a small impact on asset returns. However, in the case of illiquid assets, Heaton and Lucas (1996) find that a significant illiquidity premium is associated with large transaction costs even when markets are continuously opened. The search process leads to opportunity costs and bargaining process for immediacy. With this motivation in mind, Duffie, Garleanu, and Pedersen (2005, 2007) introduce a framework to quantify the effect of search frictions in over-the-counter markets. Their papers are part of the literature that constitutes considerable steps toward understanding illiquidity stemming from search frictions. However, only

few of these studies have considered the illiquidity problem from an optimal asset allocation perspective with all investment opportunity sets (risky assets, fixed-income securities, illiquid assets, etc.).

3.2.3 Diversity in valuations

Traditional asset pricing models start with strong assumptions about preferences, beliefs and constraints that imply asset demand is homogenous across investors.¹³ More recent models based on heterogeneous beliefs, information or constraints imply heterogeneous portfolios in equilibrium. The role of investor heterogeneity for the functioning markets is widely recognized (see for example Sadka and Scherbina, 2007). Even though we only observe a single transaction price, it captures all the heterogeneity from investors. That is:

- *Divergence of information (information asymmetry)*: Differences in the amount and quality of assets as well as the processing and interpretation of information. Some investors have information advantage over others (Daley and Green, 2016). Moreover, Glosten (1989) argues that well-informed traders can achieve maximum return from having information advantage.¹⁴
- *Divergence in expectations*: Different forecasts of the future cash flow and the associated risk concerning the future of an investment can be obtained even with the same dataset. For example, Jankowitsch, Nashikkar, and Subrahmanyam (2011) derive a quantitative measure of liquidity based on the price uncertainty distribution around a consensus valuation.
- *Divergence in used possibilities*: Possibilities to own an asset depend on individual needs and valuations; for example some arts/collectible assets are worthless for certain class of investors but priceless for others.

Before detailing the tools to measure illiquidity, it proves useful to briefly summarize the general concept of it.

3.3 Concept of illiquidity

The implementation of illiquidity into investment decisions entails relevant definition, method and framework for measuring illiquidity and assessing its economic consequences. The con-

¹³For example, the celebrated portfolio separation theorem of Tobin, 1958 implies that all investors hold identical portfolios up to leverage.

¹⁴A micro foundation of asymmetric information can be grounded in a model with adverse selection. Daley and Green (2016) document that adverse selection leads to important delays in trading frequency and time-varying cost of liquidity.

cept of illiquidity can be analyzed as a function of time¹⁵ and realized sale/trading value. Both Hicks (1962) and Miller (1965) have recognized these two dimensions of illiquidity. In general, an illiquid asset is one that can either be sold promptly in which case a large discount has to be accepted, or one has to be willing to accept a long liquidation period in order to avoid or minimize the large discount.

This tradeoff has to be considered with caution. It is not the typical tradeoff between expected return and volatility. It rather is the tradeoff between two different dimensions. When analyzing this kind of situation we need to proceed with caution, not omitting either dimension, otherwise we might find a relation that does not really exist, or find no relation when one does actually exist. Investors who care more about the time aspect will prefer assets that can be quickly sold albeit at discount, while those attaching more importance on the value aspect will prefer an asset that would sell at its fair price albeit rather later.

To sum up, when dealing with an illiquid asset, the following characteristics should be considered:

- *Duration*: the time it takes to liquidate the position, it can either be deterministic or uncertain (waiting time plays a significant role in portfolio decisions; Brigo and Nardio, 2010 account for illiquidity by introducing randomness into the holding period);
- *Quantity*: the size of the position to trade,¹⁶ (allows assets to be freely traded at posted prices, but in limited quantities as shown by Longstaff, 2001);
- *Cost*: the applied discount on fair value to enable transaction (liquidity can always be generated by paying costs as shown by Constantinides, 1986).

Now that we understand the origins as well as the concept of illiquidity, we can review the tools to measure it. This allows us to analyze portfolio choice implications in a final step.

4 Measuring illiquidity

In order to deal with illiquidity in a portfolio context, the optimization engine needs to be supplied with the magnitude of illiquidity. But that ingredient is unobservable, hard to measure, and difficult to estimate from actual data. Despite its importance, there exists no universally agreed or adopted measure that adequately quantifies the magnitude and accounts for the whole dimension of illiquidity. When considering a list of common illiquidity indicators, there is a wide array of illiquidity metrics. However, some metrics will be more or

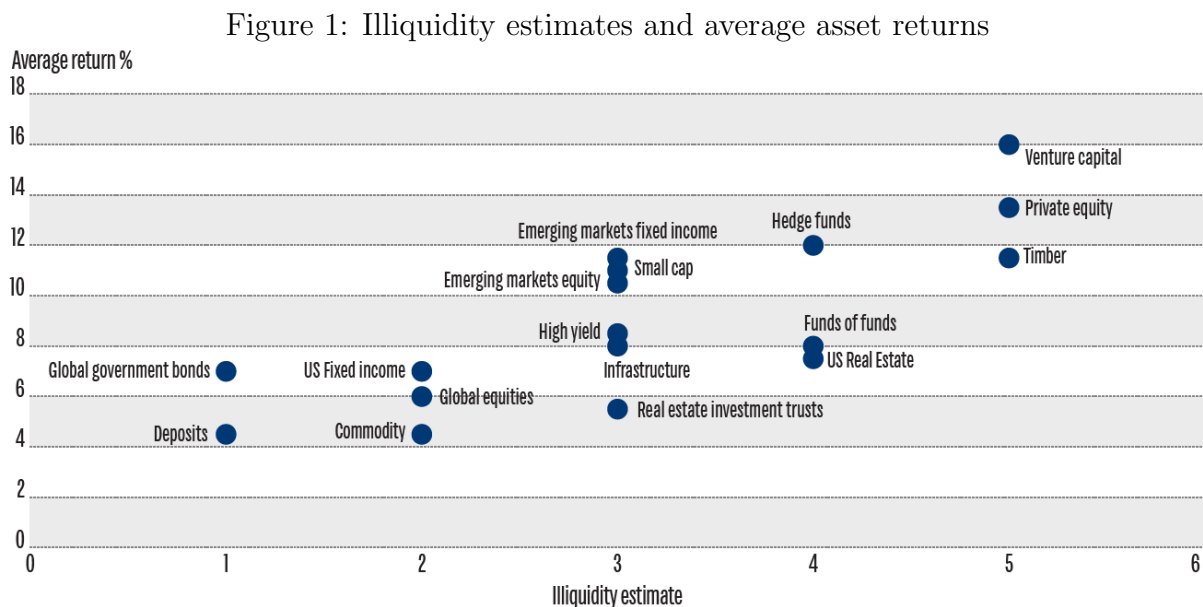
¹⁵The time effect takes into account the entire duration of the selling process from the moment of the decision to sell to the moment of cash receipt (including time necessary to find a trade partner, to deal with all formalities and to realize the payment).

¹⁶During the 1998 LTCM crisis for example, hedge fund positions had grown so large that it was impossible to liquidate them without a significant price impact.

less relevant for a given asset class. Thus, a starting point would be to develop a maximum harmonized list of illiquidity measures. This is the aim of this section. We start by providing a list of survey estimates. We further, for the sake of completeness, look at alternative investment measures.

4.1 Survey estimates

Figure 1 provides intuitive estimates of the degree of illiquidity and average returns for the corresponding asset classes. The figure confirms the intuition that less liquid assets provide



Source: Ilmanen (2011) Expected returns. Average asset returns 1990-2009. Subjective illiquidity estimates.

a higher average return. However, estimates from rule of thumb and valuation models would be flawed, biased and misleading, but from practitioner experience, having numbers seems to be more important than whether these numbers are meaningful and purposeful.

4.2 Alternative investment measures

Despite fundamental differences, a unique construct of principles can, and should be used, to measure illiquidity in alternative investments. Dealing with different classes of illiquid asset is very challenging. The reasons for this include:

- Lack of liquid comparable of an illiquid asset;
- Availability of timely and accurate data;

- Challenge to isolate the illiquidity effect from the other characteristics.

While standard measures of illiquidity relevant to other asset classes (equity, fixed-income, foreign exchange) are derived under assumptions that trade is always possible up to some level of transaction costs, trading opportunities are very limited for most illiquid assets even with considerable costs. The search of trading partner is the key driver of illiquidity and related to the search-based frictions literature. Since the seminal work of Diamond (1982), search-based models are widely adopted in alternative asset illiquidity models. The more practical measures for illiquidity in alternative investments are firmly grounded on Lippman and McCall (1986) measures that are inherently related to the (deterministic or random) waiting time until trade occurs.

4.2.1 Random trading opportunities

In this setting, searching for trading opportunities is modeled by a Poisson process. Agents have a transaction opportunity only on the arrival of the Poisson events. This leads to stochastic or predetermined periods¹⁷ where no trade occurs. David and O’Hara (1992) proposed a model to study the links between the existence of information, the timing of trades, and the stochastic process of prices. In their model, a trader chooses whether she wants to trade given the quotes that are offered. There can be several periods of no trade which implies a price adjustment from the one who is holding the asset.

