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Factor Investing in Currency Markets: Does it Make Sense?

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Abstract

The concept of factor investing emerged at the end of the 2000s and has completely changed the landscape of equity investing. Today, institutional investors structure their strategic asset allocation around five risk factors: size, value, low beta, momentum and quality. This approach has been extended to multi-asset portfolios and is known as the alternative risk premia model. This framework recognizes that the construction of diversified portfolios cannot only be reduced to the allocation policy between asset classes, such as stocks and bonds. Indeed, diversification is multifaceted and must also consider alternative risk factors. More recently, factor investing has gained popularity in the fixed income universe, even though the use of risk factors is an old topic for modeling the yield curve and pricing interest rate contingent claims. Factor investing is now implemented for managing portfolios of corporate bonds or emerging bonds.

In this paper, we focus on currency markets. The dynamics of foreign exchange rates are generally explained by several theoretical economic models that are commonly presented as competing approaches. In our opinion, they are more complementary and they can be the backbone of a Fama-French-Carhart risk factor model for currencies. In particular, we show that these risk factors may explain a significant part of time-series and cross-section returns in foreign exchange markets. Therefore, this result helps us to better understand the management of forex portfolios. To illustrate this point, we provide some applications concerning basket hedging, overlay management and the construction of alpha strategies.

Keywords: Foreign exchange rates, factor investing, carry, value, momentum, risk premium.

JEL classification: C50, F31, G11

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1 Introduction

The concept of risk factors has not been developed in the same manner across asset classes in the investment industry. For instance, equity risk was mainly defined in the context of the capital asset pricing model (CAPM) developed by Sharpe (1964). Since the nineties, it has also been popular to add other systematic risk factors (Fama and French, 1992; Carhart, 1997). Therefore, the risk of a stock is measured by means of betas, more precisely the traditional beta (the market or CAPM beta) and alternative betas (size, value, momentum, etc.). This framework has been largely adopted by portfolio managers and investors, and is the backbone of equity investing. In particular, it has structured the distinction between active and passive management.

In a similar way, the yield curve model proposed by Litterman and Scheinkman (1991) has highlighted the importance of common risk factors in the asset class of fixed-income securities:

> "*Market participants have long recognized the importance of identifying the common factors that affect the returns on U.S. government bonds and related securities. To explain the variation in these returns, it is critical to distinguish the systematic risks that have a general impact on the returns of most securities from the specific risks that influence securities individually and hence a negligible effect on a diversified portfolio*" (Litterman and Scheinkman, 1991, page 54).

However, contrary to the equity framework which is based on market risk factors, the analysis of sovereign bonds focuses more on statistical risk factors. Indeed, principal component analysis is certainly the statistical tool which is the most used to generate risk factors in the fixed-income space. Alongside the huge development of econometric models for modeling the yield curve, academics and professionals have also applied risk factors for pricing interest rate contingent claims. Since the seminal paper of Vasicek (1977), factor models of interest rates have been extensively used and are the backbone when we consider caps, floors, swaptions and fixed-income derivatives. In particular, these factor models have been popularized with the HJM model (Heath *et al.*, 1992) and its several extensions (Brigo and Mercurio, 2006).

In the case of corporate bonds, Fama and French (1993) have identified duration and credit as the main drivers of the cross-section of bond returns. During the 2000s, research on corporate bonds was mainly focused on liquidity issues. Thus, Longstaff *et al.* (2005) found that "*the majority of the corporate spread is due to the default risk*", whereas the non-default component is related to bond market liquidity. Their conclusion contrasts with the previous result obtained by Huang and Huang (2012) – a paper published in 2012, but written in 2003 – who found that the liquidity component explains more than half of the corporate bond spread. Other academic studies have followed (Chen *et al.*, 2007; Bao *et al.*, 2011; De Jong and Driessen, 2012) and have confirmed that liquidity is another systematic risk factor. This is why the traditional model of corporate bond returns is a three-factor model based on duration, credit and liquidity. More recently, Houweling and van Zundert (2017) proposed replacing these traditional risk factors by alternative risk factors (size, low-risk, value and momentum). In fact, it appears that some of these risk factors (e.g. low-risk) can be viewed as a repackaging of traditional risk factors. Nevertheless, value and momentum have improved the diversification of traditional portfolios, especially since the 2008 Global Financial Crisis (Ben Slimane *et al.*, 2019).

The concept of alternative risk premia may be seen as an extension of the concept of equity factor investing. It concerns multi-asset investing and assumes that there are many risk factors that may be rewarded by the market (Roncalli, 2013). Its roots are typically the arbitrage pricing theory (APT) formulated by Ross (1976). The underlying idea is that carry, momentum or value strategies do not only concern the stock market, but all the markets. For instance, carry strategies may be implemented in volatility or commodity asset classes, while momentum strategies are very popular in commodity markets. Although risk premia and risk factors are highly related, they cover two different concepts. To be more precise, a risk premium is associated with a return to compensate for it, because it generally rewards a risk that cannot be hedged or diversified. A risk factor is a common pattern that helps us to understand the return dispersion of a group of securities. Basically, risk premia explain time-series returns while risk factors explain cross-section returns. Since time-series returns are related to cross-section returns, many risk premia are also risk factors[1].

Currencies are a specific case. First, currencies are not considered as a traditional risk premium, contrary to stocks and bonds. For example, institutional investors never define their strategic asset allocation by considering a currency bucket[2]. Second, the economic forces that drive foreign exchange rates are not necessarily the same as the ones that drive other financial assets. For instance, foreign exchange rates may be monetary policy instruments, and play the role of unit of account. This is why a foreign exchange rate is never measured in units, like stocks, bonds or indices are for example. Therefore, the concept of market losses or gains is not appropriate when we consider the currency market. Indeed, a loss for an investor corresponds to a gain for another investor[3].

The traditional approach for analyzing foreign exchange rates is the economic approach based on monetary policy, inflation, trade movements and capital flows. Even if this framework based on economic risk factors seems to be natural, it presents some important drawbacks when it is cast into the APT model. Indeed, economic risk factors are measured on a monthly or quarterly basis, and their publications are not synchronous across countries and regions. This is why building an APT-based model to analyze currency returns is not straightforward when manipulating low frequency and asynchronous data. An alternative approach is to consider market risk factors, as in the case of the stock market for example. The main advantage is that these risk factors are observed on a daily basis. Moreover, if we assume that the financial market is efficient, market prices contain all the available information, which also includes the economic risk factors. This is why it makes no sense to oppose market and economic risk factors. For instance, the five-factor model of Fama and French (1993) has its roots in the economic model of Chen *et al.* (1986).

In this paper, we propose analyzing foreign exchange rates using three main risk factors: carry, value and momentum. The choice of these market risk factors is driven by the economic models of foreign exchange rates. For instance, the carry risk factor is based on the uncovered interest rate parity, the value risk factor is derived from equilibrium models of the real exchange rate, and the momentum risk factor benefits from the importance of technical analysis, trading behavior and overreaction/underreaction patterns. Moreover, analyzing an asset using these three dimensions helps to better characterize the financial patterns that impact an asset: its income, its price and its trend dynamics. Indeed, carry is associated with the yield of the asset, value measures the fair price or the fundamental risk and momentum summarizes the recent price movements.

By using carry, value and momentum risk factors, we are equipped to study the cross-section and time-series of currency returns. In the case of stocks and bonds, academics present their results at the portfolio level because of the large universe of these asset classes. Since the number of currencies is limited, we can show the results at the security level.

[1]This is not always true. For instance, cat bonds must incorporate a risk premium, because the investor takes a large risk that cannot be diversified. However, the cat bond risk premium is not a risk factor that helps to explain the cross-section returns of stocks, bonds or other financial assets.

[2]By contrast, some of them may include commodities in order to have protection against inflation.

[3]For example, a loss on the USD/EUR rate is equivalent to a gain on the EUR/USD rate.

For each currency, we can then estimate the sensitivity with respect to each risk factor, the importance of common risk factors, when specific risk does matter, etc. We can also connect statistical figures with monetary policies and regimes, illustrating the high interconnectedness of market risk factors and economic risk factors. The primary goal of building an APT model for currencies is to have a framework for analyzing and comparing the behavior of currency returns. This is the main objective of this paper, and a more appropriate title would have been "*Factor Analysis of Currency Returns*". By choosing the title "*Factor Investing in Currency Markets*", we emphasize that our risk factor framework can also help to manage currency portfolios as security analysis always comes before investment decisions.

This paper is organized as follows. Section Two is dedicated to the economics of foreign exchange rates. We first introduce the concept of real exchange rate, which is central for understanding the different theories of exchange rate determination. Then, we focus on interest rate and purchasing power parities. Studying monetary models and identifying the statistical properties of currency returns also helps to define the market risk factors, which are presented in Section Three. These risk factors are built using the same approach in terms of portfolio composition and rebalancing. Section Four presents the cross-section and time-series analysis of each currency. We can then estimate a time-varying APT-based model in order to understand the dynamics of currency markets. The results of this dynamic model can be used to manage a currency portfolio. This is why Section Five considers hedging and overlay management. Finally, Section Six offers some concluding remarks.

2 The economics of exchange rates

In this section, we review the different approaches for modeling foreign exchange rates. First, it is important to understand the difference between nominal and real exchange rates. Second, we present the two pillars of currency modeling, which are the theories of interest rate parity and purchasing power parity. Using these two concepts, academics have developed monetary models of determination of exchange rates, which are useful to understand their short-run and long-run dynamics. Finally, we complement this analysis by considering the main statistical properties of foreign exchange rates.

2.1 The real exchange rate

2.1.1 Definition

The real exchange rate is a very important concept in economics, because of the crucial role it has on the trade relationship between countries. It aims to evaluate the value of a country's goods against those of other economies at current exchange rates. As such, it is a measure of how competitive an economy is relative to its trading partners. Therefore, the real exchange rate is expressed as the product of the nominal exchange rate and relative price levels in each country:

$$Q_{i,t} = S_{i,t} \times \frac{P_{i,t}^{\star}}{P_t} \tag{1}$$

where P_t is the price level of the domestic country, $P_{i,t}^{\star}$ is the price level of the foreign country i, and $S_{i,t}$ is the direct nominal exchange rate[4] of the domestic country with respect to the foreign country i.

[4] It is the home currency price of one unit of foreign currency so that $S_{i,t}$ rises as the home-country currency depreciates. For instance, USD/EUR = 0.80 is a direct quotation for a European since we have 0.80 euros with one dollar. Similarly, the indirect quotation is when one unit of domestic currency is expressed in terms of foreign currency. In this case, the indirect quotation for a European is EUR/USD = 1.25.

The above definition is the definition of the real exchange rate between two countries, but sometimes it might be useful to convert the bilateral indices into a multilateral real exchange rate index, also known as the real effective exchange rate or REER. To measure the value of the currency against a basket of other currencies, we generally use the weighted geometric mean[5]:

$$Q_t = \prod_{i=1}^{n} Q_{i,t}^{w_i} \tag{2}$$

where w_i is the weight of the i^{th} currency. By construction, the weights used in Equation (2) are kept constant over time, meaning that the multilateral real exchange rate does not reflect the dynamics of trade relationships. But these weights can also vary over time. This is actually the approach adopted by several institutions (e.g. OECD, FMI, BoE and BIS) to accommodate the fast changes of trading partners[6], especially since the 2000s. According to Klau and Fung (2006), "*one benefit of using time-varying weights rather than a static updating of the base period is that this procedure not only incorporates recent changes in trade patterns, but also better reflects the contemporaneous situation over all past periods. The resulting indices give a more accurate picture of medium- to long-term exchange rate movements by taking into account the varying importance of different trading partners at different times*". One of the most common methods used for calculating an index with changing weights is the Laspeyres index[7]. If we suppose that the weights for each bilateral real exchange rate change from w_i^1 to w_i^2 at time t_2, then the spliced real exchange rate is expressed as:

$$Q_t = Q_{t_2-1} \times \frac{\prod_{i=1}^{n} Q_{i,t}^{w_i^2}}{\prod_{i=1}^{n} Q_{i,t_2-1}^{w_i^2}} \tag{3}$$

for $t \geq t_2$. A good application of the spliced Laspeyres index is provided by Erlandsson and Markkowski (2006) for the case of Swedish krona. In their study, they allow for changes in the weights based on the importance of trade partners. However, they conclude that the weighting scheme (chained or not chained) has minor effects.

2.1.2 Practical considerations

There are essentially three issues to be taken into consideration when calculating a real exchange rate: Which price measure should be chosen? Which bilateral rates should be included? Which weights should be used?

The choice of price index matters because real exchange rates can move in very different ways depending on the price measure (Marsh and Tokarick, 1994; Chinn, 2006). The most widely used price measures are consumer price indices (CPI), because they are available for many developed countries and have a long data history. Moreover, due to the fact that it includes a broad group of goods and services, Genberg (1978) argued that it provides a comprehensive measure of changes in competitiveness. But, like all the other indices, CPI has the disadvantage of being manipulated by possible price controls or taxes. Among alternative price indices, we can use wholesale price indices, GDP deflators, producer price indices, relative unit labor costs, and relative export and import unit values. Wholesale price

[5]If we use the arithmetic mean, we have $Q_t = \sum_{i=1}^{n} w_i Q_{i,t}$.

[6]There is no specific methodology for calculating time-varying weights. While the BIS follows a three-year average trade weighted approach (it is then a kind of fixed-weight, but discretely adjusted), other institutions such as the OECD, the EC and the FED normally update their weights on a yearly basis.

[7]Another way to calculate a multilateral real exchange rate index is to use the Törnqvist index. Ellis (2001) provided several reasons why the Törnqvist index is less preferred than the Laspeyres index. For instance, the Törnqvist index requires next-period weights, and not current-period weights.

indices and relative export and import unit values have the advantage of including tradable goods but they do not have a long data history and are not available for many countries. The disadvantage of data availability is also true for unit labor costs, even though it has been considered as stable indices (Edwards, 1989). In addition, producer price indices (PPI) are not widely comparable among countries due to the different methods of computations across countries. On the contrary to PPI, the GDP deflator is comparable among countries but it does not include the import prices of the final goods. Therefore, the difficulty of obtaining comparable data between countries over a reasonable length of time explains why CPI is the most widely used measure when calculating real exchange rates in practice.

When defining a multilateral index, it is advantageous to include all currencies with significant weights. However, if one currency has huge movements, this might lead to an index that provides misleading indications of overall changes in competitiveness. So, the exclusion of some currencies might be convenient in some cases. For example, Ellis (2001) stated that:

> [...] "*the dramatic depreciation of some east Asian currencies, particularly the Indonesian rupiah, in 1997 and 1998 resulted in the published TWI remaining at roughly the same level in June 1998 as it had been a year earlier, despite the A$'s depreciation against other currencies*" (Ellis, 2001, page 10).

Excluding the Indonesian rupiah from this index could then be justified by the fact that Indonesia will no longer be a potential export market, and, as a result, further movements in its exchange rate do not impact other countries' competitiveness.

The appropriate weighting scheme depends on the objective when computing the multilateral exchange rate. For instance, one objective could be to capture the bilateral trade between two countries. In this case, the appropriate weighting scheme is calculated as the share of total trade with each trading partner. In order to capture the effect on the exchange rate of more than two countries, one can use the home country's share of world trade[8] or its share of world GDP.

2.2 The two pillars of exchange rate modeling

2.2.1 Uncovered interest rate parity

Covered interest rate parity (CIP) is an arbitrage condition between domestic and foreign deposit markets. It states that the return on domestic deposit is equal to the return on foreign deposit, implying that the forward exchange rate F_t satisfies the following equation:

$$(1 + i_t) = \frac{1}{S_t} (1 + i_t^\star) F_t \tag{4}$$

where i_t and $i_t^\star$ are the domestic and foreign interest rates. It follows that:

$$\frac{F_t}{S_t} = \frac{1 + i_t}{1 + i_t^\star} \tag{5}$$

or:

$$f_t - s_t \simeq i_t - i_t^\star \tag{6}$$

where $f_t = \ln F_t$ and $s_t = \ln S_t$. Equations (5) and (6) are the well-known CIP relationships.

If the currency markets are efficient and under the assumption of rational expectations, the forward exchange rate F_t is equal to the expectation of the future spot rate: $F_t =$

[8] This method is also used by the International Monetary Fund.

$\mathbb{E}_t[S_{t+1}]$. Combining this relationship with Equation (5) leads to uncovered interest rate parity (UIP) theory:

$$\frac{\mathbb{E}_t[S_{t+1}]}{S_t} = \frac{1+i_t}{1+i_t^\star} \tag{7}$$

or:

$$\mathbb{E}_t[s_{t+1} - s_t] \simeq i_t - i_t^\star \tag{8}$$

Since this condition is generally considered as too strict, one generally assumes that the expected change in the nominal exchange rate is determined by the interest rate differential and a risk premium φ_t:

$$\mathbb{E}_t[s_{t+1} - s_t] = i_t - i_t^\star + \varphi_t \tag{9}$$

If the foreign interest rates are above the domestic interest rates, we expect the exchange rate to appreciate. This will be true only under the assumption of no risk premium $\varphi_t = 0$.

Remark 1 *In order to express the uncovered interest rate parity in real terms, we subtract the expected inflation differential from both sides of Equation (9) and we get the following relationship*[9]*:*

$$\mathbb{E}_t[q_{t+1} - q_t] = r_t - r_t^\star + \varphi_t \tag{10}$$

where q_t is the logarithm of the real exchange rate, and r_t and $r_t^\star$ are respectively the domestic and foreign real interest rates. If the purchasing power parity is valid — $\mathbb{E}_t[q_{t+1} - q_t] = 0$ — and if the risk premium is equal to zero, we obtain the real interest rate parity or the Fisher effect: $r_t - r_t^\star = 0$.

There is a huge body of empirical literature that has studied whether ex-post changes in exchange rates can be explained by interest rate differentials[10]. A common problem for academic researchers, who have tried to demonstrate the UIP, is the fact that risk premia are unobservable and that the expectations are also unknown. Based on the economic theory, one expects that a positive interest rate differential should lead to home currency appreciation, but for Lewis (1995) this was not the case. Generally, empirical models based on the UIP theory have not been very successful for predicting exchange rate movements. For instance, Meese and Rogoff (1988) considered out-of-sample forecasting and found little evidence of a stable relationship between real exchange rates and real interest rates. Additionally, they concluded that a simple random walk may be a better forecasting model. Similar results are found by Cheung *et al.* (2005) and Alquist and Chinn (2008). Slightly more positive findings have been reported by Clark and West (2006) over short-term horizons, and Molodtsova and Papell (2009). According to McCallum (1994), one reason for these rejections could be that the monetary policy behavior is inconsistent with UIP. In particular, there is a debate whether the UIP theory holds better in short-term or long-term horizons. For instance, Cheung and Chinn (2001), Alexius (2001), and Chinn and Meredith (2004) reported that UIP holds better in long-term horizons than short-term horizons. On the other hand, Chaboud and Wright (2005) found out that UIP holds for short-term horizons but not for long-term horizons.

[9] We have:

$$S_t = Q_t \frac{P_t}{P_t^\star}$$

meaning that:

$$s_{t+1} - s_t = q_{t+1} - q_t + \underbrace{(p_{t+1} - p_t)}_{\pi_{t+1}} - \underbrace{(p_{t+1}^\star - p_t^\star)}_{\pi_{t+1}^\star}$$

where π_{t+1} and $\pi_{t+1}^\star$ are the domestic and foreign inflation rate. By noting $r_t = i_t - \mathbb{E}_t[\pi_{t+1}]$ and $r_t^\star = i_t^\star - \mathbb{E}_t[\pi_{t+1}^\star]$, we obtain Equation (10).

[10] Its focus has been on the change in the exchange rate (Equation 9) rather than the level of the exchange rate (Equation 7) because of econometric considerations on non-stationary time series.

2.2.2 Purchasing power parity

Another well-known arbitrage condition is the purchasing power parity (PPP), which suggests that two currencies will be in equilibrium when similar goods are priced the same in the two countries[11]. If that won't be the case then we will observe a demand switch from the expensive good to the cheaper one. It is not guaranteed that this switch in the demand will last forever, but it will remain as long as supply and demand equalize the prices (based on the law of one price). At the economy-wide level, PPP deviations will then lead to supply/demand changes which will move the real exchange rate to the unity:

$$Q_t = \frac{S_t P_t^\star}{P_t} = 1$$

Let p_t and $p_t^\star$ be the logarithm of the commodity price index in home and foreign countries. We consider the following linear regression:

$$s_t = \alpha + \beta \left(p_t - p_t^\star \right) + \varepsilon_t \tag{11}$$

Then PPP implies the null hypothesis $\mathcal{H}_0 : \alpha = 0$ and $\beta = 1$, meaning that the real exchange rate is a constant stationary process:

$$q_t = \varepsilon_t \sim I\left(0\right) \tag{12}$$

The debate whether PPP holds or not, seems to be still open among researchers. Hakkio (1984) argued that the failure of the PPP might be due to the fact that the tests are usually done on a bilateral framework. This is why this author has tested the PPP in a multilateral framework by using time-series cross-sectional estimation procedures and found out that PPP holds in the long-run. In a similar way, using non-stationary panel techniques, Frankel and Rose (1996a), MacDonald (1996), Oh (1996), Coakley and Fuertes (1997), Papell (1997) and Taylor and Sarno (1998) found evidence in favor of PPP. On the other hand, Cheung *et al.* (2005) focused on both short-term and long-term horizons. They found that PPP forecasts better than the random walk in the long-run, but the forecasting ability is not as good as the random walk for the short-run. Whether PPP holds in-sample and out-of-sample continues to be debated in the literature (Haug and Basher, 2011), two empirical facts seem to be accepted. First, exchange rates tend toward purchasing power parity in the long-run with a slow speed of convergence. Second, deviations from the PPP in the short-run are large.

Following Taylor *et al.* (2001), the rejection of the unit root does not automatically mean that PPP holds in the long run, so the first PPP puzzle could be formalized as a lack of strong evidence for long-run PPP. On top of this, Rogoff (1996) raised the following question:

> "*How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?*" (Rogoff, 1996, page 647).

Indeed, he confirmed that the half-life adjustment from PPP takes three to five years[12], and concluded that the slow speed of adjustment was related to nominal wages and prices, which usually take a shorter time to adjust. For Murray and Papell (2002), the problem of the PPP puzzle came from the techniques used for measuring persistence. For instance, Imbs *et al.* (2005) demonstrated that the half-life adjustment of PPP might be around

[11] PPP does not take into account that the quality of goods might be different between the two countries.

[12] While Chen and Engel (2005) claimed that the half-life adjustment take even more than five years.

11 months when the heterogeneity of accounting data is to be considered. In addition, another possible explanation of this puzzle could be the distinction between tradable and non-tradable goods. Indeed, PPP might not hold when non-traded goods and services are included in the definition of the real exchange rate. This conclusion is drawn by the use of the Balassa-Samuelson effect[13] (Driver and Westaway, 2003). The third possible solution is that the adjustment of the exchange rate follows a non-linear trend, which could be translated in allowing the autoregressive parameter to vary in empirical works (Taylor and Taylor, 2004).

2.3 Monetary models

Monetary models emerged in the nineties in order to complement the original theoretical framework of Frenkel (1976) by distinguishing the time horizon. Among the long list of models published between 1990 and 2010, we can cite CHEER, ITMEER and BEER for short-run equilibrium, FEER and DEER for medium-run equilibrium and PEER and NATREX for long-run equilibrium.

2.3.1 The basics of monetary models

Monetary models were first introduced in the seventies in order to improve the ability of PPP to explain the behavior of exchange rates (Dornbush, 1976; Frenkel, 1976). They have their roots in the Mundell-Fleming model by emphasizing the role of money supply and demand. From a theoretical point of view, they express the exchange rate as a function of macro-economic variables (price, income, interest rate, etc.). For instance, the flexible-price monetary model of Frenkel (1976) and Bilson (1978) can be tested using the following linear regression:

$$s_t = (m_t - m_t^\star) + \beta_1 (y_t - y_t^\star) + \beta_2 (i_t - i_t^\star) + \varepsilon_t \tag{13}$$

where s_t is the nominal exchange rate, m_t and $m_t^\star$ are the money supply for domestic and foreign countries respectively, y_t and $y_t^\star$ denotes real income for domestic and foreign countries, and i_t and $i_t^\star$ are the nominal interest rate for domestic and foreign countries. In this model, the null hypothesis is then $\mathcal{H}_0 : \beta_1 < 0$ and $\beta_2 > 0$. In the case of the sticky-price monetary model, Frankel (1979) showed that the linear regression can be expressed as:

$$s_t = (m_t - m_t^\star) + \beta_1 (y_t - y_t^\star) + \beta_2 (i_t - i_t^\star) + \beta_3 (\pi_t - \pi_t^\star) + \varepsilon_t \tag{14}$$

where π_t and $\pi_t^\star$ are the inflation rate in domestic and foreign countries. The null hypothesis becomes $\mathcal{H}_0 : \beta_1 < 0$, $\beta_2 < 0$ and $\beta_2 + \beta_3 > 0$. The findings on these monetary models are mixed. While Meese and Rogoff (1983) found no evidence that monetary models forecast better than a random walk, MacDonald and Taylor (1993) found a cointegration relationship between exchange rates and macroeconomic fundamentals.

Another famous monetary model is the Taylor rule established by Taylor (1993). The underlying idea is that the interest rate is set by the monetary authority as a function of the output gap and an inflation target level $\bar{\pi}$. If two countries apply the Taylor rule, their

[13] The Balassa-Samuelson effect assumes that prices of tradable goods will converge at the international level, but not non-tradable goods. Let α be the proportion of non-traded goods within the economy. The logarithm of the real exchange rate is then equal to:

$$q_t = \left(s_t + p_t^{(T)^\star} - p_t^{(T)}\right) + \alpha \left(p_t^{(T)} - p_t^{(NT)}\right) - \alpha^\star \left(p_t^{(T)^\star} - p_t^{(NT)^\star}\right)$$

where the superscripts T and NT refer to tradable and non-tradable goods respectively. Therefore, the real exchange rate is a combination of the real exchange rate for traded goods and the ratio of the relative prices of traded to non-traded goods in the two economies.

bilateral real exchange rate satisfies the following expectation model (Rossi, 2013):

$$\mathbb{E}_t \left[q_{t+1} - q_t \right] = \beta_0 + \beta_1 \left(r_t - r_t^\star \right) + \beta_2 \left(\text{gap}_t - \text{gap}_t^\star \right) \tag{15}$$

where gap_t and $\text{gap}_t^\star$ are the output gap in domestic and foreign countries. The empirical evidence of Taylor-rule fundamentals is again mixed, with Rogoff and Stavrakeva (2008) finding that the Taylor rule is not robust. Furthermore, Chinn (2006) noticed that the signs of the coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$ are not as we should expect based on the theory. On the other hand, studies such as Engel and West (2005, 2006) supported the link between exchange rates and Taylor-rule fundamentals. Moreover, Molodtsova and Papell (2009) found that the Taylor rule forecasts exchange rates better than the random walk in the out-of-sample, and more encouraging results were further provided by Giacomini and Rossi (2010), Molodtsova *et al.* (2010) and Inoue and Rossi (2012).

2.3.2 Short-run to medium-run equilibrium concepts

After having tested in practice the arbitrage conditions (UIP and PPP) and the classical monetary approaches (such as the Taylor rule), econometricians started to mix the approaches together in order to see whether they could obtain more consistent results in terms of exchange rate forecasting. For instance, Johansen and Juselius (1992) and Juselius (1995) combined the UIP and the PPP by creating in this way a new concept of the equilibrium of exchange rates, which was named by MacDonald (2000) as the Capital Enhanced Equilibrium Exchange Rate or CHEER[14]. This empirical model consists in estimating cointegration relationships between exchange rates, relative prices and interest rate differentials. MacDonald and Marsh (1997) found out that the CHEER model forecasts better than the random walk model for short-term horizons (three months). Furthermore, Johansen and Juselius (1992) and Juselius and MacDonald (2004) showed that the speed of convergence for CHEER is faster than for the PPP.

Another widely used method to evaluate the fair value of a currency is the Behavioral Equilibrium Exchange Rate or BEER, which was first introduced by Clark and MacDonald (1999). They considered the BEER as a function of the interest rate differential, net foreign assets as a ratio of GDP (NFA), terms of trade (ToT), the relative price of non-traded to traded goods (TnT), and a risk premium that was considered as the relative domestic to foreign government debt[15]:

$$\text{BEER}_t = f \left(r_t - r_t^\star, \text{NFA}_t, \text{ToT}_t, \text{TnT}_t, \frac{\text{GDB}_t}{\text{GDB}_t^\star}, \right) \tag{16}$$

In particular, the partial derivatives are negative for terms of trade and positive for net foreign assets. The value BEER_t is an estimate of the real exchange rate at the equilibrium and then must be compared to q_t.

Many researchers have estimated modified versions of BEER. A good example is Wadhwani (1999), who used the Intermediate Term Model-based Equilibrium Exchange Rate or ITMEER. It is a variant of the BEER approach with the inclusion of the relative unemployment rates[16]. Different from the CHEER, this approach also takes into account the risk

[14] MacDonald (2000) assumed CHEER as a medium-run equilibrium concept while Driver and Westaway (2003) considered it as a short-term concept.

[15] Later, Clark and MacDonald (2003) have omitted the risk premium to estimate the BEER model.

[16] MacDonald (2000) provided two justifications for why the unemployment is an important variable. First, if the level of unemployment in a country is high, then this might be a sign of deterioration of the current account. Secondly, the foreign direct investments might be higher in countries experiencing low unemployment.

premium, by considering it as a hidden variable driven by the returns on assets (stocks and bonds). Moreover, the focus of the model is to forecast nominal exchange rates s_t rather than real exchange rate movements q_t.

2.3.3 Medium-run equilibrium concepts

The medium-run equilibrium exchange rate concept is related to a consistent level of activity and a sustainable balance of payments. The internal balance is reached when the economy is at full employment and low inflation, while the external balance results in an economy spending and investing abroad no more than other economies spend and invest in it. One concept that satisfies the medium-run equilibrium is the so-called Fundamental Equilibrium Exchange Rate or FEER, which has been developed by Williamson (1985, 1991).

The external balance can be expressed as:

$$\mathrm{ca}_t + \mathrm{ka}_t = 0 \tag{17}$$

where ca_t is the current account and ka_t is the capital account. Given that the current account is a function of the real effective exchange rate and the output in the home and foreign country, we can express Equation (17) as follows:

$$\overline{\mathrm{ca}}_t = \beta_0 + \beta_1 \bar{q}_t + \beta_2 \bar{y}_t + \beta_3 \bar{y}_t^{\star} = -\overline{\mathrm{ka}}_t \tag{18}$$

So, the FEER will be the value of the real effective exchange rate $\bar{q}_t$ that will equalize the current account with the '*underlying*' or '*sustainable*' capital account, where the determinants of the current account were set at their full employment levels and under the assumption that the actual value of q_t will converge to the FEER (Wren-Lewis, 1992; Clark and MacDonald, 1998). A comparison of the current real exchange rate q_t and the current FEER $\bar{q}_t$ leads to the following valuation rules:

- If $q_t > \bar{q}_t$, the current exchange rate is overvalued.
- If $q_t < \bar{q}_t$, the current exchange rate is undervalued.

Implementing in this way, the computation of the FEER requires information on many parameters: current account, capital account, potential home and foreign output as well as the equilibrium value of the capital account. In order to simplify the estimation, Isard and Faruqee (1998) and Faruqee *et al.* (1999) proposed that the equilibrium of the current account is the difference between desired aggregate saving and investment at full employment. This method is also used by the IMF, providing the medium-run equilibrium and the long-run equilibrium. The first one is considered as the time horizon when the domestic and foreign output gaps are eliminated, while the long-run equilibrium compares the current account with a proxy of the stock equilibrium (MacDonald, 2000).

Driver and Wren-Lewis (1999) acknowledged the fact that FEER is becoming more widely used than the PPP for understanding whether a currency is overvalued or undervalued. Further, Barisone *et al.* (2006) found a cointegration relationship between real exchange rates and FEER values using panel estimation, and concluded that "*the FEER approach represents an improvement over PPP in explaining medium- to long-term trends in the real exchange rates of the major industrialized countries*". Nevertheless, the FEER approach is subject to some criticism, which is related to the fact that the stock-flow equilibrium is not always reached[17]. Furthermore, a second concern arises from the link between FEER and fiscal policy (Wren-Lewis and Driver, 1998). To overcome these issues, Bayoumi *et al.* (1994) developed the so-called Desired Equilibrium Exchange Rate or DEER, where the exchange rate is also explained by a proxy for optimal fiscal policy.

[17] See Baldwin and Krugman (1989) for a detailed theoretical explanation of the hysteresis effect.

2.3.4 Long-run equilibrium concepts

In economic theory, long-term equilibrium is associated with the achievement of stock-flow equilibrium. Permanent Equilibrium Exchange Rate or PEER is considered as a statistical long-run model of exchange rates rather than an economic model (Driver and Westaway, 2003). As suggested by the name, this model breaks down the real exchange rate into permanent and transitory components. Cumby and Huizinga (1991) used the multivariate Beveridge-Nelson decomposition[18] and find that the deviations of the real exchange rates from their permanent values are very often and considerable in magnitude. Clarida and Gali (1994) showed that these results depend on the currency. Clark and MacDonald (2004) extracted the PEER from the BEER approach by separating the fundamentals into permanent and transitory dynamics. In particular, they estimated the PEER by using the cointegration method of Johansen (1988) and found that the deviation between the BEER and the PEER again depend on the currency. For instance, the PEER provides better results compared to the BEER for the case of the pound sterling.

Another method that explains the dynamic of exchange rates is the Natural Real Exchange Rate or NATREX. Stein (1995) was the first one to coin this concept and provided the following definition[19]:

> [...] "*the equilibrium real exchange rate that would prevail if speculative and cyclical factors could be removed and the unemployment rate is at its natural rate*" (Stein, 1995, page 39).

The NATREX satisfies two conditions: the balance of payments (goods markets) and the portfolio equilibrium (asset markets). As one can notice, the first condition is satisfied by the FEER as well, which makes the NATREX and the FEER relatively similar. The second condition concerns the balance between assets held denominated in domestic and foreign currencies. The difference between the FEER and the NATREX comes essentially from this second pillar. In his paper, Stein (1995) focused on the US relative to the G-10 and tried to find the fundamentals of the exchange rate. He included in the model a measure of home and foreign capital productivity, a proxy for the time preference (ratio between the sum of the consumer consumption and government spending to GNP) and the difference between long-term interest rates. The empirical results of Stein (1995) show that the NATREX is a long-run concept, implying that short-run deviations can persist. Another important observation is that the fundamentals are not necessarily the same across large, medium and small-size countries (Stein and Allen, 1995; Stein and Paladino, 1998; Driver and Westaway, 2003).

2.4 Statistical properties of foreign exchange rates

Besides the previous theoretical body, economists have also developed a large body of empirical research on exchange rate modeling. It mainly concerns four directions that are highly related: fat tail, non-linearity, heteroscedasticity and long memory. The seminal paper of Hsieh (1988) investigated the fat tail properties of daily foreign exchange rate returns. He noticed that a heavy tail probability distribution is unable to explain the kurtosis, and preferred the assumption of a time-varying probability distribution. However, he was not able to explain the leptokurtic property of foreign exchange rates. Hsieh (1989) was more successful when investigating non-linearity dependence and suggested that the main factor is the conditional heteroscedasticity of currency returns. Although this conclusion has

[18] While Huizinga (1987) used the univariate decomposition.

[19] While Allen (1995) formulated the NATREX as "*a moving equilibrium real exchange rate, responding to continual changes in exogenous and endogenous real fundamentals*".

been supported by many other publications[20], another direction has emerged by assuming that foreign exchange rates may exhibit long memory (Booth *et al.*, 1982; Baillie *et al.*, 1996; Cheung, 1993). This last assumption is an important stage for understanding the mean-reverting property of exchange rates and their associated monetary models. This also justifies that exchange rates can present different time-scale behaviors.

The above-mentioned studies raise the question of the empirical validation of the theoretical models. Indeed, most of them are tested against the random walk hypothesis. Based on the evidence that foreign exchange rates incorporate GARCH effects and are non-linear, it is obvious that the right benchmark is not the random walk model. Nevertheless, it could be an endless race to find a theoretical model that better fits the distribution of foreign exchange rates than a pure statistical model. By construction, these statistical models have a short-term horizon (less than one month) while monetary models are more long-term oriented. However, the concept of long-term is so vague in economics that prediction is a biased exercise by definition.

3 Currency market risk factors

In the previous section, we have reviewed the different theories that explain the dynamics of exchange rates. In this section, we build market risk factors that are related to the macroeconomic factors of these theoretical models. We mainly focus on three main risk factors which are carry, value and momentum. Since these risk factors are defined by currency portfolios, they can be used in the day-to-day management of foreign exchange rates. However, the construction of these market risk factors is not straightforward since there is a gap between theoretical concepts and investible portfolios.

3.1 From macroeconomic risk factors to market risk factors

For a long time, the capital asset pricing model of Sharpe (1964) has been the universal model used to determine asset prices. This theory states that there is a unique systematic risk factor that is rewarded, and this risk factor is the return of the market portfolio. In this case, the risk premium of one asset is the product of its beta and the market risk premium. Even if this theory is global, it has been exclusively used for the stock market. Indeed, the concept of market beta is not necessarily appropriate for sovereign bonds, corporate bonds, currencies and commodities.

An alternative approach to the CAPM is the arbitrage pricing theory (APT), which has been developed by Ross (1976). In this model, the return on asset i is driven by a standard linear factor model:

$$R_i = \alpha_i + \sum_{j=1}^{n_{\mathcal{F}}} \beta_i^j \mathcal{F}_j + \varepsilon_i \tag{19}$$

where α_i is the intercept, β_i^j is the sensitivity of asset i to factor j and $\mathcal{F}_j$ is the (random) value of factor j. ε_i is the idiosyncratic risk of asset i, implying that $\mathbb{E}[\varepsilon_i] = 0$, $\operatorname{cov}(\varepsilon_i, \varepsilon_k) = 0$ for $i \neq k$ and $\operatorname{cov}(\varepsilon_i, \mathcal{F}_j) = 0$. Using arbitrage theory, we can show that the expected return of asset i is a linear function of the expected returns of the factors:

$$\mathbb{E}[R_i] - R_f = \sum_{j=1}^{n_{\mathcal{F}}} \beta_i^j \left(\mathbb{E}[\mathcal{F}_j] - R_f\right) \tag{20}$$

[20] Among the profusion of published papers, we can cite the most significative research, by Andersen *et al.* (2001), Baillie and Bollerslev (2002), Bollerslev (1990), Diebold and Nerlove (1989), and Kroner and Sultan (1993).

The underlying idea of APT is that systematic risks are not entirely captured by a single market risk factor. Unlike CAPM which relies on the validity of the Markowitz model[21], APT does not assume a specific utility function. However, it assumes that it is possible to build a portfolio from a large number of assets, that is sufficiently diversified with no specific risk with respect to individual assets.

In the equity market, APT has been popularized by Fama and French (1992, 1993) and Carhart (1997). The four-factor model, which is based on market, size, value and momentum risk factors, has become the standard asset pricing model for stocks. In the bond market, the works of Litterman and Scheinkman (1991) is the first attempt to build a risk factor model for sovereign bonds, and they found that most of the variation in sovereign bond returns can be explained by a three-factor model. The three factors, which are deduced from the principal component analysis of the yield curve, are the level, the steepness and the curvature. In the case of corporate bonds, professionals prefer to consider duration, credit and liquidity risk factors (Gebhardt *et al.*, 2005; Huang and Huang, 2012; Ben Slimane *et al.*, 2019).

APT theory does not necessarily assume market risk factors. For instance, the macro-financial model of Chen *et al.* (1986) can be seen as an asset pricing theory that includes expected and unexpected inflation, and the industrial production growth to explain stock market returns. However, this approach based on macroeconomic risk factors has many limits in practice. Indeed, it can only produce long-run asset prices, but the concept of long-run period is vague and not well-defined in finance. As long as market prices have not converged to theoretical prices, the long term has not been achieved. This implies the long-term changes for each period, each asset and each model. Since the macroeconomic approach is not relevant in the short term and depends on the low-frequency publication of fundamentals, many professionals have switched to market risk factors that can be observed on a daily basis.

