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Net Zero Investment Portfolios
Part 1. The Comprehensive Integrated Approach

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Amundi ASSET MANAGEMENT

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Part 1. The Comprehensive Integrated Approach

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Abstract

The emergence of net zero emissions policies is currently one of the most important topics among asset owners and managers. It considerably changes portfolio allocation and the investment framework of both passive and active investors. The academic literature generally concludes that implementing net zero portfolios and sustainable investing is not costly. Moreover, some investors have chosen to implement highly dynamic decarbonization pathways with a continuous reference to business-as-usual benchmarks. The goal of this paper is to participate in the debate on climate investing by showing that it is not a free lunch. Net zero investment portfolios may involve some substantial costs in terms of tracking, diversification, and liquidity risks.

The decarbonization pathway requires the net zero emissions scenario to be defined. Transforming this absolute scenario into an intensity-based scenario is not straightforward because it involves a carbon budget. Once the scenario is established, it is important to assess the metrics that capture the different dimensions of a net zero emissions policy, particularly the self-decarbonization and the green intensity of issuers. Then we can combine these different figures to define the objective function involved in optimizing net zero portfolios by considering the asset class. For instance, bond portfolios and equity portfolios are not constructed in the same way. The objective of this comprehensive integrated approach is to deal with the multi-faceted dimensions of net zero investing. Another method establishes a core-satellite portfolio, where decarbonization and transition dimensions are segregated.

If we focus on the comprehensive integrated approach, our results show that net zero investing goes beyond the simple exercise of dynamic decarbonization. Compared to a business-as-usual benchmark, the tracking error cost may be relatively high, especially for equity portfolios. Moreover, the diversification risk is critical for equities and bonds because we see significant deformation of investment universes. Of course, these results depend on the parameter values we use.

Nevertheless, they clearly indicate that climate investing is not just a tilt of traditional investing. In this context, the reference to business-as-usual benchmarks is not always relevant. Of course, this situation is transitory until the world is on the right track to becoming a net zero economy, but at that time, we will again observe a convergence between business-as-usual and climate investing, and a growing correlation between the market and net zero portfolios.

Keywords: Climate change, net zero emissions scenario, decarbonization, transition, greenness

JEL classification: G11, Q5

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1 Introduction

Climate risk is the biggest challenge to humanity in the 21st century. Indeed, climate change implies higher temperatures that increase the likelihood of extreme weather events and impact living patterns. Beyond the direct effect on natural hazards, climate change may also result in a new economic order because of the transition to a low-carbon economy. Physical and transition risks question the resilience of the financial system to climate-related risks. This explains why climate change has become the top priority for financial institutions, supervisors, and policymakers. The asset management industry is primarily concerned because of the transmission channel on asset prices. Therefore, portfolio decarbonization, temperature alignment, net zero investment, and Paris-aligned benchmarks are the day-to-day reality for both asset owners and managers. Since 2014, interest in climate-related financial risks has been boosted by the development of ESG investing in Europe (Bennani *et al*., 2018; Drei *et al*., 2019). While environmental issues have lagged behind social issues during the Covid-19 crisis, the net zero carbon race and the Glasgow COP 26 event have recently changed the equation, and climate risk is now the hottest topic in asset management. This explains why climate investing is the new investment theme for asset owners and managers. Initially, this mainly involved decarbonizing portfolios, constructing low-carbon indices, and investing in climate-related securities such as green bonds. However, the concept of net zero has accelerated the scope of climate investing and we may wonder if it has profoundly changed its nature. Before the Covid-19 crisis, climate investing could be viewed as an investment strategy or a thematic strategy. But the proliferation of net zero alliances[1] (GFANZ, NZAOA, NZAM, NZBA, etc.) and their commitments imply new dynamics in climate investing that cannot be compared to the dynamics of a thematic investment. As such, considering net zero portfolios as a tilt of a business-as-usual portfolio is not obvious. This was not the case with low-carbon portfolios and indices, because a low-carbon strategy consists in removing issuers with the highest carbon footprints. With net zero portfolios, it is another story because the goal is also to green the economy, and, here, there is a long way to go (Fankhauser *et al*., 2022; Philipponnat, 2022). For instance, focusing on equities and corporate bonds, Alessi and Battiston (2022) estimated "*a greenness of about 2.8% for EU financial markets*" according to the existing European green taxonomy (European Commission, 2020, 2021a,b). The current greenness of the economy and the financial market is therefore a real challenge for net zero investment policies.

If we read reports from international bodies on the feasibility of net zero emissions by 2050, we notice that the decarbonization pathway of the net zero scenario has two statuses. It is the exogenous pathway that the economy must follow to limit the probability of reaching 1.5°C. However, it is not the solution to the problem, because we have to take some action to reach this objective. If the world and its economic stakeholders make the right decisions, the decarbonization pathway then becomes the endogenous pathway that the economy can follow to limit the probability of exceeding 1.5°C. What are these right decisions? They are very diverse, and the purpose of this research paper is not to list them, but they share a common feature. Indeed, they all require massive financing and involve new investments:

> "*Capital spending on physical assets for energy and land-use systems in the net-zero transition between 2021 and 2050 would amount to about* \$275 *trillion, or* \$9.2 *trillion per year on average, an annual increase of as much as* \$3.5 *trillion from today*" (McKinsey, 2022, page viii).

This figure of \$3.5 trillion is approximately equal to 1/2 of global corporate profits, 1/4 of

[1]GFANZ = Glasgow Financial Alliance for Net-Zero, NZAOA = Net Zero Asset Owner Alliance, NZAM = Net Zero Asset Managers initiative, NZBA = Net Zero Banking Alliance.

total tax revenue, or 4.1% of world GDP. Therefore, the gap between current and expected investments is huge. It does not only concern the private sector, but that should still drive us to better define a net zero carbon commitment. Indeed, when asset owners and managers speak about net zero investing, they mainly focus on portfolio decarbonization. Reducing the portfolio's carbon footprint is important, but net zero investing goes beyond a simple portfolio decarbonization exercise. As shown by the McKinsey report, the real challenge of net zero is the transition dimension, in particular how to finance the transition to a low-carbon economy.

Building a net zero investment portfolio is more complex than building a decarbonized portfolio, because the objective function encompasses at least two goals: decarbonizing the portfolio and financing the transition. Moreover, the decarbonization dimension is no longer static. It becomes dynamic. Most investors have solved this issue by considering a time-varying reduction rate of their carbon footprint. In this case, we could wonder whether the decarbonization dimension of net zero investing could be summarized by a sequence of decarbonization rates or a yearly reassessment exercise. Indeed, if net zero investing consists in building successive independent portfolios, there is no mechanism that respects the endogenous aspect of the decarbonization pathway. In particular, if the time-varying decarbonization is only due to the rebalancing process, it is clear that the portfolio cannot claim to be net zero. Indeed, the endogenous aspect of the decarbonization pathway implies the self-decarbonization of the portfolio. Therefore, we must introduce an incentive mechanism to reach a minimum level of self-decarbonization. The objective of carbon temperature ratings is precisely to assess the capacity of an issuer to be aligned with a carbon emissions scenario. Carbon temperature can be viewed as a synthetic scoring system based on the $\mathcal{PAC}$ framework (Le Guenedal *et al.*, 2022), which measures the issuer's (past) participation, ambition and credibility. Since a rating system of carbon temperature is often perceived as a black box, we may consider a simplified approach that is more transparent. For instance, we can use net zero targets that are approved and validated by a third party. By using a linear interpolation model, we can compute the yearly self-decarbonization rate of issuers and deduce the self-decarbonization level of portfolios. This simple approach is limited for two reasons. First, the data are not homogeneous because targeted dates and scopes could be different. Second, the self-decarbonization cannot be computed for issuers without net zero engagement or validation. Another approach consists in focusing on the first pillar, which is participation. Indeed, participation is a technical term used to identify past self-decarbonization. This explains that carbon trends and carbon momentum measures are very important metrics for a net zero investor. This is a way to introduce a dynamic approach to the carbon footprint and to go beyond the current level, which is a poor estimate of the issuer's finish line and an even poorer one of how quickly the issuer will get there.

Beside net zero carbon metrics, the portfolio manager also needs net zero transition metrics to assess the greenness of the portfolio. Therefore, the green intensity is the equivalent of the carbon intensity for the transition dimension. One of the issues is the choice of the right metric. Indeed, there are many metrics and a lack of exhaustive data. Le Guenedal and Roncalli (2022) reported some of them, but most of the time they are sector-specific, biased, difficult to compute or not meaningful for all issuers. A typical example is the amount of avoided emissions, since it is not easy to define a reference for each product. This explains why the concept of green revenues has emerged and has been developed over the last few years. Once a green taxonomy is defined, green revenues can be easily computed using detailed income statements. In three years, green revenue share has become the main factor when computing a green intensity score. Nevertheless, this metric is relatively young, which explains why we do not have enough historical data to perform a dynamic analysis. An alternative is to use green capital expenditures (capex), green operational expenses (opex) or green R&D expenses, but they are under development, implying that these metrics will not be available before 2024.

Moreover, building a net zero portfolio is not an easy task because a financial investment cannot reach net zero by itself. Only an economy, a region or a group of industries can reach net zero. Indeed, CO_2 emissions can be comprehensively measured for a relatively closed system, but not for an open system. This implies integrating scope 3 emissions in order to include the CO_2 emissions of the entire supply chain. This is another difference with low-carbon portfolios. At the same time, we know that scope 3 emissions data are of poor quality. Nevertheless, we face a critical situation where we do not have time and we have no choice. As such, the definition of a net zero investment strategy is not fixed and stabilized since we are using more of a learning-by-doing approach than a mature model. Therefore, net zero processes will evolve in the future as new metrics are adopted and data quality improves. In fact, the current situation could be transitory and may be explained because the economy's pathway is far from net zero. The consequence is the huge gap between market and net zero portfolios. Nevertheless, we believe that this situation will improve in the long run with the transition to a net zero economy, and we will observe a convergence between business-as-usual and net zero investing. In the meantime, net zero investing is a true test for ESG investors with strong ethical convictions. In the short run, the world economy is far from being on the right track and the current energy crisis is a new factor that challenges our ability to keep global warming below 1.5°C. The short-term risk is that the discrepancy between business-as-usual portfolios and net zero portfolios increases, in particular if the transition to a low-carbon economy is delayed. For a net zero portfolio, this is a micro-economic risk, but for the asset management industry, this is a macro-economic risk. Indeed, the high commitment of net zero alliances implies a large investment universe of net zero assets. However, the current investment universe is relatively small in terms of green or transition assets. This implies that the financial market and the issuers must become sufficiently green very quickly. Otherwise, the gap between traditional and climate investing would widen.

This research paper is organized as follows. In Section Two, we introduce the concept of a net zero emissions scenario, which is a physical concept based on carbon budgets. We compare it to the financial concept of a decarbonization pathway based on the carbon intensity metric, and we also illustrate the relationships between emission-based and intensity-based scenarios. Section Three is dedicated to net zero metrics and contains two parts. The first part reviews the metrics associated with the decarbonization dimension. After studying static measures of carbon footprint, we consider dynamic measures that are related to the self-decarbonization aspect. In particular, we focus on the carbon momentum metric. The second part deals with the transition dimension. After a discussion on green taxonomy, we introduce static and dynamic measures of greenness such as green revenues and green capex. The construction of net zero investment portfolios is discussed in Section Four. First, we analyze the impact of portfolio decarbonization in terms of tracking risk, sector allocation and transition metrics. We consider both equity and bond portfolios and show that the results are similar. Second, we present the integrated approach of net zero investing, which involves defining a unique optimization problem by considering all the aspects of the transition dimension. This implies adapting the original problem of portfolio decarbonization by adding many constraints. In this case, the results on equity portfolios differ from those on bond portfolios if we focus on tracking risk. Nevertheless, the results are similar in the two asset classes when we consider diversification and liquidity risks. In Section Four, we also present an alternative method for building net zero investment portfolios by using a core-satellite approach, but this method will be extensively studied in a forthcoming paper (Roncalli *et al.*, 2023). Finally, Section Five offers some concluding remarks.

2 Net zero emissions scenario

In order to implement a net zero investing policy, asset managers and owners have to define a net zero scenario, which is summarized by a decarbonization pathway.

2.1 Paris-aligned benchmark pathways

To implement the Paris agreement on climate change, the European Union has created two climate benchmark labels: climate transition benchmark (CTB) and Paris-aligned benchmark (PAB). These two labels are structured along the following common principles:

1. A year-on-year self-decarbonization $\Delta\boldsymbol{\mathcal{R}}$ on average per annum, based on scope 1, 2 and 3 emissions intensity;
2. A minimum carbon intensity reduction $\boldsymbol{\mathcal{R}}^-$ compared to the investable universe;
3. A minimum exposure to sectors highly exposed to climate change;
4. A set of exclusion rules.

We deduce that the decarbonization pathway is defined by:

$$\boldsymbol{\mathcal{R}}(t_0, t) = 1 - (1 - \Delta\boldsymbol{\mathcal{R}})^{t-t_0} \left(1 - \boldsymbol{\mathcal{R}}^-\right) \tag{1}$$

where t_0 is the base year, t is the year index, and $\boldsymbol{\mathcal{R}}(t_0, t)$ is the reduction rate of the carbon footprint between t_0 and t. For the CTB label, the minimum reduction $\boldsymbol{\mathcal{R}}^-$ is set to 30% whereas it is equal to 50% for the PAB label. Moreover, the additional reduction rate $\Delta\boldsymbol{\mathcal{R}}$ is set to 7% for the two labels. Formula (1) can be used to create other decarbonization pathways. For instance, Figure 1 compares several trajectories of $\boldsymbol{\mathcal{R}}(t_0, t)$ by assuming that the base year is 2020. We notice that if $\Delta\boldsymbol{\mathcal{R}}$ is sufficiently large, the choice of the initial reduction rate $\boldsymbol{\mathcal{R}}^-$ has little impact on the long-run reduction rate $\boldsymbol{\mathcal{R}}(2020, 2050)$.

2.2 Carbon budget constraint

While CTB and PAB are the most known pathways in finance, their construction lacks theoretical and solid foundations. Indeed, they have been created *ex nihilo* such that the carbon footprint is close to zero by 2050, but they have no physical or economic foundations.

In fact, a net zero emissions (NZE) scenario corresponds to a carbon pathway, which is compatible with a carbon budget:

> *Using global mean surface air temperature, as in AR5, gives an estimate of the remaining carbon budget of 580 $GtCO_2e$ for a 50% probability of limiting warming to 1.5°C, and 420 $GtCO_2e$ for a 66% probability* (IPCC, 2018, page 26).