4.2.2 Trade restrictions and options theory

Usually illiquid assets can be traded with a significant delay and discount. In other words, it is generally acceptable that illiquid assets are restricted to trade for a given period. This lock-in time period is also called the total legal lifetime of the fund, and private equity can be considered as closed-end fund with finite lifetime.

Chaffe III (1993) derives a discount on securities under trade restrictions. Both theoretical and empirical evidence suggest that investors attach a lower price to assets that are not frequently traded. As one can observe from Table 1, Longstaff (1995) analysis shows that the liquidity discount premium can range from 0.4% to 65%. The upper bound on the lack of marketability represents the present value of the greatest possible loss that an investor would experience by foregoing the right to sell the private equity fund at any point in time. Under realistic benchmark calibrations, Buchner (2016) shows that the typical discount on private equity can be as large as 21.5% over the typical 10-year lock-in period. This corresponds to an annual return premium of about 4%, which is consistent with the risk-adjusted excess returns around 4–5% that recent studies by Harris, Jenkinson, and Kaplan (2014) and Ang,

¹⁷Deterministic non-trading times are more appropriate for investments with lockup periods where expiry dates are publicly available. For some asset classes like private equity, the time to exit is often uncertain and/or negotiable.

Table 1: Upper Bounds for Percentage Discounts for Lack of Marketability
The standard deviations correspond to the range typically observed for equity securities.

Marketability Restriction Period	$\sigma = 0.10$	$\sigma = 0.20$	$\sigma = 0.30$
1 Day	0.421	0.844	1.268
5 Days	0.944	1.894	2.852
10 Days	1.337	2.688	4.052
20 Days	1.894	3.817	5.768
30 Days	2.324	4.691	7.100
60 Days	3.299	6.683	10.153
90 Days	4.052	8.232	12.542
180 Days	5.768	11.793	18.082
1 Year	8.232	16.984	26.276
2 Years	11.793	24.643	38.605
5 Years	19.128	40.979	65.772

Source: Longstaff (1995)

Chen, Goetzmann, and Phalippou (2017) document for private equity funds. Similarly, Lerner and Schoar (2004) investigate the transfer restrictions imposed by fund managers as a source of fund illiquidity and/or information asymmetry.

5 Illiquidity and asset allocation: the theory

Over the years, the relevance of illiquidity for investors worldwide has led academics as well as practitioners to undertake significant work in this area. Now that we have reviewed the concept of illiquidity and how to measure it, we discuss the literature that links illiquidity and investment strategies.

5.1 Illiquidity and investment objective

Illiquid assets may be used to achieve diverse goals. For long-term investors, there are multiple benefits of introducing illiquid asset classes into an investment portfolio: The key benefits include:

- *Expanded investment opportunity set.* This allows the construction of an investment portfolio with improved risk and return characteristics. In particular, illiquid asset classes provide exposure to risk and return drivers that are not accessible through listed asset classes.

Table 2: Alternatives broad categories

	PRIVATE EQUITY	PRIVATE DEBT	REAL ASSETS	HEDGE FUNDS	OTHER
Composition	Buyouts, venture capital, distressed debt	Corporate real estate, infrastructure, other	Real estate, natural resources, infrastructure, commodities	Long/short equity, long/short credit, event-driven, macro/managed futures, multi-strategy, other	Various
Purpose	Improve returns relative to public equity markets, access new sources of alpha	Generate income with some cushioning of downside risk	Diversification, generate income, provide some inflation-sensitive exposure	Generate alpha with less volatility than public equity markets, improve diversification, access strategies likely to assist in tail-risk hedging	Diversification, enhance returns by taking advantage of niche opportunities and other return drivers/risk factors not captured in other categories

Source: Mercer, "The role of alternative investments", August 2016

- *Improved diversification/lower equity market beta.* Equity risk tends to dominate within institutional investment portfolios. The introduction of illiquid asset classes can help to diversify such that equity risk is minimized.
- *Greater potential value added compared to listed markets.* Private markets are less complete and less efficient than their listed counterpart. For example, information is less spread and more difficult to acquire. Greater control of private asset markets and better governance provide the ability to add value post investment.
- *Inflation sensitivity in the case of real assets.* Given the way real assets are structured, their cash flows are often linked, directly or indirectly, to inflation.

Table 2 breaks down illiquid assets according to their role in the portfolio choice.

5.2 Challenges in quantifying illiquid asset risk

There is a vast strand of literature on asset prices and liquidity from which one can analyze the impact of illiquidity risk on portfolio choice. The relation between illiquidity and price can be stylized within the seminal work of Black (1970). A main take away of this analysis and the subsequent research paper by Bagehot (1971) is that there must exist a spread between the fair and the real price to enable transaction between buyer and seller: this spread can be thought of as illiquidity factor. Although there exists significant difference across studies that analyze the impact of illiquidity on security prices, the underlying economic intuition is the same. That is, the less liquid assets must earn a higher expected return to attract investors. Current state of practice and research regarding illiquidity risk is more advanced for listed products as opposed to non-listed products.

Table 3: Moments of illiquid asset returns

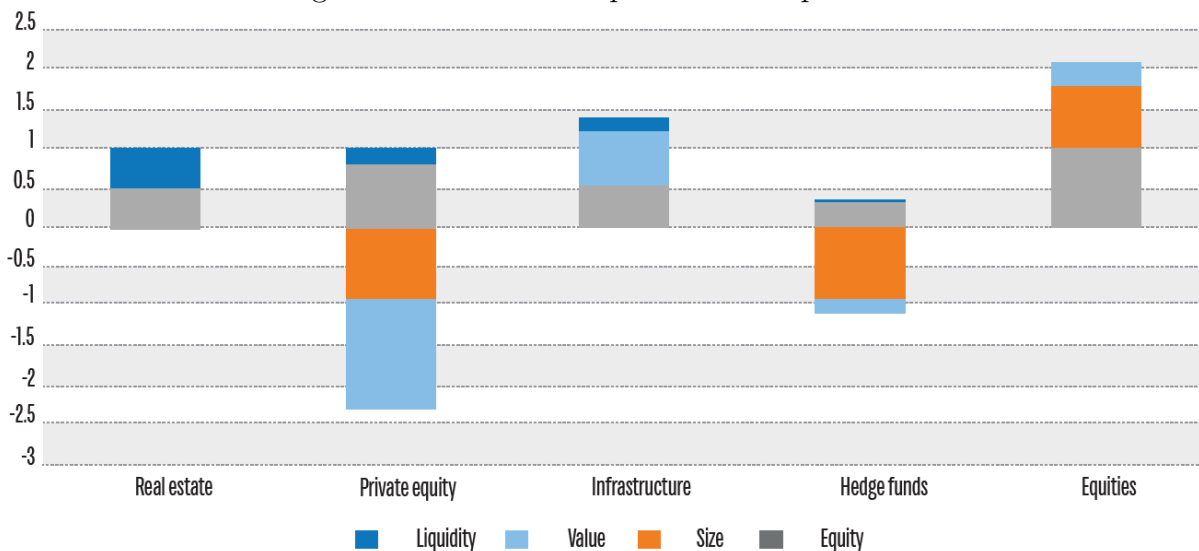
	Volatilities				Equity correlations		Equity betas	
	Reported	Adjusted	Factors	Idiosync.	Reported	Adjusted	Reported	Adjusted
Private equity and real assets								
Private equity	11%	22%	17%	5%	75%	75%	0.5	1.0
Venture capital	25%	52%	26%	26%	41%	45%	0.6	1.4
Infrastructure	15%	17%	12%	6%	56%	56%	0.5	0.6
REITs	22%	27%	20%	7%	61%	59%	0.7	0.9
Farmland	7%	14%	12%	2%	-13%	1%	-0.1	0.0
Timberland	9%	17%	11%	6%	11%	18%	0.1	0.2
Property (unlevered)	5%	13%	9%	4%	13%	52%	0.0	0.4
Real estate (core)	6%	16%	11%	5%	12%	47%	0.0	0.5
Real estate (value added)	9%	21%	14%	7%	16%	49%	0.1	0.6
Real estate (opportunistic)	12%	31%	22%	9%	31%	47%	0.2	0.9
Hedge funds								
Hedge fund index	7%	9%	8%	1%	76%	74%	0.4	0.4
Fund of funds	6%	8%	6%	1%	60%	62%	0.2	0.3
Multi-strategy	5%	7%	5%	2%	45%	39%	0.1	0.2
Emerging markets	14%	18%	16%	2%	67%	69%	0.6	0.8
Equity hedge	10%	12%	10%	2%	77%	75%	0.5	0.5
Long/short equity	10%	12%	9%	3%	66%	64%	0.4	0.5
Short bias	19%	21%	18%	3%	-73%	-69%	-0.8	-0.9
Relative value	4%	6%	5%	1%	59%	55%	0.2	0.2
Event-driven	7%	9%	7%	2%	74%	71%	0.3	0.4
Fixed income arbitrage	6%	9%	6%	3%	33%	41%	0.1	0.2
Global macro	9%	10%	5%	6%	21%	17%	0.1	0.1
Equity market neutral	3%	5%	2%	3%	29%	38%	0.1	0.1