The case of foreign exchange rates remains very specific. As seen in the previous section, the majority of the models assume macroeconomic risk factors. For example, the latest stochastic discount factor model of Sarno and Schmeling (2014) continues to use macro fundamentals (inflation, money balances, nominal GDP) for predicting foreign exchange rates. It was only recently that Lustig and Verdelhan (2007) introduced market risk factors but in an indirect way. They formed sorted portfolios based on the level of interest rates and tested different linear factor models with time-varying coefficients in order to explain currency risk premia of sorted portfolios. Our approach is different because our goal is to explain currency returns with market risk factors that are directly extracted from foreign exchange markets. Therefore, the underlying idea is to develop a '*Carhart model*' for currencies.

3.2 The design of currency risk factors

In what follows, we consider the list of currencies given in Table 1. They correspond to the most traded currencies according to the BIS (2016). We can classify them into two groups. The G11 currency group is made up of 11 developed market (DM) currencies: AUD, CAD, CHF, DKK, EUR, GBP, JPY, NOK, NZD, SEK and USD. The second group corresponds to remaining currencies and is mainly made up of emerging market (EM) currencies. However, we notice that it also contains DM currencies (Singapore, Hong Kong, Israel and Korea) according the definition of index sponsors (FTSE, MSCI or S&P).

In Figures 1 and 2, we have reported the turnover of the 40 currencies, which is defined "*as the gross value of all new deals entered into during a given period, and is measured in*

[21]This implies that investors adopt a mean-variance analysis.

Table 1: List of currencies

ARS	Argentine peso	KRW	Korean won
AUD	Australian dollar	LTL	Lithuanian litas
BGN	Bulgarian lev	LVL	Latvian lats
BHD	Bahraini dinar	MXN	Mexican peso
BRL	Brazilian real	MYR	Malaysian ringgit
CAD	Canadian dollar	NOK	Norwegian krone
CHF	Swiss franc	NZD	New Zealand dollar
CLP	Chilean peso	PEN	Peruvian new sol
CNY/RMB	Chinese yuan (Renminbi)	PHP	Philippine peso
COP	Colombian peso	PLN	Polish zloty
CZK	Czech koruna	RON	new Romanian leu
DKK	Danish krone	RUB	Russian rouble
EUR	Euro	SAR	Saudi riyal
GBP	Pound sterling	SEK	Swedish krona
HKD	Hong Kong dollar	SGD	Singapore dollar
HUF	Hungarian forint	THB	Thai baht
IDR	Indonesian rupiah	TRY	Turkish lira
ILS	Israeli new shekel	TWD	new Taiwan dollar
INR	Indian rupee	USD	US dollar
JPY	Japanese yen	ZAR	South African rand

Source: BIS (2016).

terms of the nominal or notional amount of the contacts" (BIS, 2016). The turnovers total 200% because of the bilateral aspect of currency trading. We notice that market activity and liquidity is concentrated in a small number of currencies. For example, the US dollar and the euro represent 88% and 31% of turnover, while the cumulated turnover is equal to 160% with five currencies.

Liquidity is an important criterion when building market risk factors. For instance, some currencies have insufficient turnover to be part of factor portfolios (COP, PHP, RON, PEN, ARS, BGN, BHD, LTL and LVL). Other currencies (BGN, BHD, DKK, HKD and SAR) are pegged to another currency (generally the US dollar). Three currencies (CNY, THB and MYR) are also generally excluded when defining risk factors, because they present some particularities (short history, stressed events, past convertibility problems, etc.). This is why the market practice defines two groups of currencies:

1. AUD, CAD, CHF, EUR, GBP, JPY, NOK, NZD, SEK and USD;

2. BRL, CLP, CZK, HUF, IDR, ILS, INR, KRW, MXN, PLN, RUB, SGD, TRY, TWD and ZAR.

The first group (red) corresponds to the G10 currencies, while the second group (blue) is called the EM (or G15) currencies[22]. We also define a third group G10 + EM (or G25), which is the combination of the two groups.

[22]Even if this group is not exclusively made up of EM countries.

Figure 1: Turnover (in %) of most traded currencies (April 2016)

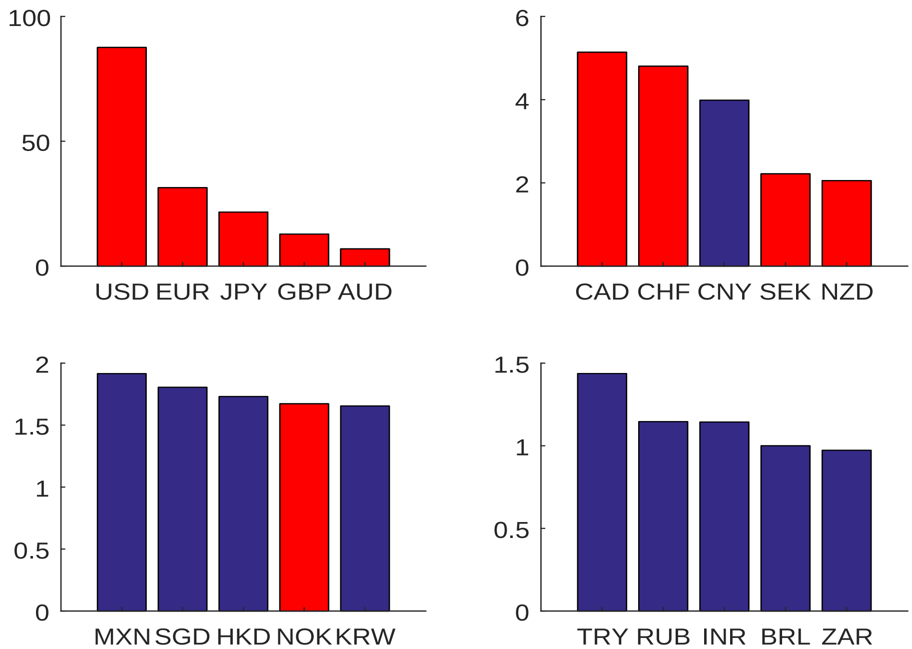
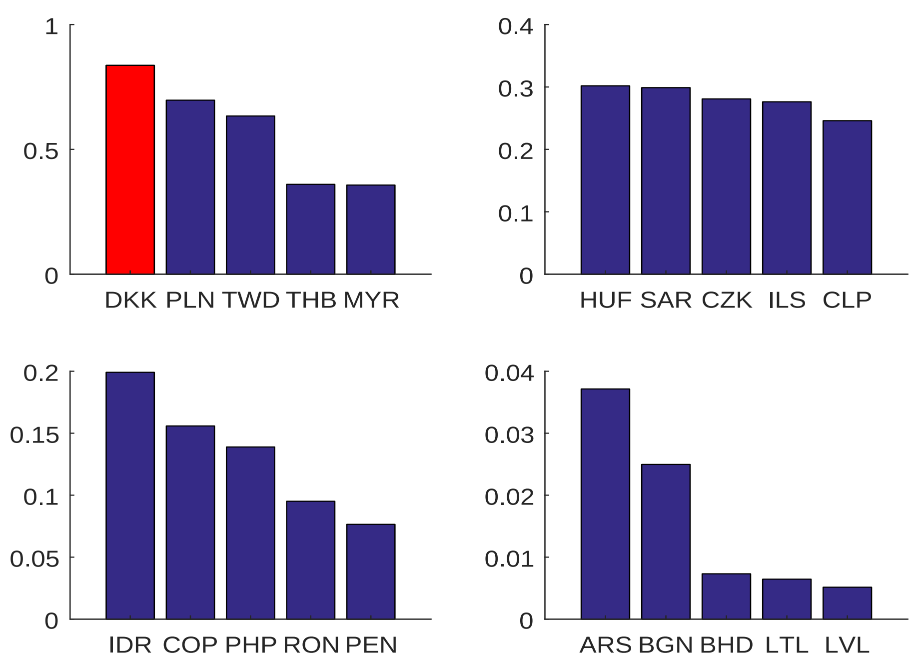


Figure 2: Turnover (in %) of most traded currencies (April 2016)



Once the universe of currencies is established, we build a market risk factor with the following rules:

- At the end of the month, we calculate a score $\mathcal{S}_{i,t}$ for each currency of the eligible universe.
- We create a sorted portfolio by being long on currencies that have the highest score and short on currencies that have the lowest score. We also impose that the number of long exposures is exactly equal to the number of short exposures: 3 for G10, 4 for EM and 7 for G10 + EM. This implies that the portfolio is not invested in all the currencies that make up the universe. Moreover, the G10 + EM risk factor is not a combination of the G10 and EM risk factors. Indeed, the portfolio is not necessarily invested in six G10 currencies and eight EM currencies.
- The performance of the risk factor during the month corresponds to the return of the equally-weighted unfunded portfolio invested in currency future contracts.
- We rebalance the portfolio every month.

Contrary to stocks and bonds, the market practice is to choose a uniform weighting scheme for two related main reasons. First, the risk factors are based on a small number of assets (25 in the case G10 + EM). The equally-weighted portfolio then reduces the (possible) idiosyncratic risk of the systematic risk factor. Second, a capitalization-weighted (or turnover-weighted) portfolio would certainly not represent the behavior of a systematic risk factor, but would correspond more to a long/short trading portfolio between the currencies with the highest turnover.

Remark 2 *The G10 + EM risk factor is not a combination of the two G10 and EM risk factors. Indeed, we only merge the two universes when we define the G10 + EM risk factor. This implies that the selected currencies of the three risk factors are not necessarily the same. By construction, the G10 + EM risk factor is composed of fourteen currencies. Therefore, we could face the situation where the G10 + EM portfolio contains a minimum of zero G10 and five EM currencies, and a maximum of nine G10 and fourteen EM currencies.*

3.3 Cross-section risk factors

3.3.1 Carry risk factor

Definition Koijen *et al.* (2018) defined the carry of a futures (or forward) contract F_t as the expected return if the spot price S_t remains the same. Let X_t be the capital allocated at time t to finance a futures position on the asset. At time $t+1$, the excess return of this investment is[23]:

$$R_{t+1}(X_t) - R_f = \mathcal{C}_t + \frac{\mathbb{E}_t[\Delta S_{t+1}]}{X_t} + \varepsilon_{t+1}$$

where $\varepsilon_{t+1} = (S_{t+1} - \mathbb{E}_t[S_{t+1}])/X_t$ is the unexpected price change and $\mathcal{C}_t$ is the carry:

$$\mathcal{C}_t = \frac{S_t - F_t}{X_t}$$

[23] Based on the assumption that the futures price expires at the future spot price ($F_{t+1} = S_{t+1}$), Koijen *et al.* (2018) showed that:

$$\begin{aligned} R_{t+1}(X_t) - R_f &= \frac{F_{t+1} - F_t}{X_t} \\ &= \frac{S_t - F_t}{X_t} + \frac{\mathbb{E}_t[S_{t+1}] - S_t}{X_t} + \frac{S_{t+1} - \mathbb{E}_t[S_{t+1}]}{X_t} \end{aligned}$$

It follows that the expected excess return is the sum of the carry and the expected price change:

$$\mathbb{E}_t\left[R_{t+1}\left(X_t\right)\right] - R_f = \mathcal{C}_t + \frac{\mathbb{E}_t\left[\Delta S_{t+1}\right]}{X_t}$$

The nature of these two components is different. The carry is an ex-ante observable quantity whereas the expected price change depends on the dynamic model of S_t. If we assume that the spot price does not change, the expected excess return is equal to the carry. This means that the carry investor will prefer asset i to asset j if the carry of asset i is higher:

$$\mathcal{C}_{i,t} \geq \mathcal{C}_{j,t} \Rightarrow i \succ j$$

The carry strategy would then be long on high carry assets and short on low carry assets.

Let us apply the carry strategy to currency markets. In the previous section, we have showed that the forward exchange rate F_t is equal to:

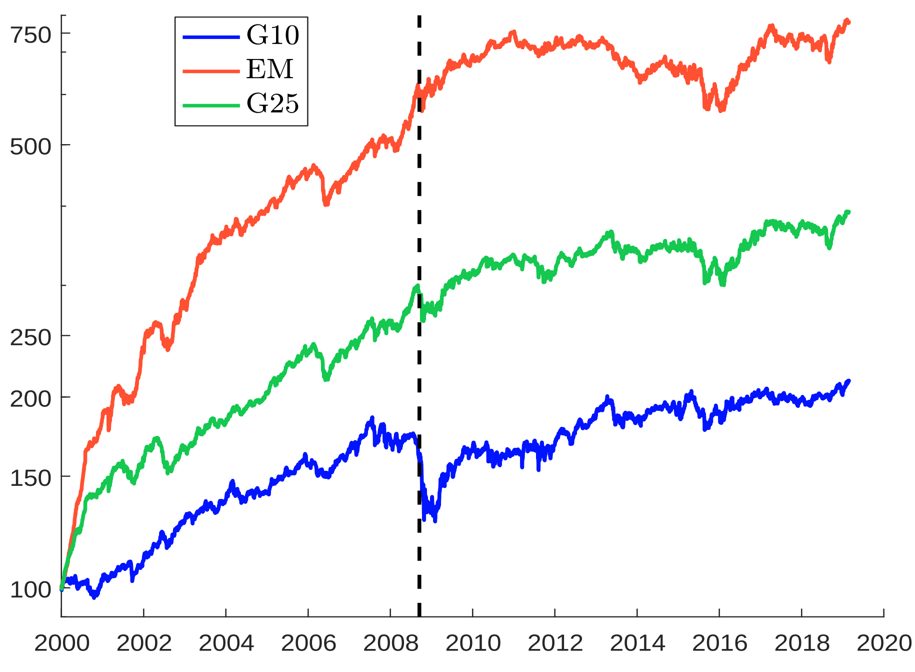
$$F_t = \frac{1 + i_t}{1 + i_t^*} S_t$$

implying that the carry is approximately equal to the interest rate differential:

$$\mathcal{C}_t = \frac{i_t^* - i_t}{1 + i_t} \simeq i_t^* - i_t$$

The carry strategy is then long on currencies with high interest rates and short on currencies with low interest rates.

Figure 3: Cumulative performance of the carry risk factor



Results In Figure 3, we report the carry risk factor for the three universes: G10, EM and G25 (G10 + EM). In Table 2, we also give the main risk/return statistics for the period

2000-2018. We notice that the EM carry risk factor presents the best performance, and the volatility of G10 and EM carry risk factors are similar. Therefore, it follows that the EM carry risk factor has a better Sharpe ratio than the G10 carry risk factor. Contrary to risk factors in other asset classes, we also notice that the EM carry risk factor does not have a larger drawdown than the G10 carry risk factor. An interesting result is the good diversification between G10 and EM carry strategies, since the volatility and the drawdown are lower when considering the G10 + EM currency universe. Indeed, the G25 carry risk factor benefits from the low correlation between G10 and EM strategies[24], which is equal to 29.2%.

Table 2: Risk/return statistics of the carry risk factor (2000-2018)

	G10	EM	G25
Excess return (in %)	3.75	11.21	7.22
Volatility (in %)	9.35	9.12	8.18
Sharpe ratio	0.40	1.23	0.88
Maximum drawdown (in %)	−31.60	−25.27	−17.89

3.3.2 Value risk factor

Definition The value measure is related to the concept of fair price. Let $S_{i,t}$ and $\hat{S}_{i,t}$ be the current and fair prices of Asset i. The value measure is defined as:

$$\mathcal{V}_{i,t} = \frac{\hat{S}_{i,t} - S_{i,t}}{S_{i,t}}$$

If $\mathcal{V}_{i,t}$ is positive (resp. negative), we anticipate that the asset is underpriced (resp. overpriced). The value strategy consists then of long assets with a positive value and short assets with negative value. One of the issues associated with this risk factor is how to precisely define the fair price. For instance, in the stock market, the most common metrics are the book-to-market ratio, the free cash-flow yield or the dividend yield. In the commodity market, the fair price may be estimated as the five-year moving average (Asness *et al.*, 2013). In the credit market, the fundamental price is calculated with a Merton-like structural model. More generally, there are two main approaches for defining a fair price: the economic approach and the historical approach. The first method uses theoretical economic models to define the long-run equilibrium price $\hat{S}_{i,t}$ of the asset. The second method consists in calculating the fair price as the long-term average of spot prices. In this case, the value risk factor is a typical mean-reverting strategy. It follows that value is less a model-free strategy than carry since it highly depends on the parameterization: the choice of the theoretical model, the econometric estimation procedure or the length of historical data.

In the case of foreign exchange rates, the economic approach is preferred since we have many economic models for computing real equilibrium exchange rates. Moreover, the historical approach is inconsistent with the occurrence of currency devaluations. Using the survey done in the previous section, we focus on the four main models: PPP, FEER, BEER and NATREX. This means that a currency does not have only one value, but several value

[24] In Appendix (Figure 29 on page 90), we have reported the weekly payoff between the two strategies. We verify that the stochastic dependence is low.

dimensions. Since we use direct exchange rates, the value measure takes the following form[25]:

$$\mathcal{V}_{i,t} = \frac{Q_{i,t} - \hat{Q}_{i,t}}{Q_{i,t}}$$

where $\hat{Q}_{i,t}$ is the real equilibrium exchange rate.

Remark 3 *Professionals may also use the alternative measure:*

$$\begin{aligned} \mathcal{V}_{i,t} &= \frac{Q_{i,t} - \hat{Q}_{i,t}}{\hat{Q}_{i,t}} \\ &\simeq \ln \frac{Q_{i,t}}{\hat{Q}_{i,t}} \\ &= q_{i,t} - \hat{q}_{i,t} \end{aligned}$$

where $q_{i,t} = \ln Q_{i,t}$ and $\hat{q}_{i,t} = \ln \hat{Q}_{i,t}$. Even if the two value measures are very close, they do not necessarily produce the same sorting.

PPP One of the most widely used methods to assess the value of exchange rates is Purchasing Power Parity (PPP). This theory essentially states that exchange rates adjust to reflect the difference between inflation rates among countries. For instance, countries experiencing higher inflation must depreciate their exchange rate. However, the PPP holds only on the long term and, as such, it is used by market participants as guidance for the long-run fair value of the exchange rate. A similar method to the PPP states that the real effective exchange rate (REER) must periodically return to its long-term average. Thus, the long-term average of the REER can be used as a proxy to infer the equilibrium REER. Therefore, we use this approach and benchmark the PPP by calculating REER deviation from its long-term average[26].

In Figure 4, we have reported the cumulative performance of the PPP risk factor. We notice that the behavior of the G10 risk factor is completely different from the EM risk factor, especially since 2010. Over the last 10 years, the performance of the G10 risk factor has been flat while it is very high for the EM risk factor. If we compare the risk of carry and PPP strategies, the volatility is similar, but the drawdown is lower for the PPP strategy. In fact, we verify that the carry strategy is highly risky and may suffer from short-term crashes.

Table 3: Risk/return statistics of the PPP risk factor (2000-2018)

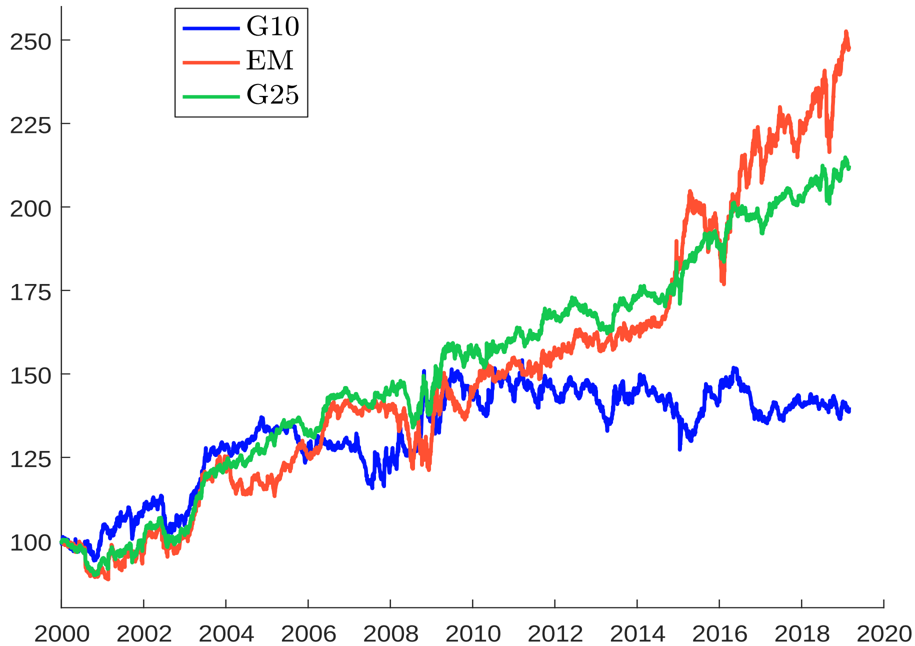
	G10	EM	G25
Excess return (in %)	1.84	4.85	4.07
Volatility (in %)	7.84	8.45	5.74
Sharpe ratio	0.23	0.57	0.71
Maximum drawdown (in %)	−17.47	−15.00	−10.70

[25] If we use indirect exchange rates, we retrieve the previous formula:

$$\mathcal{V}_{i,t} = \frac{\hat{Q}_{i,t} - Q_{i,t}}{Q_{i,t}}$$

[26] The base period is set at January 1994 and we use the REER series calculated by the Bank for International Settlements (BIS).

Figure 4: Cumulative performance of the PPP risk factor



FEER/BEER In this article, we follow Salto and Turrini (2010) to estimate the fundamental equilibrium exchange rate (FEER). Broadly speaking, the authors state that in the current account based-models, the misalignment is defined as the change in the real effective exchange rate needed to close the gap between the "*underlying current account*" and the "target *current account*":

$$\frac{Q_t - \mathrm{FEER}_t}{\mathrm{FEER}_t} = \frac{(CA/Y)_t^{\mathrm{UND}} - (CA/Y)_t^{\mathrm{TARGET}}}{e_t}$$

where $(CA/Y)_t^{\mathrm{UND}}$ and $(CA/Y)_t^{\mathrm{TARGET}}$ are underlying and target current accounts as percentage of GDP, and the denominator e_t is the current account long-term semi-elasticity[27]. Hence, the estimation requires essentially three elements: a measure of the underlying current account $(CA/Y)_t^{\mathrm{UND}}$, a notion of the target current account $(CA/Y)_t^{\mathrm{TARGET}}$ and finally the current account semi-elasticity e_t which allows us to translate the gap between underlying and target current accounts into a gap between actual and equilibrium exchange rates. In our model, the underlying current account follows the specification which is given in Saadaoui (2018):

$$\begin{aligned}(CA/Y)_t^{\mathrm{UND}} \quad = \quad & (CA/Y)_t + ((M/Y)_t \beta_M + (X/Y)_t \beta_X)(0.4\Delta q_t + 0.15\Delta q_{t-1}) + \\ & (M/Y)_t \delta_M YGAP_t - (X/Y)_t \delta_X YGAP_t^{\star}\end{aligned}$$

where $(CA/Y)_t^{\mathrm{UND}}$ is the underlying current account as a percentage of GDP, Y_t is the GDP, $(CA/Y)_t$ is the current account balance, M_t is the imports of goods and services, X_t

[27]The current account's long-term semi-elasticity is defined as:

$$e_t = \frac{\Delta CA_t / Y_t}{\Delta Q_t / Q_{t-1}}$$

where CA_t is the current account, Y_t is the GDP and Q_t is the real effective exchange rate.

is the exports of goods and services, q_t is the logarithm of the real exchange rate, $YGAP_t$ is the domestic output gap and $YGAP_t^\star$ represents the foreign output gap, which is the average output gap of the main trade partners[28]. β_M and β_X are the long-run import and export price elasticity coefficients while δ_M and δ_X are the long-run export and import volume elasticity coefficients[29]. We note that a drawback of the FEER model is that it is a normative model and the '*desirable*' or '*target*' current account is arbitrary. We thus decide to follow Cline (2017) and estimate the target current account $(CA/Y)_t^{\text{TARGET}}$ using the IMF's current account forecasts on a 5-year horizon since it is supposed to take into account the evolution of economic fundamentals. However, given the arbitrary nature of this choice, we also make a second estimation by considering the average of the current account of the past 10 years as the target. Finally, the long-term elasticity is calculated following the IMF's methodology (Phillips *et al.*, 2013).

The behavioral equilibrium exchange rate (BEER) model tries to find a long term relationship between the REER and different macroeconomic fundamentals. In this paper, we use four macroeconomic variables to measure the exchange rate's fair value:

1. Productivity (ϱ) which is approximated using PPP-based GDP per capita. Changes in the REER are also linked to changes in a country's relative productivity. The "*Balassa-Samuelson*" effect suggests that higher productivity growth in the trade-goods sector tends to cause the REER to appreciate. However, we note that while total-factor productivity is seemingly very important in accounting for changes in REERs, it is hard to measure and it is difficult to arrive at a uniform approximation for a large set of countries. In our model, PPP-based GDP per capita is taken as a log and as a difference relative to trade partners' GDP per capita (weighted by the share in the trade of the country being considered).

2. Terms of trade (ToT) which is measured as the ratio of export prices to import prices. Fluctuations in the REER are primarily linked to terms of trade. For instance, in countries producing commodities, upward pressure on commodities prices leads to an appreciation of the REER. This is especially the case for the "*commodity currencies*" such as the Australian, Canadian or New Zealand dollar or the Chilean or Colombian peso. Terms of trade are considered relative to the terms of trade of the trade partners (weighted by the share in the trade of the country being considered).

3. Debt-to-GDP ratio (Debt). Rising public debt increases the risk of default and the risk of inflation, which is usually negative for the currency. Debt is considered relative to the debt of trade partners (weighted by the share in the trade of the country being considered).

4. Open trade as a percentage of GDP (Open) which is the sum of imports and exports divided by two. The most open economies are usually the regions where the prices of traded goods are the lowest. Therefore, an increase in open trade should lead to a fall in REER.

The model consists in estimating a panel cointegration relationship (with fixed effects) between the logarithm of the real effective exchange rate and the previous four macroeconomic variables:

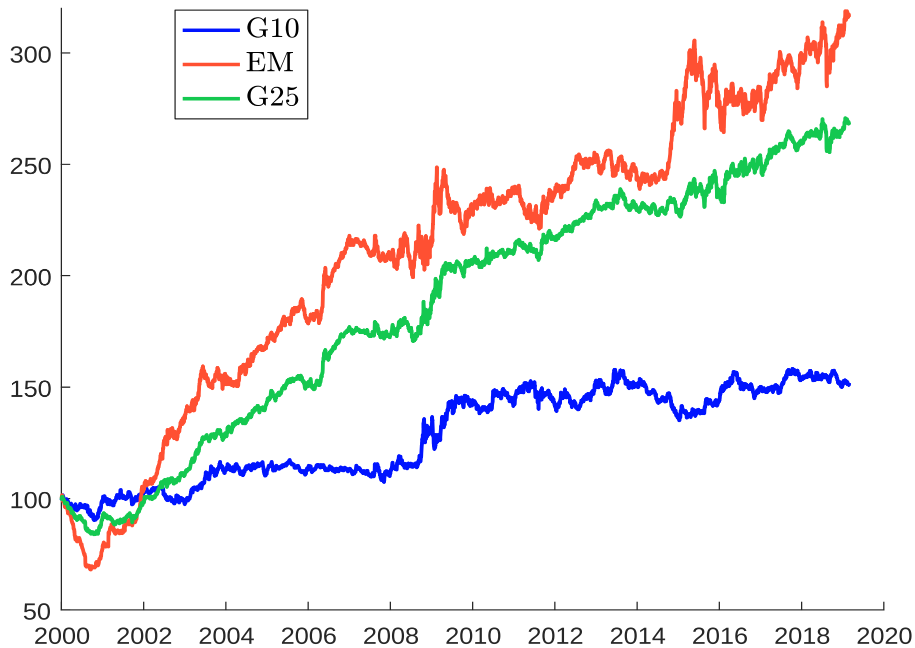
$$q_{i,t} = \beta_0 + \beta_1 \varrho_{i,t} + \beta_2 ToT_{i,t} + \beta_3 Debt_{i,t} + \beta_4 Open_{i,t} + \alpha_i + \varepsilon_{i,t} \tag{21}$$

[28] It is weighted by the share in trade of the different countries being considered.

[29] They are estimated following the external balance assessment (EBA) methodology of IMF (Phillips *et al.*, 2013).

where β_0 is the intercept, $(\beta_1, \ldots, \beta_4)$ is the vector of slope coefficients, α_i is the individual intercept (or the fixed effect) and $\varepsilon_{i,t}$ is the residual. This model has been estimated quarterly since 2000 in order to not be impaired by the Asian financial crisis.

Figure 5: Cumulative performance of the BEER risk factor



We have calibrated the two models. However, the FEER estimates only begin in 2011 because of the available data. This is why we focus on the BEER estimates[30]. Results are given in Figure 5 and Table 5. Not surprisingly, we observe some similarity between PPP and BEER risk factors. First, there is a clear difference in terms of behavior between G10 and EM risk factors. The performance of the EM risk factor is better, whereas the performance of the G10 risk factor has been flat since 2010. The volatility is also similar between PPP and BEER risk factors. However, the drawdown of the EM BEER risk factor is twice the drawdown of the EM PPP risk factor. This is because the dynamics of the macroeconomic fundamentals are more chaotic than the dynamics of the PPP variable in emerging countries, especially the open trade and the debt-to-GDP ratio.

Table 4: Risk/return statistics of the BEER risk factor (2000-2018)

	G10	EM	G25
Excess return (in %)	2.19	6.15	5.28
Volatility (in %)	7.29	8.48	5.68
Sharpe ratio	0.30	0.73	0.93
Maximum drawdown (in %)	−14.49	−32.97	−16.85

[30] We also observe a correlation larger than 70% between the returns of the two risk factors.

NATREX Let Q_t, τ_t and ϱ_t be the real exchange rate, the time preference and the productivity[31]. We note $y_t = (\ln Q_t, \ln \tau_t - \ln \tau_t^\star, \ln \varrho_t - \ln \varrho_t^\star)$. We assume that y_t is integrated of order one — $y_t \sim I(1)$, and there is a cointegration relationship between these variables — $y_t \sim CI(1,0)$. This is equivalent to say that the disequilibrium z_t is integrated of order zero:

$$z_t = \ln Q_t - \beta_0 - \beta_1 \ln \frac{\tau_t}{\tau_t^\star} - \beta_2 \ln \frac{\varrho_t}{\varrho_t^\star} \sim I(0)$$

In order to estimate the NATREX model, we consider the linear regression:

$$\ln Q_t = \beta_0 + \beta_1 \ln \frac{\tau_t}{\tau_t^\star} + \beta_2 \ln \frac{\varrho_t}{\varrho_t^\star} + u_t$$

It follows that the NATREX rate is equal to:

$$\mathrm{NATREX}_t^\star = \exp\left(\hat\beta_0 + \hat\beta_1 \ln \frac{\tau_t}{\tau_t^\star} + \hat\beta_2 \ln \frac{\varrho_t}{\varrho_t^\star}\right) \tag{22}$$

The drawback of this approach is that the long-run period is not very-well defined, meaning the previous scoring does not take into account the time of convergence. Let us consider two currencies that are undervalued by 20%. If one currency returns to the equilibrium in one year and the second currency returns to the equilibrium in ten years, the first currency has a better value than the second currency. This is why we consider a second version of the NATREX rate, which is defined for a given horizon h. Since $y_t \sim CI(1,0)$, we know that the dynamics of y_t is given by a vector-error correction model (VECM). We assume that the optimal lag is one:

$$\Delta y_t = \Phi \Delta y_{t-1} - \alpha z_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim \mathcal{N}(0, \Omega)$. Using the method of maximum likelihood, we estimate the vector of parameters $\theta = (\operatorname{vec}\Phi, \alpha, \operatorname{vech}\Omega)$. It follows that the NATREX rate for the given time horizon h is equal to[32]:

$$\begin{aligned} \mathrm{NATREX}_t(h) &= \exp\left(\mathbb{E}\left[\ln Q_{t+h} \mid \mathcal{F}_t\right]\right) \\ &= \exp\left(\mathbf{e}_1^\top \left(C^h \begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix} - \sum_{k=0}^{h-1} C^{k-1} \begin{pmatrix} \hat\alpha\hat\beta_0 \\ \mathbf{0}_3 \end{pmatrix}\right)\right) \end{aligned} \tag{23}$$

where $\mathbf{e}_1 = (1,0,0,0,0,0)$, $\hat\Pi = \hat\alpha\gamma^\top$, $\gamma = \left(1, -\hat\beta_1, -\hat\beta_2\right)$ and:

$$C = \begin{pmatrix} I_3 + \hat\Phi - \hat\Pi & -\hat\Phi \\ I_3 & \mathbf{0}_{3\times 3} \end{pmatrix}$$

In this case, we define the value measure as follows:

$$\mathcal{V}_t = \frac{Q_t - \mathrm{NATREX}_t(h)}{Q_t} \tag{24}$$

The difference between Equations (22) and (23) is the time horizon. Equation (22) defines a static long-run NATREX rate by considering that time preference and productivity are

[31]Following Stein and Allen (1997), the time preference is defined as the ratio of private consumption and government purchases to GDP, whereas the productivity is the GDP per capita.

[32]We apply the results given in Appendix A.2.1 on page 75.

exogenous and are already at their equilibrium level. On the contrary, Equation (23) assumes that time preference and productivity are endogenous, and produces a dynamic NATREX rate. At the time horizon $h = 0$, we verify that the NATREX is equal to the current REER:

$$\text{NATREX}_t\,(0) = Q_t$$

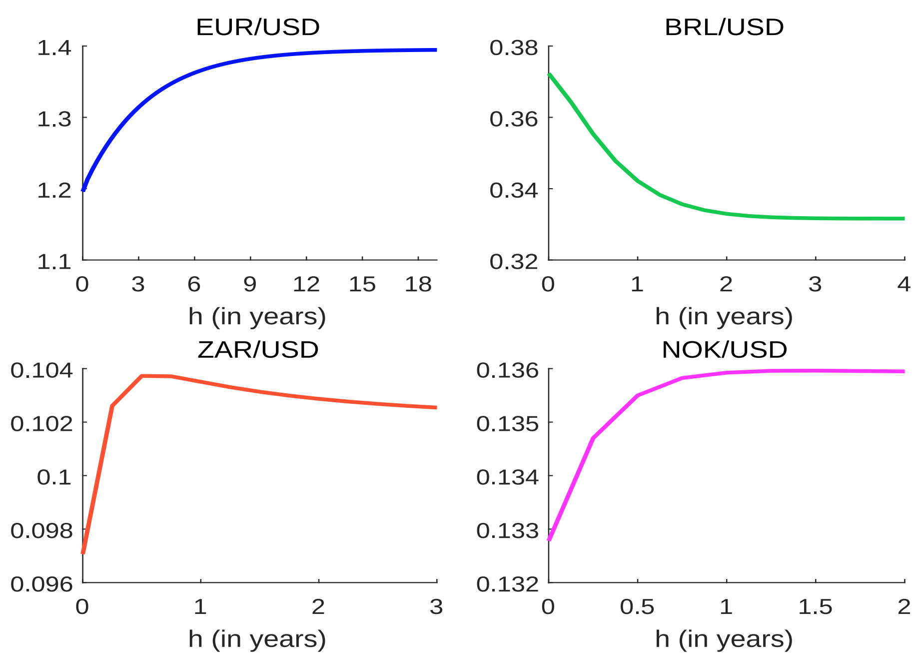
When $h \to \infty$, we obtain the long-run NATREX:

$$\text{NATREX}_t\,(\infty) = \lim_{h \to \infty} \text{NATREX}_t\,(h)$$

The VECM approach is then more interesting since we obtain the NATREX path between the current REER and the long-run equilibrium. If the long-run values of τ_t, $\tau_t^\star$, ϱ_t and $\varrho_t^\star$ coincide with the current values, we verify the identity between static and dynamic long-run NATREX rates:

$$\text{NATREX}_t\,(\infty) = \text{NATREX}_t^\star$$

Figure 6: NATREX dynamics (March 2018)



Source: Hervé and Roncalli (2018)

In order to illustrate the difference between the two approaches, we have reported the dynamics of $\text{NATREX}_t\,(h)$ for several currencies at the end of March 2018 in Figure 6. According to the NATREX model, EUR, ZAR and NOK are undervalued while BRL is overvalued. The convergence to the long-run equilibrium differs from one currency to another one. For instance, it will take a long time for EUR while NOK will converge very soon. In Table **??**, we show the REER variation forecast for several time horizon. We also report the convergence horizon $h^\star$ defined as follows:

$$h^\star = \arg\inf \left\{ h : \left| \frac{\text{NATREX}_t\,(h) - \text{NATREX}_t\,(\infty)}{\text{NATREX}_t\,(\infty)} \right| \leq 5 \text{ bps} \right\}$$

Table 5: Real exchange rate variation forecasts (March 2018)

Currency	Q1 2019	Q1 2020	Q1 2023	∞	$h^\star$ (in year)
EUR	4.43%	7.55%	12.96%	16.65%	11.25
CHF	−2.66%	− 5.06%	− 7.86%	− 8.67%	6.50
NOK	2.37%	2.39%	2.36%	2.36%	0.75
SEK	6.58%	9.64%	12.32%	12.69%	4.75
CAD	0.67%	0.85%	1.05%	1.10%	1.00
AUD	5.36%	6.33%	6.62%	6.62%	2.00
JPY	6.09%	8.24%	9.91%	10.09%	4.00
ZAR	6.64%	5.98%	5.36%	5.22%	0.50
MXN	2.22%	4.70%	7.72%	8.59%	6.25
BRL	−8.09%	−10.58%	−10.93%	−10.93%	2.25

Source: Hervé and Roncalli (2018).

It is equal to 11 years for EUR whereas it is less than one year for ZAR.

These results pose the problem of choosing the right NATREX rate to define the value of the currency: $\mathrm{NATREX}_t^\star$, $\mathrm{NATREX}_t(h)$ with a fixed horizon period h (e.g. two years) or $\mathrm{NATREX}_t(\infty)$. Finally, we consider the dynamic long-run equilibrium and we have:

$$V_t = \frac{Q_t - \mathrm{NATREX}_t(\infty)}{Q_t}$$

This choice is motivated by two reasons. First, the assumption that time preference and productivity are at their equilibrium is unrealistic. Second, we have selected the NATREX model in order to define a long-term value component. For example, choosing $\mathrm{NATREX}_t(2)$ will certainly be more coherent with PPP and BEER, but it does not correspond to a long-term equilibrium.

Results are given in Figure 7 and Table 6. Contrary to PPP and BEER risk factors, we observe that the G10 NATREX risk factor performs well while the EM NATREX risk factor is the worst performer. We also notice that the NATREX risk factor is the value risk factor that presents the more positive skewness.