Let $\boldsymbol{\mathcal{CE}}(t)$ be the global carbon emissions at time t and $\boldsymbol{\mathcal{CB}}(t_0, t)$ be the global carbon budget between t_0 and t (Le Guenedal *et al.*, 2022):

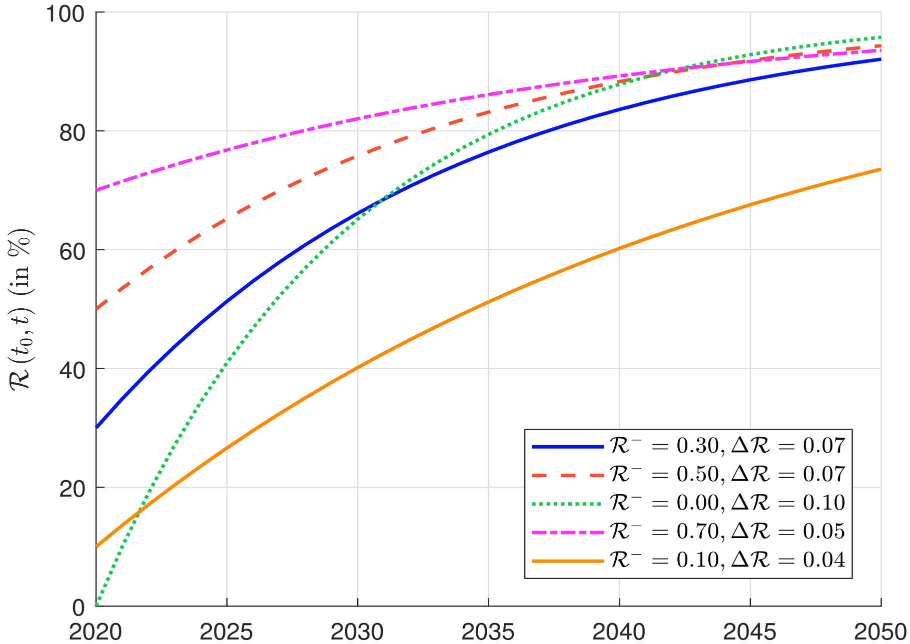
$$\boldsymbol{\mathcal{CB}}(t_0, t) = \int_{t_0}^{t} \boldsymbol{\mathcal{CE}}(s)\, \mathrm{d}s \tag{2}$$

A NZE scenario can be defined by a carbon pathway that satisfies the following constraints:

$$\begin{cases} \boldsymbol{\mathcal{CB}}(t_0, 2050) \leq \boldsymbol{\mathcal{CB}}^+ \text{ GtCO}_2\text{e} \\ \boldsymbol{\mathcal{CE}}(2050) \approx 0 \text{ GtCO}_2\text{e} \end{cases} \tag{3}$$

where t_0 is the base date and $\boldsymbol{\mathcal{CB}}^+$ is the maximum carbon budget.

Figure 1: Examples of decarbonization pathway (in %)



Remark 1. *If we consider the AR5 results of IPCC (2018), we can set* $t_0 = 2019$ *and* $\boldsymbol{\mathcal{CB}}^+ = 580$. *If we would like to increase the probability that the global warming remains under* $1.5°\text{C}$, *the maximum carbon budget* $\boldsymbol{\mathcal{CB}}^+$ *can be replaced by a lower figure. Over the years, the budget constraint is moving, especially if the decarbonization pathway of the economy is not satisfied. For instance, the previous constraint* $\boldsymbol{\mathcal{CB}}(2019, 2050) \leq 580$ *$GtCO_2e$ is generally updated and has become* $\boldsymbol{\mathcal{CB}}(2021, 2050) \leq 500$ *$GtCO_2e$.*

If we consider the decarbonization pathway given in Equation (1), we have:

$$\begin{aligned} \boldsymbol{\mathcal{CE}}(t) &= \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \boldsymbol{\mathcal{CE}}(t_0) \\ &= (1 - \Delta\boldsymbol{\mathcal{R}})^{t-t_0} \left(1 - \boldsymbol{\mathcal{R}}^-\right) \boldsymbol{\mathcal{CE}}(t_0) \end{aligned} \tag{4}$$

Using the analytical expression given in Le Guenedal *et al.* (2022, Equation (105), page 56), we obtain:

$$\boldsymbol{\mathcal{CB}}(t_0, t) = \left(\frac{(1 - \Delta\boldsymbol{\mathcal{R}})^{t-t_0} - 1}{\ln(1 - \Delta\boldsymbol{\mathcal{R}})} \right) \left(1 - \boldsymbol{\mathcal{R}}^-\right) \boldsymbol{\mathcal{CE}}(t_0) \tag{5}$$

By considering several values of $\boldsymbol{\mathcal{R}}^-$ and $\Delta\boldsymbol{\mathcal{R}}$, and assuming that $\boldsymbol{\mathcal{CE}}(2020) = 36$ GtCO_2e we obtain the figures given in Table 1. For instance, the carbon budget $\boldsymbol{\mathcal{CB}}(2020, 2050)$ is equal to 308 GtCO_2e if $\boldsymbol{\mathcal{R}}^- = 30\%$ and $\Delta\boldsymbol{\mathcal{R}} = 7\%$.

2.3 The IEA scenario

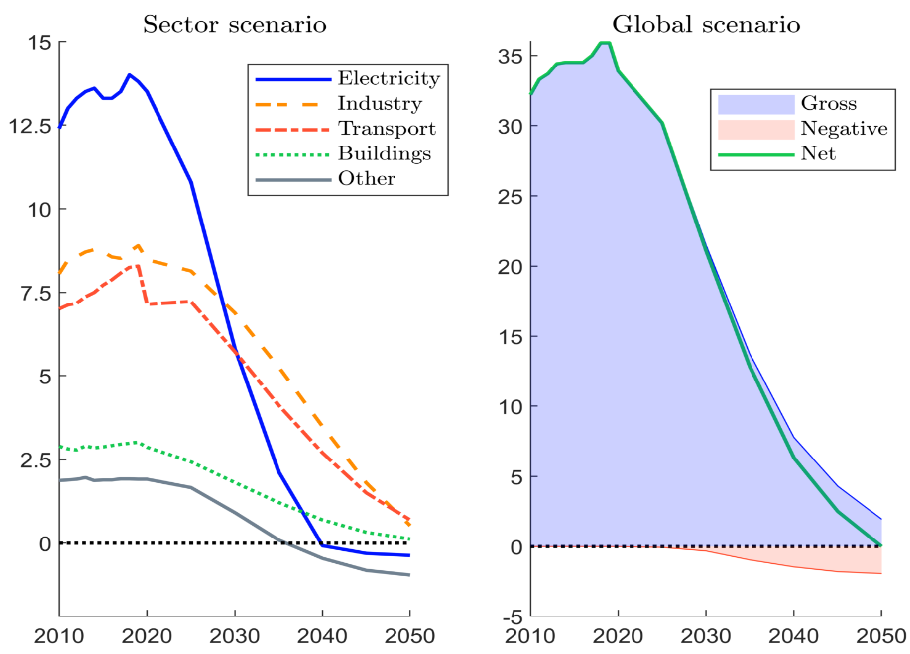
We must be careful with the specification of a decarbonization pathway, because its interpretation may differ from one application to another. Indeed, a decarbonization pathway is generally valid for an economy or a country. In this case, it is defined with respect to absolute carbon emissions. However, portfolio decarbonization uses carbon intensity, and not carbon emissions.

Table 1: Carbon budget $\mathcal{CB}(2020, 2050)$ of decarbonization pathways (in $GtCO_2e$)

	$\mathcal{R}^-$	0%	10%	20%	30%	50%	75%
	5%	551	496	441	386	276	138
	6%	491	442	393	344	245	123
$\Delta\mathcal{R}$	7%	440	396	352	308	220	110
	8%	396	357	317	277	198	99
	9%	359	323	287	251	180	90
	10%	327	294	262	229	164	82

Let us consider the International Energy Agency (IEA) net zero scenario (IEA, 2021). IEA has analyzed each important sector to list the existing technologies and the future innovations that can help to reach net zero by 2050. For each sector, they have computed the resulting decarbonization pathway represented in the first panel in Figure 2. We notice that the power generation sector is the main contributor followed by the industry and transport sectors. The global decarbonization pathway[2] can then be deduced by summing all the sector trajectories and is reported in the second panel in Figure 2. We observe an acceleration of the decarbonization rate after 2025.

Figure 2: CO_2 emissions by sector in the IEA NZE scenario (in $GtCO_2e$)



Source: IEA (2021) & Authors' calculations.

To compute the carbon budget $\mathcal{CB}(2019, 2050)$, we consider that the carbon pathway is a piecewise linear function. Therefore, we assume that $\mathcal{CE}(s)$ is known for $s \in$

[2] The IEA scenario gross CO_2 emissions in $GtCO_2e$ are equal to:

Year	2019	2020	2025	2030	2035	2040	2045	2050
$\mathcal{CE}(t)$	35.90	33.90	30.30	21.50	13.70	7.77	4.30	1.94

These figures are used to calibrate several pathways in the sequel.

$\{t_0, t_1, \ldots, t_m = t\}$ and $\mathcal{CE}(s)$ is linear between two consecutive dates:

$$\mathcal{CE}(s) = \mathcal{CE}(t_{k-1}) + \frac{\mathcal{CE}_i(t_k) - \mathcal{CE}_i(t_{k-1})}{t_k - t_{k-1}}(s - t_{k-1}) \qquad \text{if } s \in [t_{k-1}, t_k] \tag{6}$$

Le Guenedal *et al.* (2022, Equation (112), page 57) has demonstrated that:

$$\begin{aligned}\mathcal{CB}(t_0, t) &= \frac{1}{2}\sum_{k=1}^{m}\left(\mathcal{CE}(t_k) - \mathcal{CE}(t_{k-1})\right)(t_k + t_{k-1}) + \\ &\quad \sum_{k=1}^{m}\left(\mathcal{CE}_i(t_{k-1})\, t_k - \mathcal{CE}(t_k)\, t_{k-1}\right)\end{aligned} \tag{7}$$

Using the IEA scenario, we obtain $\mathcal{CB}(2019, 2050) = 512.35$ $GtCO_2e$. Since the two equations of the system (3) are satisfied[3], the IEA scenario can be considered as a 2050 net zero emissions scenario.

2.4 Relationships between carbon intensity and carbon emissions pathways

2.4.1 Relationship between reduction rates

Analytical method By definition, the carbon intensity $\mathcal{CI}(t)$ is defined as the ratio between the carbon emissions $\mathcal{CE}(t)$ and the normalization variable $Y(t)$:

$$\mathcal{CI}(t) = \frac{\mathcal{CE}(t)}{Y(t)} \tag{8}$$

Let $\mathcal{R}_{\mathcal{CI}}(t_0, t)$ and $\mathcal{R}_{\mathcal{CE}}(t_0, t)$ be the reduction rates of carbon intensity and emissions between t_0 and t. We have the following relationship:

$$\begin{aligned}\mathcal{R}_{\mathcal{CI}}(t_0, t) &= \frac{\mathcal{CI}(t_0) - \mathcal{CI}(t)}{\mathcal{CI}(t_0)} \\ &= \frac{g_Y(t_0, t) + \mathcal{R}_{\mathcal{CE}}(t_0, t)}{1 + g_Y(t_0, t)}\end{aligned} \tag{9}$$

where $g_Y(t_0, t)$ is the growth rate of the normalization variable. Generally, we assume that[4] $g_Y(t_0, t) \geq 0$ and $0 \leq \mathcal{R}_{\mathcal{CE}}(t_0, t) \leq 1$. Therefore, we can show the following property:

$$\left\{\begin{array}{l} g_Y(t_0, t) \geq 0 \\ 0 \leq \mathcal{R}_{\mathcal{CE}}(t_0, t) \leq 1 \end{array}\right. \Rightarrow \mathcal{R}_{\mathcal{CI}}(t_0, t) \geq \mathcal{R}_{\mathcal{CE}}(t_0, t) \tag{10}$$

We conclude that the reduction rate of the carbon intensity is always greater than the reduction rate of the carbon emissions.

Remark 2. *The emissions and intensity decarbonization pathways $\mathcal{R}_{\mathcal{CE}}(t_0, t)$ and $\mathcal{R}_{\mathcal{CI}}(t_0, t)$ are also called the 'economic' and 'financial' decarbonization pathways.*

Most of the time, we consider that the annual growth rate of the normalization variable is constant: $Y(t) = (1 + g_Y)\, Y(t-1)$. We deduce that the compound growth rate is equal to:

$$g_Y(t_0, t) = (1 + g_Y)^{t - t_0} - 1 \tag{11}$$

[3] We assume that $\mathcal{CB}^{+} = 580$.

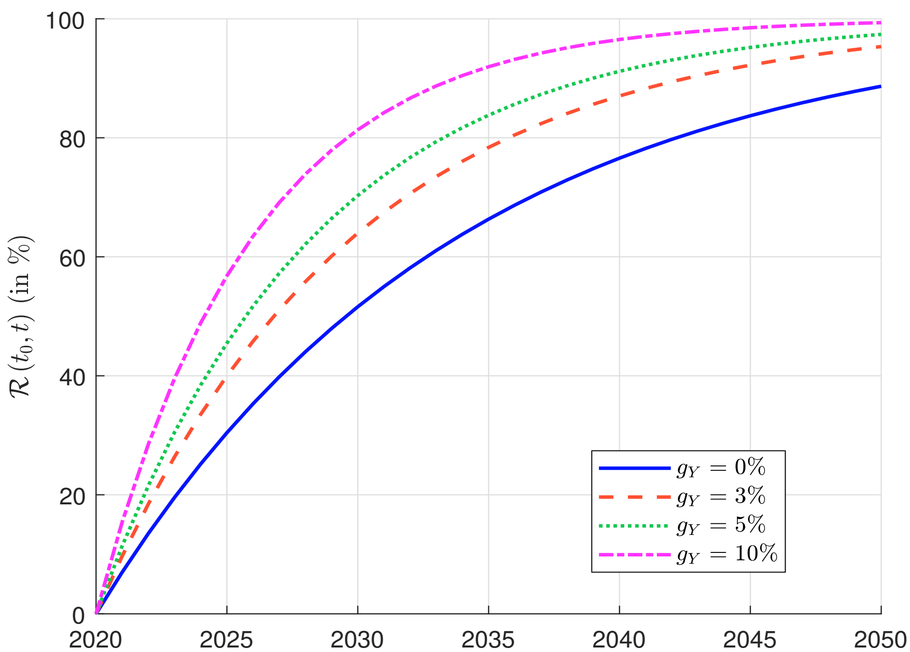
[4] For example, we anticipate that the sales or the revenues are increasing over time.

If we also assume that the annual reduction rate of carbon emissions is constant – $\mathcal{CE}(t) = (1-\mathcal{R}_{\mathcal{CE}})\,\mathcal{CE}(t-1)$, we obtain $\mathcal{R}_{\mathcal{CE}}(t_0,t) = 1-(1-\mathcal{R}_{\mathcal{CE}})^{t-t_0}$ and:

$$\mathcal{R}_{\mathcal{CI}}(t_0,t) = 1-\left(1-\frac{(g_Y+\mathcal{R}_{\mathcal{CE}})}{1+g_Y}\right)^{t-t_0} \tag{12}$$

Equation (12) is the mirror formula of Equation (9) in the case of constant rates. Therefore, the annualized reduction rate of carbon intensity is approximatively equal to $g_Y + \mathcal{R}_{\mathcal{CE}}$. This implies that the intensity decarbonization pathway must be more aggressive than the emissions decarbonization pathway, as illustrated in Figure 3.

Figure 3: Impact of the growth rate g_Y on the intensity decarbonization pathway (in %) — $\mathcal{R}_{\mathcal{CE}}$ is set to 7%



Estimation method Let us consider a given economic decarbonization pathway $\{\mathcal{R}_{\mathcal{CE}}(t_0,t), t = t_1,\ldots,t_m\}$ and a given trajectory of the normalization variable growth $\{g_Y(t_0,t), t = t_1,\ldots,t_m\}$, we can use Equation (9) to compute the resulting financial decarbonization pathway $\{\mathcal{R}_{\mathcal{CI}}(t_0,t), t = t_1,\ldots,t_m\}$. If we assume that the functional form of the carbon intensity reduction is equal to:

$$f_1\left(t;\mathcal{R}^-_{\mathcal{CI}},\Delta\mathcal{R}_{\mathcal{CI}}\right) = 1-(1-\Delta\mathcal{R}_{\mathcal{CI}})^{t-t_0}\left(1-\mathcal{R}^-_{\mathcal{CI}}\right) \tag{13}$$

we can postulate the following regression model:

$$\mathcal{R}_{\mathcal{CI}}(t_0,t) = f_1\left(t;\mathcal{R}^-_{\mathcal{CI}},\Delta\mathcal{R}_{\mathcal{CI}}\right)+\varepsilon(t) \tag{14}$$

and estimate the parameters $\left(\mathcal{R}^-_{\mathcal{CI}},\Delta\mathcal{R}_{\mathcal{CI}}\right)$ by least squares.