Source: PIMCO, Cambridge Associates, DJCS, NCREIF, Bloomberg, "Asset Allocation: Risk Models for Alternative Investments", May 2013

Measuring illiquidity in one issue. Quantifying the risk of an illiquid asset constitutes yet another challenge. Empirical research on the risk-return performance of non-traditional assets has been hampered by data availability. For years, both practitioners and academics have been struggling with one of the thorniest on how to integrate illiquid assets into the traditional mean-variance framework. The key challenge lies in the computation of the main ingredients (means, variances, correlations) to feed their investment decisions engine. As mentioned before, in the absence of an organized market (with daily pricing, standard performance metrics and disclosure requirements), return data are smoothed, leading to an artificial reduction in volatility and the alteration of correlation estimates. This is well-illustrated in Table 3 that compares moments computed from published illiquid index returns with those obtained from "un-smoothed" returns estimated via an econometric factor model developed by Pedersen, Page, and He (2014). Their data covers the period from December 1991 to December 2012 and shows that volatility is indeed higher than reported.

Recent estimates of the illiquidity risk premium range from around 0.50% to 0.75% per

Figure 2: Risk factor exposures of illiquid assets



Source: Schroders. Pedersen, Page, He (2014) Asset Allocation: Risk Models for Alternative Investments

annum for assets such as senior infrastructure debt, to around 3% or more for assets further up the risk spectrum such as private equity (Franzoni, Nowak, and Phalipou, 2012). Ang, Papanikolaou, and Westerfield (2014) benchmark the illiquidity risk premium demanded by investors with the observed returns received when investing in these assets. They measure the (il)liquidity risk premium for investment horizons ranging from 6 months to 10 years. Their main findings show that the premium required above an identical liquid asset ranges from 0.7% to 6% depending on how illiquid the asset is. Along the same lines, Pedersen, Page, and He (2014) attempt to isolate the underlying factor exposures of alternative investments using an econometric analysis. As illustrated in Figure 2, their analysis shows that the amount of illiquidity risk depends on the alternative asset class considered. Estimates of (il)liquidity premium reported by Terhaar, Staub, and Singer (2003) and displayed in Table 4 shows the same ranking among alternative investments. In a nutshell, illiquid assets constitute valuable investment opportunities as they carry additional premium due to their illiquidity.

Being able to quantify risk premium as well as illiquity premium allows us to determine the optimal share of illiquid asset holdings in a portfolio.

5.3 Optimal share of illiquid assets in portfolios

This section addresses the following questions:

- How can illiquidity constraints be accounted for in investment decisions?

Table 4: Liquidity premium of conventional and alternative asset classes

	Risk Premium (%)	Excess Return (%) (Risk Premium + Liquidity Premium)
GIM	1.84	1.97
U.S. Equity	3.59	3.59
Ex-U.S. Equity	3.42	3.42
U.S. Bonds	0.80	0.80
Ex-U.S. Bonds	0.75	0.75
Private Equity	7.88	9.92
Real Estate	1.82	3.06
Natural Resources	2.07	3.88
Hedge Funds	1.82	2.57

Source: Terhaar, Staub, and Singer (2003)

- What are the reasonable share of illiquid assets/factors in investment portfolios?
- What are the significant factors affecting the share of illiquid assets?

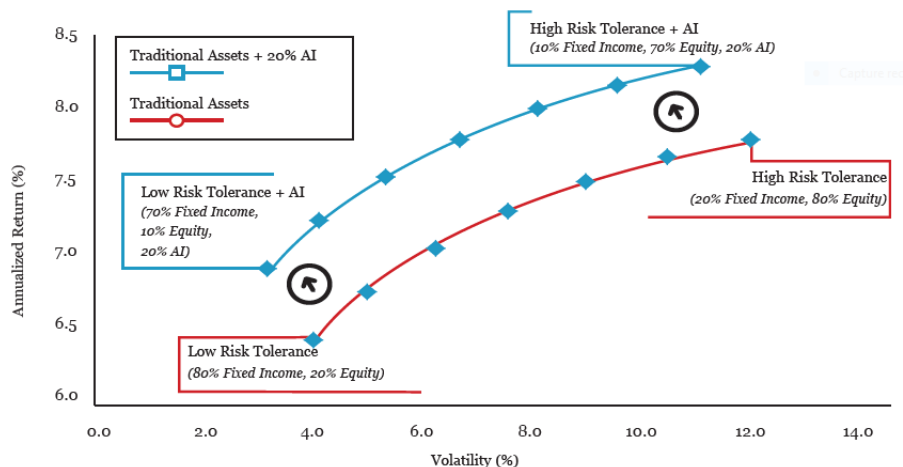
Due to their inherent complexity, there are only few asset allocation models that account for illiquidity constraints and the ubiquitous standard mean-variance framework widely used doesn't integrate illiquidity dimension in investment decisions. We review some major liquidity studies from the asset allocation perspective.

5.3.1 Benchmark studies

This literature starts with Mayers (1972) who extends the classic CAPM model of capital market equilibrium of Sharpe (1964), Lintner (1965), Black (1972), and others to include non-marketable assets such as human capital. The model adapted the expected return-risk relationship to redefine the benchmark model to include all marketable assets as well as the total payoff (income) on all non-marketable assets. Later, Brito (1977) incorporates non-marketability into the mean-variance framework and formulates a “three-fund separation theorem”. The theorem implies that in addition to risk-free and market portfolios, investors also hold a modified portfolio influenced by non-marketable holdings. Brito (1978) further shows that the investment decision is embedded in this three-dimensional space, and that the optimal portfolio weights depend on the correlation between illiquid and liquid assets.¹⁸ Along the same lines, Kahl, Liu, and Longstaff (2003) express the optimal solution as a

¹⁸The correlation coefficient can be expressed as beta-coefficient of illiquidity shock. High beta values enable to hedge illiquid asset by allocating the associated negative weights on risky liquid assets.

Figure 3: Efficient frontier with illiquid assets



Source: Blackstone (2017)

function of the risk-aversion coefficient, risky asset volatility and investment horizon. Silber (1991) shows that inability to trade either temporarily or permanently affects the tradeoff between risky securities and illiquid assets. Longstaff (2009) uses benchmark calibrations and simulations to obtain the same results. In a nutshell, illiquidity is an important dimension of investment decisions that is complementary to the traditional mean-variance framework.

5.3.2 Methodological framework and main findings

The common methodology to include illiquidity constraints in investment decisions is to alter the mean-variance framework for illiquidity considerations. Lo, Petrov, and Wierzbicki (2003) suggest three different settings to adjust the mean-variance optimization:

- Filter out illiquid assets before optimization, then only include assets with illiquidity lower than a given threshold in the portfolio and treat them equally;
- Enforce illiquidity constraints after portfolio construction and choose portfolio on the frontier that satisfy the illiquidity requirements;
- Add illiquidity constraints to mean-variance optimization: Illiquidity is included in the utility function and risk preferences are adjusted to account for by the weight placed on illiquidity constraints. Adding illiquid assets to the investment universe can significantly improve the locus of the efficient frontier as shown in Figure 3.

Table 5: Asset Classes with Illiquidity Index, Expected Real Returns and Risk

Asset Class	Illiquidity Index (l)	Expected Returns (μ)	Standard Deviations (σ)	Correlations					
				PubEq	PE	HF	RE	FI	Cash
Public Equity	0.25	6.0%	20%	1.0	0.7	0.5	0.4	-0.4	-0.1
Private Equity	1.00	9.0%	25%	0.7	1.0	0.5	0.5	-0.4	0.1
Hedge Funds	0.45	4.5%	11%	0.5	0.5	1.0	0.5	-0.3	0.1
Real Estate	1.00	5.0%	14%	0.4	0.5	0.5	1.0	-0.2	0.2
Fixed Income	0.00	2.0%	6%	-0.4	-0.4	-0.3	-0.2	1.0	0.0
Cash	0.00	0.0%	1%	-0.1	0.1	0.1	0.2	0.0	1.0

Source: Hayes, Primbs, and Chiquoine (2015)