Table 6: Risk/return statistics of the NATREX risk factor (2003-2018)

	G10	EM	G25
Excess return (in %)	4.99	4.14	4.81
Volatility (in %)	6.24	7.89	5.32
Sharpe ratio	0.80	0.52	0.90
Maximum drawdown (in %)	−16.61	−16.50	−13.14

Composite factor We define the composite value risk factor as the equally-weighted portfolio of the previous approaches (PPP, BEER and NATREX). The excess performance of this composite value risk factor is reported in Figure 8, whereas the risk/return statistics are given in Table 7. We notice that the Sharpe ratio of the composite value risk factor is improved with respect to the other value risk factors. It benefits from the low correlation

Figure 7: Cumulative performance of the NATREX risk factor

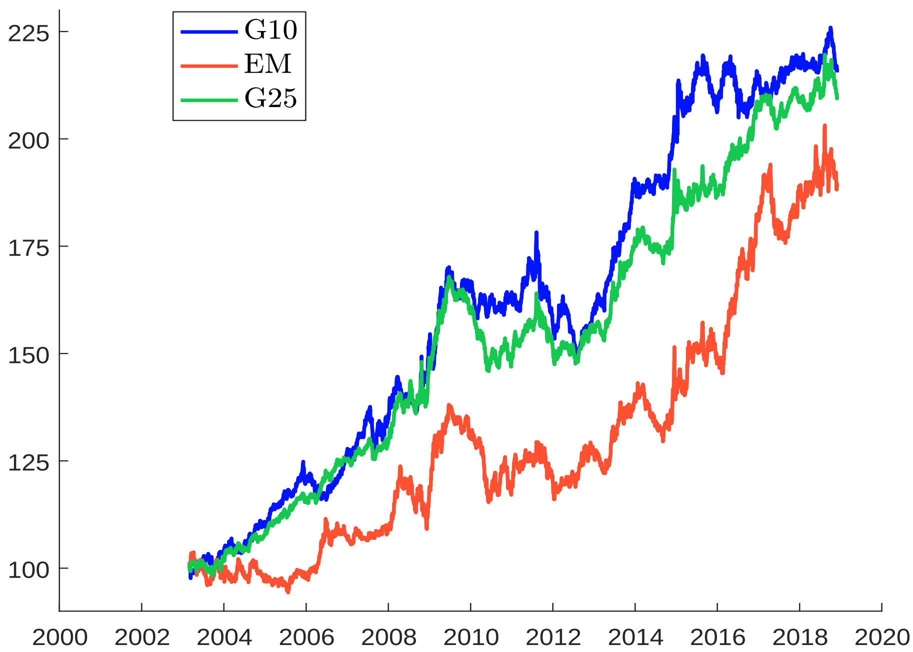
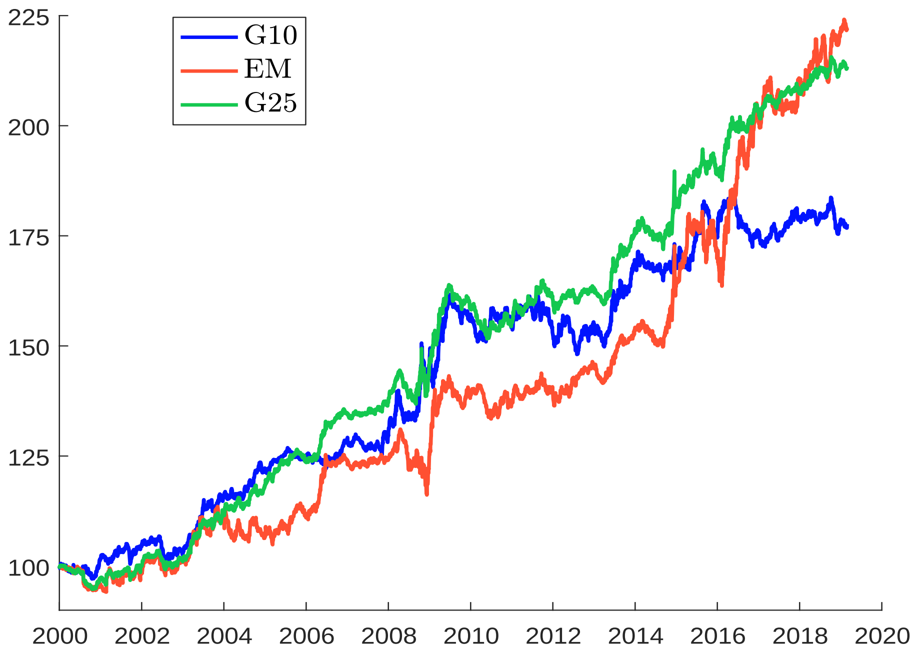


Figure 8: Cumulative performance of the value risk factor



between the NATREX and the two other risk factors (see Table 8). On the contrary, PPP and BEER are highly correlated.

Table 7: Risk/return statistics of the value risk factor (2000-2018)

	G10	EM	G25
Excess return (in %)	3.09	4.27	4.08
Volatility (in %)	4.82	6.01	4.05
Sharpe ratio	0.64	0.71	1.01
Maximum drawdown (in %)	−8.45	−11.42	−7.52

Table 8: Correlation between the value risk factors (2000-2018)

	G10			EM			G25		
	PPP	BEER	NAT- -REX	PPP	BEER	NAT- -REX	PPP	BEER	NAT- -REX
PPP	100.0	66.7	−1.7	−14.5	−5.5	12.9	51.7	33.7	12.0
BEER	66.7	100.0	13.7	−11.6	−9.1	10.9	31.5	46.5	16.6
NATREX	−1.7	13.7	100.0	−8.8	−13.6	9.3	−6.7	−3.5	54.2
PPP	−14.5	−11.6	−8.8	100.0	66.7	7.3	60.8	47.4	−3.1
BEER	−5.5	−9.1	−13.6	66.7	100.0	−3.8	46.1	68.7	−15.0
NATREX	12.9	10.9	9.3	7.3	−3.8	100.0	13.9	3.1	77.0
PPP	51.7	31.5	−6.7	60.8	46.1	13.9	100.0	64.1	8.6
BEER	33.7	46.5	−3.5	47.4	68.7	3.1	64.1	100.0	−1.0
NATREX	12.0	16.6	54.2	−3.1	−15.0	77.0	8.6	−1.0	100.0

Remark 4 *We have decided to not include the FEER risk factor as it is highly correlated to the PPP and BEER risk factors. Ideally, the composite value risk factor should reflect three different time horizons: short-term, medium-term and long-term. We reiterate that BEER is more a short-term to medium-term risk factor while NATREX is definitively a long-term risk factor. We think that adding FEER will reduce the long-term contribution in the composite risk factor, and then increase the correlation with the carry risk factor.*

3.3.3 Momentum risk factor

The momentum risk factor has been extensively documented both for equities (Jegadeesh and Titman, 1993; Carhart, 1997) and commodities (Erb and Harvey, 2006; Miffre and Rallis, 2007). Moskowitz *at al.* (2012) and Asness *et al.* (2013) also discuss the presence of momentum in other asset classes, for instance in currencies and fixed-income instruments. The momentum strategy is well-known in investment management and has been used by hedge funds and CTAs for many years (Lempérière *et al.*, 2014). It corresponds to the trend-following strategy and is called "*time-series momentum*" by Moskowitz *at al.* (2012). Nevertheless, a variant of this strategy was proposed a long time ago by Carhart (1997) for the purposes of analyzing the return on equity portfolios. This second strategy is known as "*cross-section momentum*".

The momentum of Asset i at time t corresponds to its past return:

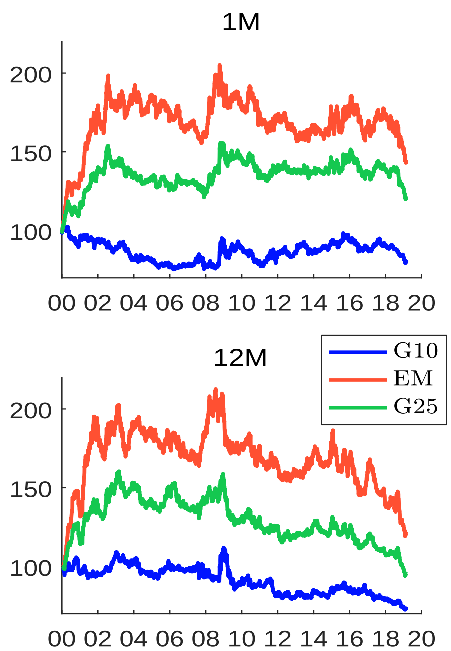
$$\mathcal{M}_{i,t} = \frac{S_{i,t} - S_{i,t-h}}{S_{i,t}}$$

where $S_{i,t}$ is the asset price and h is the momentum period. Using cross-section (CS) momentum, we have:

$$\mathcal{M}_{i,t} > \mathcal{M}_{j,t} \Longrightarrow i \succ j$$

This means that the portfolio is long on currencies that present a higher momentum than the other currencies.

Figure 9: Cumulative performance of the cross-section momentum risk factor

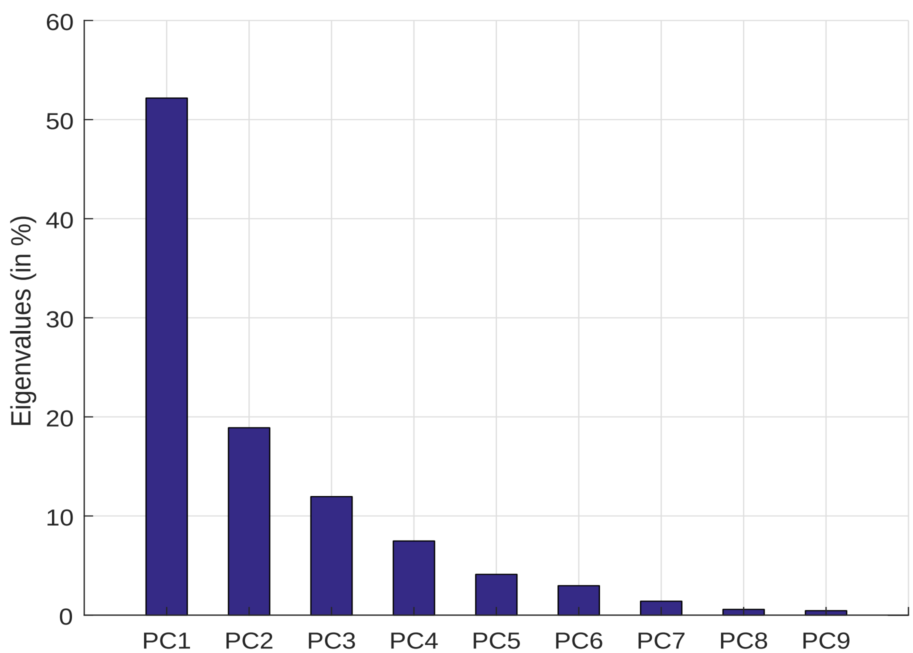


We consider different time-horizons for defining the momentum risk factor. In practice, three periods are generally used: a one-month (1M) horizon for a short-term momentum, three-months (3M) for a medium-term momentum and one-year (12M) for a long-term momentum. We have reported the risk/return statistics in Table 9. We notice that G10 momentum strategies post a negative performance, while EM momentum strategies post a positive performance. However, we should be cautious with these results, because the good performance of EM momentum strategies is mainly due to the 2000-2004 period. When analyzing the cumulative performance of these strategies in Figure 9, we have the feeling that the different strategies are highly dependent or correlated. In fact, this is not true. Indeed, a principal component analysis reveals that only 52% of the weekly return variance is explained by the first principal factor (Figure 10), meaning that the other statistical factors play a significant role. For example, the second and third factors explain respectively 19% and 12% of the variance. Therefore, this diversification pattern is larger than that observed in stock and sovereign bond markets. This diversification effect also explains the large reduction of the drawdown when considering the G10 composite momentum risk factor.

Table 9: Risk/return statistics of the cross-section momentum risk factor (2000-2018)

		G10	EM	G25
1M	Excess return (in %)	−0.95	2.24	1.25
	Volatility (in %)	8.43	9.23	7.46
	Sharpe ratio	−0.11	0.24	0.17
	Maximum drawdown (in %)	−26.58	−26.16	−21.47
3M	Excess return (in %)	−1.92	2.38	0.25
	Volatility (in %)	8.71	9.19	7.60
	Sharpe ratio	−0.22	0.26	0.03
	Maximum drawdown (in %)	−37.05	−23.41	−35.94
12M	Excess return (in %)	−1.58	1.24	−0.06
	Volatility (in %)	8.49	9.36	7.55
	Sharpe ratio	−0.19	0.13	−0.01
	Maximum drawdown (in %)	−34.03	−40.93	−38.56
Composite	Excess return (in %)	−1.35	2.11	0.58
	Volatility (in %)	6.78	7.34	6.12
	Sharpe ratio	−0.20	0.29	0.09
	Maximum drawdown (in %)	−26.00	−28.60	−30.48

Figure 10: Principal component analysis of cross-section momentum risk factors (weekly returns)



3.4 Time-series risk factors

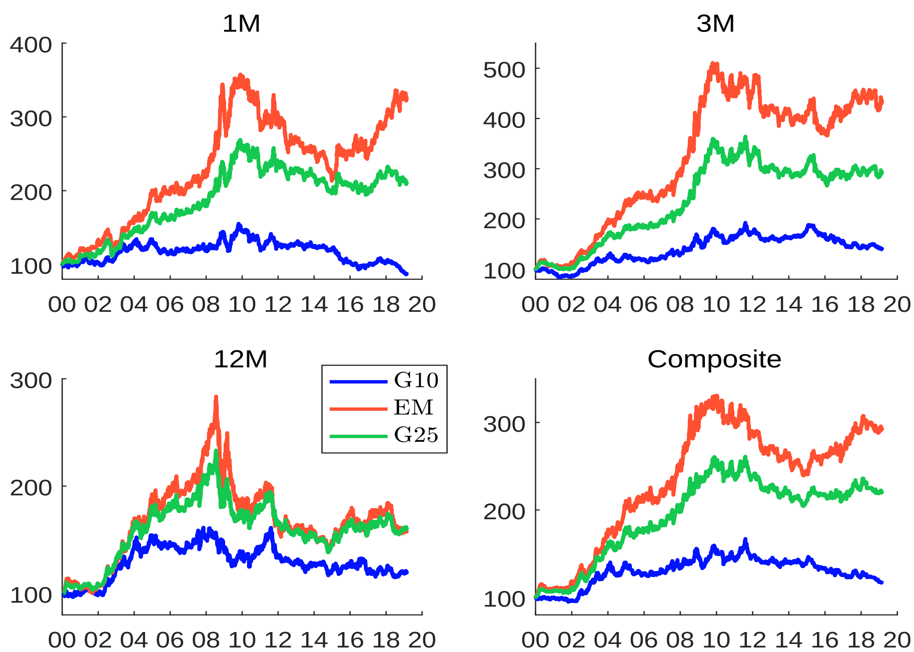
3.4.1 Momentum risk factor

The time-series momentum strategy is defined by:

$$\begin{cases} \mathcal{M}_{i,t} > 0 \Longrightarrow i \succ 0 \\ \mathcal{M}_{i,t} < 0 \Longrightarrow i \prec 0 \end{cases}$$

In this case, the portfolio is long on the currency i if it has positive momentum (and, conversely, short on the currency i if it has negative momentum). This strategy is also called the "*trend continuation*", because it assumes that the past trend is a predictor of the future trend. This contrasts with the cross-section momentum strategy, where currencies with negative trends can make up the long exposure. Indeed, cross-section momentum is based on relative momentum while time-series momentum is based on absolute momentum.

Figure 11: Cumulative performance of the time-series momentum risk factor



As expected, time-series and cross-section momentum present different patterns if we compare Figures 9 and 11. In particular, the performance is better for time-series momentum, especially during the period 2000-2008. Moreover, we notice that time-series momentum risk factors are less dependent on the window parameter than cross-section momentum risk factors. Indeed, the principal component analysis shows that the first statistical factor explains more than 70% of the variance of time-series momentum risk factors.

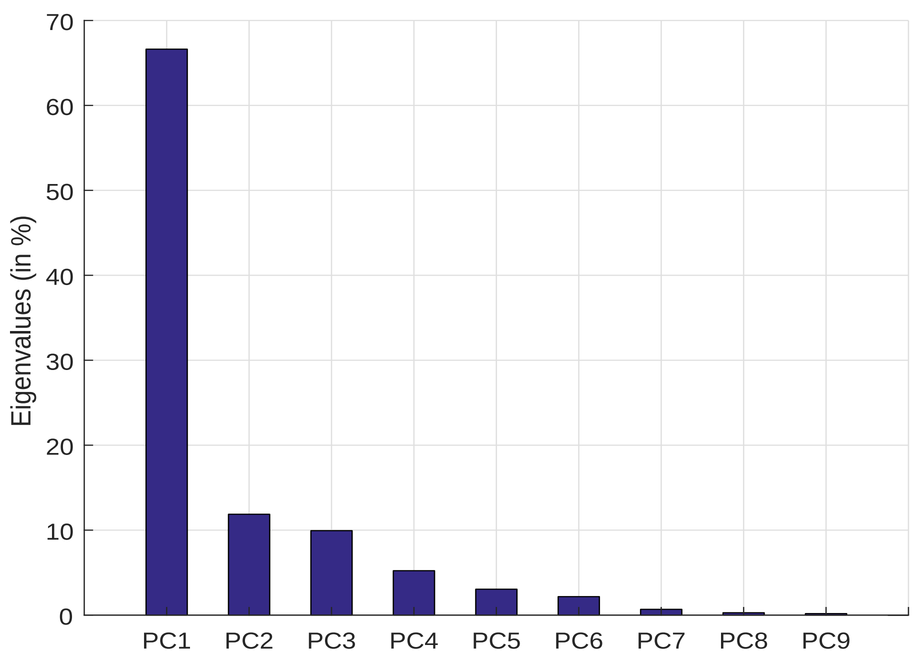
3.4.2 Other time-series risk factors

Many other time-series strategies are implemented in the hedge fund industry and replicated by commercial and investment banks. For example, Anand *et al.* (2019) considers four signals: momentum spill-over, DTCC positioning, CFTC momentum and CFTC reversal.

Table 10: Risk/return statistics of the time-series momentum risk factor (2000-2018)

		G10	EM	G25
1M	Excess return (in %)	−0.61	6.43	4.07
	Volatility (in %)	9.12	10.38	8.92
	Sharpe ratio	−0.07	0.62	0.46
	Maximum drawdown (in %)	−42.85	−40.46	−27.92
3M	Excess return (in %)	1.83	7.91	5.71
	Volatility (in %)	9.03	10.16	8.91
	Sharpe ratio	0.20	0.78	0.64
	Maximum drawdown (in %)	−27.12	−28.44	−27.08
12M	Excess return (in %)	0.89	2.43	2.51
	Volatility (in %)	9.12	10.18	8.78
	Sharpe ratio	0.10	0.24	0.29
	Maximum drawdown (in %)	−28.88	−51.00	−40.87
Composite	Excess return (in %)	0.85	5.77	4.23
	Volatility (in %)	7.24	8.10	7.14
	Sharpe ratio	0.12	0.71	0.59
	Maximum drawdown (in %)	−29.77	−27.79	−21.83

Figure 12: Principal component analysis of time-series momentum risk factors (weekly returns)



The momentum spill-over (MSO) signal assumes that time-series variation of interest rates has a predictive power in currency markets. The rationale of the DTCC signal is based on the relationship between order flows and foreign exchange dynamics (Evans and Lyons, 2002). Therefore, Anand *et al.* (2019) exploit the information available on the DTCC options flow data, in particular option volumes with respect to several strikes. The underlying idea is that these figures reveal a summary of investor positioning. Anand *et al.* (2019) considers the open interest data on currency futures which are reported to the CFTC. They build a short-term momentum signal that is based on long and short positions within a month. They also propose a reversal signal based on long and short positions, but with larger time horizons: 1, 2 and 3 months. Academics have also extensively studied the dynamics of foreign exchange rates, and exhibited trading strategies based on technical analysis, short-term reversal or volatility patterns (Taylor and Allen, 1992; LeBaron, 1999; Lustig *et al.*, 2014; Della Corte *et al.*, 2016). We do not consider these strategies in this paper, because our goal is to define some common risk factors that help to explain the behavior of currency markets. There is a gap between trading strategies and common risk factors. In general, the trading strategies that have been listed above are short-term, and we do not believe that they can explain the dynamics of a group of securities. They can perhaps explain the dynamics of one currency at a given time period, meaning that these strategies are more related to idiosyncratic risk factors and not to common risk factors.

4 Factor analysis of foreign exchange rates

4.1 Time-series regression model

We note $R_{i,t}$ the return of Currency i at time t:

$$R_{i,t} = \frac{S_{i,t} - S_{i,t-h}}{S_{i,t-h}}$$

where $S_{i,t}$ is the nominal exchange rate of the domestic currency with respect to the dollar and h is the time horizon. We consider the standard linear factor model:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{n_{\mathcal{F}}} \beta_{i,t}^{j} \mathcal{F}_{j,t} + \varepsilon_{i,t} \tag{25}$$

where $n_{\mathcal{F}}$ is the number of risk factors, $\beta_{i,t}^{j}$ is the factor loading, $\mathcal{F}_{j,t}$ is the value of the j^{th} risk factor and $\varepsilon_{i,t} \sim \mathcal{N}\left(0, \sigma_i^2\right)$ is a white noise process. It follows that we can break down currency returns into two parts:

$$R_{i,t} = R_{i,t}^{\text{Systematic}} + R_{i,t}^{\text{Specific}}$$

where $R_{i,t}^{\text{Systematic}}$ represents the systematic part:

$$R_{i,t}^{\text{Systematic}} = \sum_{j=1}^{n_{\mathcal{F}}} \beta_{i,t}^{j} \mathcal{F}_{j,t}$$

and $R_{i,t}^{\text{Specific}}$ is the specific or idiosyncratic part:

$$R_{i,t}^{\text{Specific}} = \alpha_i + \varepsilon_{i,t}$$

Since Model (25) is estimated with ordinary least squares, the explanatory power of the systematic part is given by the centered coefficient of determination $\mathfrak{R}_c^2$ of the linear regression.

In what follows, we estimate a one-factor model, where $\mathcal{F}_{j,t}$ is respectively the carry, value, cross-section momentum and time-series momentum risk factor, and a four-factor model, which is defined as:

$$\begin{aligned} R_{i,t} &= \alpha_i + \beta_{i,t}^{\text{Carry}} R_t^{\text{Carry}} + \beta_{i,t}^{\text{Value}} R_t^{\text{Value}} + \\ &\quad \beta_{i,t}^{\text{CS}-\text{MOM}} R_t^{\text{CS}-\text{MOM}} + \beta_{i,t}^{\text{TS}-\text{MOM}} R_t^{\text{TS}-\text{MOM}} + \varepsilon_{i,t} \end{aligned}$$

We also distinguish two modeling approaches. The first one, also known as the static case, assumes that the factor loadings are constant over time:

$$\beta_{i,t}^{j} = \beta_i^{j}$$

This approach is the standard model when testing the asset pricing model (Fama and McBeth, 1973). However, as we will see, it is more realistic to assume that the factor loadings are time-varying. This second approach is called the dynamic case.

4.1.1 Cross-section analysis of systematic risk factors

We consider both weekly and monthly returns. For the exogenous variables, we pool the risk factors of the three universes (G10, EM and G25). We first begin by estimating the linear factor model for the entire observation period from January 2000 to December 2018. We report the average $\mathfrak{R}_c^2$ for all 40 currencies[33] in Table 11. The static case corresponds to the top panel whereas the bottom panel presents the results of the dynamic approach. In the static case, the carry risk factor explains 11.64% and 16.80% of the cross-section variance in the currency market when we respectively use monthly and weekly returns. Value and cross-section momentum have a low explanatory power below 10%, whereas the $\mathfrak{R}_c^2$ coefficient of the time-series momentum is close to the value obtained for the carry risk factor. If we combine the four factors, the average $\mathfrak{R}_c^2$ coefficient is equal to 28.83% and 36.48%. As expected, the explanatory power of the factor model increases when we consider a lower frequency (monthly instead of weekly) because the number of observations decreases.

Table 11: Return decomposition (in %) between common and idiosyncratic risk factors

Frequency	Estimation	One-factor				Four-factor	
		Carry	Value	CS-MOM	TS-MOM	Systematic	Specific
Weekly	Static	11.64	4.15	1.90	12.30	28.83	71.17
Monthly	Static	16.80	6.93	3.73	15.05	36.48	63.52
Weekly	Dynamic	18.92	15.49	9.56	22.33	47.91	52.09
Monthly	Dynamic	30.60	25.84	20.14	32.86	73.51	26.49

However, these preliminary results are not satisfactory because the estimation is performed using the full observation period. This implies that the sensitivity of a currency with respect to a given risk factor is constant for the entire period from January 2000 to December 2018. By construction, this assumption is difficult to verify. Let us for example consider the value risk factor. Assuming a constant sensitivity of the currency implies that the currency is always overvalued or undervalued for the entire period. Therefore, it is better to assume that the sensitivities are time-varying. During one period, we can imagine that

[33] All the currencies are expressed with respect to the US dollar. For the USD, we consider the Bloomberg Dollar Spot Index (BBDXY), which tracks the performance of a basket of leading global currencies versus the US dollar. Since it is only available from January 2005, we retropolate it using the ICE US Dollar Index (DXY).

Figure 13: Coefficient of determination $\mathfrak{R}_c^2$ in % (weekly returns, static estimation)

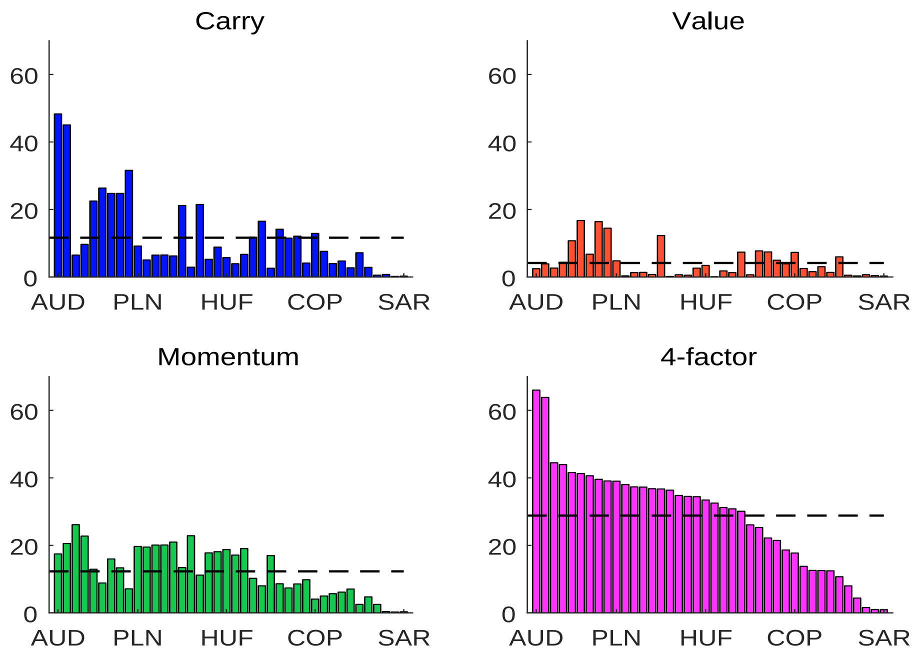
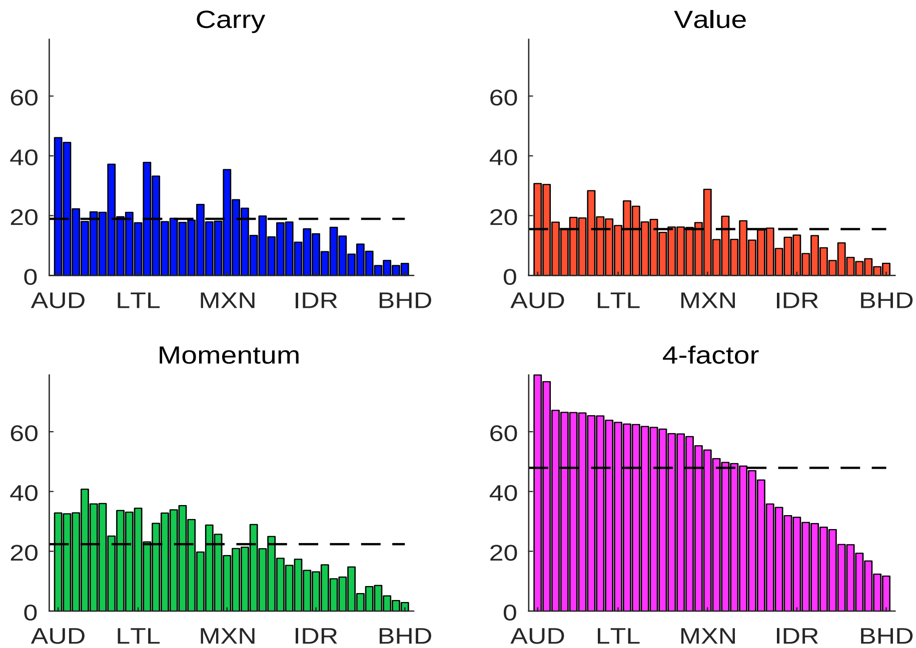


Figure 14: Coefficient of determination $\mathfrak{R}_c^2$ in % (weekly returns, dynamic estimation)



the value sensitivity of the currency is positive, whereas it may be negative in a subsequent period. This is why we consider the dynamic approach based on the estimation with a two-year rolling window. Results are given in the bottom panel in Table 11. The dynamic approach improves the systematic part explained by risk factors with respect to the static approach. For instance, the $\mathfrak{R}_c^2$ coefficient becomes 18.92% for the carry risk factor whereas it was previously equal to 11.64% when we use weekly returns. If we calculate the ratio between the systematic part explained by time-varying factor loadings and the systematic part explained by constant factor loadings, we obtain the following figures:

Frequency	Carry	Value	CS-MOM	TS-MOM	Four-factor
Weekly	1.6	3.7	5.0	1.8	1.7
Monthly	1.8	3.7	5.4	2.2	2.0

It is remarkable that the cross-section momentum presents the largest ratio. Indeed, this is the risk factor with the largest turnover. Therefore, it is difficult to assume that the factor loading of a currency is constant over time. It would mean that the currency is systematically among the worst or the best performers. It is more realistic to assume that the currency has positive and negative sensitivities to the cross-section momentum risk factor. The second largest ratio is observed for the value risk factor. In Figures 13 and 14, we compare the coefficient of determination $\mathfrak{R}_c^2$ for the different currencies in the static and dynamic cases. We notice a significant improvement when factor loadings are time-varying and not constant.

Even if results are better using monthly returns, we prefer to focus on weekly returns because we think that they are more robust. Results for individual currencies are reported in Table 12 when factor loadings are time-varying and in Table 32 on page 85 when factor loadings are constant. In the static case, specific risks explain the largest part of currency returns (71% vs 29% for systematic risks). Time-series momentum and carry are the two most important factors since each of these risk factors explains about 12% of currency returns on average. On the contrary, the value risk factor contribution is low (about 4%), whereas the cross-section momentum is not significant (less than 2%). However, we observe heterogenous results among currencies:

- Commodity currency returns are mainly explained by the carry factor. For instance, the coefficient of determination of the carry risk factor model is larger than 20% for AUD, NZD, ZAR, BRL, MXN, CLP and CAD. NOK is an exception since it is a currency commodity, but it is more sensitive to the time-series momentum. The first factor of RUB is carry, but its explanatory power is low (11.51%).

- For the other major currencies, the time-series momentum is more significant. For instance, USD, EUR, DKK, SEK and LTL have a $\mathfrak{R}_c^2$ coefficient larger than 20%.

- These results confirm that JPY[34] and TRY[35] are carry currencies, even if they are not commodity currencies.

- JPY and CHF are generally considered as two safe-haven assets. However, their factor decomposition is different. Curiously, the carry risk factor is less important than the time-series momentum for CHF. We can explain that because JPY is a global safe-haven asset and is the pillar of the carry strategy when building the short exposure of the portfolio. CHF is more a local safe-haven asset with respect to the European market. This is why it reacts more to the time-series momentum risk factor.

[34]Certainly, because it is generally used to form the short leg of the carry strategy.
[35]Turkish inflation is very sensitive to oil price and currency shocks, implying that Turkey has structurally high interest rates.

Table 12: Return decomposition (in %) between common and idiosyncratic risk factors (weekly returns, dynamic estimation)

Currency	One-factor				Four-factor	
	Carry	Value	CS-MOM	TS-MOM	Systematic	Specific
AUD	46.10	30.73	14.25	32.82	78.97	21.03
NZD	44.49	30.42	12.03	32.54	76.73	23.27
CHF	22.28	17.81	12.75	32.86	67.15	32.85
USD	18.03	15.61	12.66	40.76	66.48	33.52
EUR	21.25	19.41	13.56	35.84	66.41	33.59
DKK	21.11	19.22	13.46	35.97	66.28	33.72
BRL	37.20	28.35	14.31	25.08	65.34	34.66
CZK	19.63	19.58	13.53	33.63	65.28	34.72
NOK	21.10	18.88	10.65	33.08	63.83	36.17
LTL	17.62	16.69	12.04	34.40	63.12	36.88
TRY	37.81	24.93	16.68	23.13	62.56	37.44
ZAR	33.22	23.13	14.30	29.34	62.39	37.61
PLN	17.99	17.88	10.90	32.78	61.75	38.25
HUF	19.11	18.73	12.45	33.86	61.44	38.56
SEK	17.74	14.38	11.23	35.27	60.87	39.13
BGN	18.48	16.19	10.08	30.61	59.35	40.65
JPY	23.74	16.19	11.12	19.74	59.25	40.75
LVL	17.91	16.04	9.21	28.76	58.36	41.64
GBP	18.14	17.67	11.07	25.68	55.29	44.71
MXN	35.41	28.80	13.88	18.55	53.87	46.13
CAD	25.33	11.99	8.57	20.91	50.98	49.02
CLP	22.48	19.79	12.14	21.36	49.72	50.28
SGD	13.37	12.06	9.00	28.94	49.32	50.68
RUB	19.87	18.27	12.57	20.85	48.50	51.50
RON	12.89	11.80	6.87	24.94	46.95	53.05
KRW	17.59	15.20	7.50	17.65	43.84	56.16
MYR	17.86	15.82	8.40	15.28	35.82	64.18
TWD	11.11	9.00	6.47	17.33	34.66	65.34
INR	15.59	12.75	7.51	13.61	31.91	68.09
IDR	13.92	13.49	6.96	13.11	31.37	68.63
THB	7.96	7.31	4.50	15.47	29.64	70.36
COP	16.09	13.32	5.97	10.79	29.25	70.75
PHP	13.17	9.24	5.36	11.36	28.05	71.95
ILS	7.12	4.98	4.82	14.75	27.24	72.76
PEN	10.48	10.88	4.74	5.83	22.27	77.73
HKD	8.07	6.02	4.76	8.18	22.19	77.81
CNY	3.30	4.63	5.17	8.56	19.33	80.67
ARS	5.00	5.58	5.08	3.34	16.75	83.25
SAR	3.30	2.90	3.11	3.50	12.35	87.65
BHD	4.00	4.04	2.63	2.84	11.69	88.31
Average	18.92	15.49	9.56	22.33	47.91	52.09

Rolling estimation significantly improves the results as the systematic part of the return gets closer to 50% on average. The ordering of factors remains the same, but the most important changes involve the value and cross-section momentum risk factors, whose $\mathfrak{R}_c^2$ coefficient is substantially increased. More interesting is the impact on the ordering of the currencies. For example, the CHF was ranked 19$^{\text{th}}$ in the static approach whereas it is ranked third in the dynamic approach in terms of $\mathfrak{R}_c^2$. Again, the results are quite heterogeneous:

- On average, the four-factor model explains 66% of G10 currency returns and 48% if we consider the 40 currencies.
- Most commodity currency returns are still largely explained by the carry risk factor, except NOK and RUB. Also note that the carry factor clearly explains the returns (above 40%) of AUD and NZD. This corroborates evidence of the very popular use of these currencies in carry trade strategies[36].
- JPY and KRW remain carry currencies[37], whereas GBP turns out to be a momentum currency.

In fact, the factor model gives interesting information if we want to perform clustering between the different currencies. In particular, we distinguish two main clusters:

1. The first cluster corresponds to carry-based currencies and is made up of AUD, BRL, CAD, CLP, JPY, MXN, NZD, TRY and ZAR.
2. The second cluster corresponds to momentum-based DM currencies and is made up of CHF, DKK, EUR, GBP, NOK, SEK and USD.

4.1.2 On the importance of specific risks

We note that the higher explanatory power of the specific factors is reached for the bloc of emerging currencies. For instance, specific risk factors dominate systematic risk factors for ARS, BHD, CNY, HKD and SAR, which correspond to currencies of countries that have adopted fixed or intermediate exchange rate regimes[38] according to the IRR classification of Ilzetzki *et al.* (2017) reported in Table 13. These authors classify FX regimes attributing codes from 1 to 13, depending on the degree of foreign exchange regime flexibility[39]. Higher values correspond to more flexible currency regimes whereas lower values correspond to fixed and less flexible currency regimes. In Appendix A.3.1 on page 86, Table 33 gives the IRR classification[40] of the 40 currencies between 2000 and 2016. In Figure 15, we draw the scatter plot between the average IRR code and the idiosyncratic risk for the different currencies. We observe mainly four blocs of currencies.

The case of ARS, BHD, CNY, HKD and SAR naturally raises the question of whether fixed or intermediate exchange rate regimes could explain the differences among the factors influencing currencies' returns and, especially, whether these regimes would somehow limit

[36]A common strategy is to be long on currency pairs like AUD/JPY or NZD/JPY as the interest rate differential in these pairs is often high. Note that the unconventional monetary policies adopted by major economies such as the US, Euro area and Japan while Australia kept positive interest rates might have helped to promote carry strategies. This was a problem for the Reserve Bank of Australia that saw the resurgent carry trade in the AUD as a major issue in 2015 when the currency was appreciating.

[37]See Footnote 34 on page 43.

[38]In late 2018, as part of an agreement with the IMF, Argentina adopted a floating FX rate regime with limited foreign exchange interventions.

[39]Codes 14 and 15 identifies economies suffering from major structural issues.

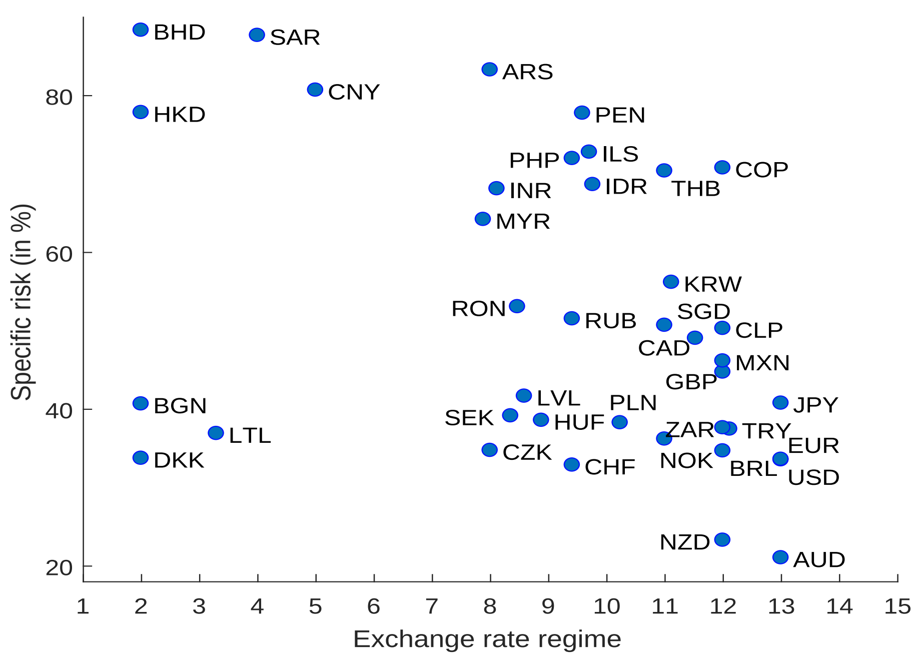
[40]For EUR, we take into account the IMF classification which categorizes it as '*freely floating*' rather than '*no separate legal tender or currency union*'. In the IRR classification, the authors define the euro as '*currency union*' a category at the bottom of exchange rate flexibility rank.

Table 13: Fine exchange rate arrangement classification

Code	Definition
1	No separate legal tender or currency union
2	Pre announced peg or currency board arrangement
3	Pre announced horizontal band that is narrower than or equal to ±2%
4	De facto peg
5	Pre announced crawling peg; de facto moving band narrower than or equal to ±1%
6	Pre announced crawling band or de-facto horizontal band that is narrower than or equal to ±2%
7	De facto crawling peg
8	De facto crawling band that is narrower than or equal to ±2%
9	Pre announced crawling band that is wider than or equal to ±2%
10	De facto crawling band that is narrower than or equal to ±5%
11	Moving band that is narrower than or equal to ±2%
12	De facto moving band ±5% / Managed floating
13	Freely floating
14	Freely falling
15	Dual market in which parallel market data is missing

Source: Ilzetzki *et al.* (2017), Table 2, page 17.