Table 2: Intensity decarbonization pathways (in %) deduced from the IEA net zero emissions scenario

t	$\mathcal{R}_{\mathcal{CE}}(t_0,t)$	$\mathcal{R}_{\mathcal{CI}}(t_0,t)$				EU labels	
		$g_Y=3\%$	$g_Y=5\%$	$g_Y=10\%$	$g_Y=20\%$	CTB	PAB
2020	0.0	0.0	0.0	0.0	0.0	30.0	50.0
2021	3.2	6.0	7.8	12.0	19.3	34.9	53.5
2022	6.3	11.7	15.0	22.6	35.0	39.5	56.8
2023	9.5	17.2	21.8	32.0	47.6	43.7	59.8
2024	12.7	22.4	28.2	40.4	57.9	47.6	62.6
2025	15.8	27.4	33.1	47.7	66.2	51.3	65.2
2026	20.7	33.6	40.8	55.2	73.5	54.7	67.7
2027	25.6	39.5	47.1	61.8	79.2	57.9	69.9
2028	30.5	45.1	52.0	67.6	83.8	60.8	72.0
2029	35.4	50.5	58.4	72.6	87.5	63.6	74.0
2030	40.3	55.6	63.3	77.0	90.4	66.1	75.8
2035	61.9	75.6	81.7	90.9	97.5	76.4	83.2
2040	78.4	88.0	91.9	96.8	99.4	83.6	88.3
2045	88.1	94.3	96.5	98.9	99.9	88.6	91.9
2050	94.6	97.8	98.8	99.7	100.0	92.1	94.3
$\mathcal{R}^{-}_{\mathcal{CI}}$	−12.6	−8.7	−6.8	−3.7	−1.3	30.0	50.0
$\Delta\mathcal{R}_{\mathcal{CI}}$	7.1	9.2	10.6	13.9	20.3	7.0	7.0

By using the IEA net zero emissions scenario and considering linear interpolation scheme[5], we compute the emissions decarbonization pathway $\mathcal{R}_{\mathcal{CE}}(t_0,t)$ between 2020 and 2050 in Table 2. We also deduce the intensity decarbonization pathway $\mathcal{R}_{\mathcal{CI}}(t_0,t)$ for different values of the constant growth rate g_Y. The comparison with CTB and PAB labels clearly shows that these last ones are very aggressive pathways for the next ten years. For instance, if we consider that $g_Y=5\%$, $\mathcal{R}_{\mathcal{CI}}(2020,2025)$ is equal to 33.1% for the IEA NZE scenario, whereas this figure is equal to 51.3% and 65.2% for CTB and PAB labels. In Table 2, we have also reported the estimated values[6] $\mathcal{R}^{-}_{\mathcal{CI}}$ and $\Delta\mathcal{R}_{\mathcal{CI}}$.

2.4.2 The carbon budget approach

Since we have $\mathcal{CE}(t)=Y(t)\,\mathcal{CI}(t)$, we obtain:

$$\begin{aligned}\mathcal{CB}(t_0,t) &= \mathcal{CE}(t_0)\int_{t_0}^{t}\left(1+g_Y(t_0,s)\right)\left(1-\mathcal{R}_{\mathcal{CI}}(t_0,s)\right)\,\mathrm{d}s \\ &= \underbrace{(t-t_0)\,\mathcal{CE}(t_0)}_{\mathcal{CB}_1(t_0,t)}+\underbrace{\mathcal{CE}(t_0)\int_{t_0}^{t}g_Y(t_0,s)\,\mathrm{d}s}_{\mathcal{CB}_2(t_0,t)}- \\ &\qquad\underbrace{\mathcal{CE}(t_0)\int_{t_0}^{t}\left(1+g_Y(t_0,s)\right)\mathcal{R}_{\mathcal{CI}}(t_0,s)\,\mathrm{d}s}_{\mathcal{CB}_3(t_0,t)}\end{aligned} \tag{15}$$

We can break-down the carbon budget into three components. The first component $\mathcal{CB}_1(t_0,t)$ corresponds to the total carbon emissions if nothing is done[7]. The second component

[5] We assume that the current carbon emissions $\mathcal{CE}(2020)$ are equal to 36 $GtCO_2e$.
[6] We use a yearly partition between 2020 and 2050.
[7] This means that the emitted carbon emissions are stable.

$\mathcal{CB}_2(t_0, t)$ corresponds to the extra carbon budget if the carbon intensity remains unchanged[8]. The third component $\mathcal{CB}_3(t_0, t)$ is the removed carbon budget due to the intensity reduction.

Let us assume that the annual growth rate of $Y(t)$ is constant and we use the PAB/CTB formula for the intensity decarbonization pathway. We deduce that:

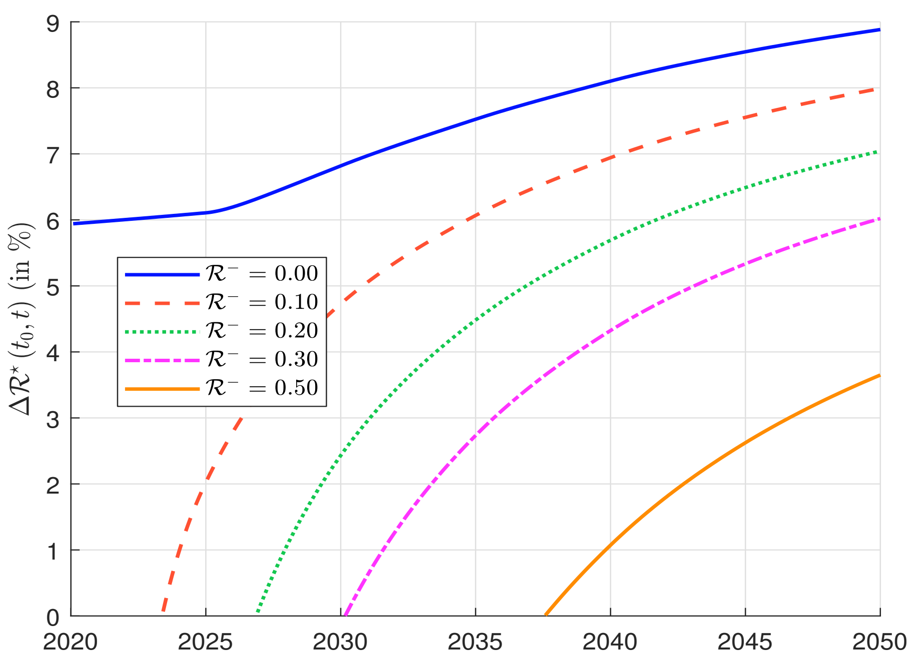
$$\begin{aligned} \mathcal{CB}(t_0, t) &= \frac{(1+g_Y)^{t-t_0}(1-\Delta\mathcal{R}_{\mathcal{CI}})^{t-t_0}-1}{\ln(1+g_Y)+\ln(1-\Delta\mathcal{R}_{\mathcal{CI}})}\left(1-\mathcal{R}^-_{\mathcal{CI}}\right)\mathcal{CE}(t_0) \\ &= f_2\left(t; \mathcal{R}^-_{\mathcal{CI}}, \Delta\mathcal{R}_{\mathcal{CI}}, g_Y\right) \end{aligned} \tag{16}$$

If we consider a given carbon budget $\mathcal{CB}(t_0, t)$ and we assume a value for the growth rate g_Y, it is possible to estimate the parameters $\mathcal{R}^-_{\mathcal{CI}}$ and $\Delta\mathcal{R}_{\mathcal{CI}}$ by using the least squares approach. Another method consists in fixing the initial reduction rate $\mathcal{R}^-_{\mathcal{CI}}$ and to find the optimal value $\Delta\mathcal{R}_{\mathcal{CI}}$ such that the carbon budget is satisfied[9]:

$$\Delta\mathcal{R}^\star(t_0, t) = \inf\left\{\theta : f_2\left(t; \mathcal{R}^-_{\mathcal{CI}}, \theta, g_Y\right) \le \mathcal{CB}(t_0, t)\right\}$$

By construction, $\Delta\mathcal{R}^\star(t_0, t)$ depends on the time horizon t because it is valid for the period $[t_0, t]$.

Figure 4: Estimated value $\Delta\mathcal{R}^\star(2020, t)$ (in %) from the IEA NZE scenario — $g_Y = 3\%$



[8]If the carbon intensity is constant, this implies that the carbon budget increases and we have $\mathcal{CB}(t_0, t) = \mathcal{CB}_1(t_0, t) + \mathcal{CB}_2(t_0, t)$.

[9]This is equivalent to solve this non-linear inequation:

$$\frac{(1-\Delta\mathcal{R}_{\mathcal{CI}})^{t-t_0}-1}{\ln(1+g_Y)+\ln(1-\Delta\mathcal{R}_{\mathcal{CI}})} \le \frac{\mathcal{CB}(t_0, t)}{(1+g_Y)^{t-t_0}\left(1-\mathcal{R}^-_{\mathcal{CI}}\right)\mathcal{CE}(t_0)} \tag{17}$$

We use the IEA NZE scenario and estimate $\Delta\boldsymbol{\mathcal{R}}^{\star}_{\boldsymbol{\mathcal{CI}}}(2020,t)$ for several values of $\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}}$ when the gross rate g_Y is set equal to 3%. Results are reported in Figure 4. When $\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}}$ is equal to zero, the optimal reduction rate is close to 6% if the time horizon is short (less than 2025), whereas it reaches 9% if the time horizon is 2050. If the investor uses an initial reduction rate ($\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}} > 0$), the additional reduction rate is implemented later. For instance, it is implemented after 2030 if $\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}} = 30\%$. These results illustrate the aggressive behavior of the PAB pathway compared to the IEA pathway since the decarbonization velocity will increase only in the last 12 years with an additional rate lower than 4%.

In the previous approach, the optimal decarbonization rate $\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)$ could be viewed as the average value of $\Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}$ that must be implemented between t_0 and t. It does not give the reduction rate we must consider after the time horizon. This is why we consider a third calibration approach, whose goal is to estimate the instantaneous decarbonization rate that must be implemented at time t. For that, we use the Chasles decomposition:

$$\boldsymbol{\mathcal{CB}}(t_0,t+h) = \boldsymbol{\mathcal{CB}}(t_0,t) + \int_t^{t+h} \boldsymbol{\mathcal{CE}}(s)\,\mathrm{d}s \tag{18}$$

where:

$$\begin{aligned}\int_t^{t+h} \boldsymbol{\mathcal{CE}}(s)\,\mathrm{d}s &= \left(1-\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}}\right)\boldsymbol{\mathcal{CE}}(t_0)\int_t^{t+h}(1+g_Y)^{s-t_0}\left(1-\Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}\right)^{s-t_0}\,\mathrm{d}s \\ &= \frac{x^{t-t_0}\left(x^h-1\right)}{\ln x}\left(1-\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}}\right)\boldsymbol{\mathcal{CE}}(t_0) \\ &= f_3\left(t,h;\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}},\Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}},g_Y\right)\end{aligned} \tag{19}$$

and:

$$x = (1+g_Y)\left(1-\Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}\right) \tag{20}$$

Therefore, the instantaneous decarbonization rate is the optimal value $\Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}$ that satisfies the following equation:

$$\boldsymbol{\mathcal{R}}^{\star}(t) = \lim_{h\to 0}\inf\left\{\theta : \boldsymbol{\mathcal{CB}}(t_0,t) + f_3\left(t,h;\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}},\theta,g_Y\right) \le \boldsymbol{\mathcal{CB}}(t_0,t+h)\right\} \tag{21}$$

By construction, $\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)$ and $\boldsymbol{\mathcal{R}}^{\star}(t)$ may differ substantially. Indeed, we have:

$$1-\boldsymbol{\mathcal{R}}(t_0,t) = \left(1-\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)\right)^{t-t_0}\left(1-\boldsymbol{\mathcal{R}}^{-}\right) \tag{22}$$

and:

$$\begin{aligned}1-\boldsymbol{\mathcal{R}}(t_0,t+h) &= \left(1-\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t+h)\right)^{t+h-t_0}\left(1-\boldsymbol{\mathcal{R}}^{-}\right) \\ &\approx \left(1-\boldsymbol{\mathcal{R}}(t_0,t)\right)\left(1-\boldsymbol{\mathcal{R}}^{\star}(t)\right)^{h}\end{aligned} \tag{23}$$

We deduce that:

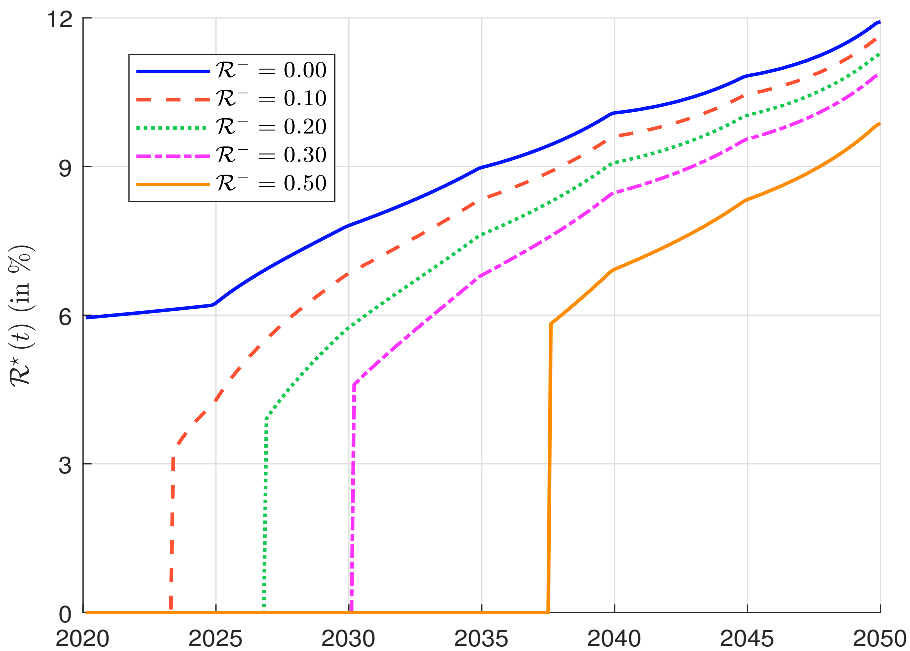
$$\begin{aligned}1-\boldsymbol{\mathcal{R}}(t_0,t+\mathrm{d}t) &= \left(1-\boldsymbol{\mathcal{R}}(t_0,t)\right)\left(1+\ln\left(1-\boldsymbol{\mathcal{R}}^{\star}(t)\right)\,\mathrm{d}t\right) \\ &\approx \left(1-\boldsymbol{\mathcal{R}}(t_0,t)\right)\left(1-\boldsymbol{\mathcal{R}}^{\star}(t)\,\mathrm{d}t\right) \\ &= \left(1-\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)\right)^{t-t_0}\left(1-\boldsymbol{\mathcal{R}}^{-}\right)\left(1-\boldsymbol{\mathcal{R}}^{\star}(t)\,\mathrm{d}t\right)\end{aligned} \tag{24}$$

In the case $\boldsymbol{\mathcal{R}}^{-} = 0$, we have the following approximation:

$$\begin{aligned}\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t) &\approx -\frac{1}{t-t_0}\int_{t_0}^{t}\ln\left(1-\boldsymbol{\mathcal{R}}^{\star}(s)\right)\,\mathrm{d}s \\ &\approx \frac{1}{t-t_0}\int_{t_0}^{t}\boldsymbol{\mathcal{R}}^{\star}(s)\,\mathrm{d}s\end{aligned} \tag{25}$$

More generally, $\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)$ can be viewed as an averaging function of $\boldsymbol{\mathcal{R}}^{\star}(t)$. If $\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)$ is an increasing function of t, we then expect that $\boldsymbol{\mathcal{R}}^{\star}(t) > \Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)$.