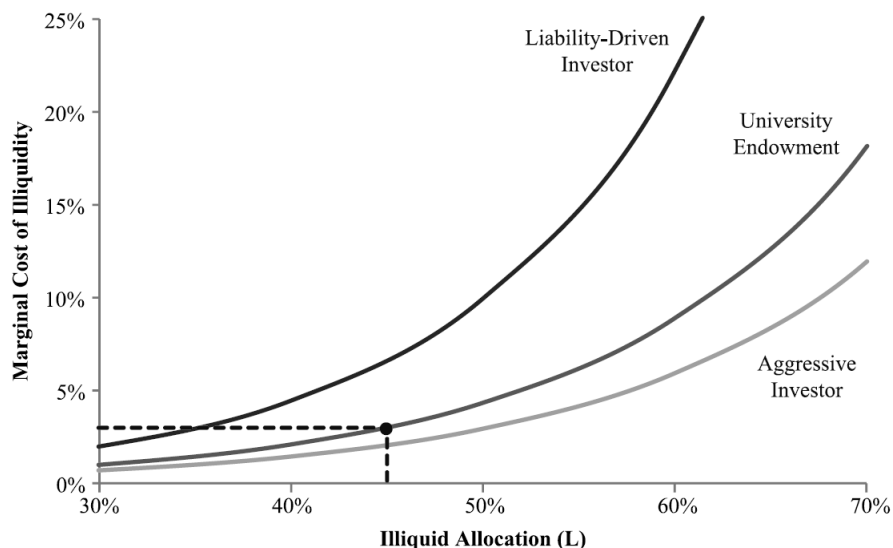
Hayes, Primbs, and Chiquoine (2015) address the limitations of the conventional mean-variance framework in incorporating liquidity constraints in the context of asset allocation. They measure the magnitude of each asset illiquidity as an index between 0 and 1 (where 0 represents perfectly liquid assets and 1 represents perfectly illiquid assets). The illustrative inputs are given in Table 5. Further, the authors introduce the portfolio marginal illiquidity penalty function that can be considered as the excess return premium over an otherwise identical liquid asset. For each typical investor (pension funds, endowment, etc.) with different type of contractual obligations, the authors derive the required premium over the liquid asset counterpart and obtain highly nonlinear functions¹⁹ as displayed in Figure 4. For example, for an endowment targeting 45% of portfolio illiquidity level, the illiquid portfolio must earn 3% of excess return at the margin.²⁰ The obtained curve is very meaningful for investment decisions as it not only serves as a relevant input in the portfolio optimization, but can be a very useful tool in providing decisions for managers in the context of illiquidity. The main findings from Hayes, Primbs, and Chiquoine (2015) are the following:

- *Efficient frontier*: A penalized efficient frontier, i.e. accounting for illiquidity, shifts the initial efficient frontier, i.e. ignoring illiquidity, to the right as shown in Figure 5. The effects are less significant for lower risk/variance with low weight on illiquid assets. The frontier expected returns are an increasing function of portfolio illiquidity level.
- *Asset allocation*: A noteworthy feature is that illiquid assets that receive significant allocations in many classical portfolios now have rather small holdings when accounting for significant illiquidity penalties.
- *Marginal illiquidity*: Marginal illiquidity penalty is an increasing function of investors' contractual obligations (for example, liability-driven investors such as pension funds

¹⁹In contrast to Kinlaw, Kritzman, and Turkington (2013), the marginal benefit of illiquidity is not constant.

²⁰See Sheikh and Sun (2012) for more on endowment models and their investment implications.

Figure 4: Marginal Illiquidity Penalty Curve for Various Asset Allocators



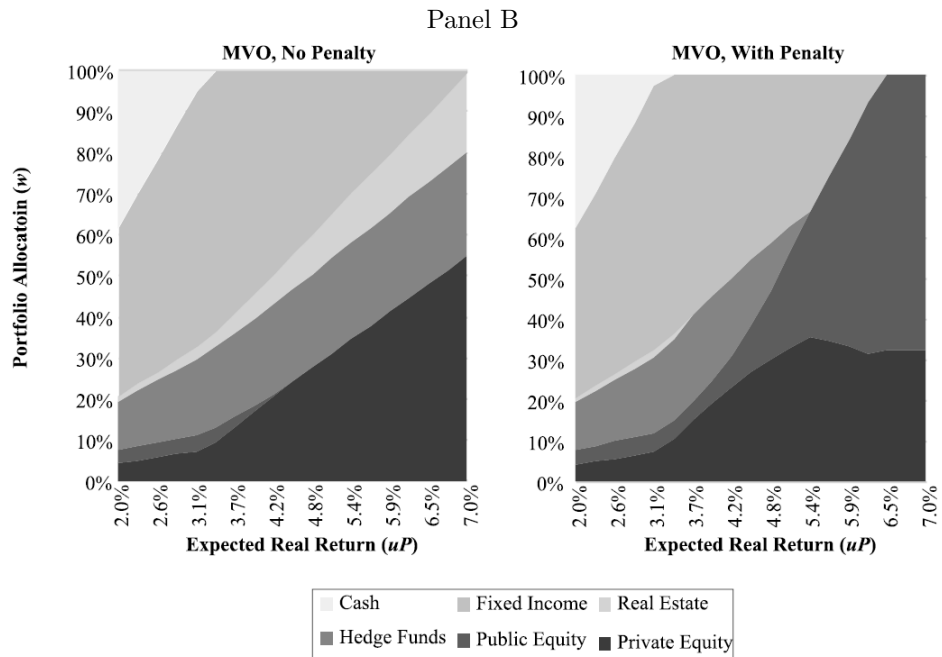
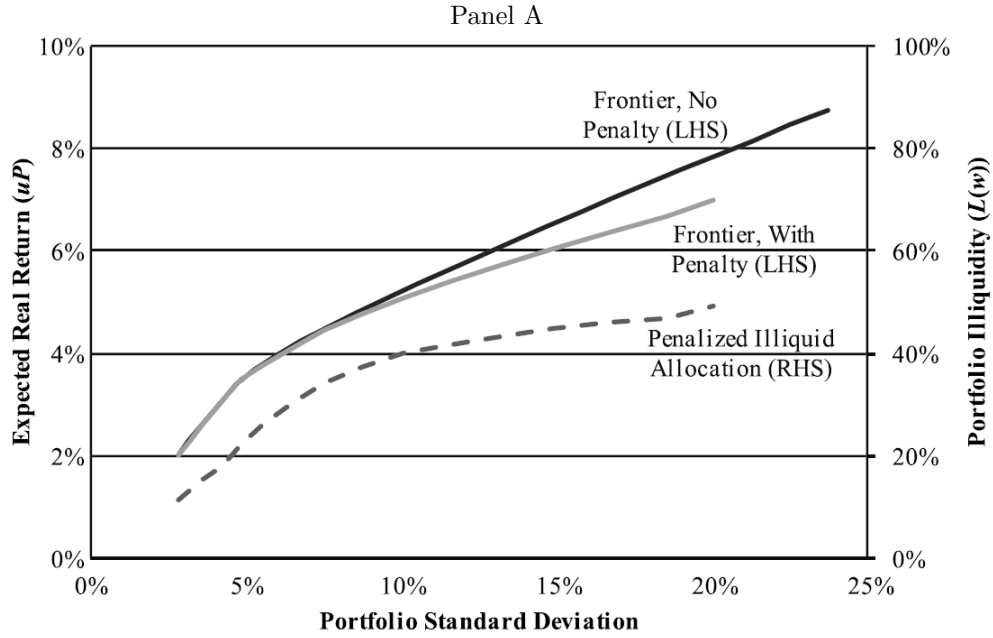
Source: Hayes, Primbs, and Chiquoine (2015)

have more penalties while an aggressive investor with a minimal liability requirement such as family offices can reap the extra premium of bearing illiquidity risk. At the middle, is a typical endowment investor). In other words, the allocations to illiquid assets increase with the likelihood of fulfilling contractual obligations.

Other frameworks that integrate illiquidity in asset allocation include Kinlaw, Kritzman, and Turkington (2013) who use shadow assets/liabilities as embedded illiquidity constraints and derive the optimal portfolio, Takahashi and Alexander (2002) who tackle the problem using multiperiod stochastic programming, and some empirical studies such as Ma and Pirone (2014) and Asl and Etula (2012) who use the robust optimization algorithm (Scherer, 2007) for its rigorous theoretical foundation and its ability to handle different investment objectives. Concretely, Asl and Etula (2012) map asset risk factors to macroeconomic indicators (see Figure 6) and solve for the corresponding mean-variance optimization on factors.

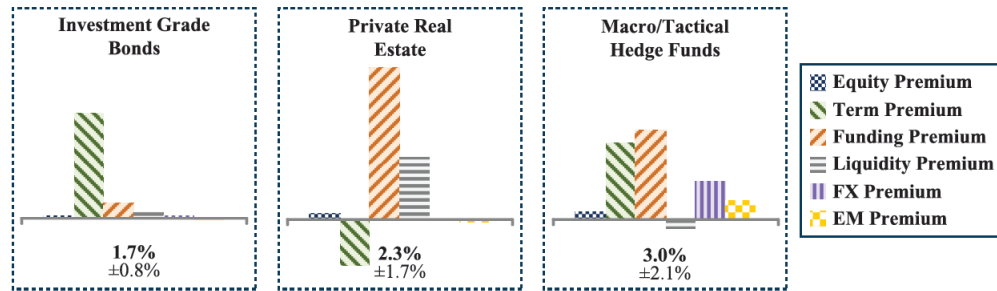
In that sense, Figure 7 shows the association between asset classes and risk factors. As one can observe, although it seems the resulting portfolio is not diversified from the perspective of asset classes, allocation based on factors are well-diversified across risk factors. The resulting efficient frontier is given in Figure 8. The implied asset allocation based on the illiquidity factor is shown to be around 10-20%.