Figure 15: Relationship between exchange rate arrangement and specific risk



the influence of systematic factors. However, analyzing the exchange rate arrangements of the whole sample, we observe that other countries also apply (or have used until recently) such less flexible regimes. For example, BGN, DKK, LTL and LVL are primarily explained by systematic factors rather than by specific ones[41]. In the same way, countries following more flexible currency regimes can also have idiosyncratic factors explaining an important part of their returns (e.g. CLP and COP). Hence, fixed or intermediate exchange rate regimes do not seem to be directly correlated with higher explanatory power of specific factors.

As a simple exercise, we extend the analysis to other country-specific characteristics such as the degree of capital account openness, size of government and control of corruption[42]. We also find that such parameters do not help to justify the lower explanatory power of the systematic factors in explaining returns of some currencies. Liquidity does not seem to be a determining factor either. For instance, despite being one the most liquid emerging market currencies, systematic and specific factors have nearly the same importance in explaining returns for the Mexican peso. Clearly, deeper analysis is needed and other idiosyncratic elements might justify such results, but finding the specific factor driving returns for each currency or group of currencies is beyond the scope of this paper.

Systematic factors greatly explain returns for high-carry or high-yield currencies (AUD, NZD, BRL, TRY and ZAR), international dominant currencies (EUR and USD) and safe havens (CHF, JPY and USD). Apart from these groups, we observe that systematic factors also play an important role in explaining returns for the currencies of countries located in Europe, regardless of the foreign exchange regime (BGN, CZK, DKK, GBP, HUF, LTL, LVL, NOK, PLN and SEK). Going further, it is very interesting to note that these currencies have quite similar loading factors and they are especially driven by the time-series momentum with similar patterns, independently of being part of emerging or developed Europe. Very likely, the common group characteristics '*belonging to the Europe*' and '*having strong ties with the Euro area*' are the determining factors for explaining such similar patterns[43]. This means that the euro would be leading the factors in the region[44].

One could also think that the CNY could exert the same influence over Asian economies as the euro exerts over European economies. However, despite the important trade linkages between China and its neighbors, we do not observe common factor patterns among Asian currencies. Exchange rates in this region are essentially driven by specific factors. This is an intriguing result as literature suggests that the CNY 's influence over most Asian currencies

[41] Note that Latvia replaced its currency with the euro in 2014 while Lithuania adopted the euro in 2015.

[42] For the degree of capital account openness, we use the Chinn-Ito financial openness index (Chinn and Ito, 2006). The control of corruption index is part of Worldwide Governance Indicators (`www.govindicators.org`) while the size of government measure is calculated using economic freedom indices (`www.fraserinstitute.org/economic-freedom`).

[43] Despite not being part of the EU, Norway is associated with the region through membership agreements in the European Economic Area and is one of the EU's most important trade partners. Note that the NOK, which is a commodity currency, actually follows similar fashion as EU currencies rather than the commodity ones. Switzerland is also not an EU member state, but it has several agreements with the group to ensure participation in its single market.

[44] Indeed, European countries not adopting the euro tend to behave as satellites around the Euro area, highlighting important connections within the region. In particular, ECB monetary policy decisions tend to spillover to small open economies neighboring the Euro area, especially to the ones with strong financial and trade relations with the region such as Denmark and Sweden. Consequently, central banks in these countries are inclined to take the ECB's monetary policy decisions into account when setting their own policy goals. As explained by Stefan Ingves, governor of Riskbanken during a press conference in October 2017, "*we are neighbors with an elephant and when the elephant moves, we are affected*". The case of Switzerland is no exception, being also affected by the diverse events related to the Euro area. Besides the traditional linkages with the region, its currency has been long perceived as a safe haven. This status is particularly observed when there are rising risks in Europe, implying that CHF would act as a kind of regional safe haven asset.

is increasing, as well as over currencies in the Latam bloc, especially after the Chinese monetary reforms in 2015 and the subsequent central bank announcement expressing its wish for more flexibility for the yuan (Drut and Fortes, 2016). Going further, Tovar and Nor (2018) recently pointed out the transformation in the international monetary system from a bi-polar system to a tri-polar one including the yuan.

Taking all this together, our results highlight that systematic factors are highly explanatory for the traditional currency blocs, namely the commodity bloc and the high-yield bloc, on which the main explanatory factor is carry. We also identify the bloc of '*dominant*' currencies, comprised of the US dollar and the euro, and on which the most explanatory factor is the time-series momentum. Despite its growing influence, CNY still does not seem to belong to the latter category. Actually, we observe that Asian currencies returns or, more broadly speaking, returns of currencies that are (1) outside one of the above-mentioned blocs or (2) out of their direct influence, are essentially driven by idiosyncratic risk factors and this goes beyond the foreign exchange regime classification.

Remark 5 *In what follows, we only consider the time-varying model (i.e. the two-year rolling estimation) with weekly returns.*

4.1.3 Time-series analysis

When we consider rolling estimation, we compute the centered coefficient of determination $\mathfrak{R}_c^2$ at each date. Therefore, we do not obtain one value of $\mathfrak{R}_c^2$, but a time-series sequence of $\mathfrak{R}_c^2$. Let $\boldsymbol{R}_{i,t}^2$ be the calculated value of $\mathfrak{R}_c^2$ for Currency i at time t. In Figure 16, we have reported the dynamics of $\boldsymbol{R}_{i,t}^2$ for four carry currencies (AUD, BRL, NZD and TRY). If we compare the carry one-factor model with the four-factor model, we observe that the two statistics do not necessarily move in a similar way. For instance, $\boldsymbol{R}_{i,t}^2$ is flat or decreasing during the 2008 crisis for the four-factor model whereas it increases if we consider the carry one-factor model. Since 2017, the variation are also not correlated for AUD, BRL and NZD. If we consider momentum currencies (CHF, EUR, SEK and USD), we obtain Figure 17. In this case, changes are more correlated between the values $\boldsymbol{R}_{i,t}^2$ of two models than for the carry currencies. These results seem to confirm that the time-series momentum is more important than the carry risk factor to understand the dynamics of some currency returns. However, academics and professionals generally pay more attention to carry or value than momentum. In Figure 18, we calculate the average value of $\boldsymbol{R}_{i,t}^2$ for the different currencies:

$$\bar{\boldsymbol{R}}_t^2 = \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{R}_{i,t}^2$$

We notice that the four-factor model is mainly driven by the time-series momentum factor. For instance, the correlation of $\bar{\boldsymbol{R}}_t^2$ between the one-factor and four-factor models is respectively equal to 37% (carry), −10% (value), 8% (cross-section momentum) and 84% (time-series momentum).

4.1.4 Out-of-sample analysis

Let $\hat{R}_{i,t+h}$ be the predicted value of $R_{i,t+h}$ given the information $\mathcal{I}_t$ available at time t. If we consider Model (25) and assume that the best predictions of $\beta_{i,t+h}^{(\mathcal{F}_j)}$ and $\mathcal{F}_{j,t+h}$ are respectively the current estimate $\hat{\beta}_{i,t}^{(\mathcal{F}_j)}$ and the current value $\mathcal{F}_{j,t}$, we obtain:

$$\hat{R}_{i,t+h} = \mathbb{E}\left[R_{i,t+h} \mid \mathcal{I}_t\right] = \hat{\alpha}_i + \sum_{j=1}^{n_{\mathcal{F}}} \hat{\beta}_{i,t}^{(\mathcal{F}_j)} \mathcal{F}_{j,t}$$

Figure 16: Time dynamics of $\boldsymbol{R}^2_{i,t}$ (carry currencies)

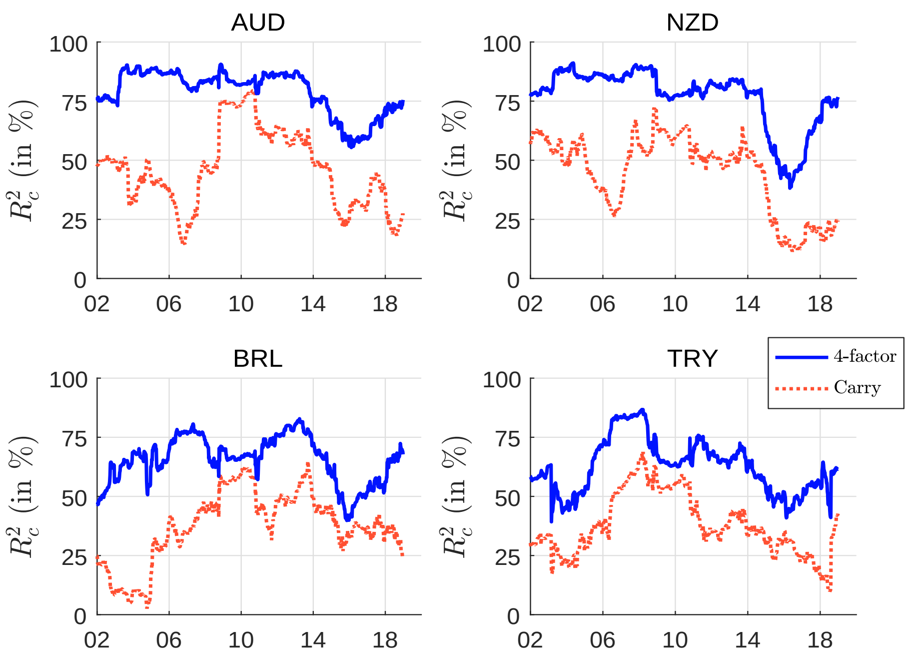


Figure 17: Time dynamics of $\boldsymbol{R}^2_{i,t}$ (momentum currencies)

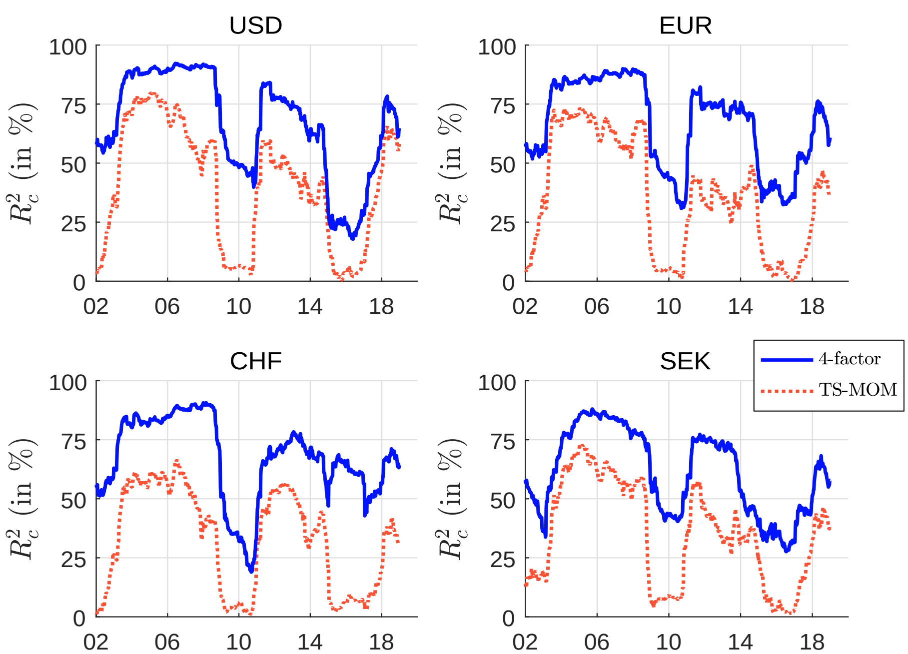
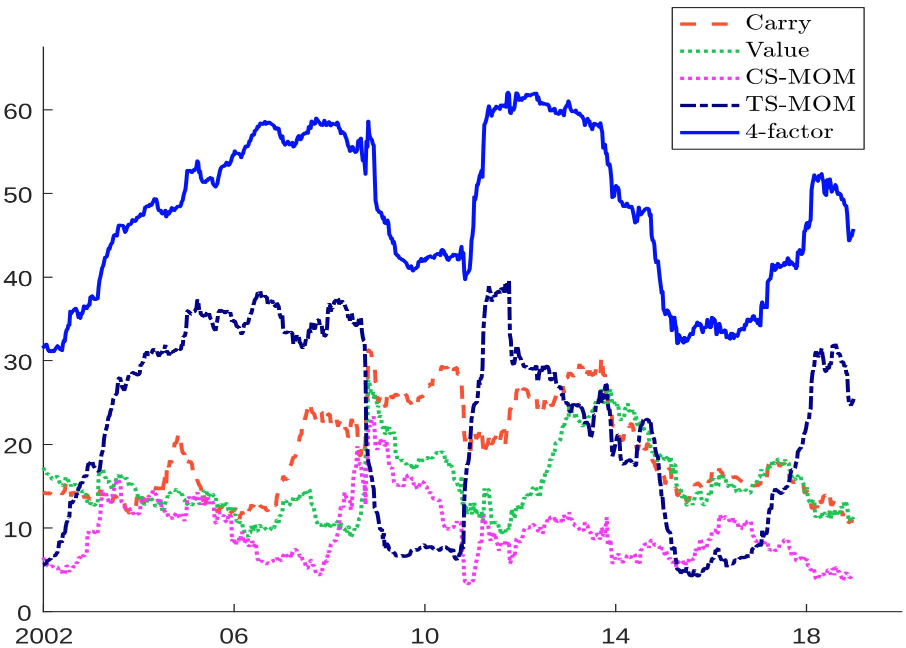


Figure 18: Average dynamics of $\bar{\boldsymbol{R}}_t^2$



We reiterate that our factor model is built to explain the cross-section variance of currency returns. In order to evaluate its out-of-sample power, we calculate the Spearman's rank correlation of currency returns:

$$\varrho_t = \varrho\left(\hat{R}_{i,t+h}, R_{i,t+h}\right) = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n\left(n^2-1\right)}$$

where $d_i = \mathcal{R}ank_i\left(\hat{R}_{i,t+h}\right) - \mathcal{R}ank_i\left(R_{i,t+h}\right)$ is the difference between the two ranks of the predicted value $\hat{R}_{i,t+h}$ and the realized value $R_{i,t+h}$. We notice that ϱ_t is a cross-section correlation measure and not a time-series correlation measure. At each date t, ϱ_t therefore measures the concordance between predicted and realized currency returns for a h-step period. If the factor model performs a perfect ranking, ϱ_t is equal to one, whereas ϱ_t is equal to zero in the case of a random ranking. Using this statistical measure, we compute the cross-section coefficient of determination ϱ_t^2, which can be compared to the time-series coefficient of determination $\bar{\boldsymbol{R}}_t^2$.

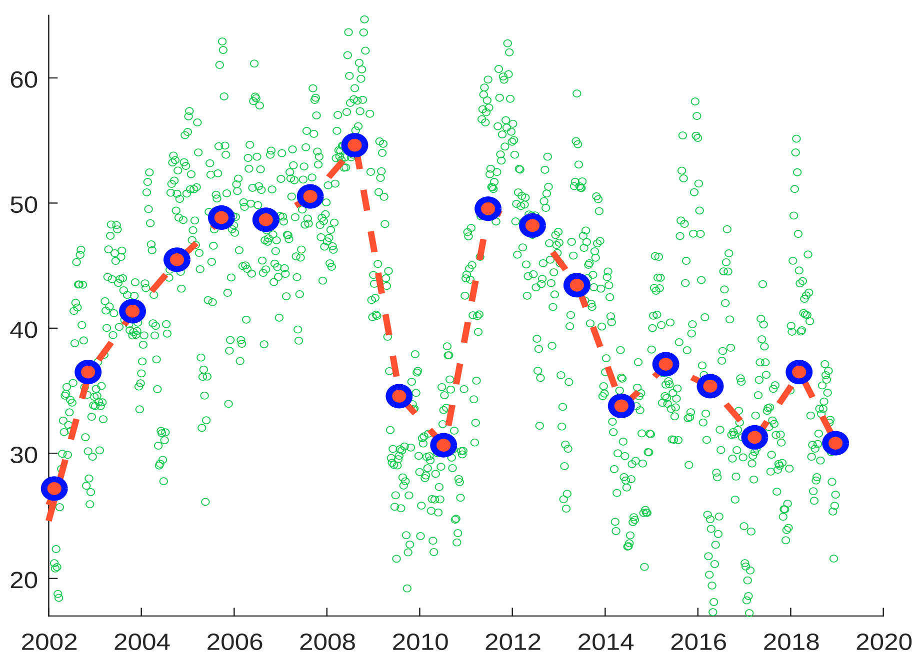
In Table 14, we report the average value of ϱ_t when we consider weekly forecasts. We obtain a surprising result. Whereas time-series momentum is better to explain the time-series dynamics of currency returns, carry is a better risk factor in order to predict cross-section currency returns. One explanation may be that the carry risk factor is more stable than the momentum risk factor. From an ex-post point of view, time-series momentum is the main risk factor because of the trends that we observe in currency markets. But from an ex-ante point of view, carry is more relevant in order to predict the cross-section ordering of currency returns. In Figure 19, we have represented the evolution of the cross-section R-squared ϱ_t^2 over time for the four-factor model. The blue line corresponds to raw values whereas the red line with circle symbols is the smoothing spline estimation. We notice that

Table 14: Out-of-sample results (average correlation)

Model	Spearman's rank correlation ϱ_t	Cross-section R-squared ϱ_t^2	Time-series R-squared $\bar{\boldsymbol{R}}_t^2$
Carry	38.82%	24.15%	18.92%
Value	32.61%	21.85%	15.49%
CS-MOM	19.90%	16.82%	9.56%
TS-MOM	31.73%	21.37%	22.34%
4-factor	59.73%	40.88%	47.92%

the cross-section R-squared increases from 2002 to 2008. Then, it dramatically decreases during the Global Financial Crisis. After 2010, it increases in particular at the end of 2011. Since 2012, the cross-section R-squared coefficient has tended to decline.

Figure 19: Cross-section R-squared ϱ_t^2 (4-factor model)



Remark 6 *The previous analysis can be done with different sets of currencies. For instance, we can consider momentum currencies, carry currencies, Latam currencies, etc.*

4.2 Cross-section regression model

Risk premia are estimated using the Fama-MacBeth approach, which accounts for the covariance structure of the risk factors. The Fama-MacBeth method is a two-step procedure described as follows:

1. For each currency i, we estimate the factor loadings by applying the time-series linear

regression model:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{n_{\mathcal{F}}} \beta_{i,t}^j \mathcal{F}_{j,t} + \varepsilon_{i,t}$$

and we note $\left(\hat{\alpha}_{i,t}, \hat{\beta}_{i,t}^1, \ldots, \hat{\beta}_{i,t}^{n_{\mathcal{F}}}\right)$ the corresponding vector of estimates.

2. For each date t, we estimate the time-varying risk premia $\pi_{j,t}$ associated with the risk factors by applying the cross-section linear regression model:

$$R_{i,t} = c_t + \sum_{j=1}^{n_{\mathcal{F}}} \pi_{j,t} \hat{\beta}_{i,t}^j + \eta_{j,t}$$

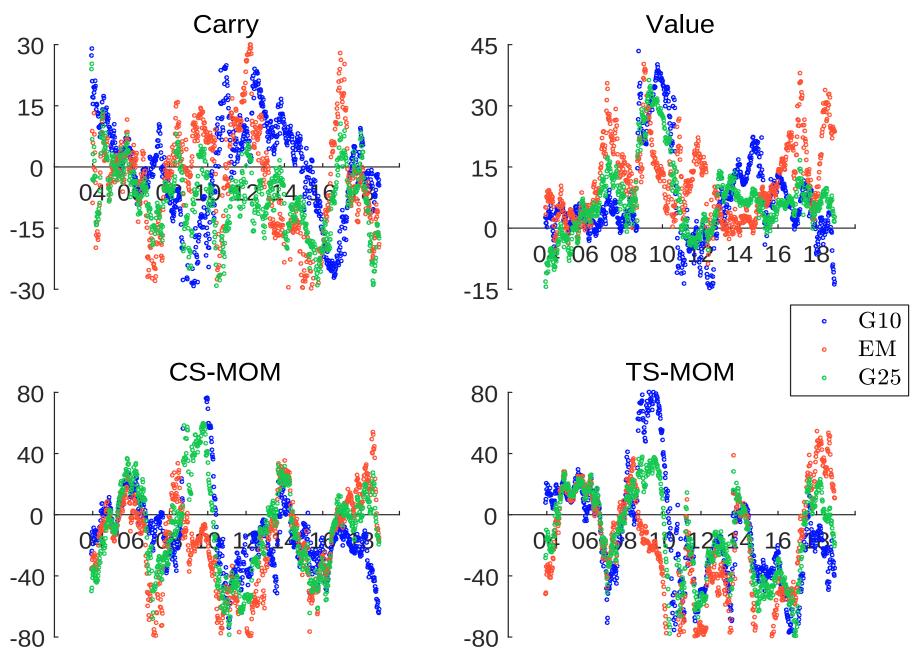
and we note $(\hat{c}_t, \hat{\pi}_{1,t}, \ldots, \hat{\pi}_{n_{\mathcal{F}},t})$ the corresponding vector of estimates.

Therefore, the Fama-MacBeth procedure estimates the implied risk premium $\hat{\pi}_{j,t}$ at time t for a given risk factor $\mathcal{F}_j$. We deduce that the long-term risk premium is given by:

$$\bar{\pi}_j = \frac{1}{T} \sum_{t=1}^{T} \hat{\pi}_{j,t}$$

Results are reported in Figure 20. We notice that the weekly risk premia $\hat{\pi}_{j,t}$ vary greatly over time. Moreover, they are not necessarily positive. For instance, the average probability of observing a positive risk premium is only 30% for the cross-section momentum and 35% for the time-series momentum. For the carry risk factor, this probability is equal to 50% except for the G25 risk factor which has a probability below 20%. Only the value risk factor exhibits recurrent positive risk premia, on average 80% of the time.

Figure 20: Estimated weekly risk premia $\hat{\pi}_{j,t}$ in bps



Results of the long-term risk premia are shown in Table 15. Except for value risk factors, the estimated risk premium is negative. Moreover, the coefficient of determination $\mathfrak{R}_c^2$ is equal to 1% for the cross-section Fama-MacBeth regression if we consider the entire observation period. We clearly conclude that these risk factors do not exhibit risk premia, and the four-factor model is not a relevant asset pricing model for foreign exchange rates.

Table 15: Estimation of long-term risk premia

	Carry			Value			CS-MOM			TS-MOM		
	G10	EM	G25	G10	EM	G25	G10	EM	G25	G10	EM	G25
$\bar{\pi}_j$ (in bps)	2	−2	−15	6	8	2	−8	−23	−17	−3	−17	−11
sign	+	−	−	+	+	+	−	−	−	−	−	−
p-value			***	***	***	*	**	***	***		***	***

The previous results are very interesting since they suggest that carry, value and momentum are important to explain the cross-section of currency returns, but the existence of a risk premium associated with each risk factor is not verified except for the value risk factor. These results can be related to the findings of the academic research. As suggested by Menkhopf *et al.* (2016), more attention is generally paid to carry and momentum than value. However, value contains interesting predictive information, since value is very different to carry and momentum (Barroso and Santa-Clara, 2015). Therefore, our results can also be related to the concept of alternative risk premia (Roncalli, 2017). By definition, carry and momentum strategies are more market anomalies whereas value is typically a skewness risk premium. In this case, it is coherent that the currency market does not price a risk premium for carry and momentum, but only for the value risk factor.

4.3 Factor loading analysis

In what follows, we analyze the factor loadings $\beta_{i,t}^j$ which are the key variables of the factor model. First, we perform lasso regressions in order to select the importance of each risk factor at each period. Second, we use the Kalman filter to model the time-varying patterns of the factor loadings.

4.3.1 Lasso selection of the relevant risk factors

The lasso regression is a powerful tool to perform variable selection. Given a study period $t \in \mathcal{T}$, we estimate the constrained least square estimator for Currency i:

$$\begin{aligned}
\hat{\beta}_i^{\text{lasso}}(\tau) &= \arg\min \sum_{t\in\mathcal{T}} \left(\frac{R_{i,t} - \hat{\mu}(R_i)}{\hat{\sigma}(R_i)} - \sum_{j=1}^{n_{\mathcal{F}}} \beta_i^j \left(\frac{\mathcal{F}_{j,t} - \hat{\mu}(\mathcal{F}_j)}{\hat{\sigma}(\mathcal{F}_j)} \right) \right)^2 \\
&\quad \text{s.t.} \quad \sum_{j=1}^{n_{\mathcal{F}}} \left| \beta_i^j \right| \leq \tau
\end{aligned}$$

where $\hat{\mu}(X)$ and $\hat{\sigma}(X)$ are the empirical mean and standard deviation of the variable X for the period $\mathcal{T}$ and τ is the target value of the ℓ_1-norm penalty function. Since the lasso method produces sparse estimators and $\hat{\beta}_i^{\text{lasso}}(0) = \mathbf{0}$, we can then rank the risk factors sequentially according to their importance. Indeed, when τ is sufficiently small, the lasso regression selects only one risk factor, meaning that all the factor loadings are equal to zero except one coefficient. This coefficient corresponds to the most important risk factor. We then increase the value of τ until the lasso procedure has selected two risk factors. We continue the iterative process until all the factors are selected, and we obtain the lasso ordering of risk factors.

We apply this approach to the four-factor model with weekly returns and a two-year rolling window. For each date (about 1 000 weeks) and each foreign exchange rate (40 currencies), we obtain the ranking between the four risk factors. In Figures 21 to 24, we have reported the results for 16 currencies.

The two first figures correspond to carry currencies. However, we notice that carry is not always the first selected factor. For instance, if we consider AUD, carry dominates all the other risk factors only between 2009 and 2015, which represents less than half of the study period[45]. If we compute the frequency when carry is the first selected factor, we obtain the first row given in Table 16: 62.7% for BRL, 52.2% for CAD, etc. For these currencies, time-series momentum is generally the second risk factor. We notice that value is not among the first two factors, except during very short periods (six months or one year). It was for instance the case of BRL in 2010, JPY between 2014 and 2017 or TRY in 2017.

Figures 23 and 24 give the two first selected risk factors for eight momentum currencies: CHF, CZK, DKK, EUR, GBP, NOK, SEK and USD. Again, we notice that the ranking of one factor is time-varying, but generally time-series momentum dominates. For instance, the frequency to observe time-series momentum as the first risk factor is 59.6% for CHF, 56.6% for CZK, etc[46]. Another important result concerns the combination of the two first selected risk factors. In Table 16, we have reported the frequency that the pairs carry/momentum, carry/value and momentum/value are in the top two[47]. In the case of carry currencies, carry/momentum is definitively the winning pair. In the case of momentum currencies, momentum/carry (or equivalently carry/momentum) is not always the winning pair. Indeed, most of the time, the trade-off is not between momentum and carry, but between momentum and value.

Table 16: Results of the lasso regression (frequency in %)

Factor	Rank	Currency							
		AUD	BRL	CAD	JPY	MXN	NZD	TRY	ZAR
Carry	1^{st}	42.7	62.7	52.2	56.9	71.8	53.3	78.3	63.0
Carry/momentum	$1^{st}/2^{nd}$	40.6	37.1	34.5	40.0	30.4	50.7	39.0	42.4
Carry/value	$1^{st}/2^{nd}$	22.2	14.8	9.9	18.8	20.8	17.9	23.2	17.4
		CHF	CZK	DKK	EUR	GBP	NOK	SEK	USD
Momentum	1^{st}	59.6	56.6	60.1	59.5	59.7	68.2	67.6	73.2
Momentum/carry	$1^{st}/2^{nd}$	31.5	27.2	23.5	23.7	6.1	19.4	35.8	23.1
Momentum/value	$1^{st}/2^{nd}$	24.2	23.3	26.4	26.0	51.1	34.4	20.4	23.3

4.3.2 Time-varying modeling with the Kalman filter

We consider our four-factor model:

$$\begin{aligned} R_{i,t} &= \alpha_i + \beta_{i,t}^{\text{Carry}} R_t^{\text{Carry}} + \beta_{i,t}^{\text{Value}} R_t^{\text{Value}} + \\ & \quad \beta_{i,t}^{\text{CS-MOM}} R_t^{\text{CS-MOM}} + \beta_{i,t}^{\text{TS-MOM}} R_t^{\text{TS-MOM}} + \varepsilon_{i,t} \end{aligned}$$

[45] Carry is the first risk factor 42.7% of the time.

[46] Results are reported in the sixth row in Table 16.

[47] We do not look at the order, meaning that we can obtain carry first and time-series momentum second, or carry second and time-series momentum first in the case of the carry/momentum pair.

Figure 21: Lasso selection of risk factors

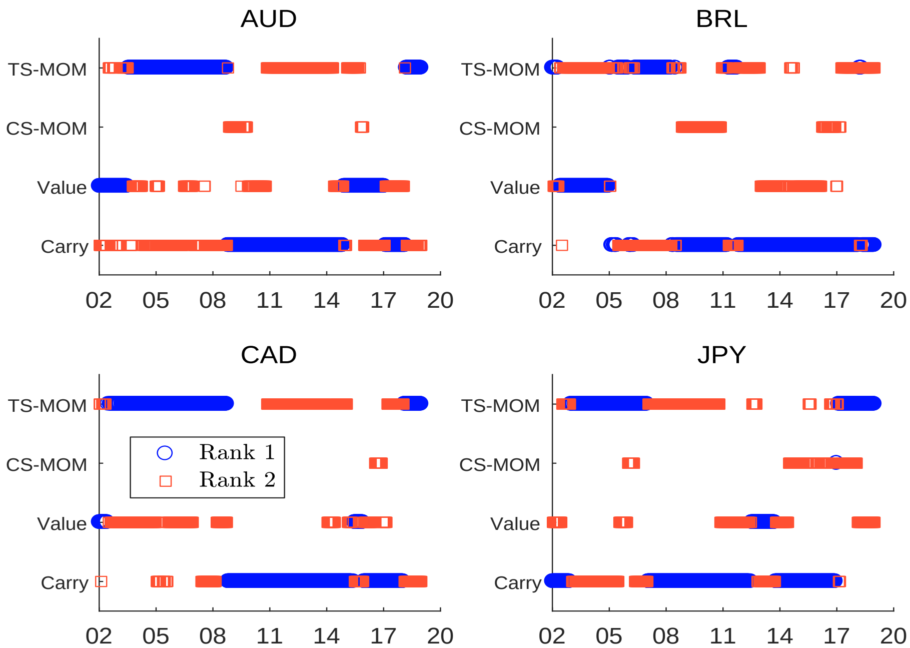


Figure 22: Lasso selection of risk factors

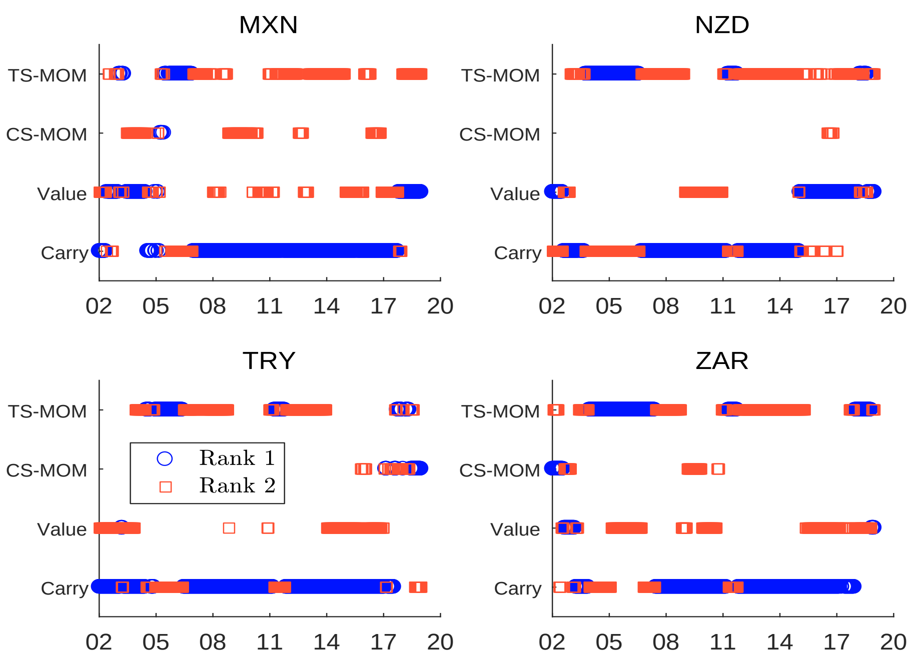


Figure 23: Lasso selection of risk factors

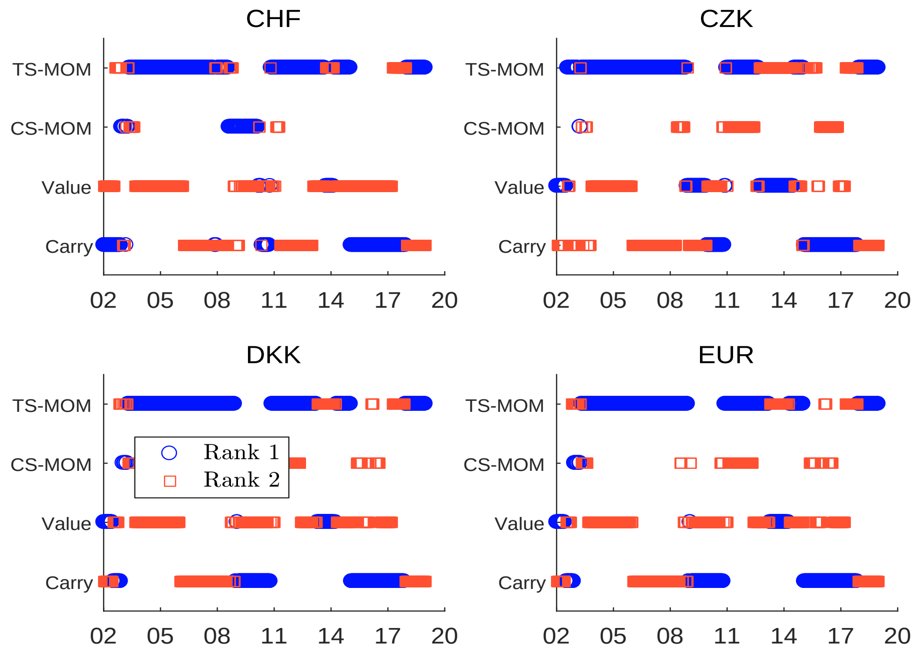
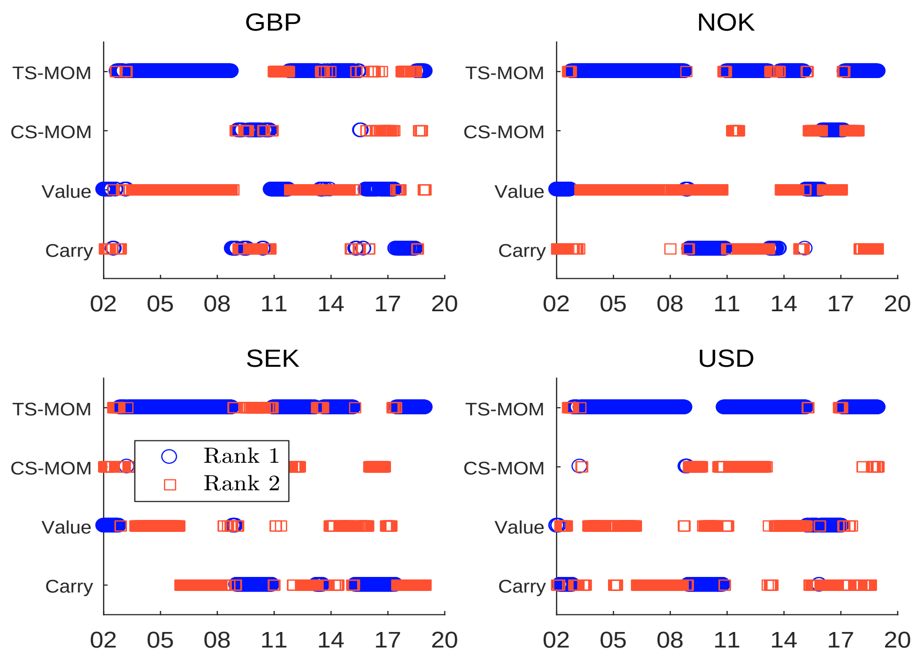


Figure 24: Lasso selection of risk factors



but we now assume that the factor loadings follow a random walk trend:

$$\begin{pmatrix} \beta_{i,t}^{\mathrm{Carry}} \\ \beta_{i,t}^{\mathrm{Value}} \\ \beta_{i,t}^{\mathrm{CS-MOM}} \\ \beta_{i,t}^{\mathrm{TS-MOM}} \end{pmatrix} = \begin{pmatrix} \beta_{i,t-1}^{\mathrm{Carry}} \\ \beta_{i,t-1}^{\mathrm{Value}} \\ \beta_{i,t-1}^{\mathrm{CS-MOM}} \\ \beta_{i,t-1}^{\mathrm{TS-MOM}} \end{pmatrix} + \begin{pmatrix} \eta_{i,t}^{\mathrm{Carry}} \\ \eta_{i,t}^{\mathrm{Value}} \\ \eta_{i,t}^{\mathrm{CS-MOM}} \\ \eta_{i,t}^{\mathrm{TS-MOM}} \end{pmatrix}$$

where $\eta_{i,t} = \left(\eta_{i,t}^{\mathrm{Carry}}, \eta_{i,t}^{\mathrm{Value}}, \eta_{i,t}^{\mathrm{CS-MOM}}, \eta_{i,t}^{\mathrm{TS-MOM}}\right)$ is a white noise process $\mathcal{N}(\mathbf{0}, \Sigma_\eta)$. This model can be estimated using the method of maximum likelihood and the Kalman filter as explained in Appendix A.2.3 on page 77.

Results are reported in Appendix from page 94 to page 113. We observe several patterns:

- The factor loadings are not significant for some currencies: BHD, CNY, HKD and SAR. This in in line with the cross-sectional analysis which showed that returns for these currencies are mostly explained by idiosyncratic factors.

- CHF and JPY have a negative sensitivity to carry, which confirms that these two currencies are used when forming the short leg of carry strategies. However, their factor loadings are not closely correlated, implying that these two currencies do not have the same carry dynamics. The carry dynamics of JPY are smoother, which is not the case for CHF. In particular, the carry dynamics of CHF present sudden changes, which might be due to frequent interventions of the Swiss central bank and the fact that CHF is less liquid than JPY.

- The carry factor loading for AUD, CAD and ILS were not significant until the 2008 Global Financial Crisis. The NOK had a negative sensitivity to carry, but there was a turning point in 2009 and a second one at the end of 2014. The explanation behind this result could be because the interest rate deposit spread against USD became significant after 2008 and less after 2014 at the time when the US rate began to rise.

- Value is the risk factor with the most frequent sign changes. This is normal since a currency cannot be systematically overvalued or undervalued.

- There is a strong connection between cross-section and time-series momentum. For instance, we observe that their factor loadings are generally opposite in sign. However, the trend in the TS-MOM factor loading is generally smoother than the trend in the CS-MOM factor. This could be explained by the fact that CS-MOM has the largest turnover among the factors. Curiously, the inverse relationship is not due to the negative correlation between CS-MOM and TS-MOM.

The previous results question the meaning of the time-series momentum risk factor. In Table 34 on page 87, we have reported the correlation of weekly returns between currencies and this risk factor. We notice that the correlation is positive except for USD. For example, the correlation between the global TS-MOM risk factor and USD is equal to -48%. On the contrary, the correlation between the global TS-MOM risk factor and European currencies is larger than 40%. If we consider the cross-section momentum, we obtain very different results with very low correlations (see Table 35 on page 88). In fact, we have the feeling that the time-series momentum is mainly a US dollar risk factor, more precisely a USD versus European currencies[48] (or DM currencies). However, from a statistical point of view, the time-series momentum cannot be reduced to this impression.

[48]See Figures 25 and 26.