Figure 5: Estimated value $\boldsymbol{\mathcal{R}}^{\star}(t)$ (in %) from the IEA NZE scenario — $g_Y = 3\%$



In Figure 5, we have reported the instantaneous rate $\boldsymbol{\mathcal{R}}^{\star}(t)$ for several values of $\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}}$. If we compare these plots with those given in Figure 4, we verify that $\boldsymbol{\mathcal{R}}^{\star}(t) > \Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)$. Let us consider the case $\boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}} = 0$. If the fund manager would like to follow the IEA NZE scenario and if we assume that $g_Y = 3\%$, he must decarbonize his portfolio with a rate of 6% at the beginning. Then, he must progressively increase the decarbonization rate to reach 12% by 2050.

Remark 3. *The previous instantaneous rate* $\Delta\boldsymbol{\mathcal{R}}^{\star}(t)$ *is different from the classic definition*[10].

Remark 4. *In Appendix B on page 90, we compare the two decarbonization rates* $\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)$ *and* $\boldsymbol{\mathcal{R}}^{\star}(t)$. *We also report the logarithmic and arithmetic mean values. This confirms that* $\Delta\boldsymbol{\mathcal{R}}^{\star}(t_0,t)$ *can be interpreted as the mean of* $\boldsymbol{\mathcal{R}}^{\star}(t)$.

To illustrate the aggressive nature of CTB and PAB pathways, we first estimate the implied growth rate g_Y that fits the intensity reduction pathway. The least square estimates are respectively equal to $\hat{g}_Y = 6.70\%$ and $\hat{g}_Y = 16.27\%$ for CTB and PAB. However, the fitted pathway is not appealing (see Figure 43 on page 91). Another approach consists in matching the carbon budget: $g_Y^{\star}(t_0,t) = \sup\left\{\theta : f_2\left(t; \boldsymbol{\mathcal{R}}^{-}_{\boldsymbol{\mathcal{CI}}}, \Delta\boldsymbol{\mathcal{R}}_{\boldsymbol{\mathcal{CI}}}, \theta\right) \leq \boldsymbol{\mathcal{CB}}(t_0,t)\right\}$. For instance, we obtain $g_Y^{\star}(2020, 2035) = 12.39\%$ for PAB. In Figure 44 on page 91, we have reported all the solutions $g_Y^{\star}(2020,t)$. These results clearly show that CTB and PAB pathways are too aggressive if we are confident in the IEA scenario.

[10]Since the relationship $\boldsymbol{\mathcal{CE}}(t) = Y(t)\,\boldsymbol{\mathcal{CI}}(t)$ can be written as $\ln\boldsymbol{\mathcal{CE}}(t) = \ln Y(t) + \ln\boldsymbol{\mathcal{CI}}(t)$, we deduce that $\mathrm{d}\ln\boldsymbol{\mathcal{CI}}(t) = \mathrm{d}\ln\boldsymbol{\mathcal{CE}}(t) - \mathrm{d}\ln Y(t)$. Let $\varrho_{\boldsymbol{\mathcal{CI}}}(t)$ be the instantaneous rate of change. We have $\mathrm{d}\boldsymbol{\mathcal{CI}}(t) = -\varrho_{\boldsymbol{\mathcal{CI}}}(t)\,\boldsymbol{\mathcal{CI}}(t)\,\mathrm{d}t$. This implies that $\varrho_{\boldsymbol{\mathcal{CI}}}(t) = \ln(1+g_Y) - \partial_t \ln\boldsymbol{\mathcal{CE}}(t)$. We verify that $\Delta\boldsymbol{\mathcal{R}}^{\star}(t) \neq \varrho_{\boldsymbol{\mathcal{CI}}}(t)$.

3 Net zero metrics

Before investigating the construction of net zero portfolios, we have to define the metrics that are useful when implementating a net zero investment policy. As explained in the introduction, we must consider two dimensions: the decarbonization dimension and the transition dimension. Therefore, we consider two types of metrics. Net zero carbon metrics are used to assess the first dimension. They are generally related to the concept of carbon footprint. Net zero transition metrics are used to assess the second dimension. They measure the capacity for financing the transition to a low-carbon economy. Since net zero carbon metrics are generally physical measures expressed in CO_2e, net zero transition metrics are rather monetary measures expressed in dollars. Another important issue is the dynamic property of net zero investing. This is the big difference from a simple portfolio decarbonization exercise. Therefore, we must distinguish between static and dynamic (or forward-looking) measures. Indeed, a net zero emissions scenario is described by a trajectory. Net zero investing cannot be reduced to the process that locates the node of the trajectory corresponding to a given date. Net zero investing must imply a dynamic pathway that corresponds to the trajectory. This is the real challenge of net zero investing.

3.1 Net zero carbon metrics

3.1.1 Static measures of carbon footprint

Scope definition The GHG Protocol corporate standard classifies a company's greenhouse gas emissions in three scopes[11]:

- Scope 1 denotes direct GHG emissions occurring from sources that are owned and controlled by the issuer.
- Scope 2 corresponds to the indirect GHG emissions from the consumption of purchased electricity, heat or steam.
- Scope 3 are other indirect emissions (not included in scope 2) of the entire value chain. They can be divided into two main categories[12]:
  - Upstream scope 3 emissions[13] are defined as indirect carbon emissions related to purchased goods and services.
  - Downstream scope 3 emissions[14] are defined as indirect carbon emissions related to sold goods and services.

Scope 1 emissions are also called direct emissions, whereas indirect emissions encompass both scopes 2 and 3 GHG emissions. Unlike scopes 1 and 2, scope 3 is an optional reporting category. Moreover, indirect emissions may present big challenges in terms of double/triple counting. For instance, a large part of scope 2 may be found in scope 1 of Utilities companies

[11]The latest version of corporate accounting and reporting standard can be found at www.ghgprotocol.org/corporate-standard.

[12]The upstream value chain includes all activities related to the suppliers whereas the downstream value chain refers to post-manufacturing activities.

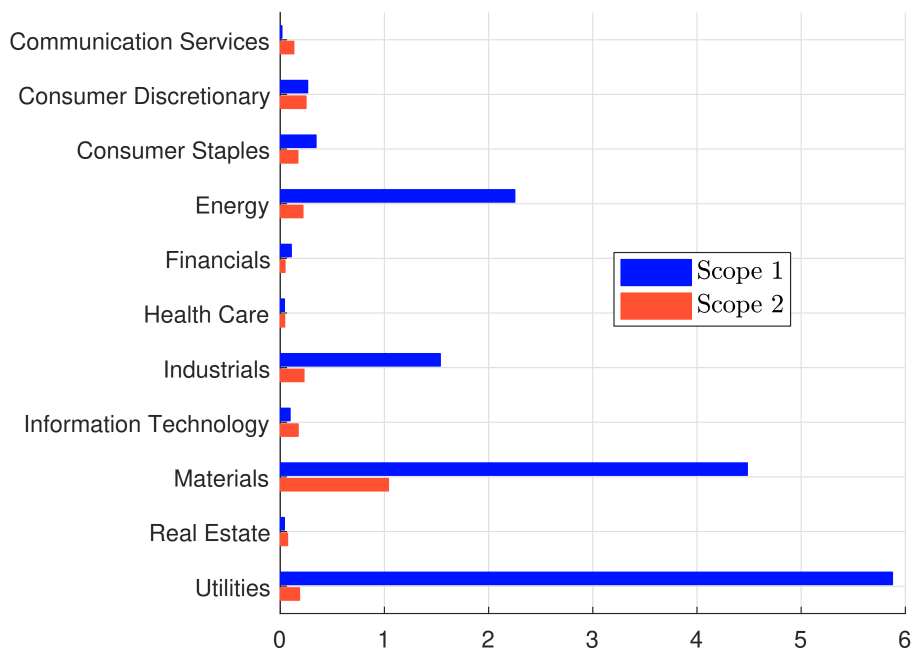
[13]In the GHG Protocol, the upstream scope 3 is based on 8 sub-categories: (1) purchased goods and services, (2) capital goods, (3) fuel and energy related activities, (4) upstream transportation and distribution, (5) waste generated in operations, (6) business travel, (7) employee commuting and (8) upstream leased assets.

[14]In the GHG Protocol, the downstream scope 3 is based on these next 7 sub-categories: (9) downstream transportation and distribution, (10) processing of sold products, (11) use of sold products, (12) end-of-life treatment of sold products, (13) downstream leased assets, (14) franchises and (15) investments.

that produce or distribute electricity. A part of upstream scope 3 is already present in Materials and Industrials companies, whereas another part of downstream scope 3 is embedded in Retailing and Distribution industries. Issues on data quality and double counting bias explain that portfolio decarbonization is generally based on scopes 1 and 2 emissions.

Carbon emissions We consider the Trucost dataset of carbon emissions as of 01/06/2022 and analyze the distribution of carbon emissions in 2019 for around 15 000 companies. We prefer to use the year 2019 instead of the year 2020, because the covid-19 crisis had a significant impact on the carbon footprint. In Figure 6, we have reported the scopes 1 and 2 carbon emissions per GICS sector. We notice that including scope 2 has a limited impact, except for some low-carbon sectors such as Consumer Services, Information Technology and Real Estate. In Table 35 on page 79, we have calculated the breakdown of carbon emissions. Scopes 1 and 2 represent 17.6 $GtCO_2e$, and the most important sectoral contributors are Utilities (34.4%), Materials (31.4%), Energy (14.0%) and Industrials (10.0%). This means that these 4 strategic sectors explain about 90% of scopes 1 and 2 carbon emissions.

Figure 6: 2019 carbon emissions per GICS sector in $GtCO_2e$ (scopes 1 & 2)



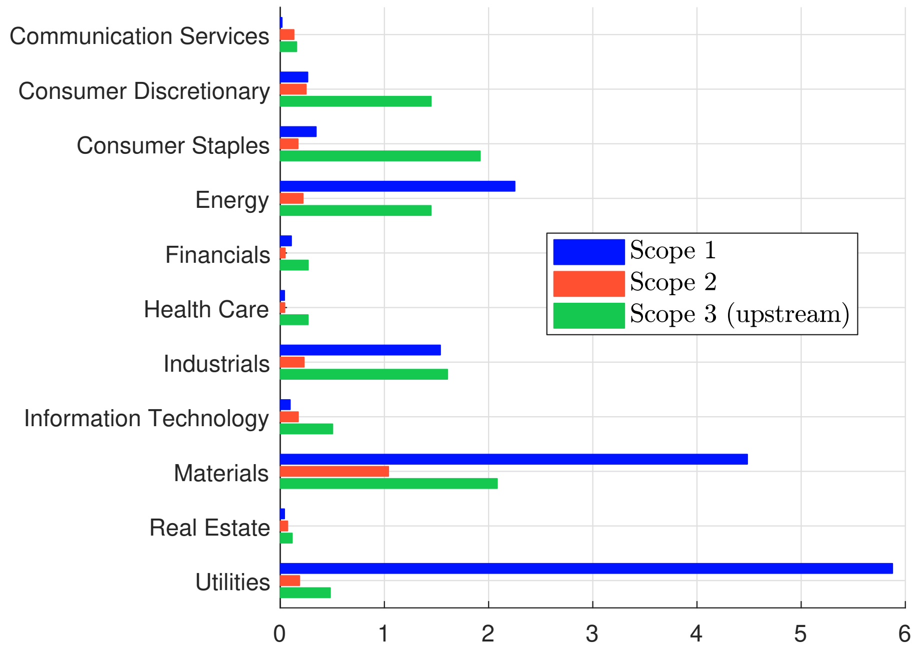
Source: Trucost (2022) & Authors' calculations.

In Figure 7, we observe that some sectors are highly impacted by the upstream scope 3 emissions. For instance, the ratio $\frac{\mathcal{SC}_3^{\text{up}}}{\mathcal{SC}_{1-2}}$ is greater than 2.5 for Consumer Discretionary, Consumer Staples and Health Care, and is close to 2 for Information Technology[15]. Among the strategic sectors, Energy and Industrials are the most penalized whereas the upstream scope 3 emissions of Utilities is relatively small compared to its scope 1 emissions.

While the impact of the upstream scope 3 is significant, the impact of the downstream scope 3 is huge as demonstrated in Figure 8. Four sectors have very large downstream carbon

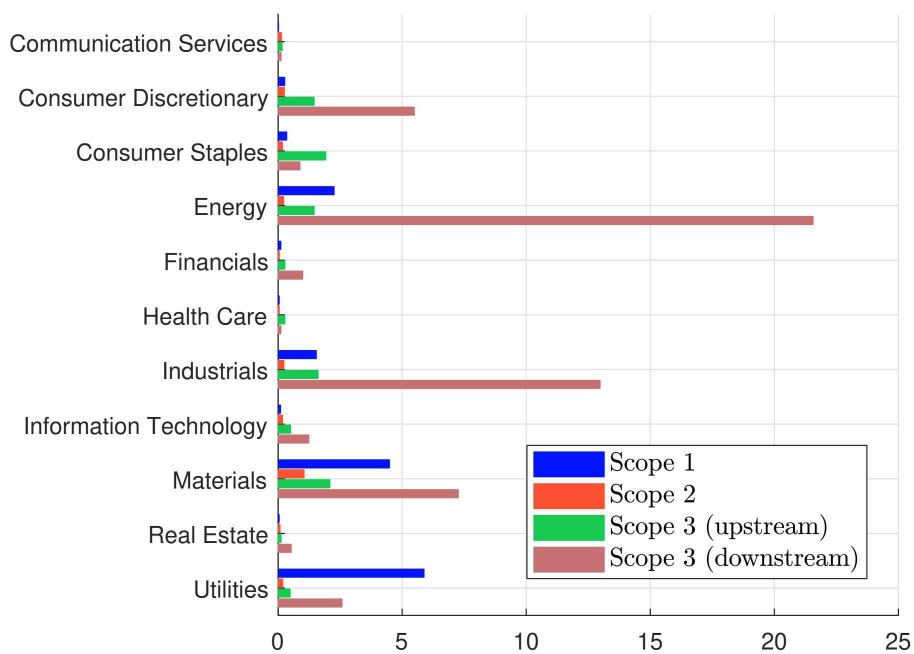
[15] See Table 36 on page 79.

Figure 7: 2019 carbon emissions per GICS sector in $GtCO_2e$ (scopes 1, 2 & 3 upstream)



Source: Trucost (2022) & Authors' calculations.