Figure 5: Efficient Frontier and Portfolio Illiquidity Level



Source: Hayes, Primbs, and Chiquoine (2015)

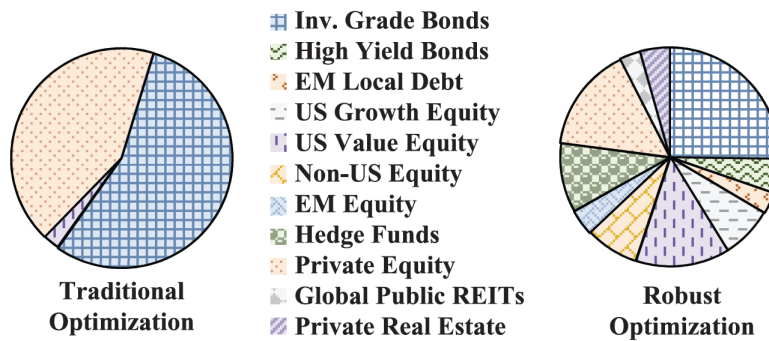
Figure 6: Examples of risk premium profiles for selected asset classes



Note: The total estimated risk premium and the associated standard error are displayed below each profile.

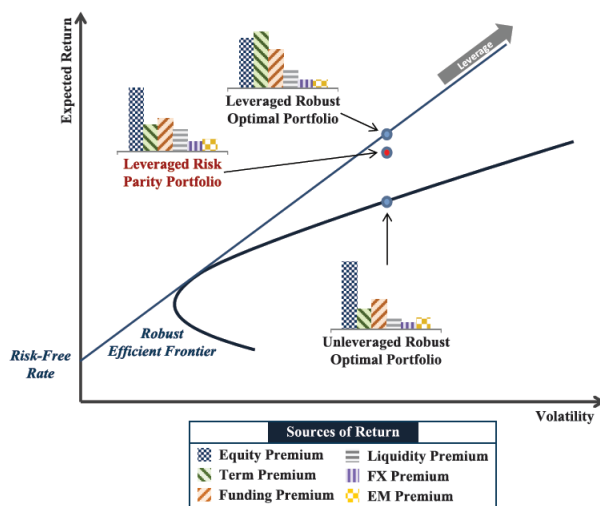
Source: Asl and Etula (2012)

Figure 7: Robust estimation acknowledges that expected returns are uncertain



Source: Asl and Etula (2012)

Figure 8: Robust factor-based approach generates “smarter” portfolios – an example for a universe of 21 asset classes



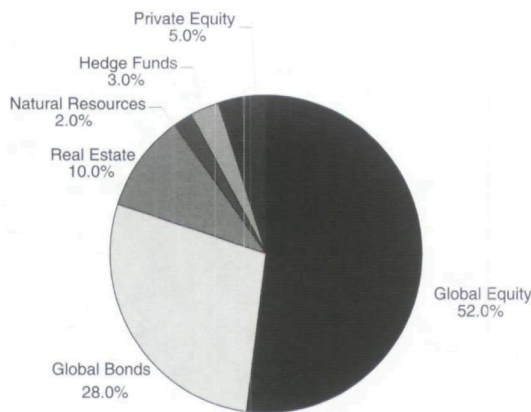
Source: Asl and Etula (2012)

5.3.3 Determinants of illiquid asset holdings

A number of studies directly relate the optimal position in illiquid assets to the trading frequency. For example, Terhaar, Staub, and Singer (2003) use simulations to integrate the absence of trading or frequent trading opportunities in a portfolio choice problem. They find that the stale prices inherent to illiquid assets can lead to unrealistic portfolio weights. Furthermore, the single-period assumption in traditional asset allocation problems results in portfolio weights that deviate significantly from policy recommendations and empirically observed weights. As illustrated in Figure 9, the investment process that accounts for the timing of cash flows together with illiquidity in the decision criteria would allocate 20% in illiquid assets (10% in real estate, 2% in natural resources, 3% for hedge funds and 5% in private equity). In terms of performance, the investor horizon should be long enough to reap the real benefit (benefits minus costs) and make investments in the illiquid assets profitable.

Likewise, Ang, Papanikolaou, and Westerfield (2014) report that optimal asset allocation to illiquid securities strongly depends on the trading frequency. It shifts from 60% with continuous rebalancing to 11% (5%) when investors are allowed to trade only once over an horizon of 5 (10) years. It is worth noting that although their analysis accounts for illiquidity events, there is no cash distribution and all risky asset returns are therefore only capital gains. Also in the proposed setting, investors are aware of the salience of liquidity and pay special

Figure 9: Policy mix for institutional investors



Source: Terhaar, Staub, and Singer (2003)

attention to the downside risk, which leads to very skewed asset allocation and polarised portfolios. This latter finding is consistent with previous research including, among others Garleanu (2009) and Longstaff (2009) who find that when dealing with illiquidity many investors give up diversification and favor more extreme positions.

In particular in the context of private equity, Ang and Sorensen (2012) argue that traditional models of asset allocation may not be suitable for investing. They find that when accounting for appropriate frictions, the optimal allocation can be reduced from 60% to 10%. Using simulations under temporary liquidity constraints, Siegel (2008) argues that the portfolio share of illiquid assets often ranges between 16% and 21% of total assets. In case of sophisticated investments, it is more polarized around 50%.

The determining factors that affect the optimal percentage of illiquid assets in a portfolio are thoroughly examined by Ma and Pirone (2014). They find that, under realistic and relevant asset information (means, variances, correlations), the optimal share of illiquid assets in the portfolio is roughly 9% with the stochastic approach²¹ and 12% for the traditional mean-variance framework. More precisely, the factors put forward by Ma and Pirone (2014) as determinants of the benchmark asset allocation are:

- *Annual spending rate*: A 4% change in the payout rate leads to significant shifts in the efficient frontier, particularly at high risk levels; it reduces the share of illiquid assets

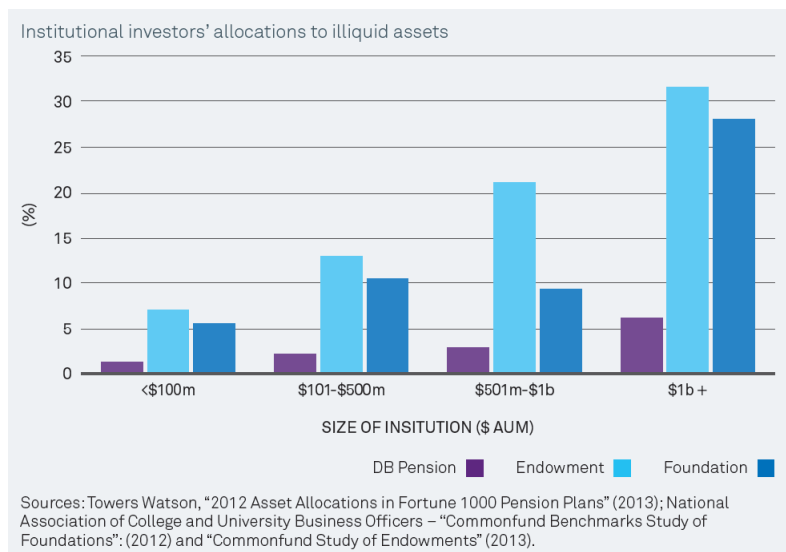
²¹The approach integrates path-dependent cash flow events (contributions, benefit payments and other assets/liabilities) over the horizon of the investment. Discounts to the NAV quantify the sales frictions in the secondary market. Discounts to the NAV are set to 50%, consistent with the empirically observed values during the financial crisis of 2008.

while increasing that of fixed income securities. Consistent with the findings of Siegel (2008), significant liability obligations decrease the capacity to invest in illiquid assets.