Figure 25: Weekly payoff between USD and TS-MOM returns

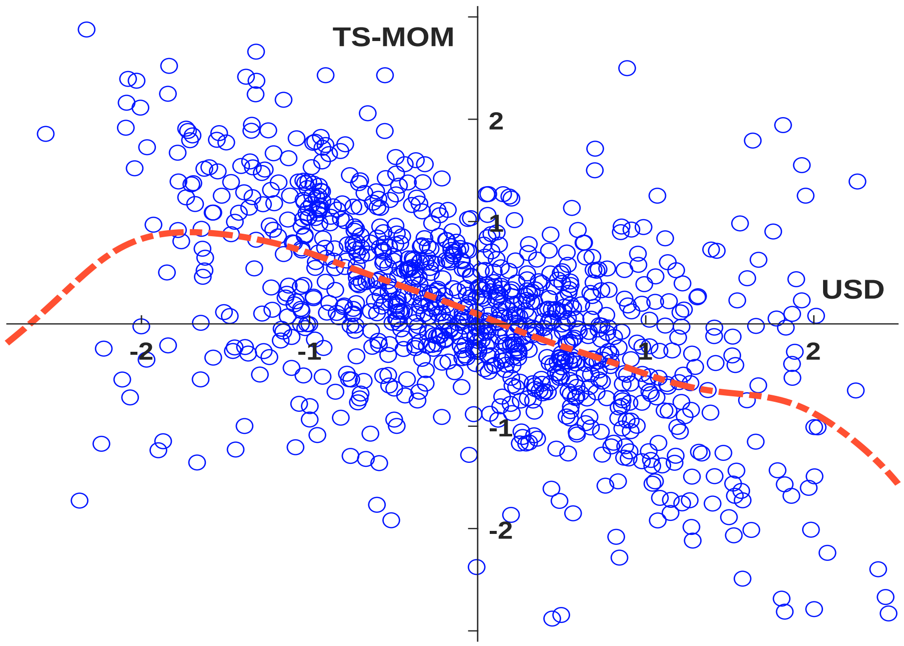


Figure 26: Weekly payoff between EUR and TS-MOM returns

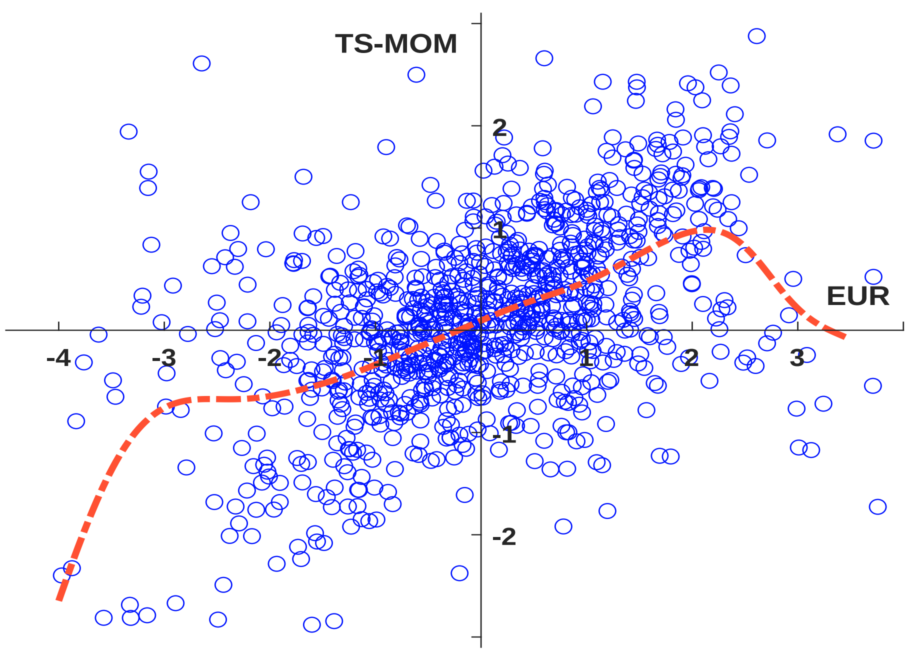


Table 17: Weekly correlation between currency returns and momentum risk factors

Currency	Time-series momentum			Cross-section momentum		
	G10	EM	G25	G10	EM	G25
USD	−46.39	−36.62	−48.04	9.34	−4.70	1.35
EUR	40.50	35.67	43.55	−6.94	5.36	1.05
JPY	34.56	13.74	25.17	11.06	8.30	13.61

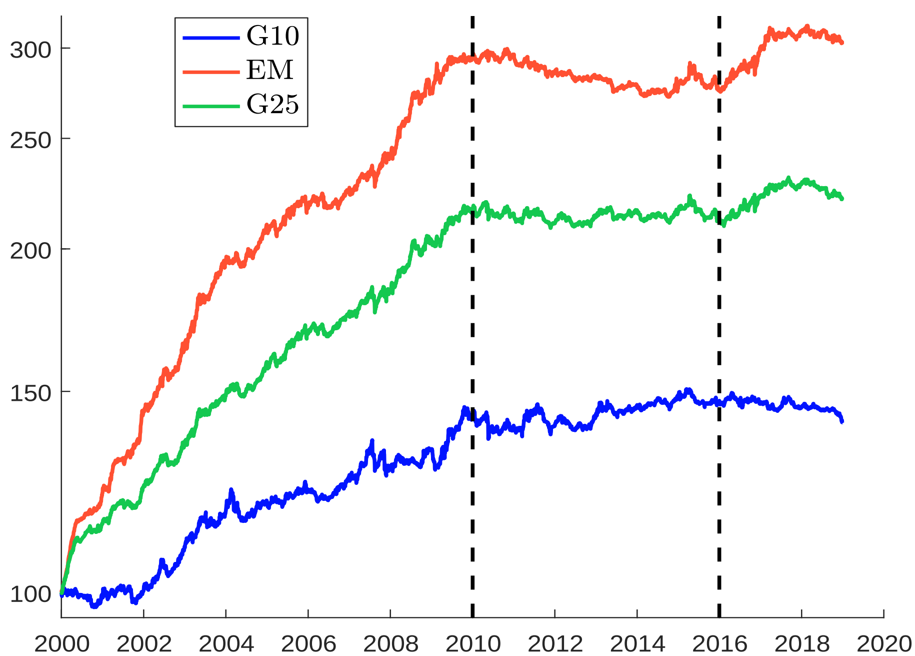
5 Currency management

With the previous four risk factors, we are equipped to manage currency portfolios. In what follows, we consider three applications: the construction of alternative risk premia portfolios, the hedging of multi-currency portfolios and the design of overlay strategies.

5.1 The alpha of foreign exchange rates

These risk factors are extensively used by hedge funds and asset managers for developing trading strategies. In particular, carry, value and momentum are well known for being candidates of alternative risk premia (Roncalli, 2017). Generally, these alternative risk premia are combined in order to build a portfolio that generates a performance which has low correlation with the traditional risk premia such as equities and bonds.

Figure 27: Cumulative performance of the currency ARP portfolio



In Figure 27, we have reported the cumulative performance of the unfunded equally-weighted portfolio, which is made up of the four risk factors: carry, value, cross-section momentum and time-series momentum. We mainly distinguish two periods. Before 2010, the ARP portfolio posted a high yearly return: 3.62% for G10, 11.43% for EM and 8.03%

for G25. Since 2010, the performance has been close to zero[49]. If we consider the annual statistics reported in Table 36 on page 89, there is however a rebound in 2016 and 2017 for the EM and G25 portfolios, but it is followed again by negative performance in 2018.

These results shed some light on the question posed by this research article: Does factor investing make sense in currency markets? From a risk factor perspective, we have seen that carry, value and momentum make a lot of sense in order to understand the time-series and the cross-section of currency returns. From an alternative risk premia perspective, it is less much clear since the performance has declined substantially in recent years. In this case, implementing factor investing in order to generate alpha remains an open question.

5.2 Basket hedging

One of the big issues when managing an asset portfolio subject to foreign exchange risk is to manage the currency risk. Generally, the portfolio manager invests in foreign financial markets in order to be exposed to the local stock or bond market. It is then interested by the local performance of the market and not by the performance of the currency. Below, we have represented four cases:

Case	Stock Market	Currency	Total
(a)	+10%	+0%	+10%
(b)	+10%	+10%	+20%
(c)	+10%	−20%	−10%
(d)	−5%	+10%	+5%

In Case (a), the local stock market posted a performance of 10% while the currency return was equal to 0%. This is the best case since the currency market has no impact on the performance of the investment portfolio. However, the currency return may be positive – Case (b) – or negative – Case (c) – implying that the portfolio performance may vary a lot depending on the currency's behavior. Case (d) corresponds to the situation where the portfolio performance was positive because the foreign exchange risk was favorable. In order to reduce this risk, we can hedge the portfolio. However, this may cost a lot of money. For instance, a full hedging can be implemented by considering a reverse forward position, meaning that the cost is approximately the difference between the foreign interest rate and the domestic interest rate. For instance, if the interest rate differential is equal to 10%, the investor earns money if the local performance of the portfolio is greater than 10%. The question of hedging is then a big issue and there is substantial literature on this topic (Black, 1989; Glen and Jorion, 1993; Campbell *et al.*, 2010; Froot, 2019).

We can formalize the hedging problem as follows. We note $R_{i,t}$ and $\mathcal{H}_{i,t}$ the returns of Currency i and its hedge. The net P&L of the hedged portfolio is equal to:

$$\Pi_{i,t} = R_{i,t} + \mathcal{H}_{i,t}$$

The underlying idea is to find the hedge $\mathcal{H}_{i,t}$ such that $\Pi_{i,t} = 0$. In this case, the solution is given by:

$$\mathcal{H}_{i,t} = -R_{i,t}$$

This hedge corresponds to a reverse forward position. Let us now consider a basket of n currencies defined by the following weights $w = (w_1, \ldots, w_n)$. The net P&L of the hedged

[49] The yearly return is respectively equal to −16 bps for G10, 30 bps for EM and 22 bps for G25 for the period 2010 – 2018.

portfolio becomes:

$$\Pi_t = \sum_{i=1}^{n} w_i R_{i,t} + \mathcal{H}_t$$

Again, one solution is to consider a basket of reverse forward contracts and we have:

$$\mathcal{H}_t = -\sum_{i=1}^{n} w_i R_{i,t}$$

In practice, considering a full hedging basket is not always efficient because of transaction costs, liquidity issues or performance motivation. Sometimes, professionals prefer to develop a currency return model and find the hedge that presents the minimum variance of the P&L subject to a given minimum return $\pi^\star$:

$$\begin{aligned}
\mathcal{H}_t^\star &= \arg\min \operatorname{var}(\Pi_t) \\
&\text{s.t.} \quad \mathbb{E}_{\mathcal{M}\text{odel}}[\Pi_t] \geq \pi^\star
\end{aligned}$$

There are of course many hedging strategies depending on the currency model and the value of $\pi^\star$. Moreover, we can change the objective function. For instance, we can choose to minimize the hedging ratio $\mathfrak{h}_t$:

$$\mathfrak{h}_t = \frac{\operatorname{var}(\Pi_t)}{\operatorname{var}(R_t)}$$

where $R_t = \sum_{i=1}^{n} w_i R_{i,t}$ is the return of the basket portfolio. A hedge ratio of 0 means that the basket is fully hedged and a hedge ratio of 1 means that the basket is not hedged at all.

In what follows, we explore how the factor investing framework can be used for building hedging strategies. We recall that the linear factor model is:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{n_{\mathcal{F}}} \beta_{i,t}^j \mathcal{F}_{j,t} + \varepsilon_{i,t}$$

It follows that:

$$\begin{aligned}
\Pi_t &= \sum_{i=1}^{n} w_i R_{i,t} + \mathcal{H}_t \\
&= \sum_{i=1}^{n} w_i \alpha_i + \sum_{i=1}^{n} \sum_{j=1}^{n_{\mathcal{F}}} w_i \beta_{i,t}^j \mathcal{F}_{j,t} + \sum_{i=1}^{n} w_i \varepsilon_{i,t} + \mathcal{H}_t
\end{aligned}$$

If we consider the following partial hedge based on the risk factor portfolios:

$$\mathcal{H}_t = -\sum_{i=1}^{n} \sum_{j=1}^{n_{\mathcal{F}}} w_i \beta_{i,t}^j \mathcal{F}_{j,t}$$

we obtain:

$$\Pi_t = \sum_{i=1}^{n} w_i \alpha_i + \sum_{i=1}^{n} w_i \varepsilon_{i,t}$$

and:

$$\begin{aligned}
\operatorname{var}(\Pi_t) &= \operatorname{var}\left(\sum_{i=1}^{n} w_i \alpha_i + \sum_{i=1}^{n} w_i \varepsilon_{i,t}\right) \\
&= w^\top D w
\end{aligned}$$

where $D = \operatorname{diag}\left(\sigma_1^2, \ldots, \sigma_n^2\right)$. It follows that the hedging ratio $\mathfrak{h}_t$ is equal to:

$$\mathfrak{h}_t = \frac{w^\top D w}{w^\top B^\top \Omega B w + w^\top D w}$$

where Ω is the covariance matrix of risk factors and $B = \left(\beta_{i,t}^j\right)$ is the loading matrix. We deduce that the hedging ratio is the specific part of the basket return R_t or:

$$\mathfrak{h}_t = 1 - \mathfrak{R}_{c,t}^2$$

where $\mathfrak{R}_{c,t}^2$ is the centered coefficient of determination of the following linear regression:

$$R_t = \alpha + \sum_{j=1}^{n_{\mathcal{F}}} \beta_t^j \mathcal{F}_{j,t} + \varepsilon_t$$

In Table 18, we have reported the average hedging ratio $\bar{\mathfrak{h}}$ for several basket portfolios when we consider weekly returns and a two-year rolling window. The basket portfolios are:

1. The basket is an equally-weighted portfolio of CHF, HKD, JPY and KRW;
2. The basket is an equally-weighted portfolio of the following illiquid currencies: ARS, CNY, COP, LTL, LVL, MYR, PEN, PHP, RON and THB;
3. The basket is an equally-weighted portfolio of AUD, BRL, CHF, DKK, EUR and NZD;
4. The basket is long on AUD, CHF and DKK and short on BRL, EUR and NZD;
5. The basket is an equally-weighted portfolio of the 40 currencies except USD.

If we consider single factors, the hedging ratio is generally minimum when we consider the time-series momentum portfolio[50]. We notice that the four-factor model may help to hedge more than 50% of the basket variance.

Table 18: Average hedging ratio $\bar{\mathfrak{h}}$ (in %)

Basket	Carry	Value	CS-MOM	TS-MOM	4F
#1	85.36	88.71	90.78	65.72	35.74
#2	86.03	85.80	92.13	69.19	47.14
#3	76.47	82.04	87.62	53.90	27.36
#4	59.75	72.83	86.89	91.02	40.26
#5	79.85	83.53	87.76	52.74	29.04

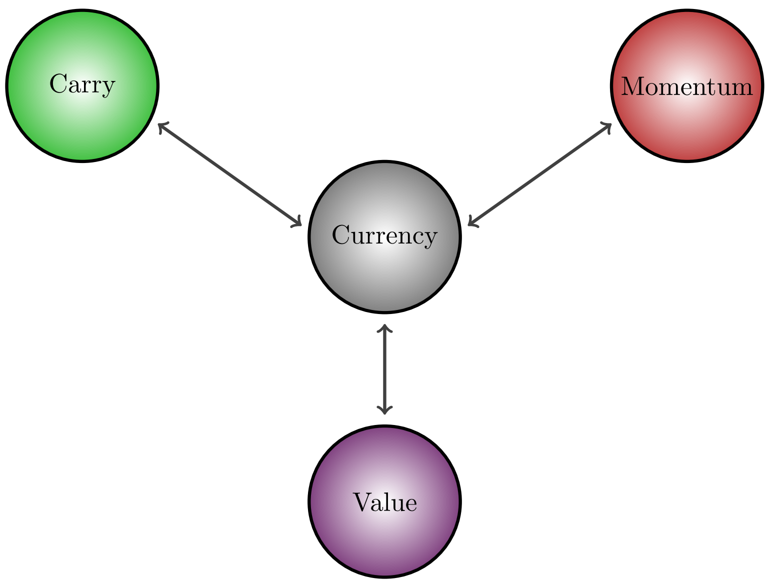
The previous approach is well known by professionals. The idea is to hedge a basket by a proxy that is easier to trade. This is particularly true for illiquid assets. In the case of currency, the four risk factors can be easily traded via investment banks with low transaction costs, and the growth of these commoditized factors may change the way currency risks are managed in the future.

[50] The exception is the fourth basket, which corresponds to a long/short portfolio with respect to the USD.

5.3 Overlay strategies

As noticed by Roncalli (2017), it is especially interesting to analyze all the assets with respect to the three dimensions: carry, value and momentum. This is also true for currencies. In particular, thinking currency returns in terms of risk factors helps to distinguish common and idiosyncratic patterns. In Figure 28, we have represented the information that is conveyed between currencies and risk factors.

Figure 28: Risk factor analysis of a currency



The traditional approach used to analyze a currency is to measure its carry $\mathcal{C}_{i,t}$, its value $\mathcal{V}_{i,t}$, its momentum $\mathcal{M}_{i,t}$ and its specific risk $\mathcal{S}_{i,t}$. Generally, the portfolio manager takes a long position in a currency that presents a positive carry with respect to the other currencies, a positive value meaning that it is undervalued, a positive momentum and a good specific risk:

Currency	$\mathcal{C}_{i,t}$	$\mathcal{V}_{i,t}$	$\mathcal{M}_{i,t}$	$\mathcal{S}_{i,t}$	Signal
1	$+$	$+$	$+$	$+$	Long
2	$-$	$-$	$-$	$-$	Short
3	$+$	$-$	$+$	$+$	?

Most of the time, the signals are not all positive or negative and the portfolio manager has to consider the trade-off between the different measures.

The alternative approach is to calculate the betas (or sensitivities) to the risk factors:

Currency	$\beta_{i,t}^{\text{Carry}}$	$\beta_{i,t}^{\text{Value}}$	$\beta_{i,t}^{\text{Momentum}}$	Signal
1	$+$	$+$	$+$	Long?
2	$-$	$-$	$-$	Short?
3	$+$	$-$	$+$	?

In this case, it is not obvious that we have to be long in a currency for which all the betas are positive. Indeed, it depends on which risk factor is rewarded by the market since our results have shown that these risk factors do not exhibit a systematic positive risk premium.

The views of the portfolio manager are then essential to take a long or short position on a currency.

The first approach typically corresponds to the traditional security analysis, where the portfolio manager analyses the current characteristics of the security. Among the different criteria, value, carry and momentum are the important metrics with the specific risk that takes into account other patterns such as economic, inflation or political risks. The second approach corresponds to a new way of thinking where common risk factors are at the center of the investment analysis. In this case, the primary role of the portfolio manager is first to analyze the risk factors, understand them and predict which risk factor will be rewarded or not. Therefore, the investment process shifts from asset picking (e.g. stock, bond or currency picking) based on security analysis to asset picking based on common factor analysis[51]. This last approach clearly redefines currency overlay strategies and complements the traditional method when implementing them.

6 Conclusion

The purpose of this paper was to develop a factor investing framework for currency markets. Factor investing has certainly been one of the most important topics in the asset management industry over the last ten years. It has changed equity investing, it has been extended to the concept of alternative risk premia and it is beginning to be implemented in the fixed-income universe and corporate bonds. Therefore, we may wonder if factor investing does make sense when considering foreign exchange rates.

Factor investing is made up of two pillars. The first pillar is to develop an asset pricing model based on common risk factors. The second pillar consists in building an investment portfolio based on the risk factors in order to generate financial performance. In this article, we have developed a risk factor model based on carry, value and momentum. This model helps us to understand the cross-section and time-series dynamics of foreign exchange rates. In particular, it helps to dissect currency returns and to distinguish carry-based and momentum-based currencies. In this case, factor investing does make a lot of sense. If we focus on the second pillar, we observe that the currency risk factors do not necessarily exhibit a positive risk premium. Moreover, the performance of an equally-weighted portfolio of carry, value and momentum is impressive until 2010, but is mixed after this date. From this point of view, it is not obvious that factor investing continues to make a lot of sense. However, this framework is appealing for building basket hedging and overlay strategies. Moreover, the factor approach clearly complements the traditional security analysis.

[51] Sometimes, this approach is inappropriately called factor picking.

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A Appendix

A.1 Notations

We use the following notations:

- S_t is the nominal exchange rate.
- F_t is the forward exchange rate.
- Q_t is the real (effective) exchange rate (REER).
- P_t is the price level in the home country.
- The lowercase variables s_t, f_t, q_t and p_t are equal to the logarithm of the corresponding uppercase variables S_t, F_t, Q_t and P_t.
- i_t is the nominal interest rate in the home country.
- $\pi_t = p_t - p_{t-1}$ is the inflation rate in the home country.
- $r_t = i_t - \pi_t$ is the real interest rate in the home country. If we consider a model of rational anticipations, we note $r_t = i_t - \mathbb{E}_t[\pi_{t+1}]$.
- We use the exponent * for indicating that the variable corresponds to the foreign country. For example, $p_t^\star$ is the logarithm of the price level in the foreign country.
- φ_t is the time-varying risk premium.
- m_t is the home-country money supply.
- y_t is the home-country income (or GDP).
- gap_t is the home-country output gap, GDB_t is the government debt, NFA_t is the net foreign assets, ToT_t is the terms of trade, and TnT_t is the relative price of non-traded to traded goods.
- ca_t is the current account, while ka_t is the capital account.

A.2 Mathematical results

A.2.1 Dynamic equilibrium in vector error-correction models

Let y_t be a n-dimensional stochastic process. We assume the following VECM:

$$\Delta y_t = \zeta + \sum_{k=1}^{p} \Phi_k \Delta y_{t-k} - \alpha z_{t-1} + \varepsilon_t \tag{26}$$

where $y_t \sim I(1)$, $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$, $z_t = \gamma^\top y_t \sim I(0)$. We have:

$$y_t = \zeta + (I_n + \Phi_1 - \Pi) y_{t-1} + \sum_{k=2}^{p} (\Phi_k - \Phi_{k-1}) y_{t-k} - \Phi_p y_{t-k-1} + \varepsilon_t$$

where $\Pi = \alpha\gamma^\top$. Let $Y_t = (y_t, y_{t-1}, \ldots, y_{t-p+1})$ be the $p \times n$ dimensional stochastic process. We can then transform the VECM (26) into the following state space model:

$$\left\{ \begin{array}{l} y_t = AY_t \\ Y_t = CY_{t-1} + c + D\varepsilon_t \end{array} \right. \tag{27}$$

where $A = \left(I_n, \mathbf{0}_{n\times n}, \ldots, \mathbf{0}_{n\times n}\right)$, $c = \left(\zeta, \mathbf{0}_{(p-1)n\times 1}\right)$, $D = A^\top$ and:

$$C = \begin{pmatrix} I_n + \Phi_1 - \Pi & \Phi_1 - \Phi_2 & \Phi_2 - \Phi_3 & \cdots & \Phi_p - \Phi_{p-1} & -\Phi_p \\ I_n & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \cdots & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & I_n & \mathbf{0}_{n\times n} & \cdots & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & I_n & & & \\ & & & \ddots & & \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & & & I_n & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & & & \mathbf{0}_{n\times n} & I_n \end{pmatrix}$$

We deduce that:

$$\begin{aligned} \mathbb{E}\left[Y_{t+h} \mid \mathcal{F}_t\right] &= C\mathbb{E}\left[Y_{t+h-1} \mid \mathcal{F}_t\right] + c \\ &= C^h Y_t + \left(\sum\nolimits_{k=0}^{p-1} C^k\right) c \end{aligned}$$

and:

$$\mathbb{E}\left[y_{t+h} \mid \mathcal{F}_t\right] = AC^h Y_t + \left(\sum\nolimits_{k=0}^{p-1} AC^k\right) c$$

A.2.2 Kalman filtering

We consider the discrete-time state space model:

$$\left\{ \begin{array}{l} Y_t = A_t X_t + a_t + B_t \varepsilon_t \\ X_t = C_t X_{t-1} + c_t + D_t \varepsilon_t^\star \end{array} \right. \tag{28}$$

where Y_t is the observed vector process and X_t is the hidden vector Markov process. Here, the time is indexed by $t \in \mathbb{N}$. We assume that $\varepsilon_t \sim \mathcal{N}\left(\mathbf{0}, Q_t\right)$ and $\varepsilon_t^\star \sim \mathcal{N}\left(\mathbf{0}, Q_t^\star\right)$ are two uncorrelated processes[52]. We note:

$$\hat{X}_t = \mathbb{E}\left[X_t \mid \mathcal{F}_t\right]$$

and:

$$\hat{X}_{t|t-1} = \mathbb{E}\left[X_t \mid \mathcal{F}_{t-1}\right]$$

The corresponding error covariance matrices are:

$$\hat{P}_t = \mathbb{E}\left[\left(X_t - \hat{X}_t\right)\left(X_t - \hat{X}_t\right)^\top\right]$$

and:

$$\hat{P}_{t|t-1} = \mathbb{E}\left[\left(X_t - \hat{X}_{t|t-1}\right)\left(X_t - \hat{X}_{t|t-1}\right)^\top\right]$$

[52] The matrix dimensions are respectively $(n\times 1)$ for Y_t, $(m\times 1)$ for X_t, $(p\times 1)$ for ε_t, $(q\times 1)$ for $\varepsilon_t^\star$, $(n\times m)$ for A_t, $(n\times 1)$ for a_t, $(n\times p)$ for B_t, $(m\times m)$ for C_t, $(m\times 1)$ for c_t, $(m\times q)$ for D_t, $(p\times p)$ for Q_t and $(q\times q)$ for $Q_t^\star$.

Let $X_0 \sim \mathcal{N}\left(\hat{X}_0, \hat{P}_0\right)$ be the initial position of the state vector. The estimates of $\hat{X}_t$ and $\hat{P}_t$ can be obtained by using the recursive Kalman filter[53]:

$$\left\{\begin{array}{l} \hat{X}_{t|t-1} = C_t \hat{X}_{t-1} + c_t \\ \hat{P}_{t|t-1} = C_t \hat{P}_{t-1} C_t^\top + D_t Q_t^\star D_t^\top \\ v_t = A_t \hat{X}_{t|t-1} + a_t - Y_t \\ V_t = A_t \hat{P}_{t|t-1} A_t^\top + B_t Q_t B_t^\top \\ \hat{X}_t = \hat{X}_{t|t-1} + \hat{P}_{t|t-1} A_t^\top V_t^{-1} v_t \\ \hat{P}_t = \left(I_m - \hat{P}_{t|t-1} A_t^\top V_t^{-1} A_t\right) \hat{P}_{t|t-1} \end{array}\right. \tag{29}$$

We notice that v_t is the innovation process at time t:

$$v_t = \mathbb{E}\left[Y_t \mid \mathcal{F}_{k-1}\right] - Y_t$$

Since we have $v_t \sim \mathcal{N}\left(\mathbf{0}, V_t\right)$, the log-likelihood function for observation t is equal to:

$$\ell_t = -\frac{n}{2} \ln 2\pi - \frac{1}{2} \ln |V_t| - \frac{1}{2} v_t^\top V_t^{-1} v_t \tag{30}$$

A.2.3 Dynamic linear factor models

The standard factor model is defined as follows:

$$R_t = \beta_0 + \sum_{j=1}^{n_{\mathcal{F}}} \beta_j \mathcal{F}_{j,t} + \varepsilon_t$$

where R_t is the return of the asset, $\mathcal{F}_{j,t}$ is the j^{th} risk factor and $\varepsilon_t \sim \mathcal{N}\left(0, \sigma^2\right)$ is a white noise process. This static linear model can be extended to the dynamic model by introducing time-varying coefficients:

$$R_t = \beta_{0,t} + \sum_{j=1}^{n_{\mathcal{F}}} \beta_{j,t} \mathcal{F}_{j,t} + \varepsilon_t \tag{31}$$

We generally assume that the coefficient $\beta_{j,t}$ is driven by a random walk process:

$$\beta_{j,t} = \beta_{j,t-1} + \eta_{j,t} \tag{32}$$

where $\eta_{j,t} \sim \mathcal{N}\left(0, \tilde{\sigma}_j^2\right)$.

Equations (31) and (32) can be written as a state space model:

$$\left\{\begin{array}{l} R_t = \mathcal{F}_t \beta_t + \varepsilon_t \\ \beta_t = \beta_{t-1} + \eta_t \end{array}\right.$$

where $\mathcal{F}_t = \left(1, \mathcal{F}_{1,t}, \ldots, \mathcal{F}_{n_{\mathcal{F}},t}\right)$ is the row vector of risk factors that include the intercept of the linear model, $\beta_t = \left(\beta_{0,t}, \beta_{1,t}, \ldots, \beta_{n_{\mathcal{F}},t}\right)$ is the vector of time-varying coefficients and $\eta_t = \left(\eta_{0,t}, \eta_{1,t}, \ldots, \eta_{n_{\mathcal{F}},t}\right)$ is the vector of white noise processes. Since we have $\eta_t \sim \mathcal{N}\left(\mathbf{0}, Q_t^\star\right)$, we need to specify the covariance matrix $Q_t^\star$. In order to avoid identification problems, we assume that $Q_t^\star$ is a diagonal matrix:

$$Q_t^\star = \begin{pmatrix} \tilde{\sigma}_0^2 & 0 & \cdots & 0 \\ 0 & \tilde{\sigma}_1^2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{\sigma}_{n_{\mathcal{F}}}^2 \end{pmatrix}$$

The estimation of the dynamic model consists in two steps:

[53] See Harvey (1990) for the derivation of these equations.

1. First, we estimate the vector of parameters $\theta = (\sigma, \tilde{\sigma}_0, \ldots, \tilde{\sigma}_{n_{\mathcal{F}}})$ by the method of maximum likelihood;

2. Second, we run the Kalman filter in order to calculate the estimated time-varying coefficients $\hat{\beta}_t$.

A.3 Additional results

A.3.1 Tables

Table 19: Annual statistics of the carry risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	2.65	84.85	43.24	6.68	6.39	6.00	−7.83	−1.72	−1.65
2001	7.61	28.97	11.26	5.49	9.26	7.63	−6.97	−7.67	−8.09
2002	15.16	18.57	4.65	5.54	8.98	6.49	−5.91	−9.83	−12.29
2003	8.12	28.94	8.92	5.29	7.65	5.12	−4.39	−3.18	−3.32
2004	2.69	8.74	12.11	6.08	6.15	4.25	−7.87	−5.88	−3.26
2005	11.27	12.69	15.57	4.46	5.00	4.61	−4.31	−4.26	−3.62
2006	4.95	1.60	1.64	5.18	7.64	6.40	−6.29	−13.65	−12.43
2007	4.27	12.69	9.68	9.81	7.53	7.74	−10.54	−6.03	−7.84
2008	−21.92	16.64	3.70	16.29	11.91	11.07	−27.11	−9.36	−12.33
2009	23.77	15.85	15.84	11.09	7.29	7.87	−8.67	−4.33	−4.80
2010	−1.17	8.43	5.77	9.16	4.83	5.77	−8.61	−2.60	−4.14
2011	2.25	−4.41	−6.22	10.10	6.12	6.43	−10.30	−8.83	−9.53
2012	11.47	−0.12	9.27	5.07	4.76	4.24	−5.54	−4.68	−4.90
2013	−1.26	−10.38	−1.13	7.05	5.12	5.88	−10.86	−13.17	−9.66
2014	4.12	2.80	2.56	5.08	6.98	5.93	−4.82	−6.87	−4.06
2015	−1.52	−12.03	−8.37	9.58	11.26	9.60	−13.33	−15.66	−15.03
2016	5.31	25.58	14.47	7.54	9.25	9.17	−4.42	−4.38	−5.52
2017	−1.05	1.19	2.51	5.47	6.90	6.14	−6.28	−8.37	−5.83
2018	1.90	2.12	0.82	4.46	8.13	6.67	−4.14	−10.07	−9.62

Table 20: Annual statistics of the PPP risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	3.05	−8.06	−5.30	8.03	6.43	4.85	−7.12	−11.08	−10.70
2001	5.40	5.62	5.67	7.13	9.73	7.02	−8.71	−7.87	−5.92
2002	−3.73	3.97	1.58	7.04	8.62	5.93	−10.52	−10.09	−8.34
2003	22.80	21.57	20.40	6.06	7.30	5.17	−3.12	−3.21	−2.09
2004	4.46	−4.91	5.56	4.00	5.92	3.86	−3.27	−9.20	−2.74
2005	−5.41	7.17	1.68	3.74	4.97	3.36	−9.05	−5.48	−4.13
2006	1.87	12.95	9.38	4.13	5.71	3.88	−3.37	−3.54	−1.85
2007	−5.14	−1.27	0.60	9.25	4.01	3.35	−12.50	−3.82	−3.06
2008	12.36	−6.27	−1.41	10.71	11.61	7.60	−10.13	−14.16	−9.52
2009	4.90	10.16	9.34	7.99	8.49	6.39	−6.94	−9.40	−4.57
2010	−1.37	7.11	3.11	7.14	4.69	5.12	−6.41	−3.90	−4.30
2011	1.17	1.95	4.63	6.77	5.81	4.04	−9.31	−4.28	−3.55
2012	0.30	3.03	−0.10	4.78	4.14	3.22	−4.40	−3.31	−3.41
2013	0.57	0.95	3.38	6.59	4.24	3.35	−8.23	−3.62	−3.54
2014	−5.62	12.67	2.59	4.86	6.47	4.25	−8.70	−5.13	−3.77
2015	3.59	2.87	5.66	8.72	8.73	5.37	−7.31	−8.99	−4.15
2016	−2.36	14.82	4.29	5.05	9.08	4.72	−10.36	−7.00	−2.95
2017	1.34	2.29	2.78	4.19	5.61	3.32	−4.05	−6.60	−2.57
2018	0.55	10.46	5.83	3.94	8.19	4.22	−5.29	−10.19	−5.43

Table 21: Annual statistics of the BEER risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−1.08	−22.69	−8.95	7.56	7.49	5.36	−11.13	−32.97	−16.85
2001	3.39	36.16	8.44	6.87	9.63	5.86	−6.35	−4.97	−5.67
2002	−4.71	29.89	11.87	5.89	8.48	5.50	−8.64	−4.01	−3.11
2003	15.41	13.65	16.38	6.00	7.18	4.67	−4.41	−6.46	−3.53
2004	−0.07	9.13	11.64	5.48	5.99	4.96	−5.35	−3.24	−2.24
2005	0.35	5.37	4.74	3.45	4.84	3.56	−5.79	−5.75	−3.94
2006	0.03	21.57	17.52	3.56	5.93	5.12	−2.85	−4.24	−3.15
2007	−1.24	−4.40	−1.98	6.21	4.30	3.85	−7.07	−5.33	−4.28
2008	16.83	3.69	8.49	10.31	10.79	7.32	−7.83	−9.12	−5.79
2009	10.81	6.16	9.90	9.48	9.00	5.92	−10.62	−12.10	−4.23
2010	−0.85	3.93	1.89	5.86	4.98	3.91	−5.27	−3.79	−3.26
2011	−0.78	−0.28	2.84	7.58	7.01	4.13	−8.27	−8.11	−4.28
2012	7.79	6.96	8.11	5.26	4.17	2.84	−6.21	−2.52	−1.21
2013	−0.99	−3.55	−1.58	5.77	4.75	3.29	−5.37	−5.27	−4.05
2014	−9.74	12.39	−0.59	4.65	6.13	2.84	−11.48	−4.62	−2.87
2015	5.64	1.40	3.77	5.14	9.54	5.80	−3.80	−12.97	−5.24
2016	3.96	1.34	6.48	4.89	8.47	5.71	−7.63	−5.09	−2.88
2017	2.33	5.14	2.59	4.18	5.25	3.44	−3.38	−5.50	−3.33
2018	−1.59	4.63	2.49	3.89	7.03	4.30	−4.78	−9.22	−5.55

Table 22: Annual statistics of the NATREX risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2003	3.74	−0.65	2.34	5.12	6.83	4.52	−3.54	−7.47	−3.60
2004	6.29	−2.46	5.52	3.91	5.96	3.32	−3.29	−5.37	−2.05
2005	9.76	0.81	7.02	4.12	4.31	3.09	−4.23	−4.25	−1.83
2006	5.08	9.91	8.55	4.27	5.81	3.66	−5.13	−5.56	−1.99
2007	6.11	1.39	3.09	6.83	3.93	3.06	−8.59	−2.73	−3.74
2008	13.16	9.29	15.13	6.69	9.06	7.28	−5.83	−11.87	−7.13
2009	9.31	10.09	7.74	6.96	7.15	5.30	−5.28	−5.71	−5.08
2010	−1.54	−9.11	−7.05	4.55	7.08	4.59	−5.27	−12.79	−9.40
2011	−4.39	0.89	1.46	6.26	6.62	4.62	−11.87	−7.21	−7.79
2012	2.28	5.71	3.24	4.63	5.52	3.64	−9.03	−3.42	−3.40
2013	18.16	10.63	13.11	5.25	5.77	4.44	−2.42	−4.28	−2.86
2014	6.14	1.44	4.62	3.95	7.77	5.50	−2.08	−9.55	−4.95
2015	2.43	4.23	1.07	7.40	8.05	5.43	−6.09	−6.42	−3.89
2016	3.71	21.79	10.85	5.17	7.52	3.97	−6.54	−4.24	−2.06
2017	1.59	3.24	0.92	3.46	6.03	3.07	−2.60	−9.48	−3.81
2018	−0.94	1.48	0.31	3.00	7.34	4.27	−4.64	−7.62	−4.67

Table 23: Annual statistics of the value risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	1.63	−4.05	−2.64	4.01	3.22	2.43	−3.54	−5.65	−5.47
2001	2.75	2.94	2.88	3.56	4.87	3.51	−4.44	−3.99	−3.00
2002	−1.80	2.10	0.85	3.52	4.31	2.97	−5.39	−5.15	−4.25
2003	12.91	9.99	11.05	4.92	5.82	3.99	−2.63	−3.89	−2.15
2004	5.40	−3.66	5.56	3.46	5.55	3.32	−2.08	−6.39	−2.33
2005	1.98	3.97	4.34	1.93	4.23	2.74	−2.23	−3.78	−2.36
2006	3.53	11.53	9.00	2.99	4.35	3.05	−2.77	−2.14	−1.31
2007	0.72	0.12	1.88	3.20	2.61	2.04	−3.63	−1.79	−0.98
2008	12.98	1.58	6.65	7.15	7.58	6.36	−5.85	−11.42	−7.27
2009	7.24	10.31	8.64	5.90	6.15	4.61	−5.99	−5.17	−3.43
2010	−1.31	−1.19	−2.02	3.81	3.88	3.37	−4.03	−5.50	−5.02
2011	−1.47	1.59	3.10	4.09	3.74	3.02	−5.31	−3.32	−2.63
2012	1.32	4.45	1.60	4.20	3.47	2.41	−5.67	−2.25	−1.93
2013	9.11	5.81	8.19	4.80	2.88	2.99	−3.38	−2.96	−2.35
2014	0.14	7.00	3.63	3.45	6.25	4.49	−3.89	−6.58	−4.36
2015	3.34	3.73	3.48	4.43	6.72	3.15	−4.68	−6.43	−3.00
2016	0.73	18.35	7.57	3.47	7.60	3.48	−6.96	−4.76	−1.73
2017	1.52	2.87	1.89	2.82	4.37	2.08	−2.23	−4.09	−1.40
2018	−0.15	6.12	3.12	2.42	4.95	2.12	−4.56	−4.85	−2.15