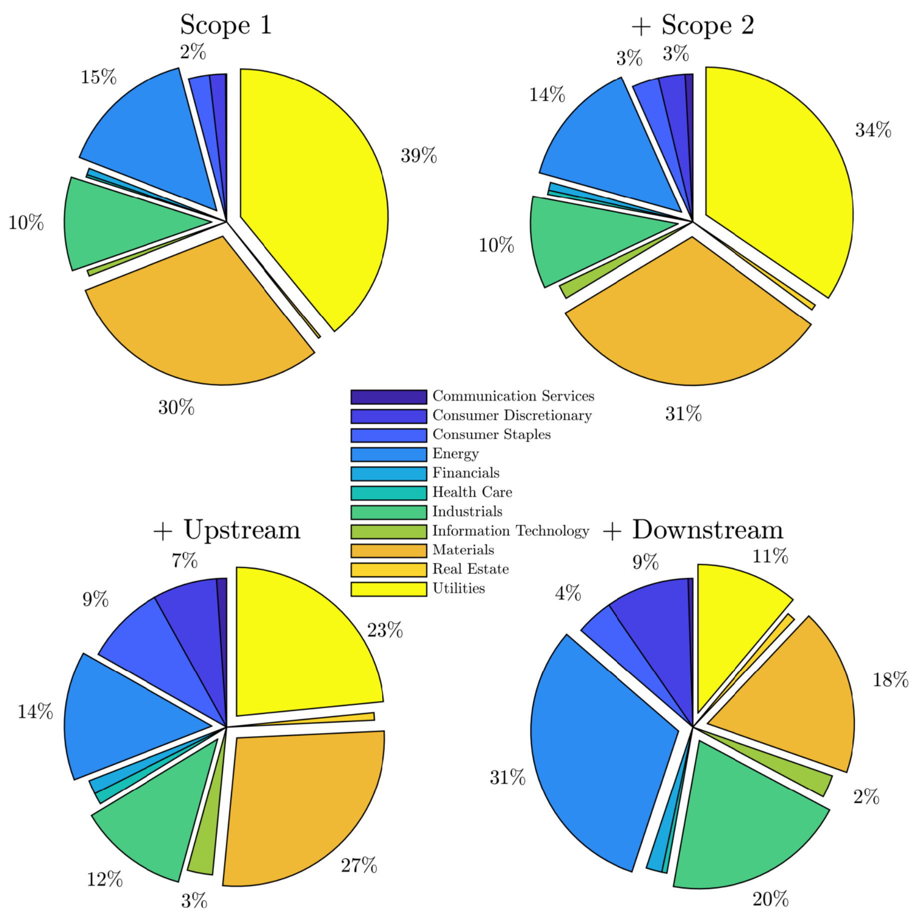
Figure 8: 2019 carbon emissions per GICS sector in $GtCO_2e$ (scopes 1, 2, 3 upstream & 3 downstream)



Source: Trucost (2022) & Authors' calculations.

emissions: Consumer Discretionary, Energy, Industrials and Materials. While Utilities has the most important contribution in terms of scopes 1 and 2 since it represents 34.4% of carbon emissions, its contribution to scope 3 is relatively modest and is equal to 4.8%. Including or not scope 3, in particular the downstream carbon emissions, changes the whole picture of the breakdown between the sectors. Figure 9 is a visualisation of the sectoral contribution by considering the addition of several scopes. At each step, the contribution of Materials and Utilities decreases whereas it increases for Consumer Discretionary, Energy, Industrials and Information Technology. Among the most significant sectors[16], the behavior of Consumer Staples is singular since its contribution increases when adding scope 2 and upstream scope 3, but decreases when considering downstream scope 3.

Figure 9: Sectoral contribution in %



Source: Trucost (2022) & Authors' calculations.

Remark 5. *When considering carbon emissions, double counting is a real issue. According to Table 35 on page 79, the total carbon emissions is* 17.6 $GtCO_2e$ *for scopes* $1+2$*, and* 81.6 $GtCO_2e$ *for scopes* $1+2+3$*, while we estimate that the world emits about* 36 $GtCO_2e$ *per year. This issue is discussed later.*

[16]They correspond to sectors that have a contribution greater than 2%.

Carbon intensity From a financial point of view, it does not make sense to compare and aggregate the carbon emissions of a large cap company with the carbon emissions of a small cap company. Therefore, portfolio managers use the concept of carbon intensity, which is a normalization of the carbon emissions. The goal is then to compare and aggregate the carbon footprint of several issuers with different business sizes. From a mathematical point of view, we have:

$$\mathcal{CI} = \frac{\mathcal{CE}}{Y} \tag{26}$$

where $\mathcal{CE}$ is the company's carbon emissions and Y is an output indicator measuring its activity. We distinguish two categories: physical and monetary intensities. In the case of physical intensity, we generally use metrics that measure the production units[17]. In the case of monetary intensity, we can consider accounting or market-based metrics. For instance, we can use revenues or sales to normalize carbon emissions. Some examples are provided in Table 3. These figures illustrate some issues in the computation of the carbon footprint at the issuer level. First, it is obvious that it is important to take into account scope 3 to have the real picture of the carbon footprint of an issuer. Indeed, we notice that some issuers have a low scope 1, because they have more or less outsourced the manufacturing of their products. Since a part of the production is located in upstream scope 3, we can not make a fair comparison between issuers if we only consider scopes 1 and 2. We face a similar issue with the distribution of the products. This implies that a part of downstream scope 3 of some issuers may be located in scope 1 of other issuers.

The magnitude of some scope 3 carbon intensities raises the question of their computation. Indeed, while scopes 1 and 2 are mandatory to report, there is no obligation for a company to report its scope 3. Moreover, while there is one unique figure for scopes 1 and 2 in the CDP reporting files, scope 3 is split into 15 categories (See Footnotes 13 and 14 on page 19), and it is extremely rare that a company reports all scope 3 categories. This explains that the frequency of estimated values is larger for scope 3. How to compare the reported value for one company with the estimated value for another company? The answer is not obvious since the estimated value depends on the statistical model of the data provider. Moreover, it seems that the GHG protocol for scope 3 is not enough precise because we may observe very large differences between two reported companies of the same industry (GICS level 3).

In Figure 10, we show the distribution of carbon intensities. Since the range may be very large (from zero to several thousand), we use a logarithmic scale. Moreover, the dotted vertical lines indicate the 5^{th} and 95^{th} percentiles. We observe that the distribution support is very large for scopes 1, 2 and 3 downstream. In this case, there are many extreme points with very low and very high carbon intensities. Therefore, it is relatively easy to reduce the carbon footprint of a portfolio. We must remove corporates with the highest carbon intensity (for instance greater than 1000) and replace them with corporates with the lowest carbon intensity (for instance less than 5). Now, if we focus on upstream scope 3, we obtain another story, because the range is not so large. Indeed, we do not have corporates with very low carbon intensity. Therefore, incorporating upstream scope 3 changes the nature of portfolio decarbonization.

Remark 6. *The question of double-counting is less important when we consider carbon intensities, especially monetary measures. Indeed, the carbon intensity can be seen as a scoring system, and portfolio managers generally use carbon intensity in a relative way, and not in an absolute way. For instance, they do not target a given carbon intensity. Their goal is then to reduce the carbon intensity relatively to a benchmark, without analyzing the*

[17] For instance, we can express the carbon intensity in CO_2e/kWh for an Electricity company.

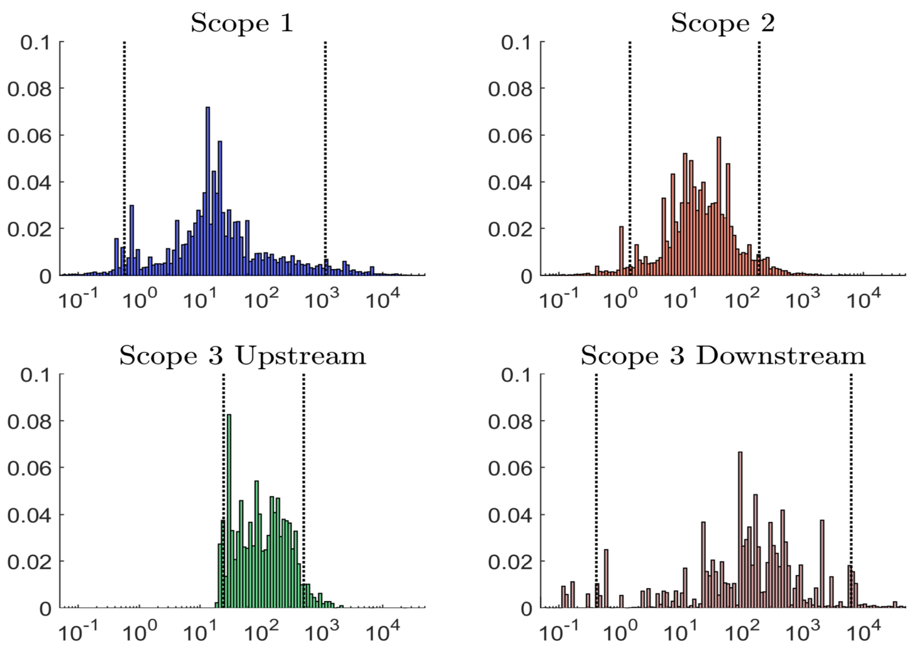
Table 3: Examples of 2019 carbon emissions and intensities

Company	Emission (in tCO_2e)				Revenue	Intensity (in tCO_2e/\$ mn)			
	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{\text{up}}$	$\mathcal{SC}_3^{\text{down}}$	(in \$ mn)	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{\text{up}}$	$\mathcal{SC}_3^{\text{down}}$
Airbus	576 705	386 674	12 284 183	23 661 432	78 899	7.3	4.9	155.7	299.9
Allianz	46 745	224 315	3 449 234	3 904 000	135 279	0.3	1.7	25.5	28.9
Amazon	5 760 000	5 500 000	20 054 722	10 438 551	280 522	20.5	19.6	71.5	37.2
Apple	50 549	862 127	27 624 282	5 470 771	260 174	0.2	3.3	106.2	21.0
BNP Paribas	64 829	280 789	1 923 307	1 884	78 244	0.8	3.6	24.6	0.0
Boeing	611 001	871 000	9 878 431	22 959 719	76 559	8.0	11.4	129.0	299.9
BP	49 199 999	5 200 000	103 840 194	582 639 687	276 850	177.7	18.8	375.1	2 104.5
Caterpillar	905 000	926 000	15 197 607	401 993 744	53 800	16.8	17.2	282.5	7 472.0
Danone	722 122	944 877	28 969 780	4 464 773	28 308	25.5	33.4	1 023.4	157.7
Enel	69 981 891	5 365 386	8 726 973	53 774 821	86 610	808.0	61.9	100.8	620.9
Exxon	111 000 000	9 000 000	107 282 831	594 131 943	255 583	434.3	35.2	419.8	2 324.6
JPMorgan Chase	81 655	692 299	3 101 582	15 448 469	115 627	0.7	6.0	26.8	133.6
LVMH	67 613	262 609	11 853 749	942 520	60 083	1.1	4.4	197.3	15.7
Microsoft	113 414	3 556 553	5 977 488	4 003 770	125 843	0.9	28.3	47.5	31.8
Nestle	3 291 303	3 206 495	61 262 078	33 900 606	93 153	35.3	34.4	657.6	363.9
PepsiCo	3 552 415	1 556 523	32 598 029	14 229 956	67 161	52.9	23.2	485.4	211.9
Pfizer	734 638	762 840	4 667 225	133 468	51 750	14.2	14.7	90.2	2.6
Roche	288 157	329 541	5 812 735	347 437	64 154	4.5	5.1	90.6	5.4
Samsung Electronics	5 067 000	10 998 000	33 554 245	60 978 947	197 733	25.6	55.6	169.7	308.4
TotalEnergies	40 909 135	3 596 127	49 817 293	456 993 576	200 316	204.2	18.0	248.7	2 280.0
Toyota	2 522 987	5 227 844	66 148 020	330 714 268	272 608	9.3	19.2	242.6	1 213.2
Volkswagen	4 494 066	5 973 894	65 335 372	354 913 446	282 817	15.9	21.1	231.0	1 254.9
Walmart	6 101 641	13 057 352	40 651 079	32 346 229	514 405	11.9	25.4	79.0	62.9

Source: Trucost (2022) & Authors' calculations.

absolute value of the benchmark itself. Moreover, the aggregation at the portfolio level is generally done thanks to the WACI[18] *measure, which indicates that the carbon intensity is more viewed as a score than a physical measure.*

Figure 10: Distribution of carbon intensities (logarithmic scale)



Source: Trucost (2022) & Authors' calculations.

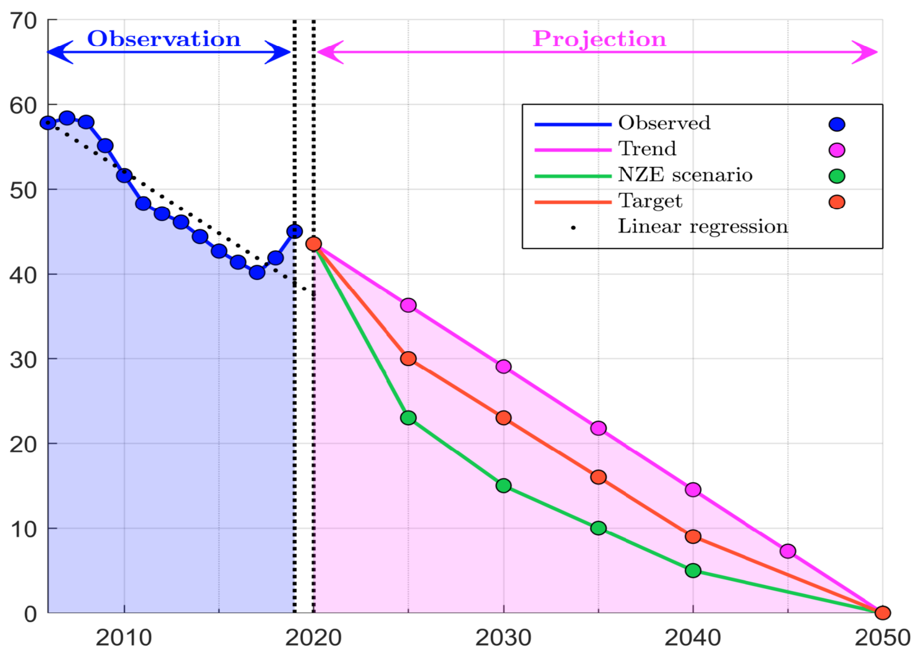
3.1.2 Dynamic measures of carbon footprint

The $\mathcal{PAC}$ framework Dynamic measures of carbon emissions (or net zero carbon metrics) are generally defined according to the $\mathcal{PAC}$ framework (Le Guenedal *et al.*, 2022). $\mathcal{PAC}$ stands for participation, ambition and credibility. Its purpose is to evaluate the decarbonization capacity and willingness of issuers. To understand this framework, we consider the example given in Figure 11. For a given issuer, we have reported the historical trajectory of carbon emissions from 2005 to 2019 (blue line). Therefore, we can estimate the associated linear trend model and project the future carbon emissions by assuming that the issuer will do the same efforts in the future than in the past (violet line). Therefore, the participation pillar measures the past efforts of the issuer. In our example, the carbon trend is negative, meaning that the issuer has globally reduced its carbon emissions in the past. Moreover, we notice that the issuer can reach net zero by 2050 if it continues its efforts. The participation of this issuer is then good and positive. The second pillar measures the ambition of the issuer, and compares the target trajectory on one side (red line) and the net zero scenario of the sector on the other side (green line). The underlying idea is to assess the announcements of the issuer concerning its net zero policy. In our case, the target trajectory being above the net zero scenario, this issuer has not a lot of ambition. Finally, we can measure the

[18] Weighted average of carbon intensity.

credibility of the targets by comparing the current trend of carbon emissions (violet line) and the reduction targets announced by the company (red line). In our case, the credibility of the issuer is good and positive. The $\mathcal{PAC}$ framework described above constitutes the backbone of temperature ratings provided by data providers.