- *Risk premium over traded securities:* The outperformance of illiquid assets over public securities leads to higher asset allocations to the illiquid investments, specially at higher target portfolio volatilities. For a target portfolio volatility of 15%, the optimal illiquid investment is 20% for the 6% spread compared to a benchmark case of 9%.
- *Discounts to Net Asset Value (NAV):* The transactions in the secondary markets are based on the discount to NAV. The discount to the NAV affects allocation to illiquid assets via the optimal balance between the incremental returns and the likelihood of the illiquidity events. Reducing the discount to NAV increases the optimal allocation to private equity, particularly at higher portfolio risk levels. For example, for a target portfolio volatility of 25%, the share of illiquid assets in the portfolio is 20% for 75% discount but increase to 40% when there is no discount to NAV.
- *Timing of the cash flow's distribution:* To adequately account for the effect of timing of distributions on the resulting asset allocation, the authors consider the alternative stream of the cash flow by lagging the benchmark case by four years. Their results reveal that the streams of the cash flow distribution meaningfully affect the shares of the illiquid assets. For example, at 15% of target portfolio volatility, the optimal asset allocation shift considerably from 9.6% to 3.7%. Then deferring cash flow distributions can prove harmful to optimal allocations. Intuitively, lagging the distributions of cash flow increases the likelihood of an illiquidity event. Conversely, shortening the distribution of the cash flow lessens the probability and the magnitude of the illiquidity events and gives rise to larger illiquid asset holdings. This is consistent with the findings of Ang, Papanikolaou, and Westerfield (2014).
- *Size:* The investor's size plays a significant role in investing in the alternative assets. As mentioned before, illiquid assets are generally specialized investments that entail subject-matter expertise to analyze and define investment strategies. Figure 10 illustrates the increasing allocation in the illiquid assets with the total amount of assets under management. This is consistent with the fact that it requires significant amount of investment to recover fixed²² and search costs inherent to illiquid assets (Dyck and Pomorski, 2011). This result is also in line with the economy of scale (Stoughton, Wu, and Zechner, 2011). Besides, the share invested in alternatives depends on the type of investor and usually, endowments invest more in illiquid assets followed by foundations.

²²Small investors are more likely to have less resources to assess specialized investments and to support intensive due diligence.

Figure 10: Investor size and capacity to allocate in illiquid assets



Source: Ma and Pirone (2014)

6 Illiquidity and asset allocation: the facts

6.1 Industry reports and surveys

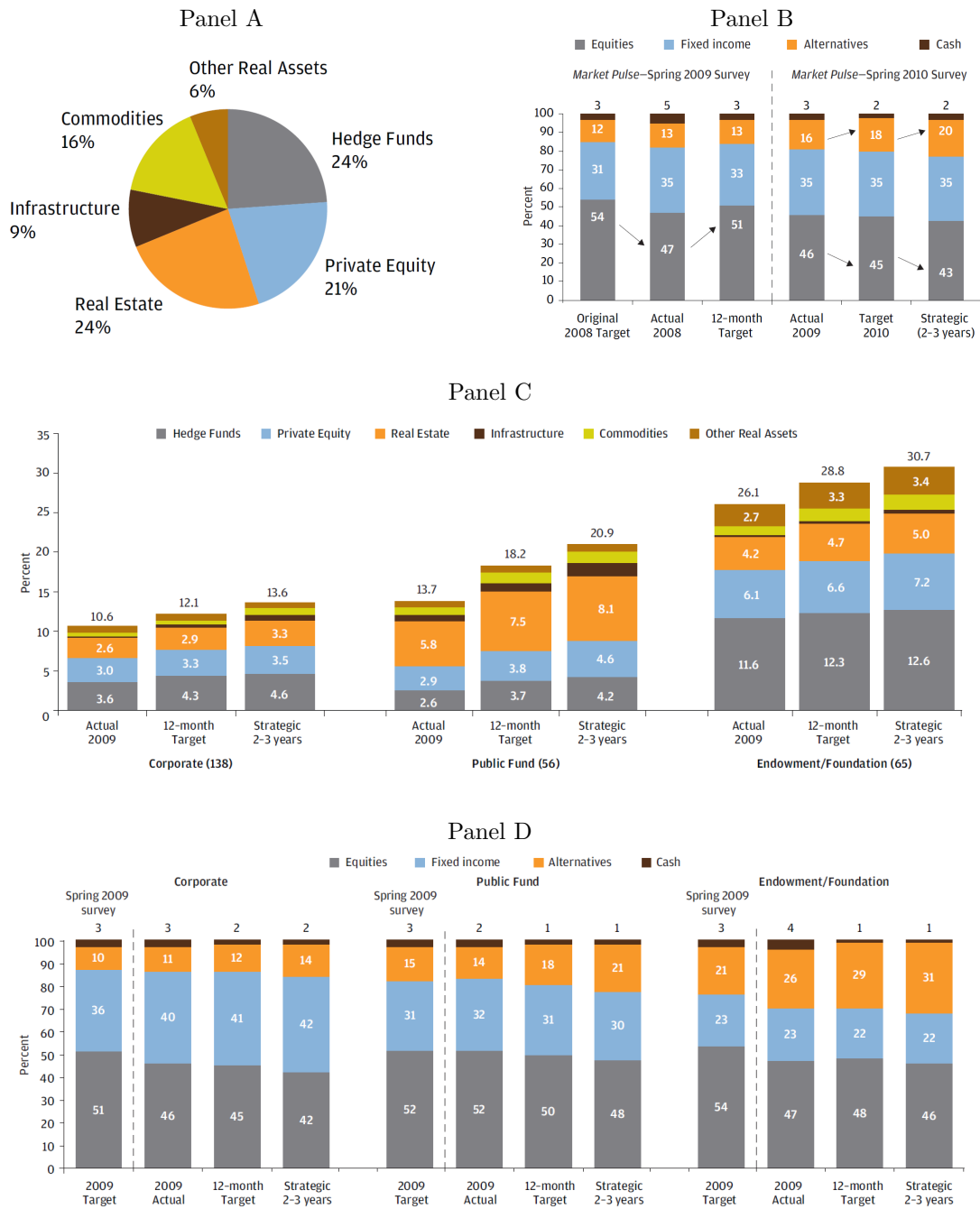
The 2010 J.P. Morgan alternative assets survey from about 3000 institutional investors shows that despite the 2007-2009 financial crisis' painful experience, investors still consider alternative investments as attractive. Panel A of Figure 11 shows the relative relevance of each alternative investment among the investigated institutional investors. Hedge funds, real estate and private equity holdings are perceived by 69% of investors as the greatest investment opportunities over a medium term horizon. Broad asset allocation from the 2009 J.P. Morgan alternative assets survey as displayed in Panel B of Figure 11 indicates that the share invested in alternative assets was 16% in 2009, with targeted level of 18% in 2010 and expected to reach 20% in the following three years.

Institutional investors are heterogenous in terms of their needs, objectives and constraints, resulting in distinct investment decisions and asset allocation. The general pattern of preferences for illiquid assets can be attributed to different investor segments such as corporate plans, public funds,²³ endowments and foundations.²⁴ That is, allocation in illiq-

²³Cost of living standards are major concerns for public pension funds and could explain their relatively high allocation in real estate in order to hedge inflation risk.

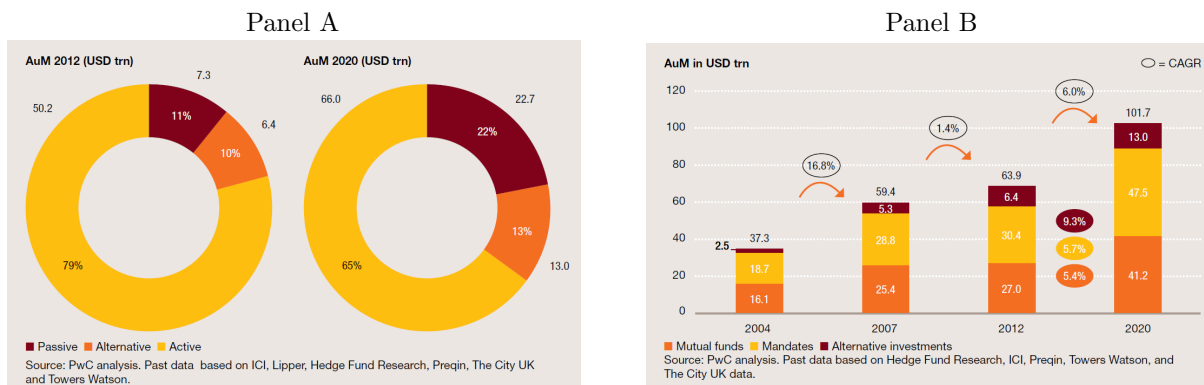
²⁴Endowments and foundations seek stable payouts while hedging the downside risk of assets as they tend

Figure 11: Investment opportunities in illiquid assets over medium-term horizon



Source: J.P. Morgan (2010), Market Pulse: Alternative Assets Survey

Figure 12: Global Asset under Management projection for 2020



Source: PwC (2014), Asset Management 2020: A Brave New World

uid assets depends on primary investment objectives and specific challenges faced by each segment. The typical strategic holdings are 13.6% for corporate investors, 20.9% for public funds and 30.7% for endowments/foundations as shown in Panel C of Figure 11. The aggressive bet of foundations/endowments on alternative investments is a good strike between exploiting alpha opportunities while insuring against downside risk over the long run.

Panel D of Figure 11 finally illustrates an expected upward trend in the portfolio share of illiquid assets which is compensated by an expected downward trend in the share of traditional assets. It is challenging for the academic literature to develop portfolio choice models that are able to explain asset allocation recommended by professional advisers or followed by institutional investors. Consequently, an interesting avenue in future research would be to examine the conditions under which models accounting for the illiquidity dimension are able to rationalize asset allocation patterns of Figure 11.