Table 24: Annual statistics of the 1M cross-section momentum risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−4.85	34.64	16.91	7.12	7.07	6.35	−12.80	−6.79	−8.35
2001	−1.00	31.30	18.65	7.25	8.48	6.98	−9.24	−7.64	−4.05
2002	−4.71	0.76	2.98	6.44	10.00	7.15	−9.76	−11.30	−9.01
2003	−8.68	4.29	−5.74	6.94	8.31	6.64	−13.00	−9.20	−8.86
2004	−0.83	−0.34	−0.84	6.17	6.90	5.17	−5.53	−10.21	−5.60
2005	−4.66	−7.05	−4.20	4.91	7.00	5.56	−6.51	−8.65	−7.52
2006	0.89	−3.72	1.67	4.76	7.01	5.23	−4.45	−9.53	−4.95
2007	−0.40	−3.42	−4.53	9.35	5.24	5.73	−11.52	−8.66	−8.62
2008	20.72	18.00	24.23	12.51	11.30	10.32	−7.69	−9.79	−5.51
2009	−6.08	−5.69	−6.56	9.15	8.66	6.81	−12.56	−7.90	−8.75
2010	−9.45	−2.99	−5.24	7.81	7.50	6.49	−11.77	−13.07	−10.92
2011	5.72	−0.86	2.97	8.33	7.87	6.24	−10.24	−6.59	−3.90
2012	2.16	−5.81	−1.65	5.10	5.61	4.85	−7.70	−8.06	−5.31
2013	1.02	1.84	−0.14	6.91	5.71	4.76	−9.01	−4.66	−6.19
2014	5.61	1.24	1.99	4.71	7.18	5.13	−3.51	−8.12	−4.42
2015	2.49	5.37	3.79	6.87	9.58	7.79	−5.53	−9.52	−5.84
2016	−6.15	−7.23	−7.24	6.86	7.98	6.45	−9.35	−15.42	−12.56
2017	1.63	7.25	2.41	4.73	6.48	4.54	−5.58	−5.99	−4.24
2018	−7.27	−12.62	−8.67	4.11	7.58	4.86	−8.22	−13.12	−9.76

Table 25: Annual statistics of the 3M cross-section momentum risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−4.66	18.54	7.69	7.59	7.27	6.49	−8.79	−16.81	−13.08
2001	−8.87	25.19	8.38	8.02	9.19	8.32	−12.37	−8.32	−8.90
2002	10.62	32.14	24.01	6.46	10.49	7.83	−5.88	−8.14	−5.48
2003	−4.89	0.97	−0.95	6.32	8.32	6.16	−11.07	−7.33	−5.80
2004	−2.88	−11.23	−9.17	6.71	6.02	5.10	−10.14	−12.89	−10.33
2005	−2.14	−2.17	3.87	4.98	6.70	5.21	−4.45	−8.62	−4.20
2006	4.65	−4.79	1.66	5.14	6.27	5.41	−4.20	−9.26	−5.64
2007	−9.14	4.55	−3.67	7.95	5.73	5.99	−17.81	−3.93	−9.05
2008	24.11	13.27	18.31	13.15	10.69	9.09	−5.96	−10.15	−6.06
2009	0.83	−4.46	−8.37	10.75	9.08	7.61	−6.40	−11.01	−12.62
2010	0.18	1.15	−4.71	7.97	7.03	6.23	−7.57	−6.47	−9.14
2011	−3.24	−4.75	1.21	7.98	8.43	6.78	−10.75	−8.43	−5.68
2012	−7.38	−3.10	−5.15	5.65	5.47	5.31	−14.88	−8.04	−11.07
2013	0.73	2.07	1.58	7.30	5.78	5.29	−9.08	−7.71	−6.80
2014	2.10	8.26	2.39	4.73	6.35	4.75	−4.92	−4.96	−4.32
2015	−6.13	−7.43	−6.93	7.03	9.94	7.87	−9.62	−12.60	−10.33
2016	−7.16	5.62	−0.29	7.34	7.98	6.30	−9.41	−8.03	−7.70
2017	−6.28	0.38	−1.18	4.83	5.89	4.51	−8.31	−6.35	−4.27
2018	−11.13	−16.83	−16.36	4.17	7.51	5.31	−11.38	−21.17	−16.47

Table 26: Annual statistics of the 12M cross-section momentum risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−4.29	33.95	15.41	8.47	7.97	7.26	−10.02	−9.68	−9.70
2001	0.35	40.13	20.43	5.83	10.44	6.96	−8.23	−7.77	−6.35
2002	13.61	4.00	11.87	5.08	10.01	8.00	−3.18	−11.50	−6.85
2003	−6.13	−3.66	−3.22	5.57	9.06	7.02	−8.87	−16.34	−10.46
2004	−6.79	−1.15	−6.48	6.08	5.78	4.90	−11.24	−8.00	−10.71
2005	5.08	−2.82	5.42	4.75	6.50	5.21	−4.50	−8.10	−4.40
2006	−2.10	−6.02	−7.72	4.59	7.53	5.68	−4.61	−9.11	−11.08
2007	−2.04	8.95	4.08	8.98	5.84	6.09	−9.80	−5.81	−6.58
2008	14.49	5.73	7.79	13.85	11.37	9.55	−8.90	−9.77	−6.52
2009	−17.33	−9.38	−14.65	11.77	7.65	6.92	−23.45	−12.96	−18.20
2010	−0.32	−1.27	−0.13	8.27	7.39	6.77	−8.76	−11.38	−9.07
2011	−9.89	−7.36	−4.14	5.95	7.20	5.86	−12.61	−8.55	−7.77
2012	−0.81	−2.20	−5.21	5.49	5.99	5.40	−5.76	−6.36	−8.54
2013	4.55	9.14	7.07	6.26	5.39	4.26	−5.38	−4.32	−3.17
2014	1.41	0.28	−1.04	4.14	6.41	4.75	−5.91	−7.67	−7.82
2015	−2.08	−4.38	−2.54	6.80	9.40	7.24	−6.96	−15.80	−10.20
2016	−2.23	−4.15	−5.18	6.69	8.81	7.27	−6.95	−16.19	−11.98
2017	−3.83	11.85	−7.02	4.98	6.08	4.46	−7.53	−17.74	−11.24
2018	−6.62	−9.96	−8.71	4.45	7.86	4.94	−7.45	−14.94	−11.95

Table 27: Annual statistics of the cross-section momentum risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−4.46	29.07	13.40	6.16	5.15	5.11	−7.42	−4.21	−4.71
2001	−3.12	32.28	15.81	5.39	8.08	6.38	−9.13	−7.85	−6.43
2002	6.29	11.72	12.74	4.78	8.41	6.73	−3.39	−6.79	−4.78
2003	−6.50	0.70	−3.21	5.21	6.51	5.24	−9.71	−9.09	−6.62
2004	−3.41	−4.24	−5.48	4.71	4.41	3.70	−8.18	−8.02	−7.57
2005	−0.61	−3.91	1.67	4.08	5.11	4.32	−4.39	−5.87	−3.78
2006	1.19	−4.72	−1.46	3.56	5.41	3.88	−2.71	−7.68	−5.10
2007	−3.68	3.32	−1.35	6.30	4.48	4.59	−12.05	−4.98	−5.28
2008	19.99	12.57	16.78	11.84	8.92	8.28	−7.52	−6.66	−5.56
2009	−7.45	−6.33	−9.79	7.35	6.39	5.39	−9.15	−9.00	−11.14
2010	−3.13	−0.89	−3.27	6.26	5.57	5.02	−7.39	−8.32	−7.90
2011	−2.50	−4.21	0.07	5.50	6.31	5.05	−9.68	−7.05	−5.30
2012	−2.01	−3.62	−3.95	4.16	4.21	3.97	−7.56	−5.83	−6.25
2013	2.17	4.36	2.84	5.99	4.75	4.15	−6.79	−4.27	−4.54
2014	3.08	3.28	1.14	3.67	5.77	4.21	−3.42	−6.08	−4.50
2015	−1.88	−2.11	−1.88	5.85	8.16	6.57	−5.36	−9.46	−6.43
2016	−5.07	−1.86	−4.17	5.42	6.12	5.29	−7.23	−11.53	−9.16
2017	−2.81	−1.62	−1.96	3.43	4.75	3.56	−4.00	−6.26	−4.66
2018	−8.31	−13.05	−11.27	3.15	6.11	4.06	−8.69	−15.07	−12.23

Table 28: Annual statistics of the 1M time-series momentum risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	7.03	20.80	12.34	7.44	4.67	5.09	−5.73	−6.04	−4.79
2001	−4.52	9.88	5.89	7.85	5.97	6.10	−9.77	−4.76	−6.51
2002	10.70	−2.97	1.85	6.69	9.95	8.47	−6.49	−20.37	−17.00
2003	15.37	24.03	20.70	7.70	8.87	7.36	−8.05	−4.14	−4.01
2004	2.56	25.30	16.19	9.01	9.29	8.27	−11.50	−6.50	−5.98
2005	−14.03	−1.91	−5.17	6.42	7.81	6.79	−15.39	−8.18	−8.81
2006	4.11	6.66	7.88	6.59	7.56	6.46	−5.31	−7.97	−4.82
2007	0.31	5.62	4.25	8.11	7.66	6.82	−8.03	−7.06	−5.51
2008	17.04	44.21	26.87	10.18	13.38	10.27	−5.29	−8.05	−7.77
2009	2.90	8.50	12.37	11.98	11.67	10.91	−16.68	−14.39	−10.81
2010	−15.46	−15.89	−13.05	9.15	9.57	9.12	−16.97	−18.12	−14.53
2011	4.25	3.91	5.72	8.74	9.49	8.86	−14.71	−12.42	−12.41
2012	0.61	−10.15	−3.01	5.93	7.99	6.48	−4.74	−16.85	−10.38
2013	−2.38	−4.68	−3.91	6.83	5.45	5.31	−8.96	−7.97	−7.37
2014	−3.25	−17.29	−11.17	4.60	5.97	4.71	−6.41	−17.82	−11.58
2015	−14.35	18.26	6.36	7.62	10.32	8.18	−18.21	−7.84	−7.57
2016	−5.46	−2.03	−6.18	7.39	10.66	8.27	−10.50	−11.84	−8.74
2017	4.77	19.84	13.86	5.42	6.70	5.96	−7.12	−3.65	−4.51
2018	−13.39	10.02	−4.37	4.94	8.41	6.20	−16.72	−8.58	−10.93

Table 29: Annual statistics of the 3M time-series momentum risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−7.55	5.86	5.68	5.45	4.62	4.12	−9.75	−11.40	−7.97
2001	−6.97	7.02	−1.83	7.84	6.86	6.76	−11.08	−5.88	−7.74
2002	24.33	25.79	26.92	7.39	8.13	7.15	−5.05	−6.30	−5.09
2003	16.47	35.94	26.79	7.54	9.39	7.71	−6.64	−4.74	−3.84
2004	3.32	21.94	11.42	8.64	8.96	7.95	−13.54	−7.07	−7.46
2005	−6.68	3.77	−1.17	6.40	8.15	6.69	−9.57	−6.02	−5.36
2006	4.73	4.06	7.72	7.02	6.67	6.02	−6.02	−8.62	−5.24
2007	2.35	14.26	7.20	8.61	9.28	8.20	−9.28	−10.07	−9.28
2008	22.03	29.85	30.49	10.39	13.45	11.22	−7.89	−11.06	−7.24
2009	7.33	30.95	24.30	11.79	10.66	10.12	−10.26	−4.46	−5.41
2010	−0.67	−7.04	−4.87	8.69	9.53	9.09	−12.64	−14.40	−13.01
2011	2.50	−1.91	−1.03	9.13	9.08	9.36	−13.23	−14.39	−16.08
2012	−5.28	−6.26	−5.92	6.03	8.10	6.46	−13.55	−17.44	−14.99
2013	0.87	−1.19	−2.15	5.72	5.74	4.86	−6.90	−9.85	−8.78
2014	8.53	−1.95	2.34	4.51	6.04	4.44	−2.54	−7.88	−5.59
2015	−4.63	−9.11	−7.20	6.09	9.25	7.86	−12.42	−16.07	−15.16
2016	−8.15	8.49	3.35	8.77	10.24	8.71	−11.33	−7.24	−6.35
2017	−8.81	8.27	−0.99	5.51	7.01	5.83	−9.99	−5.71	−6.82
2018	−0.73	−3.38	−1.28	5.02	7.61	6.06	−6.68	−9.31	−7.65

Table 30: Annual statistics of the 12M time-series momentum risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−1.04	4.33	4.63	6.58	3.71	3.54	−5.51	−8.71	−5.51
2001	0.66	4.04	3.66	7.25	5.84	5.82	−9.52	−7.11	−6.23
2002	23.37	20.68	22.42	6.88	6.63	6.15	−5.67	−5.79	−5.31
2003	16.97	25.38	22.00	8.07	9.91	7.95	−7.94	−6.97	−4.57
2004	6.78	16.88	11.54	9.87	8.73	8.48	−12.18	−8.98	−9.80
2005	−4.25	0.44	−0.94	6.75	8.38	7.29	−8.35	−9.27	−8.54
2006	−0.66	5.14	4.28	5.34	6.62	5.81	−6.57	−9.39	−8.03
2007	4.55	15.71	10.54	8.96	8.32	7.89	−11.33	−8.74	−9.64
2008	−7.27	−11.10	−10.45	11.14	15.87	11.95	−14.51	−30.98	−22.86
2009	−4.76	−11.07	−5.80	8.93	12.84	10.36	−12.75	−29.27	−19.29
2010	6.90	5.47	5.35	9.60	9.44	9.10	−10.47	−9.29	−9.73
2011	−6.51	−14.83	−9.28	9.93	10.17	9.15	−17.54	−20.06	−15.99
2012	−4.27	−2.65	−5.08	5.69	6.06	5.07	−7.26	−8.74	−8.68
2013	−0.71	−2.26	−1.26	6.59	6.14	5.78	−9.38	−6.92	−8.17
2014	−5.63	−6.61	−7.02	5.32	5.62	4.62	−10.25	−12.36	−11.57
2015	4.80	17.54	13.15	6.61	7.56	6.65	−7.98	−5.36	−6.18
2016	−6.41	−5.47	−4.39	7.99	8.65	7.21	−11.27	−13.62	−9.37
2017	1.97	7.60	6.54	5.87	7.81	6.99	−6.47	−4.52	−5.32
2018	−2.23	−11.10	−4.24	5.35	6.50	5.64	−9.07	−15.59	−11.33

Table 31: Annual statistics of the time-series momentum risk factor (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−0.49	10.17	7.59	3.54	2.78	2.57	−3.18	−5.89	−3.37
2001	−3.45	7.08	2.65	5.22	4.75	4.55	−5.54	−4.34	−3.40
2002	19.38	13.95	16.65	6.29	6.93	6.31	−5.03	−9.89	−7.96
2003	16.33	28.48	23.21	7.23	8.57	7.11	−6.42	−4.07	−3.79
2004	4.29	21.45	13.09	8.52	8.14	7.74	−11.21	−6.47	−6.71
2005	−8.31	0.80	−2.40	5.04	7.55	6.45	−10.61	−7.02	−7.32
2006	2.82	5.33	6.67	4.89	6.43	5.46	−3.53	−8.30	−5.59
2007	2.51	11.81	7.35	7.48	8.14	7.23	−7.82	−8.62	−8.08
2008	10.10	19.35	14.37	8.74	10.23	9.05	−5.07	−8.78	−7.82
2009	2.14	8.76	10.09	7.66	7.26	6.56	−10.20	−4.44	−4.57
2010	−3.35	−6.08	−4.32	7.51	8.29	7.65	−10.97	−9.79	−9.76
2011	0.15	−4.41	−1.48	7.73	7.95	6.98	−13.96	−12.25	−10.53
2012	−2.93	−6.18	−4.58	4.83	4.68	4.69	−7.11	−9.61	−9.15
2013	−0.67	−2.63	−2.38	5.39	4.42	4.41	−7.81	−6.74	−6.84
2014	−0.26	−8.77	−5.41	4.07	4.73	3.81	−5.62	−9.90	−8.01
2015	−4.85	8.50	4.05	4.07	5.51	4.07	−10.52	−6.10	−4.68
2016	−6.47	0.46	−2.30	5.78	7.46	6.10	−6.78	−7.73	−6.18
2017	−0.78	11.84	6.36	4.38	6.46	5.53	−6.69	−4.08	−5.21
2018	−5.54	−1.65	−3.18	3.61	5.04	4.17	−9.19	−6.50	−7.50

Table 32: Return decomposition (in %) between common and idiosyncratic risk factors (weekly returns, static estimation)

Currency	One-factor				Four-factor	
	Carry	Value	CS-MOM	TS-MOM	Systematic	Specific
AUD	48.31	2.47	2.94	17.46	65.99	34.01
NZD	45.05	3.89	0.68	20.55	63.82	36.18
USD	6.49	2.64	1.45	26.13	44.48	55.52
NOK	9.67	4.36	1.09	22.75	43.94	56.06
JPY	22.50	10.73	2.17	12.93	41.58	58.42
TRY	26.35	16.73	8.58	8.86	41.28	58.72
ZAR	24.76	6.74	2.19	15.98	40.63	59.37
BRL	24.76	16.40	5.84	13.35	39.57	60.43
MXN	31.59	14.45	6.22	7.13	39.09	60.91
PLN	9.17	4.80	2.72	19.68	39.01	60.99
CZK	5.05	0.34	2.05	19.49	38.02	61.98
EUR	6.49	1.32	1.08	20.09	37.34	62.66
DKK	6.52	1.37	1.03	20.11	37.27	62.73
SEK	6.24	0.74	1.44	20.97	36.78	63.22
CLP	21.18	12.27	5.01	13.43	36.71	63.29
LTL	2.90	0.17	1.27	22.86	36.35	63.65
CAD	21.49	0.67	1.93	11.19	34.79	65.21
BGN	5.24	0.52	1.11	17.78	34.50	65.50
CHF	8.85	2.62	1.74	18.12	34.42	65.58
HUF	5.74	3.42	2.08	18.77	33.44	66.56
LVL	3.93	0.06	0.94	17.13	32.52	67.48
SGD	6.71	1.81	0.62	19.05	31.22	68.78
GBP	11.78	1.31	2.64	10.23	30.83	69.17
KRW	16.51	7.36	3.66	8.02	30.10	69.90
RON	2.59	0.66	0.06	16.98	26.09	73.91
MYR	14.14	7.74	1.73	8.63	25.30	74.70
RUB	11.51	7.42	3.80	7.39	22.19	77.81
INR	12.09	4.96	1.59	8.57	21.46	78.54
TWD	4.13	3.92	1.72	9.82	18.63	81.37
COP	12.91	7.29	1.76	4.08	17.73	82.27
PHP	7.60	2.52	0.65	5.00	13.78	86.22
ILS	3.97	1.58	1.24	5.69	12.58	87.42
IDR	4.74	3.06	0.47	6.15	12.55	87.45
THB	2.72	1.36	0.33	7.06	12.47	87.53
PEN	7.18	5.97	0.85	2.52	10.72	89.28
HKD	2.85	0.52	0.41	4.75	8.01	91.99
CNY	0.54	0.32	0.14	2.51	4.38	95.62
ARS	0.75	0.68	0.39	0.23	1.59	98.41
BHD	0.18	0.40	0.24	0.26	0.99	99.01
SAR	0.26	0.28	0.04	0.33	0.95	99.05
average	11.64	4.15	1.90	12.30	28.83	71.17

Table 33: Exchange rate arrangement over the period 2000 – 2016

Currency	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	Average
ARS	2	2	14	8	8	8	8	8	8	8	8	8	8	8	8	8	14	8.0
AUD	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13.0
BGN	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2.0
BHD	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2.0
BRL	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12.0
CAD	8	8	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	11.5
CHF	11	11	11	11	11	11	11	11	11	11	11	11	2	2	2	11	11	9.4
CLP	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12.0
CNY	4	4	4	4	4	4	5	5	5	5	5	5	5	5	7	7	7	5.0
COP	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12.0
CZK	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8.0
DKK	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2.0
EUR	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13.0
GBP	12	11	11	11	11	11	11	11	11	13	13	13	13	13	13	13	13	12.0
HKD	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2.0
HUF	9	9	9	10	10	10	10	10	10	8	8	8	8	8	8	8	8	8.9
IDR	12	12	12	12	12	12	12	8	8	8	8	8	8	8	8	8	10	9.8
ILS	9	9	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10	9.7
INR	7	7	7	7	7	7	8	8	8	10	10	10	10	8	8	8	8	8.1
JPY	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13.0
KRW	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	12	12	11.1
LTL	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	13	13	3.3
LVL	10	10	8	8	8	11	11	11	11	11	2	2	2	2	13	13	13	8.6
MXN	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12.0
MYR	2	2	2	2	2	2	11	11	11	11	11	11	11	11	11	11	12	7.9
NOK	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11.0
NZD	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12.0
PEN	8	8	8	11	11	11	11	11	11	11	11	11	8	8	8	8	8	9.6
PHP	8	8	8	8	8	10	10	10	10	10	10	10	10	10	10	10	10	9.4
PLN	12	12	12	12	12	12	12	12	12	12	12	12	6	6	6	6	6	10.2
RON	14	12	12	12	12	12	12	7	7	7	7	7	7	4	4	4	4	8.5
RUB	8	8	8	8	8	8	8	8	8	10	10	10	10	10	10	14	14	9.4
SAR	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4.0
SEK	6	6	6	6	6	6	6	6	6	11	11	11	11	11	11	11	11	8.4
SGD	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11.0
THB	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11.0
TWD																		
TRY	10	14	14	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12.1
USD	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13.0
ZAR	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12.0

Table 34: Weekly correlation between currency returns and the time-series momentum

Currency	G10	EM	G25
USD	−46.39	−36.62	−48.04
LTL	43.29	36.19	45.79
NOK	43.95	35.41	45.28
DKK	40.54	35.71	43.58
EUR	40.50	35.67	43.55
CZK	37.03	36.32	43.45
PLN	32.67	37.59	43.40
SEK	42.11	32.87	43.00
HUF	34.48	37.20	42.95
BGN	37.03	32.82	40.83
RON	37.14	33.03	40.13
LVL	37.09	31.36	39.69
ZAR	26.57	34.51	38.61
AUD	36.81	27.71	38.18
SGD	38.30	25.93	38.15
CHF	39.43	26.56	37.83
NZD	41.92	24.07	37.66
GBP	27.82	26.94	31.62
CAD	30.77	23.97	31.39
BRL	13.15	29.19	30.23
CLP	17.47	23.19	29.46
TWD	23.11	23.04	29.22
TRY	15.45	23.39	26.48
THB	24.12	19.90	25.36
JPY	34.56	13.74	25.17
MYR	21.49	17.08	24.94
INR	23.26	16.57	24.90
RUB	19.54	17.77	24.11
KRW	17.87	15.74	22.95
ILS	17.31	18.76	22.74
PHP	18.51	14.55	20.20
HKD	20.59	15.30	20.17
IDR	17.14	12.22	19.51
MXN	11.09	10.89	17.52
COP	11.01	12.90	16.77
CNY	13.89	9.63	13.97
PEN	4.42	12.11	12.32
BHD	−5.05	−2.62	−4.02
SAR	5.50	1.99	3.35
ARS	−0.62	3.49	2.30

Table 35: Weekly correlation between currency returns and the cross-section momentum

Currency	G10	EM	G25
TRY	−8.48	−25.64	−26.55
BRL	−13.07	−15.63	−22.28
MXN	−12.70	−12.24	−20.82
RUB	−4.37	−18.34	−17.07
CLP	−11.90	−7.85	−17.02
ZAR	−10.26	−9.87	−14.26
JPY	11.06	8.30	13.61
COP	−7.78	−10.34	−13.03
KRW	−15.58	−2.85	−12.87
MYR	−7.21	−8.17	−12.00
AUD	−16.27	−2.16	−10.73
CHF	3.15	13.19	10.59
PLN	−13.55	0.50	−8.69
TWD	−7.72	−2.00	−8.45
PEN	−6.51	−4.90	−8.33
INR	−2.80	−4.60	−8.29
PHP	−3.12	−7.33	−7.62
GBP	−15.41	0.59	−7.18
CAD	−13.20	0.43	−6.22
IDR	−6.01	−4.73	−5.95
ILS	−8.82	0.47	−5.77
HUF	−10.69	2.56	−5.73
THB	−4.60	−3.81	−5.52
SGD	−7.07	−1.26	−5.28
LTL	−4.15	8.94	4.15
SEK	−10.43	2.47	−3.84
NZD	−7.15	0.75	−3.79
BHD	−4.76	−1.14	−3.39
NOK	−9.14	2.25	−2.96
CNY	−2.50	−0.72	−2.60
CZK	−10.82	5.29	−2.60
RON	0.02	2.28	1.80
ARS	−3.15	1.78	−1.71
BGN	−7.96	4.22	−1.37
USD	9.34	−4.70	1.35
LVL	−7.03	4.11	−1.16
DKK	−6.65	5.47	1.13
SAR	1.78	0.13	1.11
EUR	−6.94	5.36	1.05
HKD	3.21	−1.17	−0.95

Table 36: Annual statistics of the currency ARP portfolio (in %)

Year	Performance			Volatility			Drawdown		
	G10	EM	G25	G10	EM	G25	G10	EM	G25
2000	−0.58	22.11	14.80	3.02	2.20	2.67	−3.83	−0.92	−1.38
2001	0.88	17.98	7.79	3.46	3.48	2.72	−3.61	−1.38	−1.72
2002	9.18	14.75	9.56	3.47	4.28	2.97	−3.37	−2.48	−2.31
2003	7.55	17.94	9.61	4.19	4.83	3.34	−3.06	−2.12	−1.57
2004	1.93	7.40	7.03	4.28	3.64	2.74	−6.30	−2.87	−2.70
2005	2.53	3.79	6.06	2.87	3.90	3.44	−2.24	−3.34	−2.77
2006	2.64	3.82	4.27	2.35	3.19	2.53	−2.55	−3.09	−2.69
2007	1.73	7.08	4.50	5.47	4.51	4.44	−6.99	−4.28	−5.12
2008	3.82	13.51	10.57	3.19	4.56	3.92	−2.54	−2.99	−3.09
2009	7.02	7.63	6.55	5.16	3.57	3.63	−4.19	−3.92	−2.04
2010	−1.92	0.13	−0.82	4.64	3.17	3.64	−5.61	−2.38	−3.55
2011	−0.14	−3.53	−2.02	3.98	2.90	2.66	−5.98	−4.53	−4.03
2012	2.43	−0.70	1.72	2.64	2.03	2.20	−2.93	−2.50	−2.76
2013	1.88	−1.84	0.95	2.98	1.81	2.26	−3.30	−3.27	−2.59
2014	1.85	−0.17	−0.17	1.93	2.90	2.18	−2.36	−2.72	−2.53
2015	−0.97	−0.39	−1.28	2.60	3.72	2.88	−3.66	−5.56	−5.16
2016	−0.67	7.95	4.19	2.56	5.15	4.04	−3.00	−3.66	−3.41
2017	−0.71	3.90	2.71	1.92	3.49	2.87	−2.40	−2.52	−2.51
2018	−3.07	−2.20	−3.09	1.66	2.89	2.29	−3.87	−3.57	−4.07

A.3.2 Figures

Figure 29: Weekly payoff between G10 and EM carry risk factors

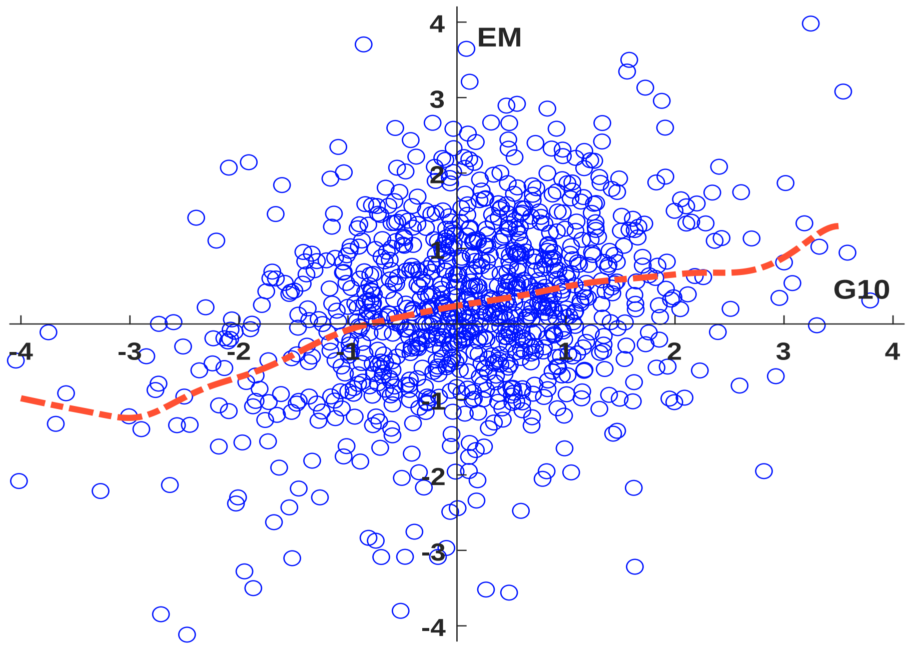


Figure 30: Weekly payoff between G10 and EM PPP value risk factors

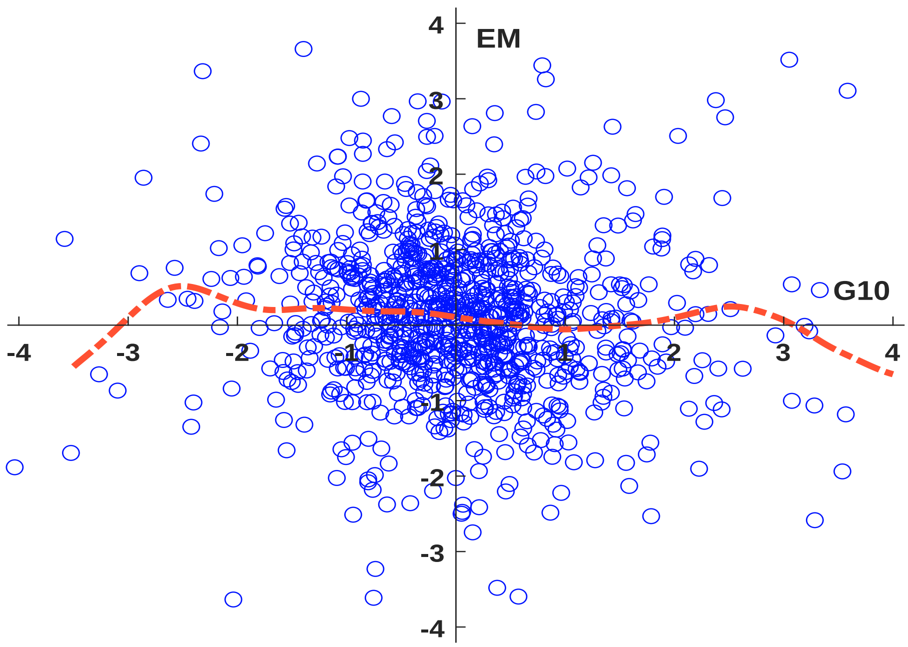


Figure 31: Weekly payoff between G10 and EM BEER value risk factors

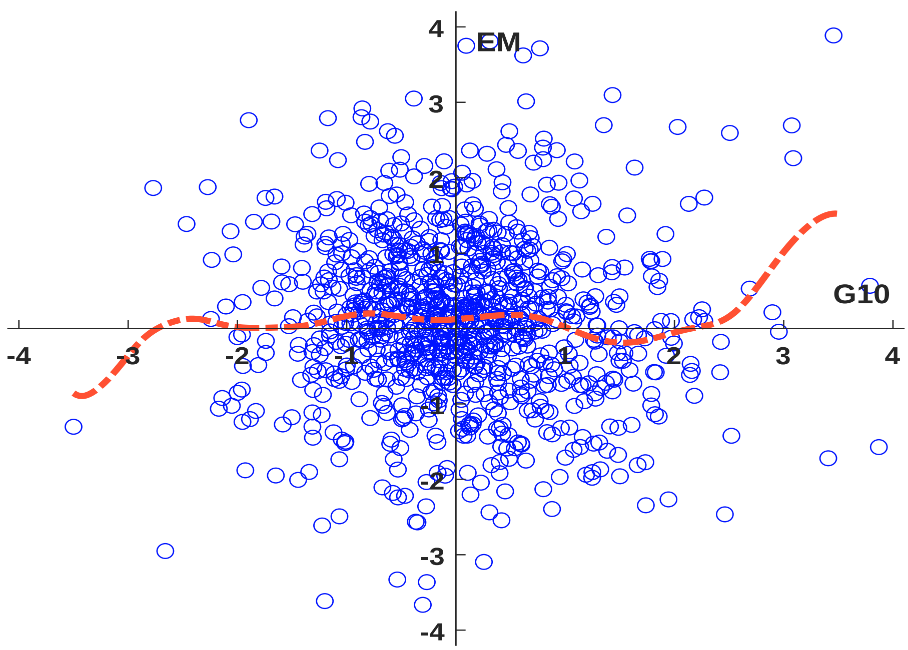


Figure 32: Weekly payoff between G10 and EM NATREX value risk factors

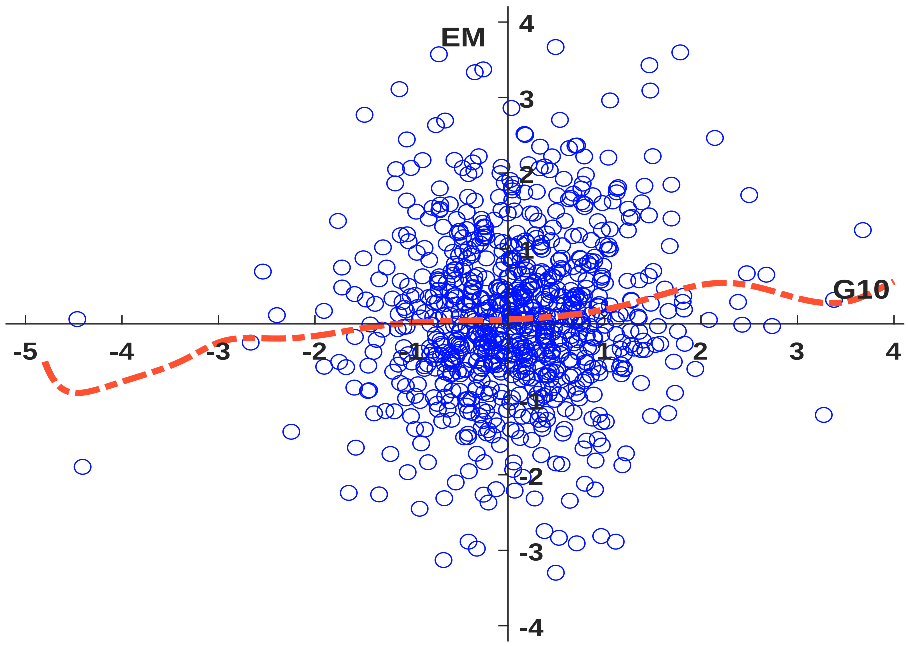


Figure 33: Weekly payoff between G10 and EM value risk factors

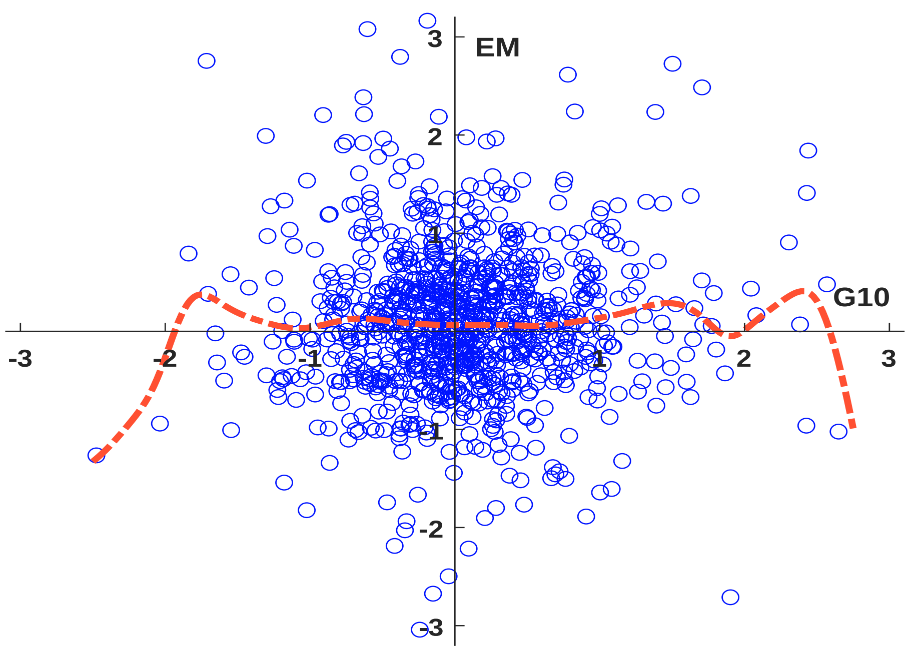


Figure 34: Weekly payoff between G10 and EM CS-MOM risk factors

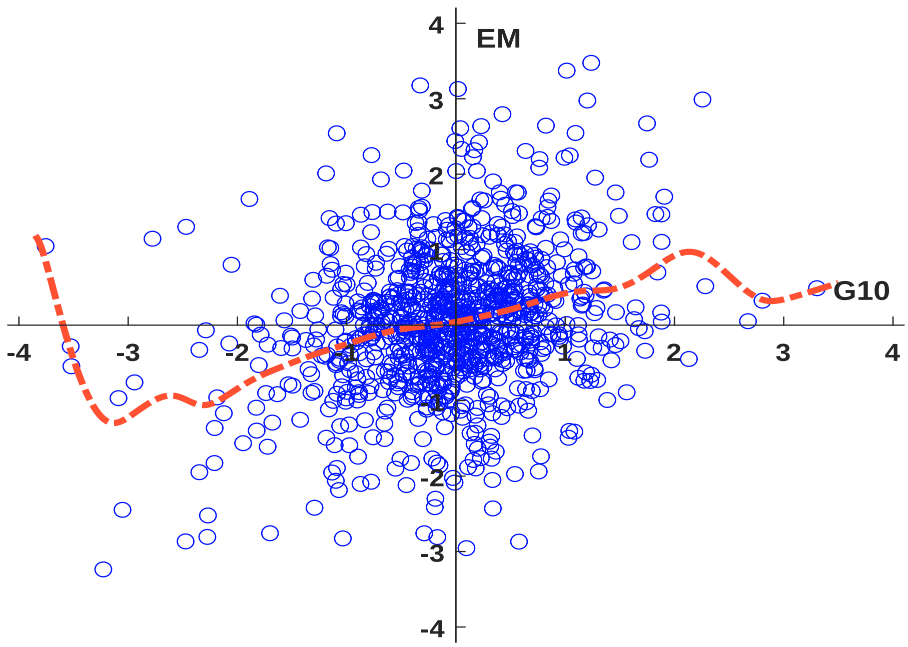


Figure 35: Weekly payoff between G10 and EM TS-MOM risk factors

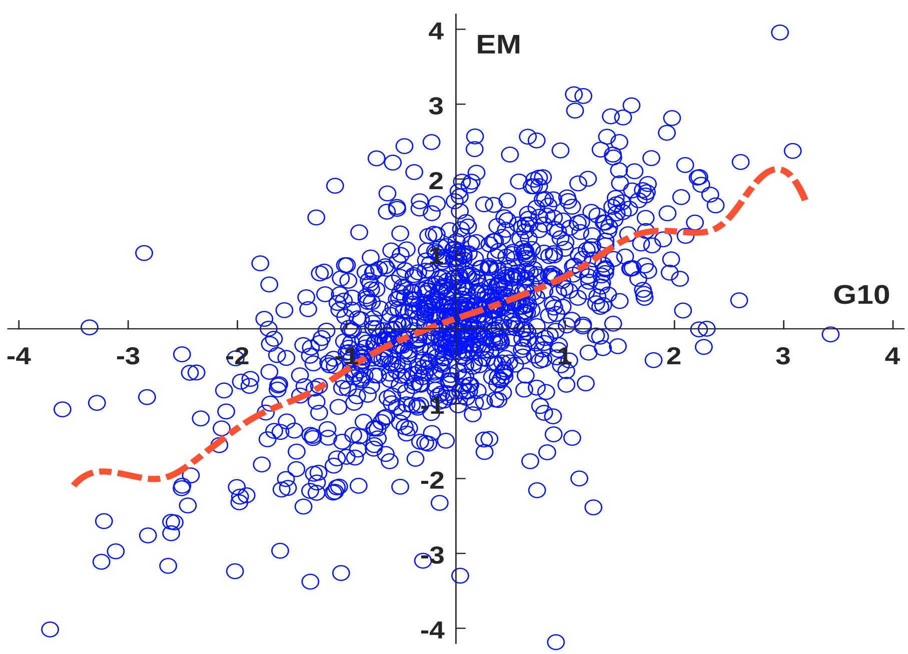


Figure 36: KF estimate $\hat{\beta}^{j}_{i,t}$ (ARS)