Figure 11: Illustration of the $\mathcal{PAC}$ framework



Carbon momentum Temperature ratings may be viewed as black-box systems. This is why some portfolio managers prefer to focus on the participation pillar since it only depends on the historical trajectory. Le Guenedal *et al.* (2022) define the carbon trend by considering the linear constant trend model:

$$\mathcal{CE}(t) = \beta_0 + \beta_1 \cdot t + u(t) \tag{27}$$

Using the least squares method, we can estimate β_0 and β_1. Let t_0 be the base year. We can build the carbon trajectory implied by the current trend by applying the projection:

$$\widehat{\mathcal{CE}}(t) = \mathcal{CE}(t_0) + \hat{\beta}_1 \cdot (t - t_0) \tag{28}$$

for $t \geq t_0$. This model is very simple since the underlying idea is to extrapolate the past trajectory. Following Le Guenedal *et al.* (2022), we can consider a dynamic version of the estimation method and we note $\hat{\beta}_1(t)$ the slope coefficient of the trend model that is estimated at time t. We define the long-term carbon momentum as the ratio between the slope and the current carbon emissions:

$$\mathcal{CM}^{\mathcal{L}ong}(t) = \frac{\hat{\beta}_1(t)}{\mathcal{CE}(t)} \tag{29}$$

Le Guenedal *et al.* (2022) also introduce the concept of carbon velocity, which measures the normalized slope change between $t - h$ and t:

$$\boldsymbol{v}^{(h)}(t) = \frac{\hat{\beta}_1(t) - \hat{\beta}_1(t-h)}{h} \tag{30}$$

The rationale for this measure is the following. A net zero emissions commitment implies a negative trend: $\hat{\beta}_1(t) < 0$. Nevertheless, it can take many years for a company to change the sign of the trend slope if it has a bad track record. Therefore, we can use the velocity to verify that the company is making significant efforts in the recent period. In this case, we must have $\boldsymbol{v}^{(h)}(t) < 0$ for low values[19] of h. Therefore, the short-term carbon momentum is defined as:

$$\mathcal{CM}^{Short}(t) = \frac{\boldsymbol{v}^{(1)}(t)}{\mathcal{CE}(t)} \tag{31}$$

Remark 7. *The previous approach can be extended to the carbon intensity measure* $\mathcal{CI}(t)$*. Moreover, we can use a logarithmic model instead of a linear model:*

$$\ln \mathcal{CE}(t) = \beta_0 + \beta_1 \cdot t + u(t) \tag{32}$$

In this case, we have:

$$\widehat{\mathcal{CE}}(t) = \mathcal{CE}(t_0)\, e^{\hat{\beta}_1 \cdot (t - t_0)} \tag{33}$$

Sequential decarbonization versus self-decarbonization For net zero investment portfolios, we remind that the decarbonization pathway is done with respect to a benchmark at a given reference year t_0. Let $\mathcal{CI}(t, x; \mathcal{F}_s)$ be the carbon intensity of Portfolio x calculated at time t with the information $\mathcal{F}_s$ available at time s. At time t, Portfolio $x(t)$ must satisfy:

$$\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right) \le \left(1 - \mathcal{R}_{\mathcal{CI}}(t_0, t)\right) \mathcal{CI}\left(t_0, b(t_0); \mathcal{F}_{t_0}\right) \tag{34}$$

where $b(t_0)$ is the benchmark at time t_0. We assume that the portfolio is rebalanced at time $t+1$. In this case, we will choose a new portfolio $x(t+1)$ such that:

$$\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) \le \left(1 - \mathcal{R}_{\mathcal{CI}}(t_0, t+1)\right) \mathcal{CI}\left(t_0, b(t_0); \mathcal{F}_{t_0}\right) \tag{35}$$

We don't have to rebalance the portfolio at time $t+1$ if and only if:

$$\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right) \le \left(1 - \mathcal{R}_{\mathcal{CI}}(t_0, t+1)\right) \mathcal{CI}\left(t_0, b(t_0); \mathcal{F}_{t_0}\right) \tag{36}$$

Therefore, the variation $\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)$ between two rebalancing dates can be breakdown into two components:

1. a self-decarbonization $\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)$ and;
2. an additional decarbonization $\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right)$.

The self-decarbonization ratio is then defined as:

$$\begin{aligned}
\mathcal{SR}(t+1) &= \frac{\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)}{\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) - \mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)} \\
&= \frac{\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right) - \mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right)}{\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right) - \mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right)}
\end{aligned} \tag{37}$$

By construction, we have:

$$\mathcal{SR}(t+1) \le 1 \tag{38}$$

The upper bound is reached when we do not have to rebalance the portfolio. If the carbon intensity of the current portfolio has not changed between t and $t+1$, the self-decarbonization ratio is equal to zero. The worst case is obtained when the carbon intensity has increased, implying that $\mathcal{SR}(t+1) < 0$.

Table 4: Backtesting of net-zero investment portfolios

s	$\mathcal{CI}^s_\star$	Case #1			Case #2		
		$\mathcal{CI}^s_x$	$\mathcal{CI}^{s+1}_x$	$\mathcal{SR}^s$	$\mathcal{CI}^s_x$	$\mathcal{CI}^{s+1}_x$	$\mathcal{SR}^s$
t	100.0	100.0	99.0		100.0	92.0	
$t+1$	93.0	93.0	91.2	14.3%	92.0	85.0	100.0%
$t+2$	86.5	86.5	91.3	27.7%	85.0	80.2	100.0%
$t+3$	80.4	80.4	78.1	−78.7%	80.2	75.0	100.0%
$t+4$	74.8	74.8	74.2	41.1%	74.8	70.0	96.3%
$t+5$	69.6	69.6	70.7	11.5%	69.6	68.9	92.3%
$t+6$	64.7	64.7	62.0	−22.4%	64.7	60.0	14.3%
$t+7$	60.2	60.2	60.0	60.0%	60.0	55.1	100.0%
$t+8$	55.9	55.9	58.3	4.7%	55.1	52.0	100.0%
$t+9$	52.0	52.0	53.5	−61.5%	52.0	47.5	100.0%
$t+10$	48.4	48.4	50.5	−41.7%	47.5	45.5	100.0%

We use the following notations for the labels: $\mathcal{CI}^s_\star$ is equal to $\left(1 - \mathcal{R}_{\mathcal{CI}}(t_0, s)\right) \mathcal{CI}\left(t_0, b(t_0); \mathcal{F}_{t_0}\right)$, $\mathcal{CI}^s_x = \mathcal{CI}\left(s, x(s); \mathcal{F}_s\right)$ is the carbon intensity of Portfolio $x(s)$ at the rebalancing date s, $\mathcal{CI}^{s+1}_x = \mathcal{CI}\left(s+1, x(s); \mathcal{F}_{s+1}\right)$ is the carbon intensity of Portfolio $x(s)$ at the end of the period $[s, s+1]$ before the next rebalancing date $s+1$, and $\mathcal{SR}^s$ is the value of the self-decarbonization ratio for the period $[s-1, s]$.

Let us consider an example to illustrate the concept of self-decarbonization. We assume that the carbon intensity of the benchmark is equal to 200 tCO_2e/\$ mn at the reference date. We begin to reduce the carbon footprint by 50%, targeting a carbon intensity of 100 tCO_2e/\$ mn at time t. Then, we use the following pathway of decarbonization rates: $53.50\%, 56.76\%, \ldots, 73.98\%, 75.80\%$. The targeted carbon intensity is reported in the second column in Table 4. We obtain 93 tCO_2e/\$ mn at time $t+1$, then 86.5, 80.4, etc. until we obtain 48.4 tCO_2e/\$ mn at time $t+10$. We consider a first portfolio. In the third column, we indicate the values taken by $\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right)$, $\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right)$, etc. The fourth column indicates the carbon intensity of the portfolio at the end of the period: $\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right)$, $\mathcal{CI}\left(t+2, x(t+1); \mathcal{F}_{t+2}\right)$, etc. For instance, we have $\mathcal{CI}\left(t, x(t); \mathcal{F}_t\right) = 100$ and $\mathcal{CI}\left(t+1, x(t); \mathcal{F}_{t+1}\right) = 99$. The carbon footprint of this portfolio has been reduced during the period $[t, t+1]$, but the self-decarbonization is not enough to reach the target 93 for the rebalancing date $t+1$. Therefore, we need to rebalance the portfolio to impose that $\mathcal{CI}\left(t+1, x(t+1); \mathcal{F}_{t+1}\right) = 93$. The self-decarbonization ratio is not high and is equal to 14.3%. Sometimes, we can also observe an increase in the carbon footprint during two rebalancing dates. This is the case of portfolio $x(t+2)$ since its carbon intensity is equal to 86.5 at the beginning of the period and 91.3 at the end of the period. Again, we need to rebalance the portfolio to match the new target, which is 80.4. Case #1 is an example where the net zero pathway is mainly obtained by sequential decarbonization. Case #2 is very interesting because we don't need to rebalance the portfolio most of the time. Indeed, the self-decarbonization is enough for 7 among 10 rebalancing dates.

Remark 8. *In Figures 45 and 46 on page 92, we have created a data visualization about the importance of self-decarbonization (green bars) with respect to sequential decarbonization (blue bars) and negative decarbonization (red bars). This last one occurs when the carbon intensity of the portfolio increases between two rebalancing dates. In Case #1, we see that self-decarbonization is secondary, whereas it dominates in Case #2.*

[19] Generally, h is equal to 1, 2 or 3 years.

Remark 9. *The computation of self decarbonization ratios is a first step towards implementing the backtesting of net zero investment portfolios. While backtesting is central to risk management and measurement, it seems that it is completely ignored by net zero processes. However, backtesting analyzes the ex-post validity of a model. Therefore, it is appropriate for validating or not net zero investment processes. Indeed, investors have the right to understand the limits of any net zero portfolio model.*

To maximize the self-decarbonization ratio, we need to model the probability distribution of the estimator $\widehat{\mathcal{CI}}\left(t+1, x\left(t\right); \mathcal{F}_t\right)$. We now understand why carbon trend, temperature rating or carbon momentum have great importance in a net zero process. For instance, the current carbon footprint gives no information about its dynamics. Indeed, if we assume that $\widehat{\mathcal{CI}}\left(t+1, x\left(t\right); \mathcal{F}_t\right) = \mathcal{CI}\left(t, x\left(t\right); \mathcal{F}_t\right)$, we have $\mathbb{E}\left[\widehat{\mathcal{CI}}\left(t+1, x\left(t\right); \mathcal{F}_t\right)\right] > \mathcal{CI}\left(t+1, x\left(t+1\right); \mathcal{F}_{t+1}\right)$ whereas we prefer to have the inequality $\mathbb{E}\left[\widehat{\mathcal{CI}}\left(t+1, x\left(t\right); \mathcal{F}_t\right)\right] \leq \mathcal{CI}\left(t+1, x\left(t+1\right); \mathcal{F}_{t+1}\right)$. Therefore, the real challenge lies in having an idea about the dynamics of the carbon footprint. Even if carbon trend or momentum seems to be simplistic at first sight from a statistical point of view, they are nevertheless relatively objective, they do not depend on sophisticated models and they are easy to understand.

Table 5: Statistics (in %) of carbon momentum $\mathcal{CM}^{\mathcal{L}ong}\left(t\right)$

Statistics	Carbon emissions			Carbon intensity		
	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$
Median	1.7	2.6	2.6	−2.3	−1.7	−1.6
Negative	43.3	37.7	34.9	69.5	66.6	72.0
Positive	56.7	62.3	65.1	30.5	33.4	28.0
$< -10\%$	22.7	17.5	13.3	21.1	14.4	6.5
$< -5\%$	30.0	24.4	19.9	31.5	22.1	13.3
$> +5\%$	34.5	37.6	35.6	11.6	13.2	7.5
$> +10\%$	17.1	17.6	15.0	5.8	6.5	3.3

Source: Trucost (2022) & Authors' calculations.

The table above gives some statistics about carbon momentum. Since we impose to have a track record of 5 years at least, we can compute the long-term carbon momentum for only 69% of issuers that are in the Trucost database. The median value of $\mathcal{CM}^{\mathcal{L}ong}\left(t\right)$ is equal to 1.7% for scope 1, 2.6% when we include scope 2, and 2.6% when we consider upstream scope 3. The median value increases when we incorporate indirect carbon emissions, for both carbon emission and carbon intensity. We cannot compute the carbon trend for scope $1+2+3$ because the data history for downstream emissions would be too short. The carbon momentum is negative for 34.9% of issuers when we consider $\mathcal{SC}_{1-3}^{\text{up}}$. This means that a majority of issuers have a positive carbon trend. For instance, 15% of issuers have a carbon momentum greater than 10%! If we consider carbon intensity instead of carbon emission, we obtain different results. Indeed, issuers with a negative trend dominate issuers with a positive trend. Therefore, it is easier to build a self-decarbonized portfolio when we consider the carbon intensity measure.

Remark 10. *Considering carbon emissions or carbon intensities gives two very different pictures of the carbon momentum. Even if meeting net zero emission in 2050 implies meeting net zero intensity as well, the pathways to meet this objective are very different.*

3.2 Net zero transition metrics

While the previous section presents the metrics associated with the decarbonization dimension, we need to specify the greenness measures for implementing the transition dimension. However, contrary to the carbon footprint, which is a well-defined concept, greenness is more difficult to assess. In fact, it is a multi-faceted concept. For instance, if one issuer changes its business model so that its new products are carbon efficient, we can measure the issuer's greenness based on the avoided emissions generated by the change of the business model. For other issuers, the greenness can be evaluated by estimating the R&D amount dedicated to green projects. Therefore, we observe a big difference between carbon and transition metrics. Indeed, while it makes sense to compute the carbon footprint of all issuers, the greenness may be indefinite for some issuers, because they have no vocation to participate in the transition. They are neutral and are not exposed to the green business. All these remarks argue in favor of considering simple and homogeneous measures of greenness. For that, we first need to specify a green taxonomy.

3.2.1 Green taxonomy

Definition The purpose of a green financial taxonomy is to define what is green, and its objective is to inform investors about the greenness of their investments. Therefore, they can evaluate whether these levels satisfy or not their expectations. A green taxonomy is all the more important as we observe a strong development of green sentiment among investors (Brière and Ramelli, 2021). Moreover, MiFID II imposes new obligations to take into account sustainable preferences. In this context, the client must determine a minimum proportion that should be invested in environmentally sustainable assets. Therefore, a green taxonomy is necessary for both asset owners and managers.

Buhr and Cormack (2020) explained that "*a taxonomy is a way of organizing knowledge*" usually in a hierarchical order. This top-down approach has many advantages and is well known by investors. For instance, sector classification systems such as GICS or ICB use this method. In a similar way, Alessi and Battiston (2022) considered the NACE classification and estimated a taxonomy alignment coefficient (TAC) for each sector of activity. In this case, we can calculate the green intensity of the portfolio by using the breakdown of the allocation with respect to the NACE classification:

$$\mathcal{GI}(w) = \sum_{j=1}^{m} w_j \cdot \mathcal{GI}_j$$

where w_j is the weight of the j^{th} sector and $\mathcal{GI}_j$ is its green intensity[20]. Nevertheless, we also know that sectoral categories are heterogeneous even when we consider industry or sub-industry levels. In the bottom-up approach, we directly estimate the green intensity at the issuer level and we have:

$$\mathcal{GI}(x) = \sum_{i=1}^{n} x_i \cdot \mathcal{GI}_i$$

where x_i is the weight of the i^{th} issuer and $\mathcal{GI}_i$ is its green intensity. Since the bottom-up approach is more informative than the top-down approach because it operates at the most granular level, it is also more complex as it requires a lot of data. Moreover, we have to estimate these data when they are missing or not mandatory to report.