More recent industry reports on illiquid assets also include the 2015 OECD report on pension funds revealing that typical asset allocation in illiquid assets has moved from 14% in 2010 to 15% in 2015. Similarly, the 2015 Towers Watson' global pensions asset study reports that the largest pension funds in the world allocate around 25% of their portfolio value in illiquid assets, compared to 5% in 1995. A comprehensive 2014 asset management survey by PwC shows the growing interest in illiquid assets among top global investment managers. As illustrated in Panel A of Figure 12, the share of alternative investments will stand at around 13% by 2020 compared to 10% in 2012. The same PwC survey shows a sustainable growth in illiquid assets under management year over year since 2004 as testifies by Panel B of Figure 12. Going forward, illiquid alternative assets are expected to grow by some 9.3% annually between 2014 and 2020 to reach a value of \$13 trillion in the financial markets.

to generate returns while keeping volatility within reasonable ranges.

All these reports advocate the same trend that illiquid assets are more and more important for investors. The ultimate question now is how important exactly?

6.2 Illiquidity as an investment style

Based on the previous section, illiquidity can be viewed as an investment style or theme that can be combined with other asset characteristics to set investment weights. Along these lines, Ibbotson, Chen, Kim, and Hu (1991) show that illiquidity can be considered as a distinct investment style and it is an economically significant indicator of long-term returns alongside size, value and momentum. Besides, the authors provide evidence that illiquidity has historically been a rewarded factor, with changes in illiquidity tracking closely changes in valuation and economic conditions. Disentangling illiquidity from other characteristics is consistent with the views expressed by Cochrane (2005) in the NBER program report, where the author suggests that illiquidity can be interpreted “as an additional feature above and beyond the usual picture of returns driven by the macroeconomic state variables familiar from the frictionless view.”

In this context, it is worth mentioning that there is a rising interest in factor investing²⁵ among investors. Consulting firm PwC recently forecasts a tripling of the assets under management in index-based investment strategies worldwide between 2012 and 2020, and suggested that factor investing would be a key part of this trend. According to PwC: “The growth of passive strategies will be fuelled by new innovations in this space, such as factor investing will cross over from the realm of active managers, through highly sophisticated institutional passive investors, and into the mass-market retail space.” Just under half (47%) of the 214 respondents to the 2015 FTSE Russell Smart Beta Global Survey, with collective assets under management of over U.S. \$2 trillion, said they were now evaluating combinations of factor strategies as part of their future asset allocation plans.

7 Conclusion

The purpose of this survey is to review portfolio investment decisions when assets differ in their degree of liquidity. Illiquidity can be an important dimension of portfolio investment decisions, complementary in nature to risk and return. As the survey focuses mainly on alternative investments, the essential dimension of liquidity refers to the ease with which an asset can be sold. Thus, an asset or an asset market is liquid if trade can take place at short notice in large quantities without a substantial price change.

²⁵Greenberg, Babu, and Ang (2016) address the question of asset allocation where weights are placed on factors rather than asset classes. Their argument is based on the fact that working with factors provides a greater freedom in asset allocation and manager selection. Their approach is closely related to that of Asl and Etula (2012).

Amid the recent financial crisis, investors can still harness the potential of alternatives over the long run if appropriate monitoring process are in place to tract those investments. The literature shows that the optimal/empirical share of investments in illiquid assets usually lies between 5-20%. Illiquid investment opportunities are subject to the constraints of each investor's risk tolerance. When investing in illiquid assets, it is well-established that investors have different long-term objectives, contractual obligations and risk tolerances which imply distinct optimal alternatives holdings. Moreover, decomposing the return drivers (coupons, dividends, and other cash ows components) into the investment process may prove useful when dealing with illiquid assets.

Despite the relevance of illiquidity for investment decisions, the practical usefulness of illiquidity is still hindered by the challenges of obtaining appropriate and unified approach to measure illiquidity, including most importantly the illiquidity score (to discriminate among assets) and the liquidity premium (compared to other risk factors).

References

- Acharya, V. V., L. H. Pedersen, 2005. Asset pricing with liquidity risk. *Journal of Financial Economics* 77(2), 375–410.
- Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets* 5(1), 31–56.
- Amihud, Y., H. Mendelson, 2012. Liquidity, the Value of the Firm, and Corporate Finance*. *Journal of Applied Corporate Finance* 24(1), 17–32.
- Ang, A., B. Chen, W. N. Goetzmann, L. Phalippou, 2017. Estimating Private Equity Returns from Limited Partner Cash Flows. Working paper Saïd Business School.
- Ang, A., W. Goetzmann, S. Schaefer, 2009. Evaluation of Active Management of the Norwegian Government Pension Fund - Global. Unpublished working paper. Norges Bank Investment Management (NBIM).
- Ang, A., W. Goetzmann, S. M. Schaefer, 2011. The Efficient Market Theory and Evidence: Implications for Active Investment Management. *Foundations and Trends(R) in Finance* 5(3), 157–242.
- Ang, A., D. Papanikolaou, M. Westerfield, 2014. Portfolio Choice with Illiquid Assets. *Management Science* 60(11), 2737–2761.
- Ang, A., M. Sorensen, 2012. Risks, Returns, and Optimal Holdings of Private Equity: A Survey of Existing Approaches. *Quarterly Journal of Finance (QJF)* 02(03), 1–27.
- Aragon, G. O., 2007. Share restrictions and asset pricing: Evidence from the hedge fund industry. *Journal of Financial Economics* 83(1), 33–58.
- Asl, F. M., E. Etula, 2012. Advancing Strategic Asset Allocation in a Multi-Factor World. *Journal of Portfolio Management* 39(1), 59–66.
- Bagehot, W., 1971. The Only Game in Town. *Financial Analysts Journal* 27(2), 12–14.
- Basel Committee, o. B. S., 2013. Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools. Unpublished working paper. .
- Beachkofski, B., N. Gallardo, C.-T. Kuo, T. Lim, J. Liu, C. Tovikkai, 2011. Liquidity Risk. .
- Black, F., 1970. Fundamental of Liquidity. Working paper University of Chicago.
- , 1972. Capital Market Equilibrium with Restricted Borrowing. *The Journal of Business* 45(3), 444–55.

- Bloomberg, 2010. Report blasts Harvard, endowment model. *Bloomberg*, May 20, 2010.
- Brigo, D., C. Nordio, 2010. Liquidity-adjusted Market Risk Measures with Stochastic Holding Period. Papers 1009.3760 arXiv.org.
- Brito, N. O., 1977. Marketability Restrictions and the Valuation of Capital Assets under Uncertainty. *Journal of Finance* 32(4), 1109–23.
- Brito, N. O., 1978. Portfolio Selection in an Economy with Marketability and Short Sales Restrictions. *The Journal of Finance* 33(2), 589–601.
- Buchner, A., 2016. Equilibrium liquidity premia of private equity funds. *Journal of Risk Finance* 17(1), 110–128.
- Chaffe III, D. B. H., 1993. Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations. *Business Valuation Review* 12(4), 182–188.
- Chen, L. H., E. A. Dyl, G. J. Jiang, J. A. Juneja, 2015. Risk, illiquidity or marketability: What matters for the discounts on private equity placements?. *Journal of Banking & Finance* 57(C), 41–50.
- Chordia, T., 1996. The structure of mutual fund charges. *Journal of Financial Economics* 41(1), 3–39.
- Cochrane, J. H., 2005. Asset Pricing: Liquidity, Trading, and Asset Prices. Unpublished working paper. .
- Constantinides, G., 1986. Capital Market Equilibrium with Transaction Costs. *Journal of Political Economy* 94(4), 842–62.
- Dai, M., P. Li, H. Liu, Y. Wang, 2016. Portfolio Choice with Market Closure and Implications for Liquidity Premia. *Management Science* 62(2), 368–386.
- Daley, B., B. Green, 2016. An Information-Based Theory of Time-Varying Liquidity. *Journal of Finance* 71(2), 809–870.
- David, E., M. O’Hara, 1992. Time and the Process of Security Price Adjustment. *Journal of Finance* 47(2), 576–605.
- Demsetz, H., 1968. The Cost of Transacting. *The Quarterly Journal of Economics* 82(1), 33–53.
- Derman, E., 2007. A Simple Model for The Expected Premium for Hedge Fund Lockups. *Journal of Investment Management* 5(3), 5–15.