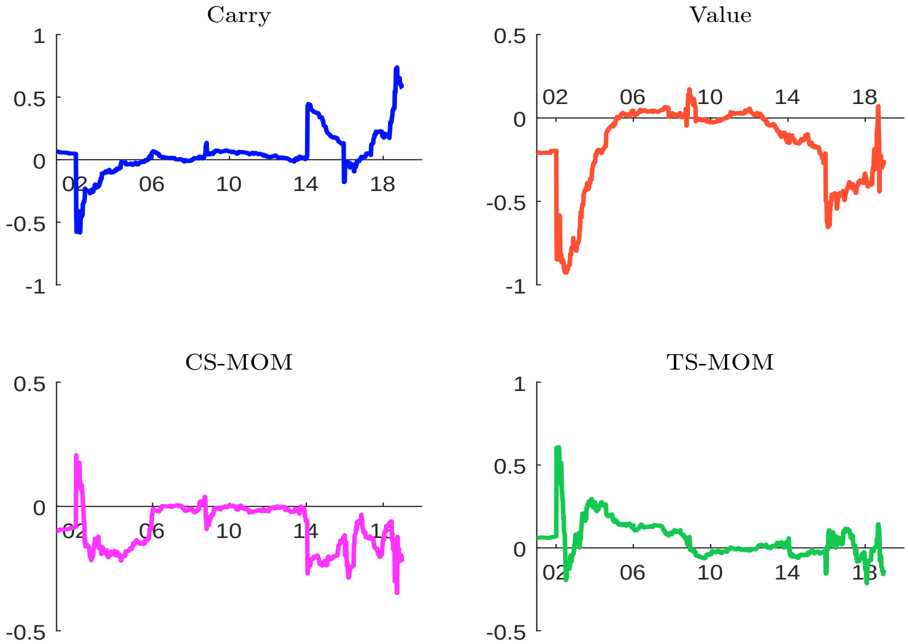


Figure 37: KF estimate $\hat{\beta}^{j}_{i,t}$ (AUD)

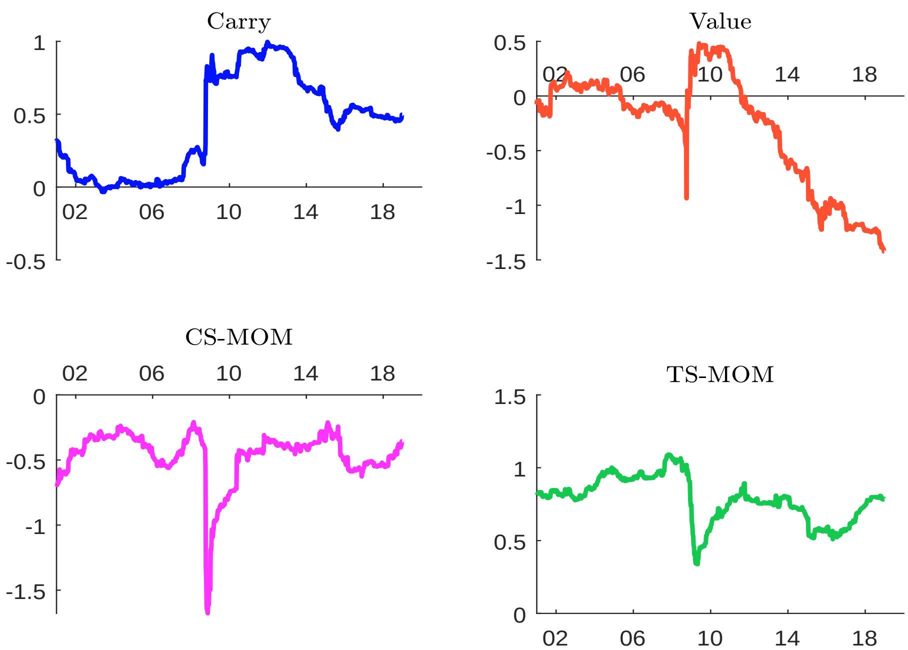


Figure 38: KF estimate $\hat{\beta}^{j}_{i,t}$ (BGN)

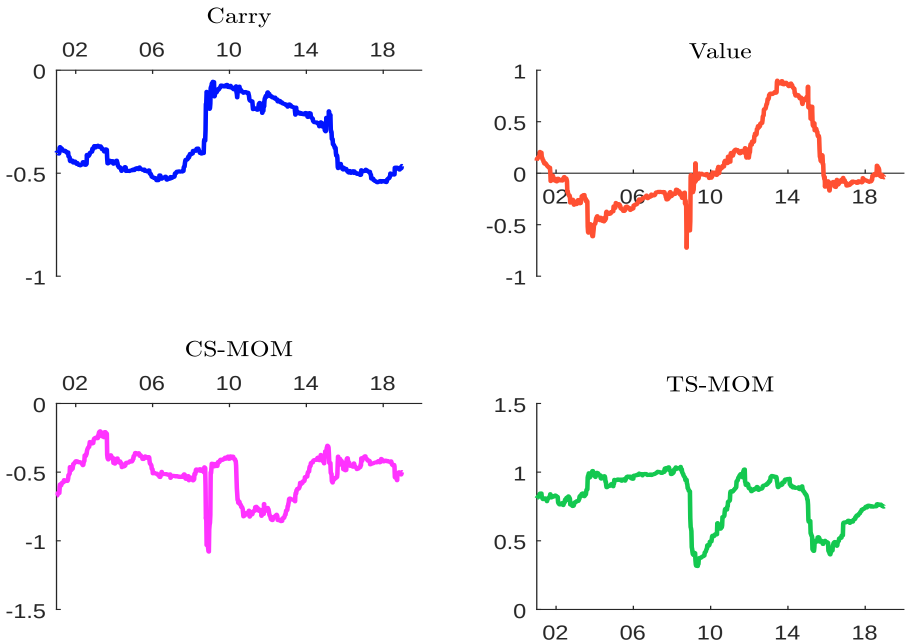


Figure 39: KF estimate $\hat{\beta}^{j}_{i,t}$ (BHD)

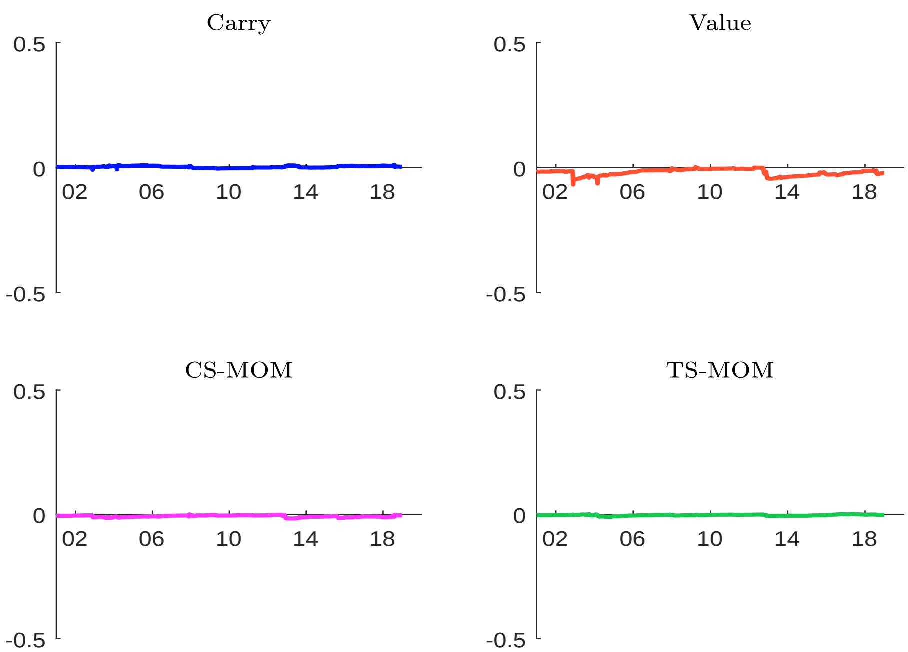


Figure 40: KF estimate $\hat{\beta}^{j}_{i,t}$ (BRL)

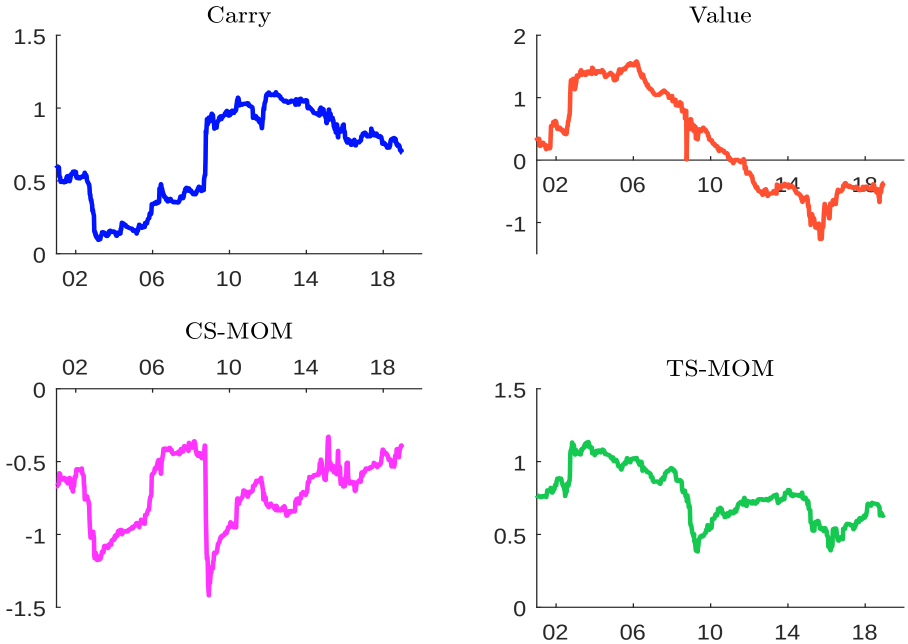


Figure 41: KF estimate $\hat{\beta}^{j}_{i,t}$ (CAD)

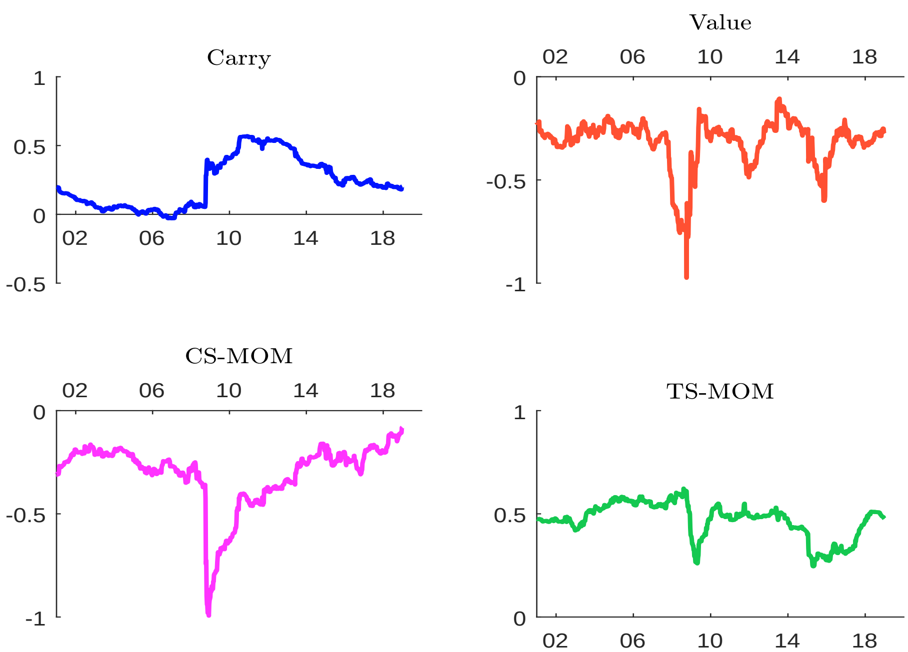


Figure 42: KF estimate $\hat{\beta}^{j}_{i,t}$ (CHF)

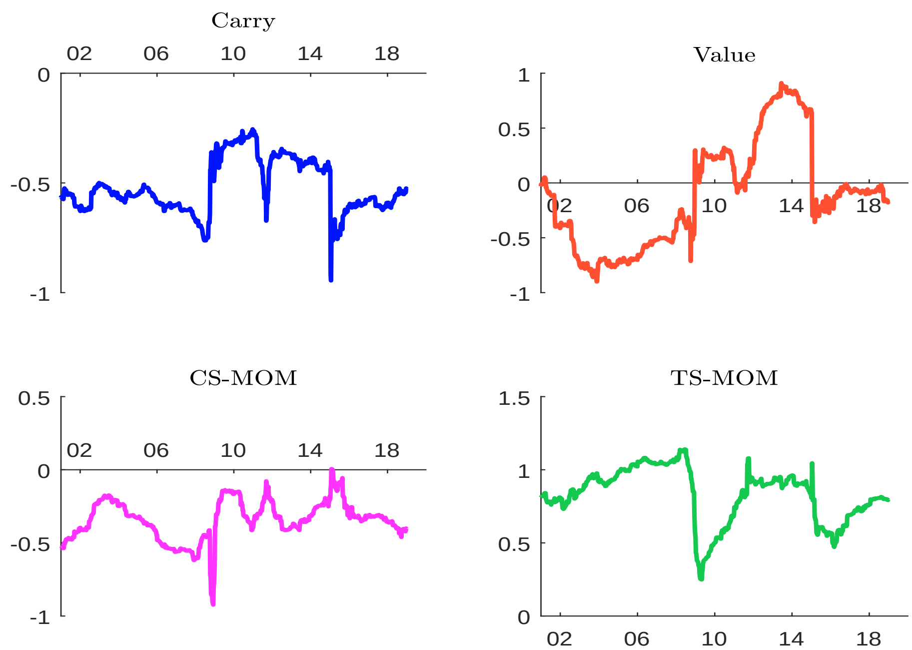


Figure 43: KF estimate $\hat{\beta}^{j}_{i,t}$ (CLP)

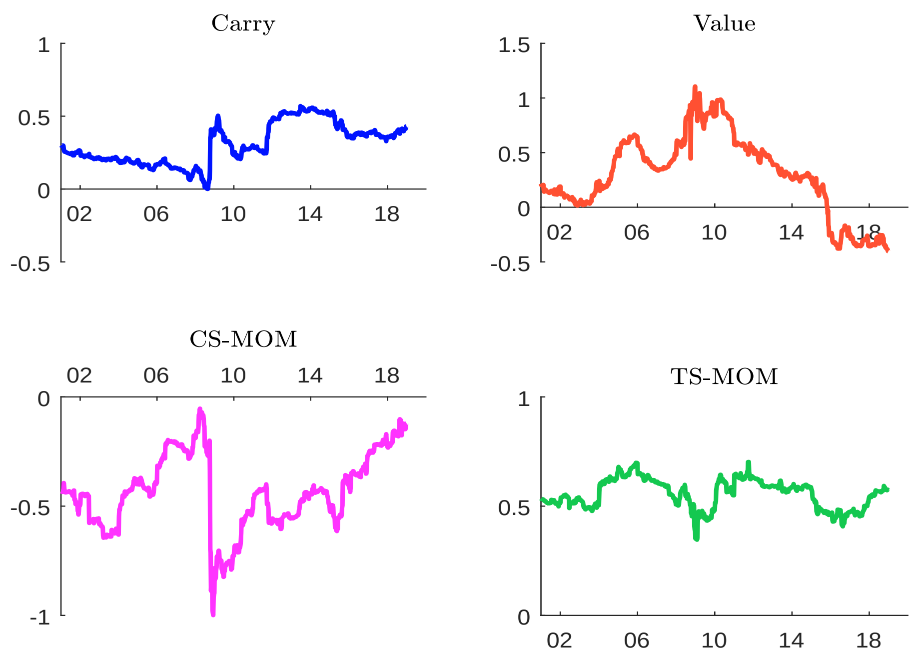


Figure 44: KF estimate $\hat{\beta}^{j}_{i,t}$ (CNY)

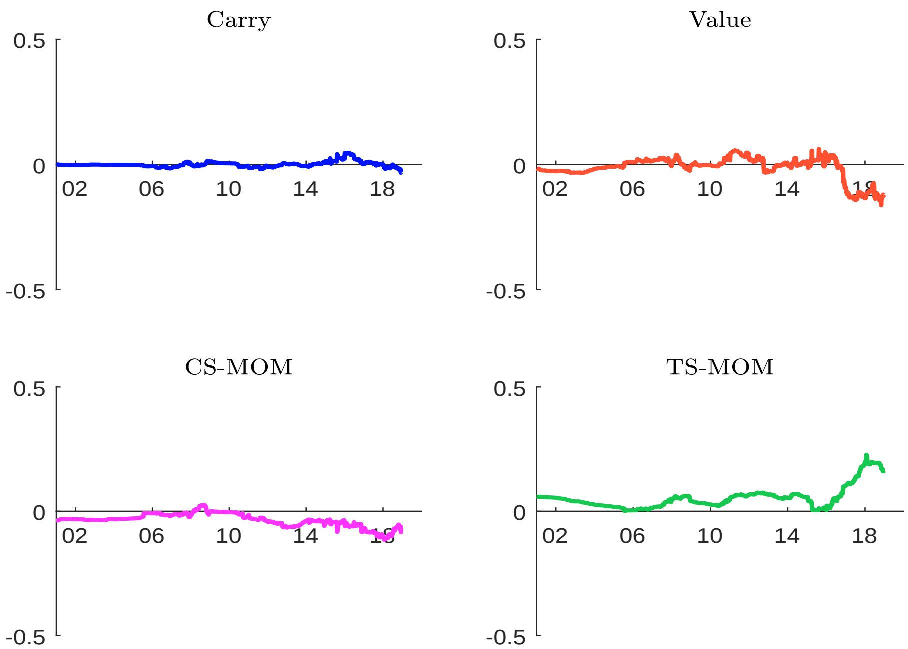


Figure 45: KF estimate $\hat{\beta}^{j}_{i,t}$ (COP)

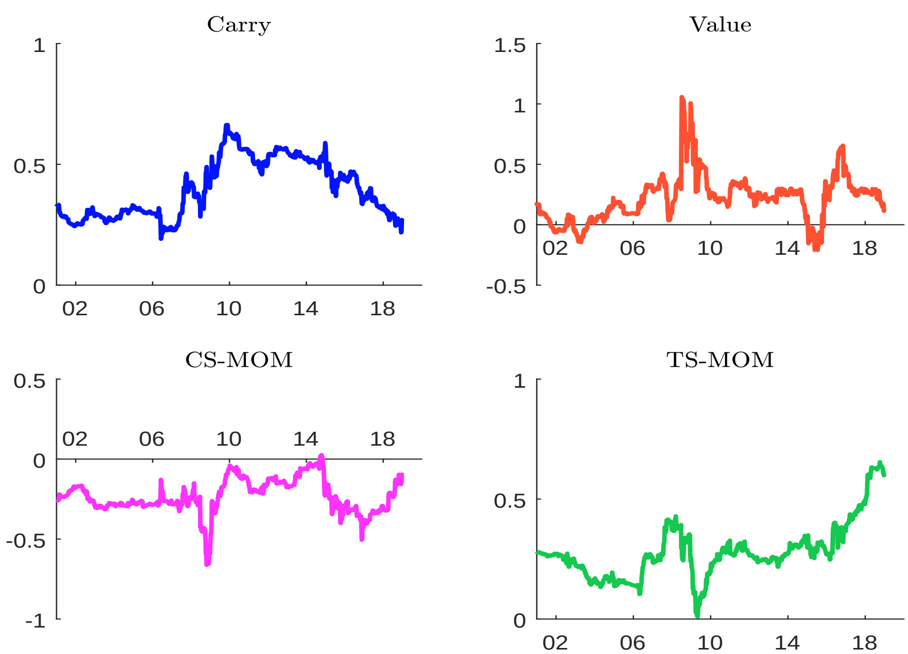


Figure 46: KF estimate $\hat{\beta}^{j}_{i,t}$ (CZK)

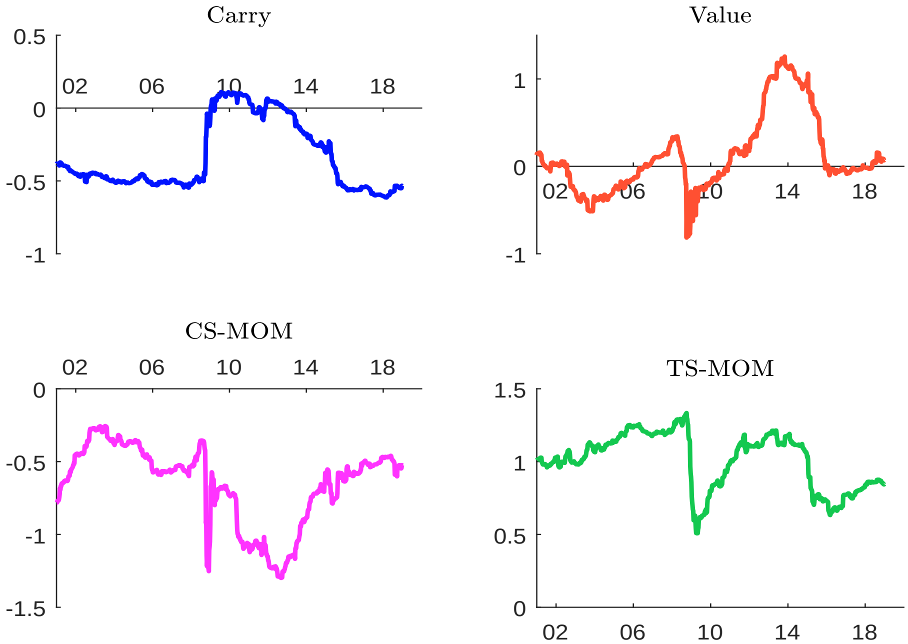


Figure 47: KF estimate $\hat{\beta}^{j}_{i,t}$ (DKK)

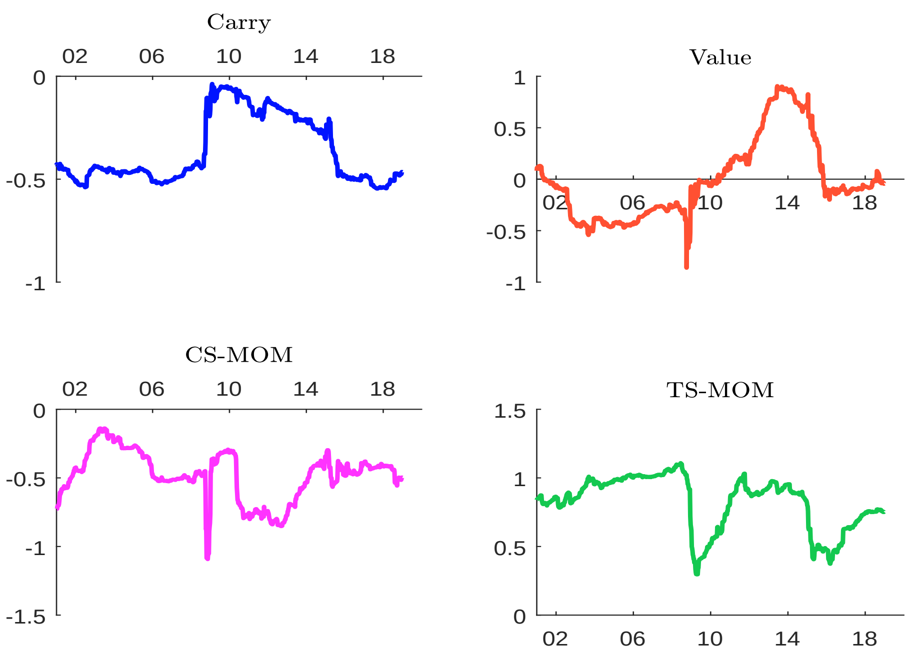


Figure 48: KF estimate $\hat{\beta}_{i,t}^{j}$ (EUR)

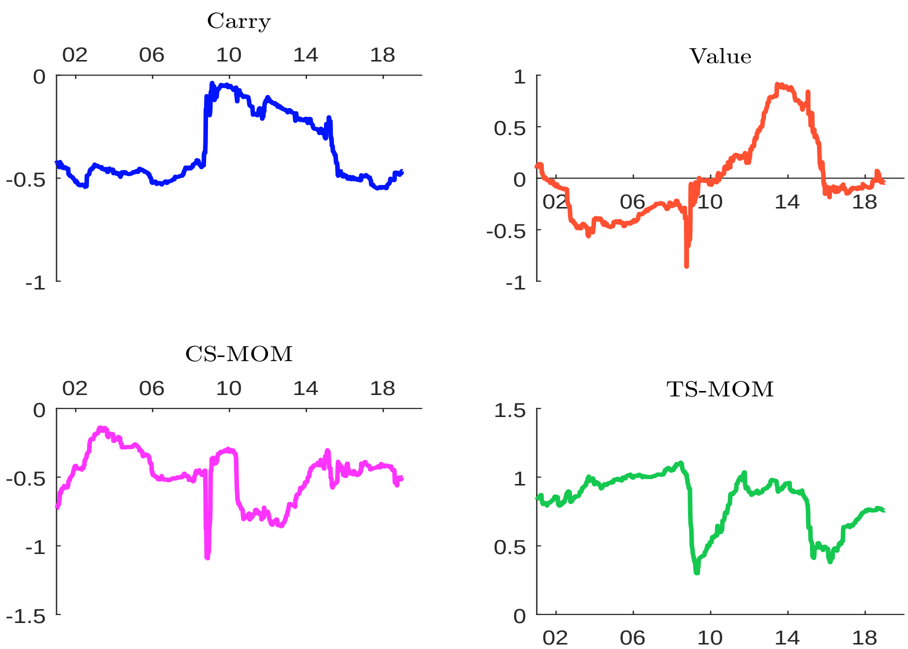


Figure 49: KF estimate $\hat{\beta}_{i,t}^{j}$ (GBP)

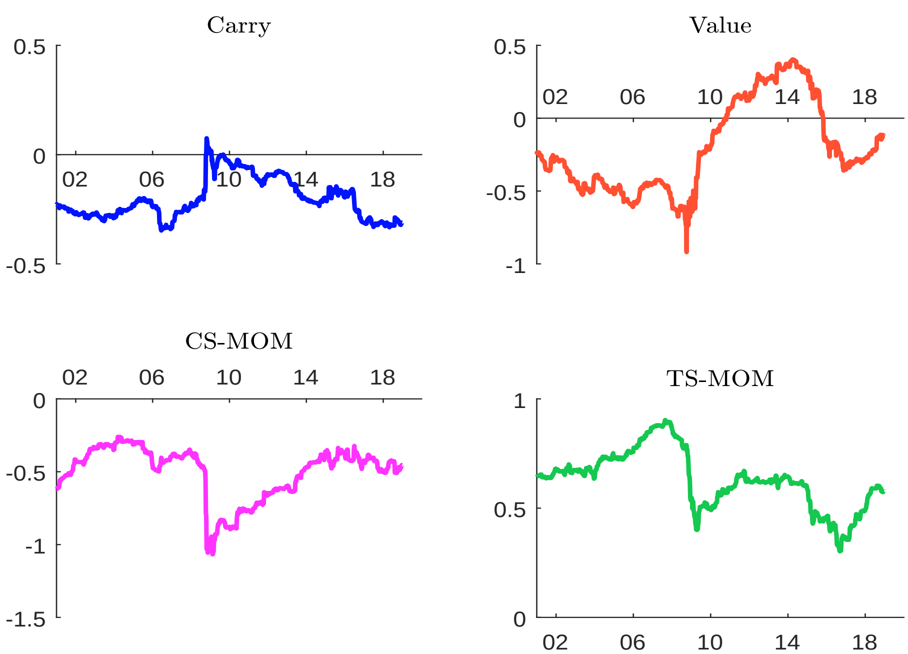


Figure 50: KF estimate $\hat{\beta}^{j}_{i,t}$ (HKD)

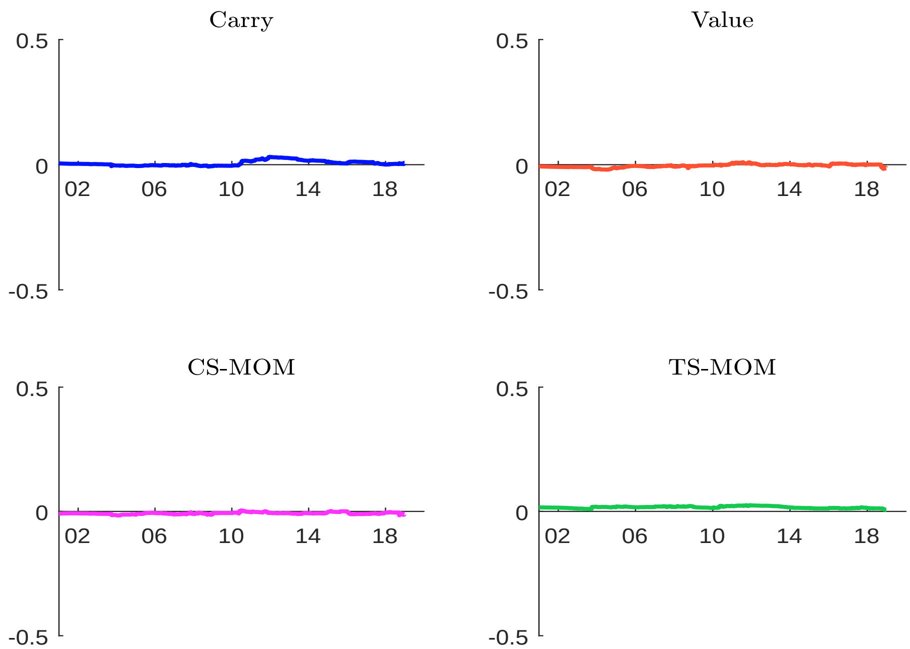


Figure 51: KF estimate $\hat{\beta}^{j}_{i,t}$ (HUF)

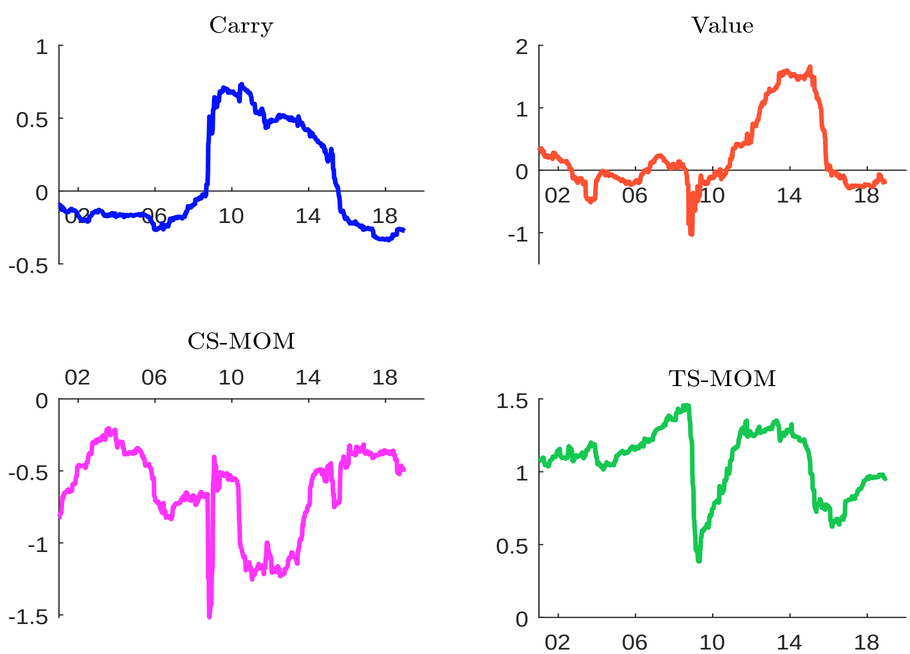


Figure 52: KF estimate $\hat{\beta}^{j}_{i,t}$ (IDR)

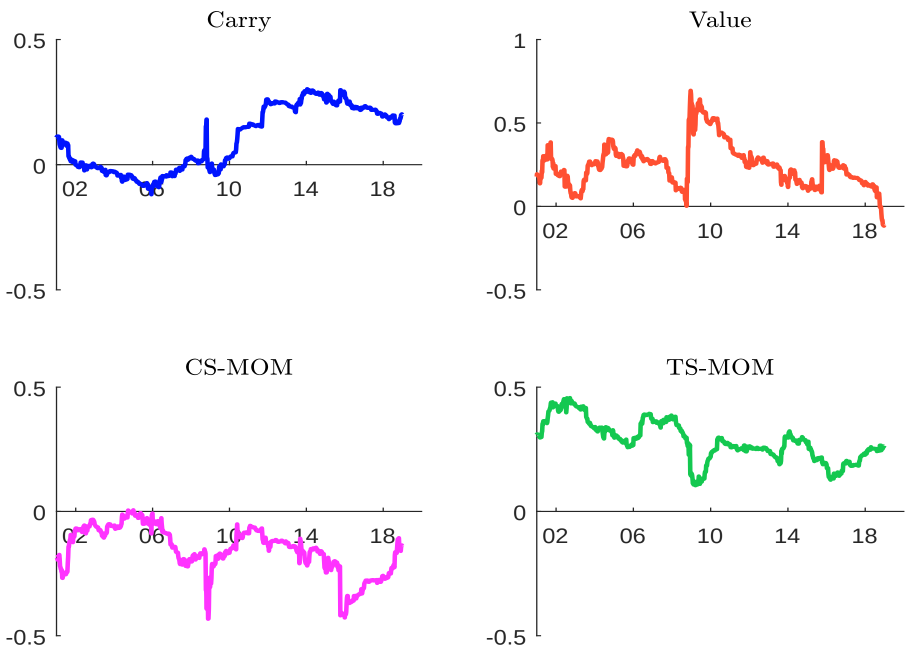


Figure 53: KF estimate $\hat{\beta}^{j}_{i,t}$ (ILS)

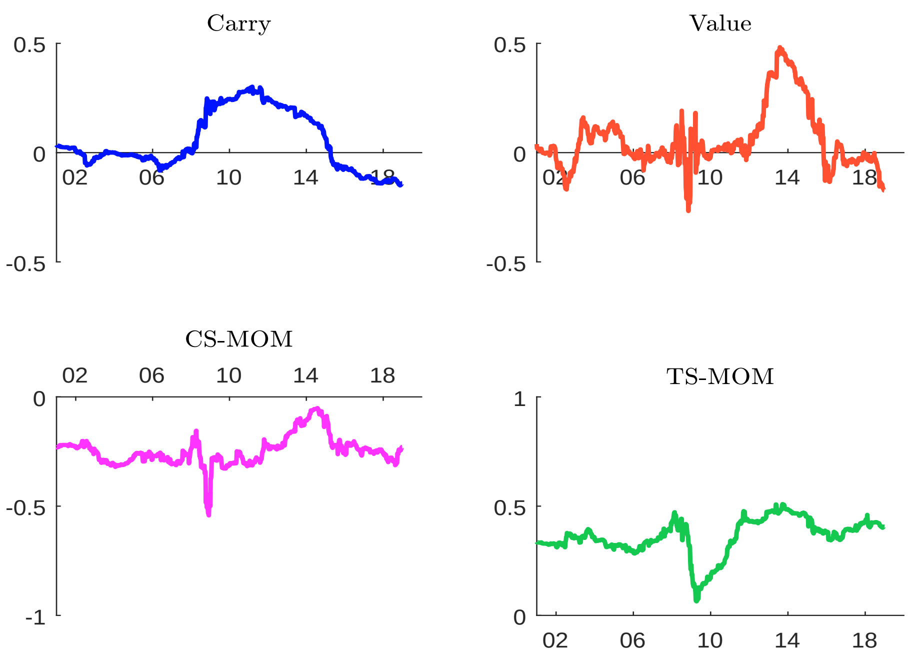


Figure 54: KF estimate $\hat{\beta}^{j}_{i,t}$ (INR)

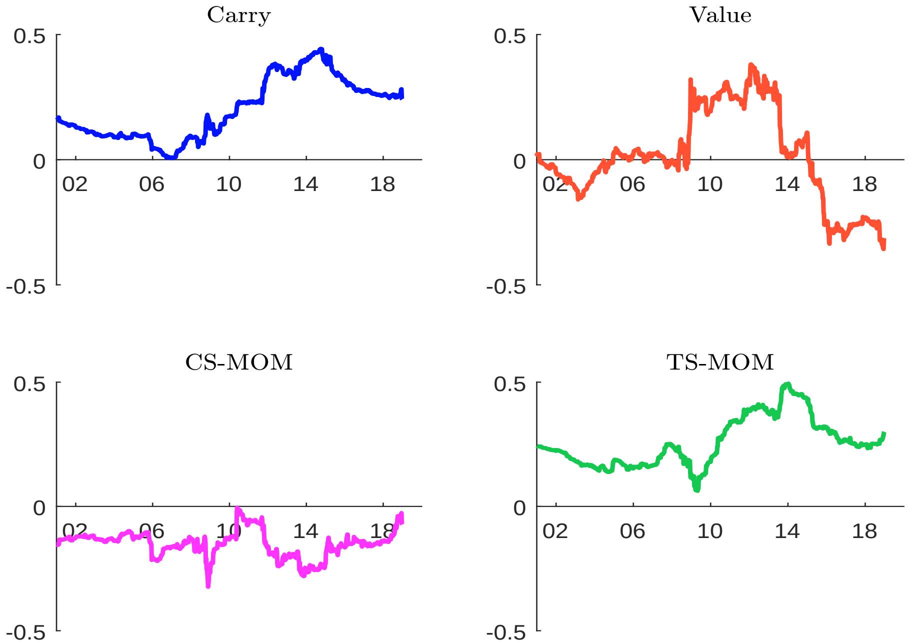


Figure 55: KF estimate $\hat{\beta}^{j}_{i,t}$ (JPY)

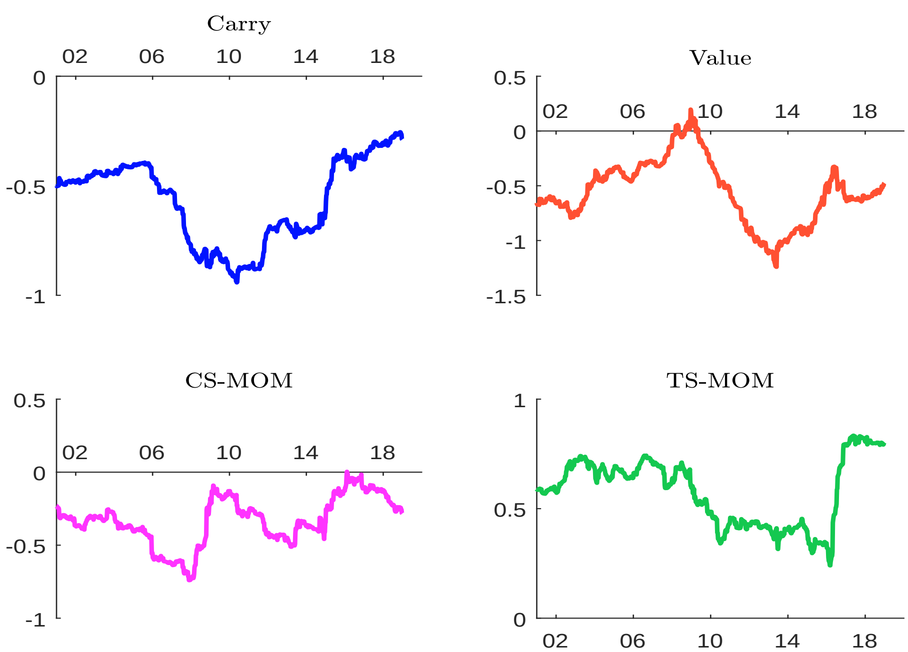


Figure 56: KF estimate $\hat{\beta}^j_{i,t}$ (KRW)