[20] The green intensity is equal to the TAC factor.

Remark 11. *From a theoretical point of view, the two approaches are equivalent if we assume that the issuer belongs to a single sector. Indeed, we have* $w_j = \sum_{i \in j} x_i$ *and:*

$$\mathcal{GI}_j = \frac{\sum_{i \in j} x_i \cdot \mathcal{GI}_i}{\sum_{i \in j} x_i}$$

We deduce that:

$$\begin{aligned} \mathcal{GI}(w) &= \sum_{j=1}^{m} \left(\sum_{i \in j} x_i \right) \cdot \left(\frac{\sum_{i \in j} x_i \cdot \mathcal{GI}_i}{\sum_{i \in j} x_i} \right) \\ &= \sum_{j=1}^{m} \sum_{i \in j} x_i \cdot \mathcal{GI}_i \\ &= \sum_{i=1}^{n} x_i \cdot \mathcal{GI}_i = \mathcal{GI}(x) \end{aligned}$$

In a multi-sector framework, the equality $\mathcal{GI}(w) = \mathcal{GI}(x)$ *does not hold because* $\sum_{j=1}^{m} x_{i,j} \cdot \mathcal{GI}_{i,j} \neq x_i \cdot \mathcal{GI}_i$ *where* $x_{i,j}$ *and* $\mathcal{GI}_{i,j}$ *are the allocation amount and the green intensity of issuer* i *in activity* j*. Another difference between the bottom-up and top-down approaches comes from the fact that the green intensities are calculated with all the issuers of the investment universe in the top-down approach. This is not the case with the bottom-up approach, which only considers the issuers that belong to the portfolio.*

As noticed by Buhr and Cormack (2020), a green taxonomy may be restrictive since it tells us nothing about the brownness of the issuer. For example, if an issuer has a green intensity of 30%, this implies that 70% is not green. It may correspond to an issuer whose brown intensity lays between 0% and 70%. Therefore, it is not possible to deduce a brown taxonomy from the green taxonomy. We can only deduce an upper bound:

$$0 \leq \mathcal{BI}_i \leq 1 - \mathcal{GI}_i$$

The advantage of having both a green taxonomy and a brown taxonomy is that we can determine the non-green-brown (or white) intensity $\mathcal{NI}_i$ of the issuer because of the following relationship:

$$\mathcal{BI}_i + \mathcal{NI}_i + \mathcal{GI}_i = 1$$

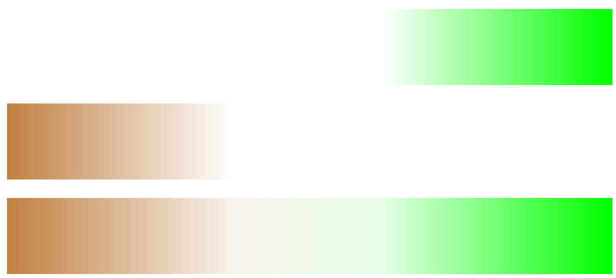
To avoid a black and white picture of greenness, another solution is to define a green taxonomy, whose range is between 0 and 200% and not between 0 and 100%. For instance, we can propose the following score:

$$\begin{aligned} \mathcal{GI}_i &= 2 \times \varpi_i^{\mathcal{G}\text{reen}} + 1 \times \left(1 - \varpi_i^{\mathcal{G}\text{reen}} - \varpi_i^{\mathcal{B}\text{rown}} \right) + 0 \times \varpi_i^{\mathcal{B}\text{rown}} \\ &= 1 + \varpi_i^{\mathcal{G}\text{reen}} - \varpi_i^{\mathcal{B}\text{rown}} \end{aligned}$$

where $\varpi_i^{\mathcal{G}\text{reen}}$ and $\varpi_i^{\mathcal{B}\text{rown}}$ are the proportion of green and brown activities. In this case, if the issuer has 50% in green activities and the remainder in white activities, its green intensity is equal to 150%, whereas the score is equal to 100% if the remainder concerns brown activities.

We have represented the different approaches of an environmental taxonomy in Figure 12. Each type differs in the objective it pursues. For example, the goal of a green-based taxonomy is to identify more strictly green activities to promote them. Therefore, with a green-based taxonomy, investors have no incentive to disinvest from brown activities. This is not the case with a brown-based taxonomy, whose objective is clearly to promote exclusion strategies. On the contrary, a mixed taxonomy recognizes many shades of green and not only one (Carney, 2019). These 3 taxonomy types are the counterpart of ESG investing strategies, that make the difference between selection, exclusion and integration.

Figure 12: Three types of environmental taxonomy



Examples of green/brown taxonomy The most famous example is the European green taxonomy. According to the European Commission[21], the EU taxonomy for sustainable activities is "*a classification system, establishing a list of environmentally sustainable economic activities. [...] The EU taxonomy would provide companies, investors and policymakers with appropriate definitions for which economic activities can be considered environmentally sustainable. In this way, it should create security for investors, protect private investors from greenwashing, help companies to become more climate-friendly, mitigate market fragmentation and help shift investments where they are most needed.*". Developed by the Technical Expert Group (TEG, 2020), the EU green taxonomy defines economic activities which make a substantive contribution to at least one of the following six environmental objectives: (1) Climate change mitigation, (2) Climate change adaptation, (3) Sustainable use and protection of water and marine resources, (4) Transition to a circular economy, (5) Pollution prevention and control, and (6) Protection and restoration of biodiversity and ecosystem. To qualify as sustainable, a business activity must also meet two other criteria. Indeed, the activity must do no significant harm to the other environmental objectives (DNSH constraint) and comply with minimum social safeguards[22] (MS constraint). Figure 12 summarizes the different steps.

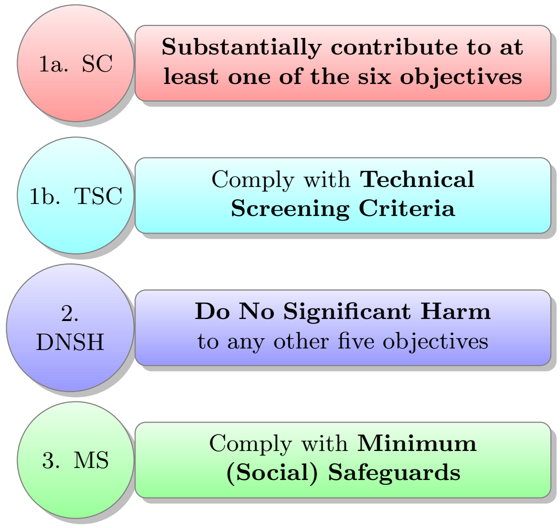
The EU taxonomy is not finalized and only concerns the first two objectives as of today (July 2022). Another drawback is that we must use reported data from the companies, implying that estimated data are prohibitive. The use of the EU taxonomy is then limited to assessing the transition dimension in the short term as long as the Corporate Sustainability Reporting Directive (CSRD) is not implemented. In the meantime, we can use proprietary taxonomies developed by data providers. For instance, MSCI has defined its taxonomy for identifying green activities. They are grouped into 6 categories: (1) Alternative energy, (2) Energy efficiency, (3) Green building, (4) Pollution prevention and control, (5) Sustainable agriculture and (6) Sustainable water. The green taxonomy of MSCI could be viewed as the first step of the green taxonomy of the European Union without including the DNSH and MS steps.

Remark 12. *In some sense, a brown taxonomy is included in the EU taxonomy since the*

[21] See the EU website: `https://ec.europa.eu/info/business-economy-euro/banking-and-finance/sustainable-finance_en`.

[22] For example, the UN guiding principles on business and human rights.

Figure 13: EU taxonomy for sustainable activities



TSC and DNSH criteria are related to brown activities. If we consider data providers, brown activities are generally limited to the following sectors: coal, oil/petroleum, and gas.

3.2.2 Static measures of greenness

There are several ways to compute the green intensity. This is why we observe some significant differences between data providers. One method is to translate the 3-step approach of the EU taxonomy into the following equation:

$$\boldsymbol{\mathcal{GI}} = \frac{\boldsymbol{\mathcal{GR}}}{\boldsymbol{\mathcal{TR}}} \cdot (1 - \mathcal{P}) \cdot \mathbb{1}\left\{\mathcal{S} \geq \mathcal{S}^-\right\} \tag{39}$$

where $\boldsymbol{\mathcal{GR}}$ is the green revenues deduced from the objectives, $\boldsymbol{\mathcal{TR}}$ is the total revenues, $\mathcal{P}$ is the penalty coefficient reflecting the DNSH constraint, $\mathcal{S}$ is the minimum safeguard score and $\mathcal{S}^-$ is the threshold. The first term is a proxy of the turnover KPI and corresponds to the green revenue share:

$$\boldsymbol{\mathcal{GRS}} = \frac{\boldsymbol{\mathcal{GR}}}{\boldsymbol{\mathcal{TR}}} \tag{40}$$

By construction, we have $0 \leq \boldsymbol{\mathcal{GRS}} \leq 1$. This measure is then impacted by the DNSH coefficient. If the penalty coefficient is equal to zero, the green activities of the issuer do not significantly harm the other objectives and we have $\boldsymbol{\mathcal{GI}} = \boldsymbol{\mathcal{GRS}}$. Otherwise, the green intensity satisfies $0 \leq \boldsymbol{\mathcal{GI}} = \boldsymbol{\mathcal{GRS}} \cdot (1 - \mathcal{P}) \leq \boldsymbol{\mathcal{GRS}}$. Finally, the indicator function $\mathbb{1}\left\{\mathcal{S} \geq \mathcal{S}^-\right\}$ is a binary all-or-nothing variable. It is equal to one if the firm complies with minimum social safeguards. Otherwise, the green intensity is equal to zero if the firm doesn't pass this materiality test. It follows that an upper bound of the green intensity is the green revenue share since we have $\boldsymbol{\mathcal{GI}} \leq \boldsymbol{\mathcal{GRS}}$. In what follows, we assume that $\boldsymbol{\mathcal{GI}} \approx \boldsymbol{\mathcal{GRS}}$, implying that our results overestimate the green taxonomy of investments. Moreover, it is easier to find gross green revenue shares than net revenue shares aligned with the EU taxonomy.

In Table 6, we report some descriptive statistics about the green revenue share based on the MSCI database. We use the MSCI ACWI IMI universe with 9283 issuers. For

Table 6: Statistics in % of green revenue share (MSCI ACWI IMI)

Category	Frequency $\mathbf{F}(x)$				Quantile $\mathbf{Q}(\alpha)$				Mean	
	0	25%	50%	75%	75%	90%	95%	Max	Avg	Wgt
(1)	9.82	1.47	0.96	0.75	0.00	0.00	2.85	100.00	1.36	0.77
(2)	14.10	1.45	0.65	0.31	0.00	1.25	6.12	100.00	1.39	3.50
(3)	4.84	1.68	1.02	0.31	0.00	0.00	0.00	100.00	1.16	0.51
(4)	4.79	0.30	0.10	0.06	0.00	0.00	0.00	99.69	0.32	0.22
(5)	1.00	0.39	0.20	0.09	0.00	0.00	0.00	98.47	0.26	0.10
(6)	4.75	0.28	0.11	0.05	0.00	0.00	0.00	99.98	0.29	0.14
Total	27.85	5.82	3.17	1.68	0.42	11.82	30.36	100.00	4.78	5.24

Source: MSCI (2022) & Authors' calculations.

each category[23], we compute the frequency $\mathbf{F}(x) = \Pr\{\mathcal{GRS} > x\}$, the statistical quantile $\mathbf{Q}(\alpha) = \inf\{x : \Pr\{\mathcal{GRS} \leq x\} \geq \alpha\}$, the average $\overline{\mathcal{GRS}} = n^{-1}\sum_{i=1}^{n} \mathcal{GRS}_i$ and the weighted mean $\mathcal{GRS}(b) = \sum_{i=1}^{n} b_i \cdot \mathcal{GRS}_i$ where b_i is the weight of Issuer i in the MSCI ACWI IMI benchmark. For instance, 9.82% of issuers have a green revenue share that concerns alternative energy. This figure becomes less than 1% if we consider a green revenue share greater than 50%. The average value is equal to 1.36% whereas the weighted value is equal to 0.77%. This indicates a small cap bias. For energy efficiency, the average is lower than the weighted mean, implying a bias towards big companies. If we consider the total green revenue share, 27.85% have a positive figure and only 3.17% have a figure greater than 50%. The 90% quintile is equal to 11.82%. Therefore, we notice a high positive skewness for the distribution. The green revenue share is then located in a small number of companies.

3.2.3 Dynamic measures of greenness

A first approach to define a dynamic measure of greenness is to estimate the trend of the green intensity (or the green revenue share).

$$\mathcal{GI}(t) = \gamma_0 + \gamma_1 \cdot t + v(t)$$

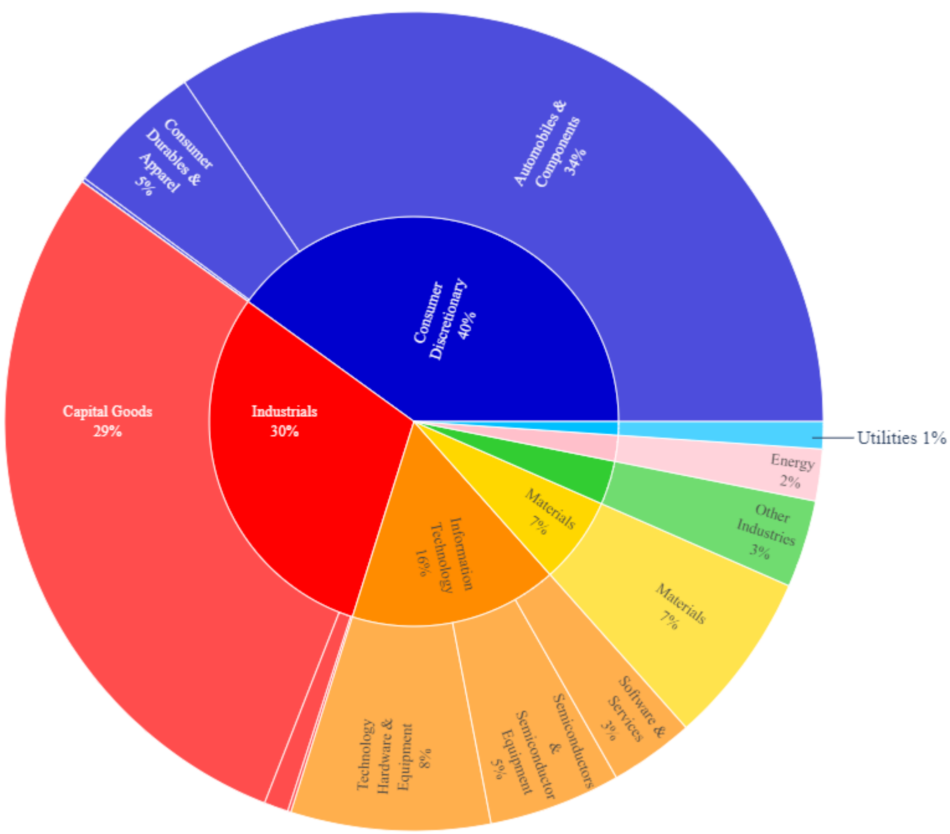
We can then build the same dynamic measures as those defined for the carbon metrics: green trend, green velocity and green momentum. The current issue is that we do not have a long historical time series of green revenue shares. Instead of estimating $\widehat{\mathcal{GI}}(t) = \mathcal{GI}(t_0) + \hat{\gamma}_1 \cdot (t - t_0)$, we can use a proxy or a KPI that contains information about the future green intensity of the issuer. A first indicator may be the green capex. The rationale is the following. According to IEA (2021), "*almost half of the emissions savings needed in 2050 to reach net zero emissions rely on technologies that are not yet commercially available*". All the climate scenarios describe the same need, that is a significant level of green investment in clean transportation, clean energy, energy storage, or carbon capture and storage to name a few. Therefore, it does make sense to assess the current green investment, which can be measured by green capex. Unfortunately, very few companies are disclosing it at this time. For example, the green capex metric provided by Reuters Eikon covers barely 100 companies in the MSCI World. There is increasing pressure on companies to disclose their green capex, and the data availability will soon be improved[24].