- Diamond, P. A., 1982. Aggregate Demand Management in Search Equilibrium. *Journal of Political Economy* 90(5), 881–894.
- Duffie, D., N. Garleanu, L. Pedersen, 2005. Over-the-Counter Markets. *Econometrica* 73(6), 1815–1847.
- Duffie, D., N. Grleanu, L. Pedersen, 2007. Valuation in Over-the-Counter Markets. *Review of Financial Studies* 20(6), 1865–1900.
- Dyck, I. J. A., L. Pomorski, 2011. Is Bigger Better? Size and Performance in Pension Plan Management. Working paper Rotman School of Management.
- Edelen, R. M., 1999. Investor flows and the assessed performance of open-end mutual funds. *Journal of Financial Economics* 53(3), 439–466.
- Eisfeldt, A. L., 2004. Endogenous Liquidity in Asset Markets. *Journal of Finance* 59(1), 1–30.
- Franzoni, F., E. Nowak, L. Phalipou, 2012. Private Equity Performance and Liquidity Risk. *The Journal of Finance* 67(6), 2341–2373.
- Garleanu, N., 2009. Portfolio choice and pricing in illiquid markets. *Journal of Economic Theory* 144(2), 532–564.
- Garleanu, N., L. Pedersen, 2011. Margin-based Asset Pricing and Deviations from the Law of One Price. *Review of Financial Studies* 24(6), 1980–2022.
- Glosten, L. R., 1989. Insider Trading, Liquidity, and the Role of the Monopolist Specialist. *The Journal of Business* 62(2), 211–235.
- Golts, M., M. Kritzman, 2010. Liquidity Options. *Journal of Derivatives* 18(1), 80–89.
- Greenberg, D., A. Babu, A. Ang, 2016. Factors to Assets: Mapping Factor Exposures to Asset Allocations. *Journal of Portfolio Management* 42(5), 18–27.
- Harris, R. S., T. Jenkinson, S. N. Kaplan, 2014. Private Equity Performance: What Do We Know?. *Journal of Finance* 69(5), 1851–1882.
- Hayes, M., J. A. Primbs, B. Chiquoine, 2015. A Penalty Cost Approach to Strategic Asset Allocation with Illiquid Asset Classes. *Journal of Portfolio Management* 41(2), 33–41.
- Heaton, J., D. J. Lucas, 1996. Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing. *Journal of Political Economy* 104(3), 443–487.

- Hechinger, J., 2009. Harvard, Yale Are Big Losers in 'The Game' of Investing. *The Wall Street Journal*, September 11, 2009.
- Hicks, J. R., 1962. Liquidity. *The Economic Journal* 72(288), 787–802.
- Holmstrom, B., J. Tirole, 2001. LAPM: A Liquidity-Based Asset Pricing Model. *The Journal of Finance* 56(5), 1837–1867.
- Ibbotson, R. G., Z. Chen, D. Y.-J. Kim, W. Y. Hu, 1991. Liquidity as an Investment Style. *Financial Analysts Journal* 69(3), 60–64.
- Jang, B.-G., H. Keun Koo, H. Liu, M. Loewenstein, 2007. Liquidity Premia and Transaction Costs. *The Journal of Finance* 62(5), 2329–2366.
- Jankowitsch, R., A. Nashikkar, M. G. Subrahmanyam, 2011. Price dispersion in OTC markets: A new measure of liquidity. *Journal of Banking & Finance* 35(2), 343–357.
- Kahl, M., J. Liu, F. A. Longstaff, 2003. Paper millionaires: how valuable is stock to a stockholder who is restricted from selling it?. *Journal of Financial Economics* 67(3), 385–410.
- Kinlaw, W., M. Kritzman, D. Turkington, 2013. Liquidity and Portfolio Choice: A Unified Approach. *Journal of Portfolio Management* 39(2), 19–27.
- Lancaster, K. J., 1966. A New Approach to Consumer Theory. *Journal of Political Economy* 74(2), 132–157.
- Lerner, J., A. Schoar, 2004. The illiquidity puzzle: theory and evidence from private equity. *Journal of Financial Economics* 72(1), 3–40.
- Li, J., R. M. Mooradian, W. D. Zhang, 2007. Is Illiquidity a Risk Factor? A Critical Look at Commission Costs. *Financial Analysts Journal* 63(4), 28–39.
- Lintner, J., 1965. Security prices, Risk, and Maximal gains from Diversification. *The Journal of Finance* 20(4), 587–615.
- Lippman, S. A., J. J. McCall, 1986. An Operational Measure of Liquidity. *American Economic Review* 76(1), 43–55.
- Liu, H., M. Loewenstein, 2002. Optimal Portfolio Selection with Transaction Costs and Finite Horizons. *Review of Financial Studies* 15(3), 805–835.
- Lo, A., C. Petrov, M. Wierzbicki, 2003. It's 11pm - Do You Know Where Your Liquidity Is? The Mean-Variance-Liquidity Frontier. *Journal of Investment Management* 1(1), 55–93.

- Lo, A. W., H. Mamaysky, J. Wang, 2004. Asset Prices and Trading Volume under Fixed Transactions Costs. *Journal of Political Economy* 112(5), 1054–1090.
- Longstaff, F., 1995. How Much Can Marketability Affect Security Values?. *Journal of Finance* 50(5), 1767–74.
- Longstaff, F. A., 2001. Optimal Portfolio Choice and the Valuation of Illiquid Securities. *The Review of Financial Studies* 14(2), 407–431.
- , 2009. Portfolio Claustrophobia: Asset Pricing in Markets with Illiquid Assets. *American Economic Review* 99(4), 1119–44.
- Ma, W., J. Pirone, 2014. Alternatives and liquidity: Incorporating liquidity constraints into portfolio construction. Unpublished working paper. .
- Mayers, D., 1972. Nonmarketable assets and capital market equilibrium under uncertainty. in *Studies in the theory of capital markets : [papers of the Conference on Modern Capital Theory, held at the University of Rochester in August, 1969, augmented by several closely related papers]*, ed. by M. C. Jensen. New York, NY [u.a.] : Praeger New York pp. 223–248.
- Miller, H. L., 1965. On “Liquidity” and “Transaction Costs”. *Southern Economic Journal* 32(1), 43–48.
- Pastor, L., R. F. Stambaugh, 2003. Liquidity Risk and Expected Stock Returns. *Journal of Political Economy* 111(3), 642–685.
- Pedersen, N., S. Page, F. He, 2014. Asset Allocation: Risk Models for Alternative Investments. *Financial Analysts Journal* 70(3), 34–45.
- Pensions&Investments, 2009. Illiquid holdings put Ivy League manager performance in worse light. *Pensions&Investments*, October 13, 2009.
- Sadka, R., A. Scherbina, 2007. Analyst Disagreement, Mispricing, and Liquidity. *Journal of Finance* 62(5), 2367–2403.
- Scherer, B., 2007. Can robust portfolio optimisation help to build better portfolios?. *Journal of Asset Management* 7(6), 374–387.
- Sharpe, W. F., 1964. Capital Asset Prices: a Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance* 19(3), 425–442.
- Sheikh, A. Z., J. Sun, 2012. Defending the “Endowment Model”: Quantifying Liquidity Risk in a Post-Credit Crisis World. *Journal of Alternative Investments* 14(4), 09–24.

- Siegel, L. B., 2008. Alternatives and Liquidity: Will Spending and Capital Calls Eat Your “Modern” Portfolio?. *Journal of Portfolio Management* 35(1), 103–114.
- Silber, W. L., 1991. Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices. *Financial Analysts Journal* 47(4), 60–64.
- Stigler, G., 1961. The Economics of Information. *Journal of Political Economy* 69.
- Stoughton, N. M., Y. Wu, J. Zechner, 2011. Intermediated Investment Management. *Journal of Finance* 66(3), 947–980.
- Sun, W., A. Fan, L.-W. Chen, T. Schouwenaars, M. A. Albota, 2006. Optimal Rebalancing for Institutional Portfolios. *Journal of Portfolio Management* 32(2), 33–43.
- Svensson, L., I. M. Werner, 1993. Nontraded assets in incomplete markets: Pricing and portfolio choice. *European Economic Review* 37(5), 1149–1168.
- Swensen, D., 2009. *Pioneering Portfolio Management: An Unconventional Approach to Institutional Investment, Fully Revised and Updated*. Free Press, .
- Takahashi, D., S. Alexander, 2002. Illiquid Alternative Asset Fund Modeling. *Journal of Portfolio Management* 28(2), 90–100.
- Terhaar, K., R. Staub, B. D. Singer, 2003. Appropriate Policy Allocation for Alternative Investments: A factor approach and simulation techniques. *Journal of Portfolio Management* 29(3), 101–110.
- Tett, G., 2010. Singapore’s lesson from Harvard model. *The Financial Times*, April 8, 2008.
- Tobin, J., 1958. Liquidity Preference as Behavior Towards Risk. *Review of Economic Studies* 25(2), 65–86.
- Vayanos, D., 1998. Transaction Costs and Asset Prices: A Dynamic Equilibrium Model. *Review of Financial Studies* 11(1), 1–58.
- Vayanos, D., T. Wang, 2007. Search and endogenous concentration of liquidity in asset markets. *Journal of Economic Theory* 136(1), 66–104.
- Vayanos, D., P.-O. Weill, 2008. A Search-Based Theory of the On-the-Run Phenomenon. *Journal of Finance* 63(3), 1361–1398.

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