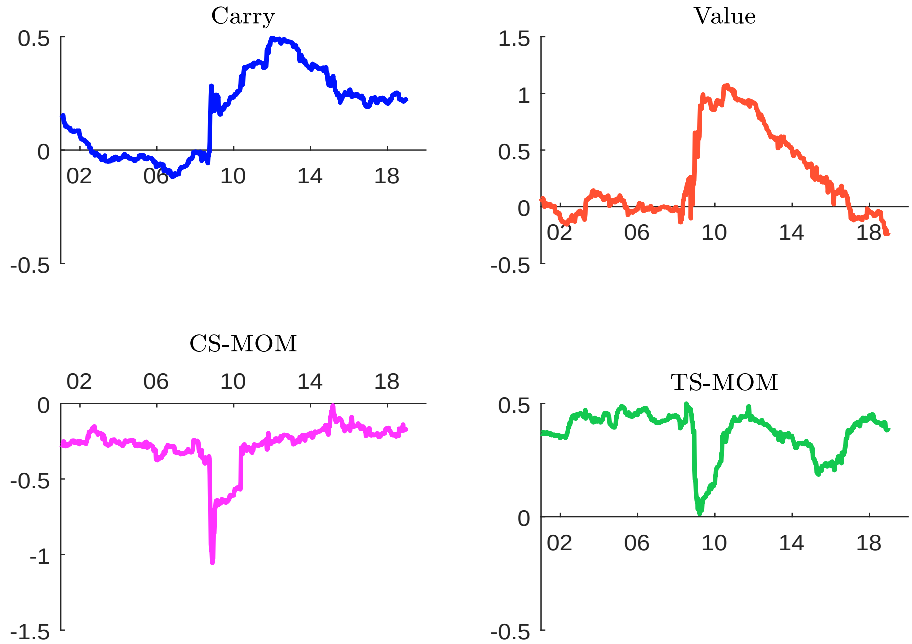


Figure 57: KF estimate $\hat{\beta}^j_{i,t}$ (LTL)

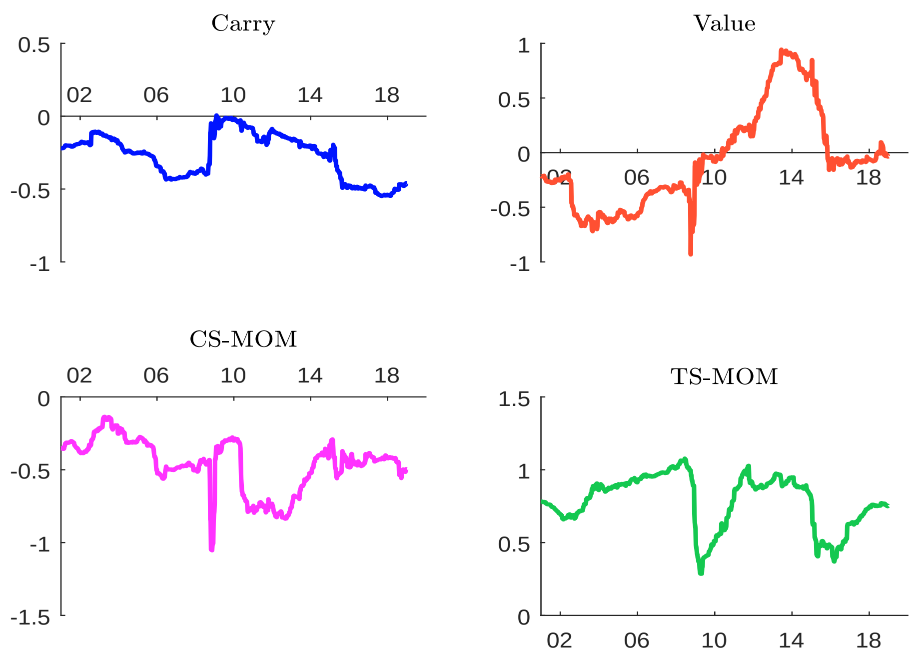


Figure 58: KF estimate $\hat{\beta}^{j}_{i,t}$ (LVL)

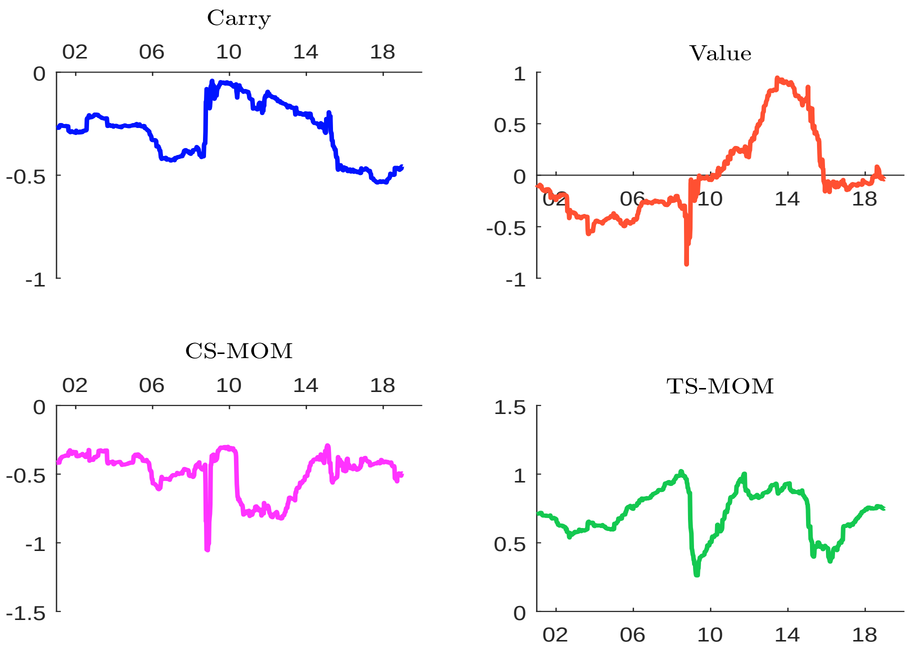


Figure 59: KF estimate $\hat{\beta}^{j}_{i,t}$ (MXN)

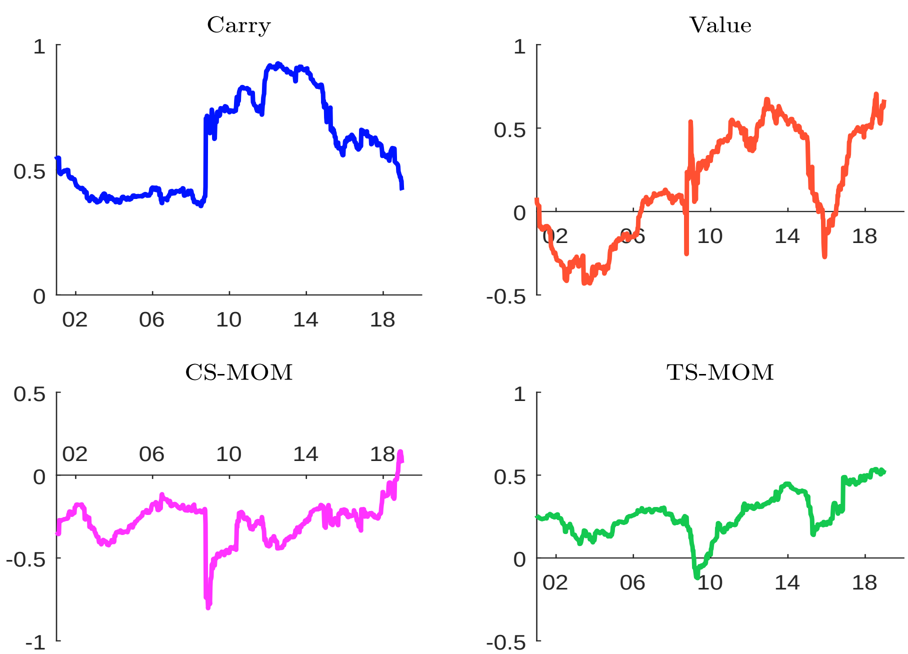


Figure 60: KF estimate $\hat{\beta}^{j}_{i,t}$ (MYR)

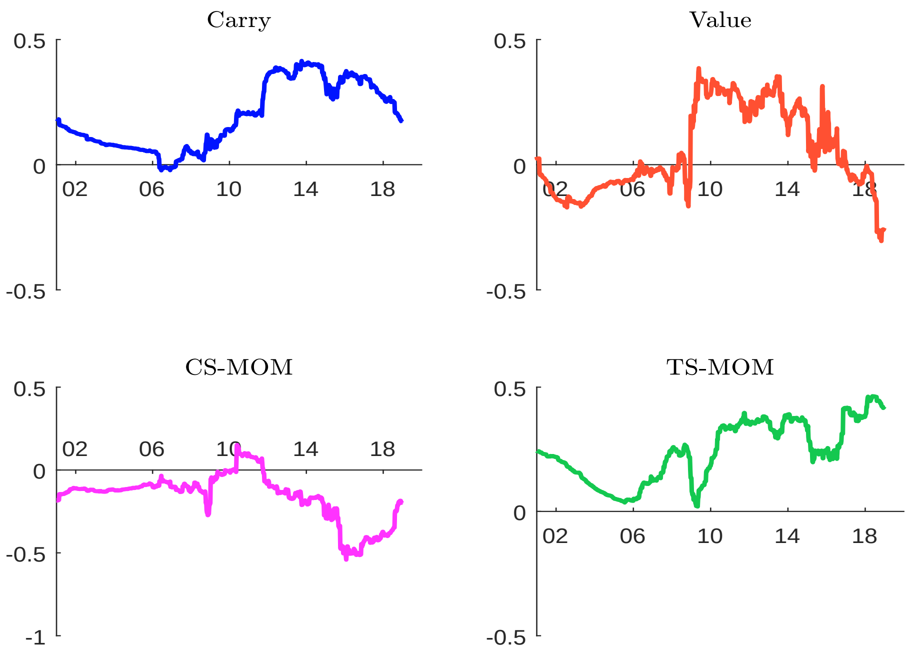


Figure 61: KF estimate $\hat{\beta}^{j}_{i,t}$ (NOK)

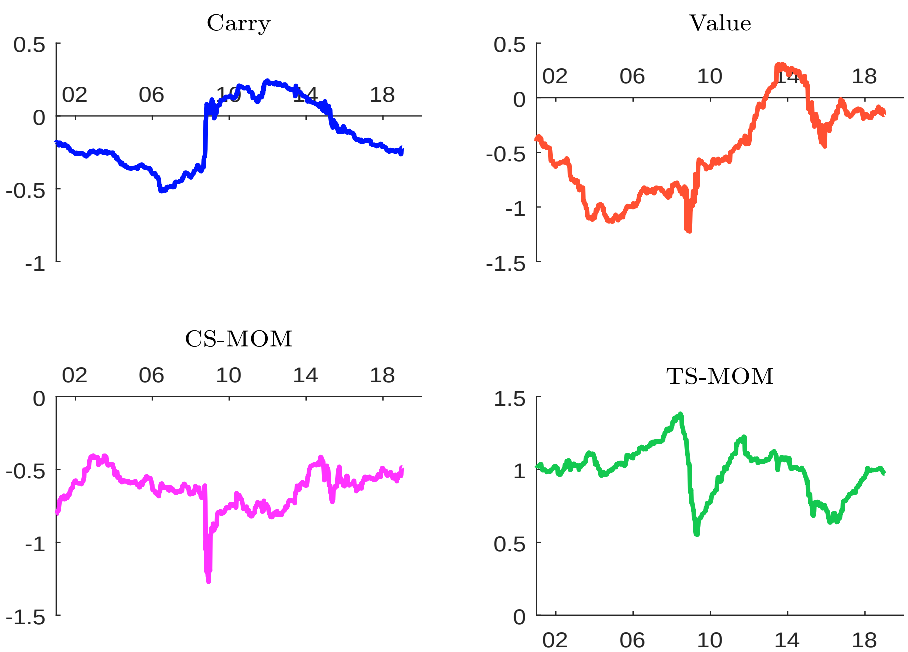


Figure 62: KF estimate $\hat{\beta}^{j}_{i,t}$ (NZD)

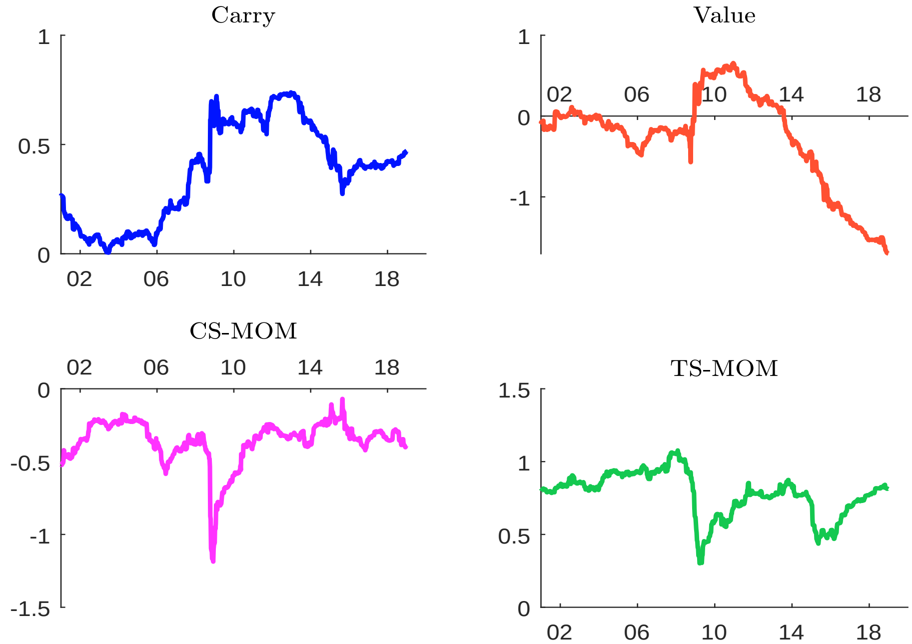


Figure 63: KF estimate $\hat{\beta}^{j}_{i,t}$ (PEN)

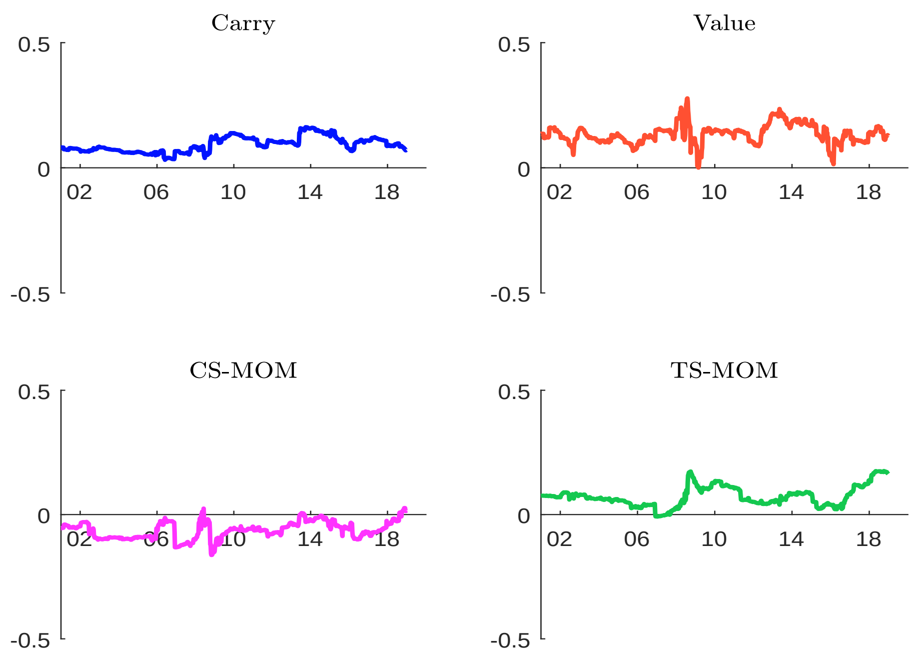


Figure 64: KF estimate $\hat{\beta}^{j}_{i,t}$ (PHP)

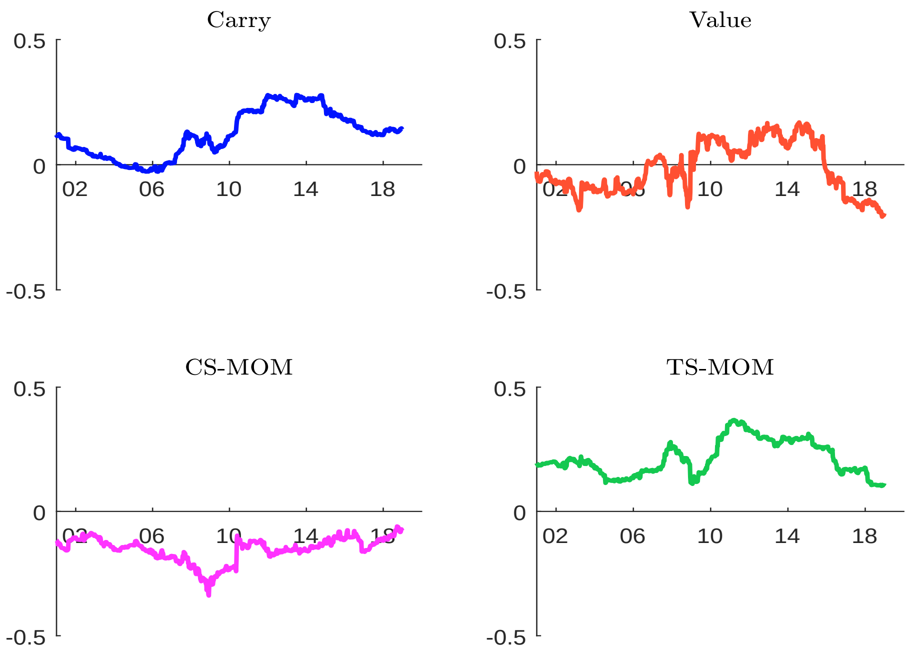


Figure 65: KF estimate $\hat{\beta}^{j}_{i,t}$ (PLN)

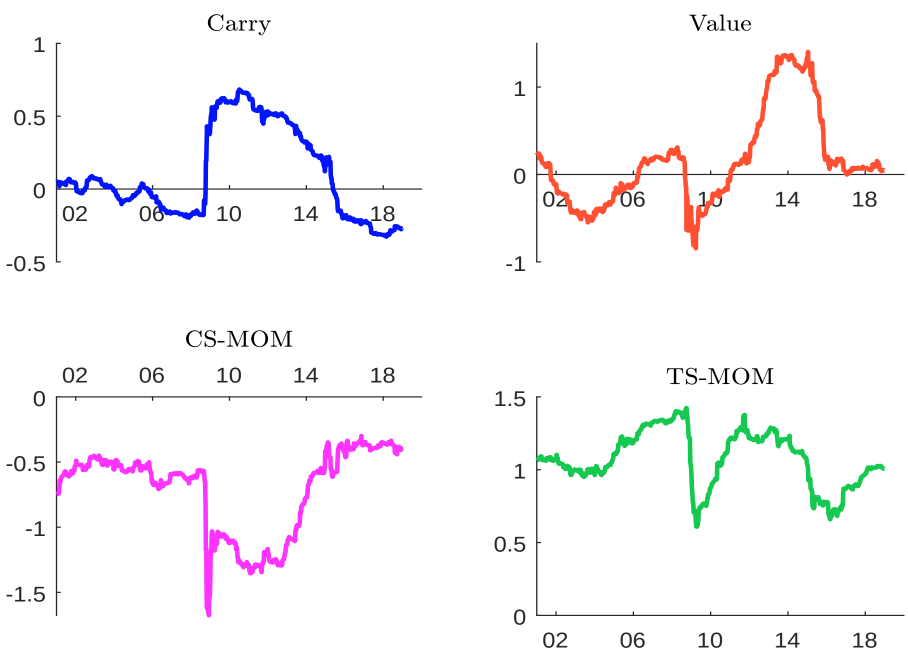


Figure 66: KF estimate $\hat{\beta}_{i,t}^{j}$ (RON)

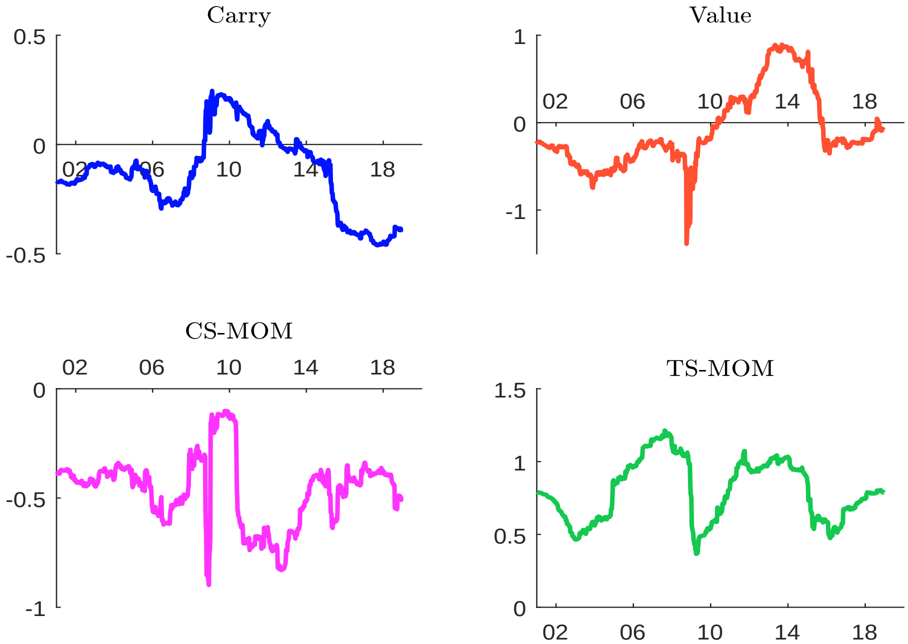


Figure 67: KF estimate $\hat{\beta}_{i,t}^{j}$ (RUB)

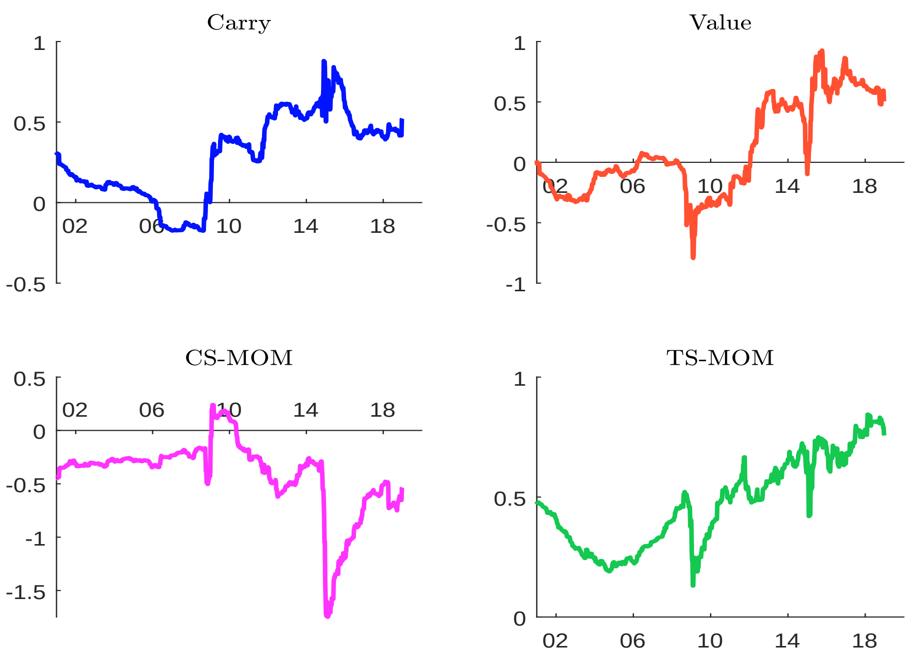


Figure 68: KF estimate $\hat{\beta}^{j}_{i,t}$ (SAR)

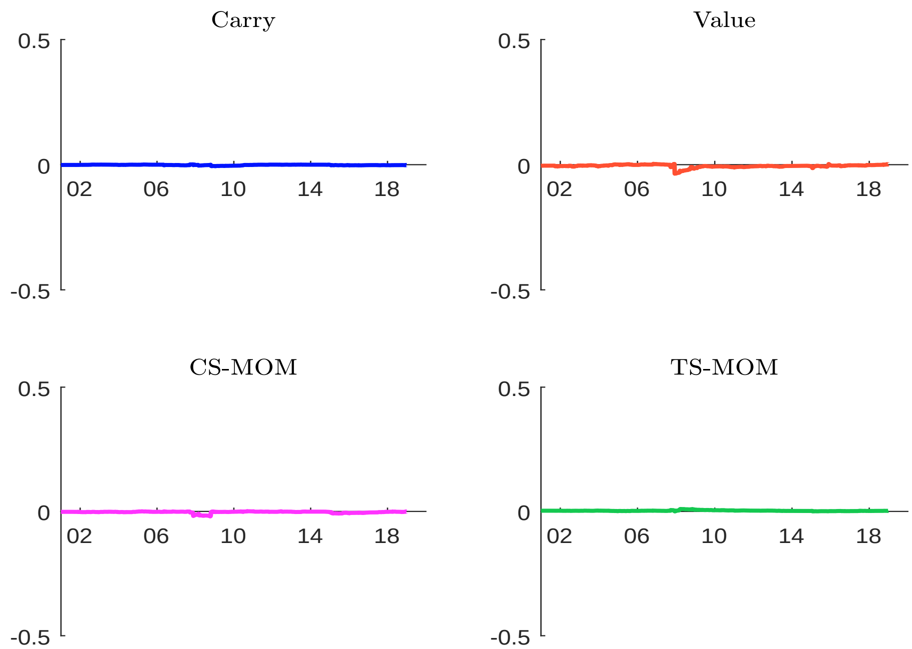


Figure 69: KF estimate $\hat{\beta}^{j}_{i,t}$ (SEK)

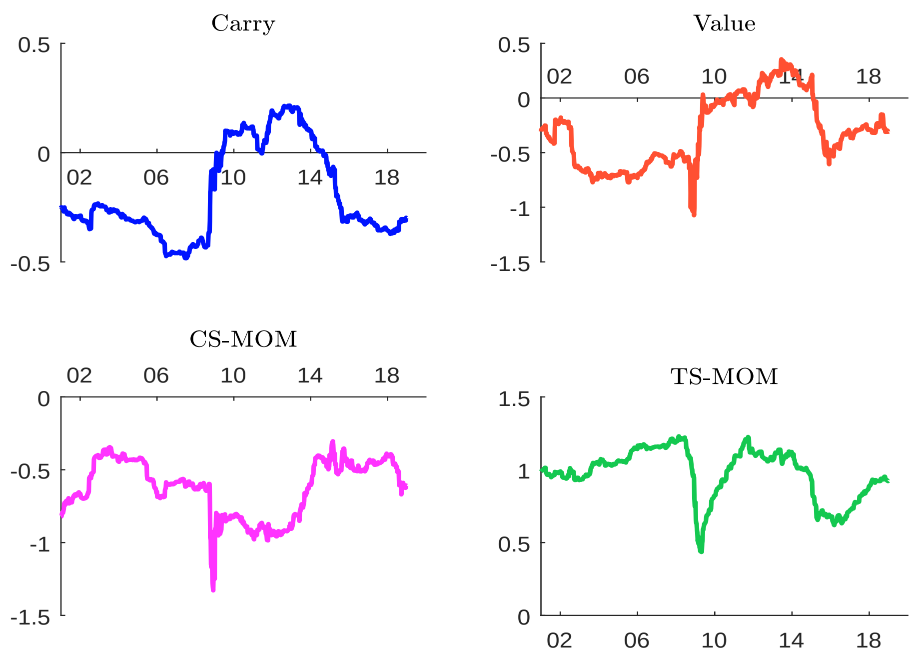


Figure 70: KF estimate $\hat{\beta}^j_{i,t}$ (SGD)

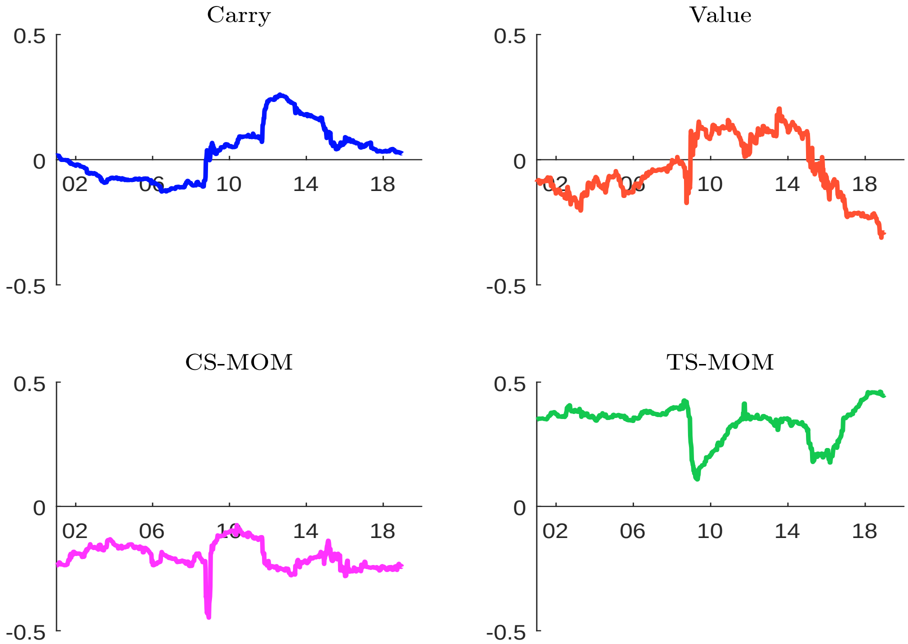


Figure 71: KF estimate $\hat{\beta}^j_{i,t}$ (THB)

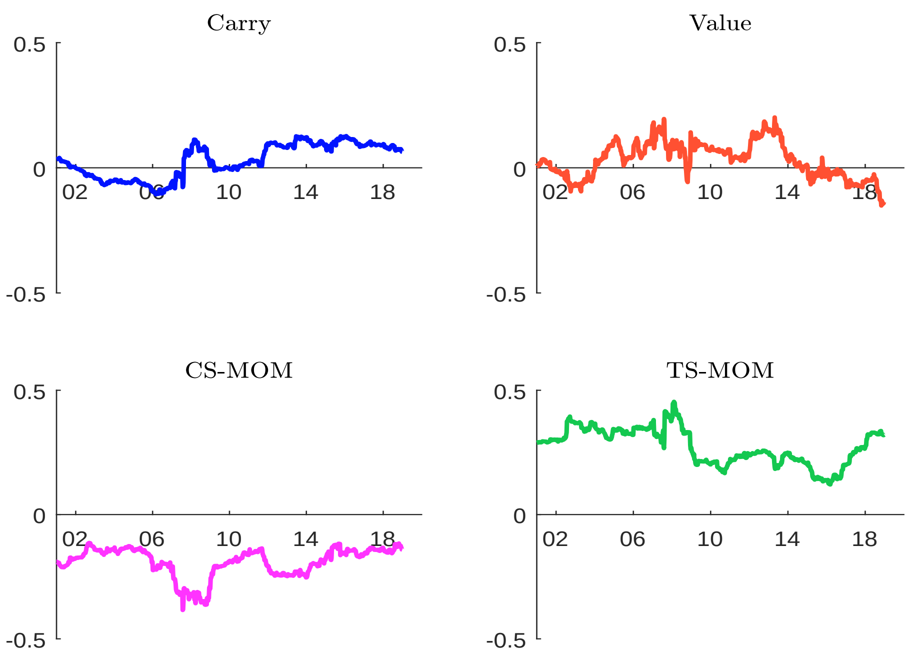


Figure 72: KF estimate $\hat{\beta}^{j}_{i,t}$ (TRY)

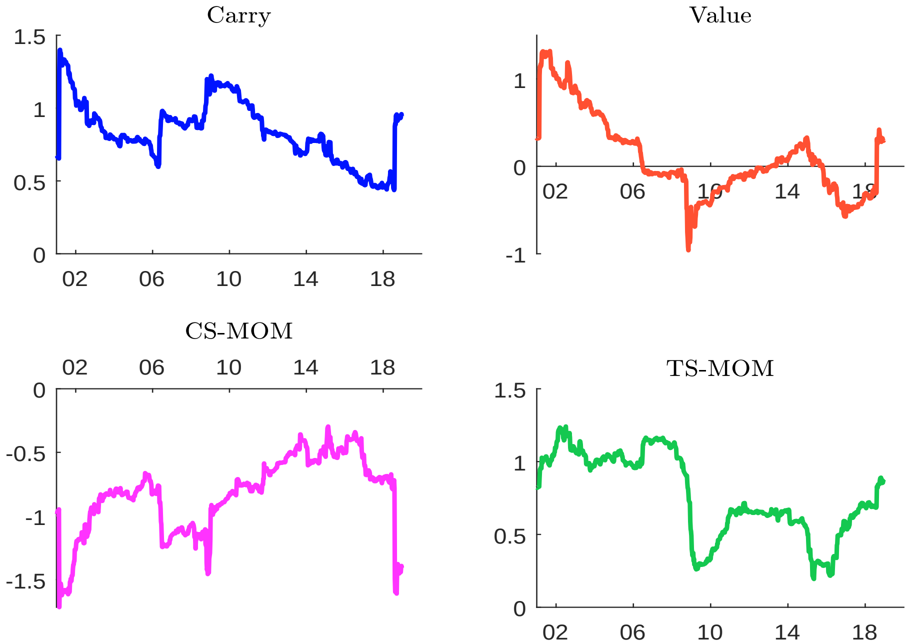


Figure 73: KF estimate $\hat{\beta}^{j}_{i,t}$ (TWD)

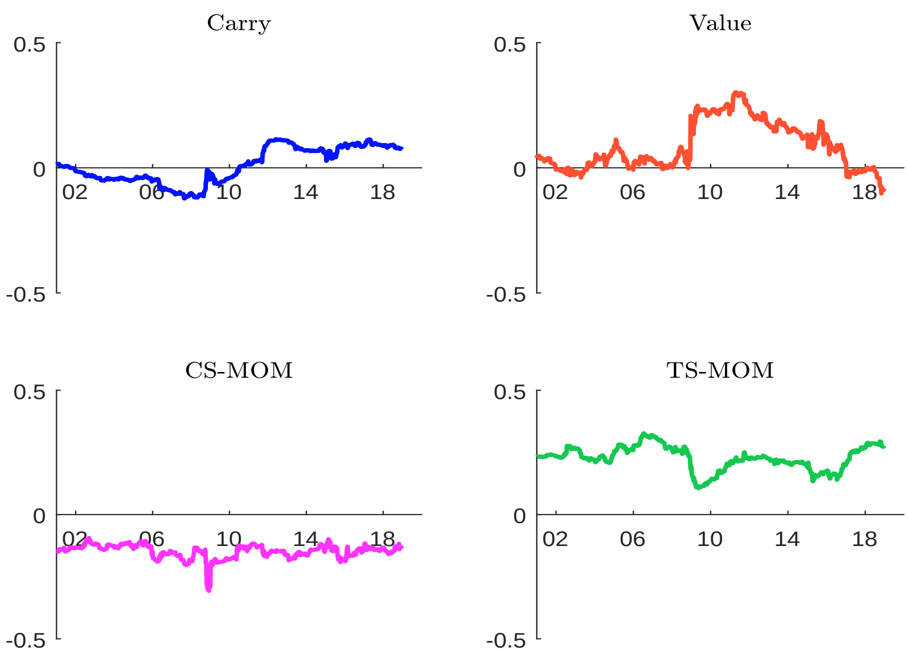


Figure 74: KF estimate $\hat{\beta}^{j}_{i,t}$ (USD)

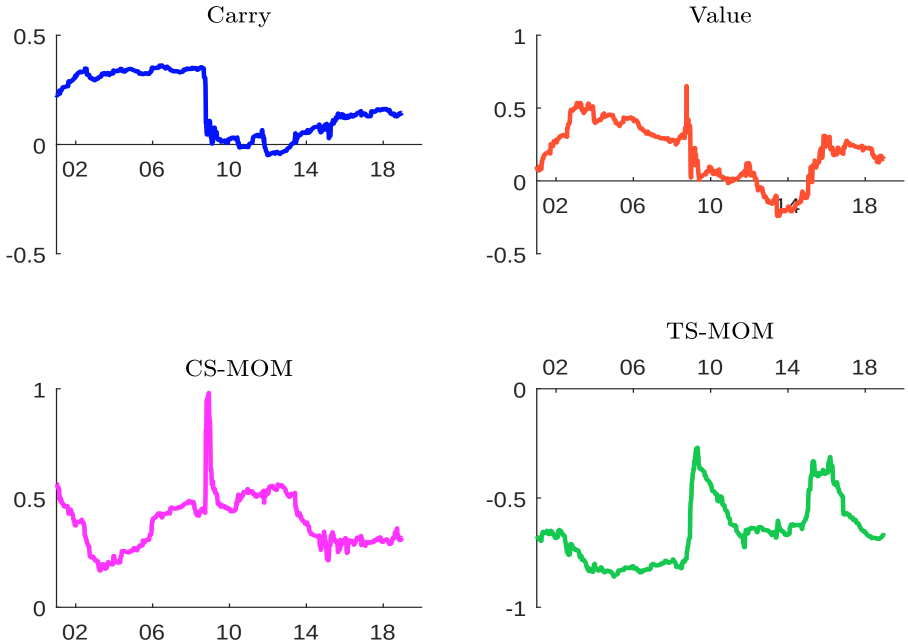
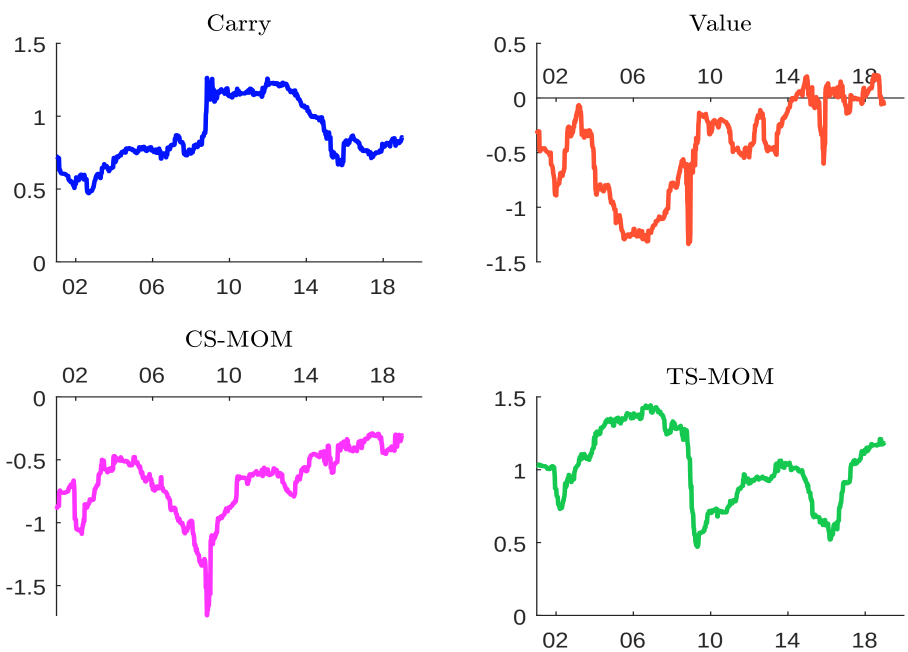


Figure 75: KF estimate $\hat{\beta}^{j}_{i,t}$ (ZAR)



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