[23] We remind them: (1) Alternative energy, (2) Energy efficiency, (3) Green building, (4) Pollution prevention and control, (5) Sustainable agriculture and (6) Sustainable water.

[24] For example, the disclosure of aligned capex is required for European companies under the EU Green Taxonomy.

Low-carbon patents are another measure of a company's research effort on climate solutions. The European Patent Office (EPO) has developed a classification scheme for climate mitigation and adaptation technologies, which allows for low-carbon patents identification. Green capex and low-carbon patents meet the same need since they provide a forward-looking measure of green revenues. For example, it took between 10 and 30 years between the prototype and the mass market for LEDs or lithium-ion batteries development (European Patent Office, 2021), leading to a large lag between forward-looking measures and green revenues. However, green capex and low-carbon patents have many dissimilarities. Green capex is a leading indicator of a company's ability to innovate, as the patent filing process takes between one and three years. On the one hand, green capex spending does not indicate whether these funds have resulted in patent registration or commercialization. On the other hand, a company may decide not to file a patent, and benefit from its innovation. Moreover, a company can hold a patent and not exploit it. These two metrics are therefore complementary. The advantage of low-carbon patents metric over green capex is data availability. It covers 80% of the companies of the MSCI World, representing 93% of the market cap. Low-carbon patents are mostly filed by companies belonging to polluting sectors (e.g., Automobile and Capital Goods), with the exception of Information Technology. If the Utilities sector represents a small share of the low-carbon patents, almost 24% of its patents are green.

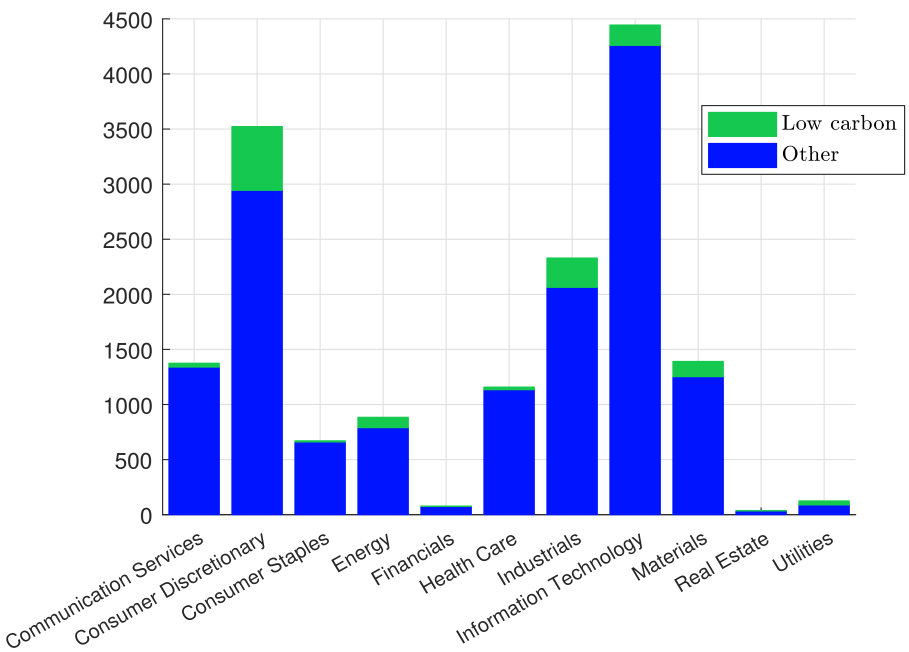
Figure 14: Patent breakdown per GICS sector and industry



Source: MSCI (2022) & Authors' calculations.

Remark 13. *Figures 14 and 15 illustrate this phenomenon. The Automotive industry files more than 85% of the low-carbon patents in the Consumer Discretionary sector, with the latter holding 40% of all low-carbon patents. The Automotive industry thereby accounts for more than one-third of the total number of low-carbon patents held by companies of the MSCI World index. However, although this sector leads by a wide margin in terms of the number of low-carbon patents, a study of the share that this represents in all of its patents paints a different picture. Indeed, despite filing the largest number of low-carbon patents, they represent only 6.3% of the sector's patents while the Utilities sector is in the exact opposite situation. Thus, one might consider these two indicators when constructing a portfolio to get a more balanced picture of companies low-carbon innovation.*

Figure 15: Average number of patents per GICS sector



Source: MSCI (2022) & Authors' calculations.

4 Net zero investing

Following Le Guenedal and Roncalli (2022), we consider the construction of net zero investment portfolios based on benchmark optimization. The underlying idea is to modify an existing benchmark portfolio by introducing net zero features. This top-down approach, which is based on asset allocation, is used extensively in passive management. However, it is not appropriate in active management, whose bottom-up approach is based on asset selection. While the top-down approach can be easily replicated, the bottom-up approach is difficult to backtest because it depends on too many discretionary choices, including the number of selected assets, the scoring system, the weighting scheme, and the timing of rebalancing. The top-down approach is more standardized and replicable. In what follows, we therefore consider the top-down approach to show how net zero investing differs from portfolio decarbonization. We also consider a core-satellite framework, which is more appropriate for bottom-up approaches and strategic asset allocation.

4.1 Decarbonization approach

In what follows, we distinguish equity portfolios from bond portfolios because the objective function is not the same due to two different definitions of the tracking risk. For equity portfolios, the benchmark is the MSCI World index, whereas we use the Bloomberg Global Investment Grade Corporate Bond index for bond portfolios.

4.1.1 Equity portfolios

Benchmark analysis Let $b = (b_1, \ldots, b_n)$ be the weights of the stocks that belong to the benchmark. Its carbon intensity is given by its weighted average:

$$\mathcal{CI}(b) = \sum_{i=1}^{n} b_i \cdot \mathcal{CI}_i \tag{41}$$

where $\mathcal{CI}_i$ is the carbon intensity of stock i. If we focus on the carbon intensity for a given sector, we use the following formula:

$$\mathcal{CI}\left(\mathcal{S}ector_j\right) = \frac{\sum_{i \in \mathcal{S}ector_j} b_i \cdot \mathcal{CI}_i}{\sum_{i \in \mathcal{S}ector_j} b_i} \tag{42}$$

In Table 7, we report the carbon intensity of the MSCI World index and its sectors. We obtain 130 tCO_2e/\$ mn for scope 1, 163 tCO_2e/\$ mn if we include scope 2, 310 tCO_2e/\$ mn if we add upstream scope 3, and finally 992 tCO_2e/\$ mn if we consider the full scope 3. We notice a large cap bias because the MSCI World equally-weighted portfolio shows higher figures. We also observe a high discrepancy between sectors. Low-carbon sectors are Communication Services, Financials, Health Care and Information Technology, whereas high-carbon sectors are Energy, Materials and Utilities. We foresee that decarbonizing a portfolio implies reducing the exposure to high-carbon sectors and increasing the exposure to low-carbon sectors. For Industrials and Consumer Staples, the sector allocation will depend on the choice of the scope.

Table 7: Carbon intensity in tCO_2e/\$ mn per GICS sector (MSCI World, June 2022)

Sector	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	2	28	134	172
Consumer Discretionary	23	65	206	590
Consumer Staples	28	55	401	929
Energy	632	698	1 006	6 823
Financials	13	19	52	244
Health Care	10	22	120	146
Industrials	111	130	298	1 662
Information Technology	7	23	112	239
Materials	478	702	1 113	2 957
Real Estate	22	101	167	571
Utilities	1 744	1 794	2 053	2 840
MSCI World	130	163	310	992
MSCI World EW	168	211	391	1 155

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

We can compute the risk contribution of each sector as follows:

$$\mathcal{RC}\left(\mathcal{S}ector_j\right) = \frac{\sum_{i \in \mathcal{S}ector_j} b_i \cdot \mathcal{CI}_i}{\mathcal{CI}(b)} \tag{43}$$

Results are reported in Table 8. For example, Consumer Services represents 7.58% of the nominal allocation, but only 0.14% of the carbon allocation if we consider scope 1. If we focus on the first two scopes, Utilities is the main contributor, followed by Energy and Materials. By including upstream scope 3 emissions, the contribution of Consumer Staples becomes significant. We also notice that the Utilities contribution has strongly been reduced whereas the Industrials contribution increases when we consider the three scopes.

Table 8: Sectoral contribution in % (MSCI World, June 2022)

Sector	Index	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	7.58	0.14	1.31	3.30	1.31
Consumer Discretionary	10.56	1.87	4.17	6.92	6.21
Consumer Staples	7.80	1.68	2.66	10.16	7.38
Energy	4.99	24.49	21.53	16.33	34.37
Financials	13.56	1.33	1.58	2.28	3.34
Health Care	14.15	1.12	1.92	5.54	2.12
Industrials	9.90	8.38	7.83	9.43	16.38
Information Technology	21.08	1.13	3.03	7.57	5.06
Materials	4.28	15.89	18.57	15.48	12.93
Real Estate	2.90	0.48	1.81	1.57	1.65
Utilities	3.21	43.47	35.59	21.41	9.24

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Table 9: Carbon (intensity) momentum $\mathcal{CM}^{\mathcal{L}ong}$ in % by sector (MSCI World, June 2022)

Sector	Average $\mathcal{CM}_x^{\mathcal{L}ong}$			Frequency $\mathcal{CM}_i^{\mathcal{L}ong} > 0$		
	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$
Communication Services	−7.3	0.7	0.9	29.5	40.9	44.3
Consumer Discretionary	−0.1	−0.3	−1.1	16.3	23.5	15.7
Consumer Staples	−5.0	−4.4	−2.2	17.8	17.8	15.8
Energy	2.3	2.3	1.3	75.9	77.8	68.5
Financials	−0.9	−0.9	−0.9	27.8	35.7	24.4
Health Care	−10.0	−7.8	−3.1	13.7	17.3	12.9
Industrials	−0.4	−0.7	−1.4	19.0	25.5	19.4
Information Technology	−6.0	−0.9	−0.7	30.9	31.4	17.7
Materials	−0.4	−0.8	−0.1	32.1	39.1	31.8
Real Estate	0.9	4.4	2.4	34.7	47.4	47.4
Utilities	−7.4	−6.9	−6.3	16.7	24.4	23.1
MSCI World	−3.0	−2.4	−1.7	25.5	31.5	25.0

Source: Trucost (2022) & Authors' calculations.

It is also important to take into account the carbon momentum metric, as shown in Table 9. We use the aggregation method described in Appendix A.2.1 on page 78. On average, the carbon momentum of the MSCI World index is negative and only 25% of issuers have positive momentum. Nevertheless, we observe a lot of discrepancies between sectors. While Utilities and Energy are the two major contributors to the MSCI World's carbon intensity, Utilities exhibits a negative carbon momentum, but Energy has a positive carbon momentum. We have also reported the share of each sector's constituents exhibiting positive carbon momentum. If we consider scope $\mathcal{SC}_{1-3}^{\text{up}}$, 68.5% of the companies belonging to the Energy sector have increased their carbon intensities these last years. This figure is 44.3% for Communication Services, and 47.4% for Real Estate. It is also interesting to notice that the Real Estate sector has a low-carbon allocation but a positive carbon momentum. Introducing a carbon momentum constraint is thus crucial in the optimization to avoid overweighting companies with positive carbon momentum.

Optimization problem Le Guenedal and Roncalli (2022) describe several mathematical approaches to formulating the portfolio decarbonization problem. We focus on the max-threshold solution since it is the most accepted method among professionals. Let x be a portfolio and Σ the covariance matrix of stock returns. The objective function is to minimize the tracking error variance of Portfolio x with respect to Benchmark b subject to a carbon reduction constraint:

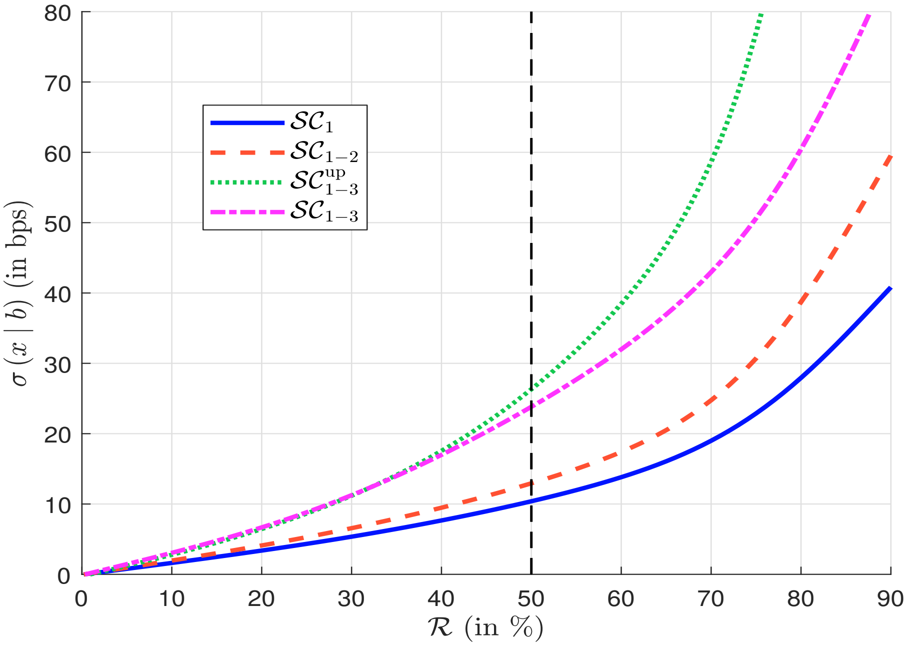
$$\begin{aligned} x^{\star}(\boldsymbol{\mathcal{R}}) &= \arg\min \frac{1}{2}(x-b)^{\top}\Sigma(x-b) \\ & \text{s.t.} \left\{ \begin{array}{l} \boldsymbol{\mathcal{CI}}(x) \leq (1-\boldsymbol{\mathcal{R}}) \cdot \boldsymbol{\mathcal{CI}}(b) \\ x \in \Omega_1 \cap \Omega_2 \end{array} \right. \end{aligned} \tag{44}$$

where $\boldsymbol{\mathcal{R}}$ is the reduction rate and $\Omega = \Omega_1 \cap \Omega_2$ is a set of constraints. The first set $\Omega_1 = \left\{x : \mathbf{1}_n^{\top} x = 1, \mathbf{0}_n \leq x \leq \mathbf{1}_n\right\}$ implies that we obtain a long-only portfolio, whereas the second set Ω_2 controls the weight deviation between Portfolio x and Benchmark b. For instance, we can use $\Omega_2 = \left\{x : m_w^- b \leq x \leq m_w^+ b\right\}$ where $m_w^- \in [0,1[$ and $m_w^+ \in [1,\infty[$. In this case, the portfolio's weight x_i can only deviate from the benchmark's weight b_i by lower and upper ratios m_w^- and m_w^+. Typical figures are $m_w^- = 1/2$ and $m_w^+ = 2$. Another approach consists in controlling the sector deviations. In this case, we can use a relative deviation allowance — $\Omega_2 = \left\{\forall j : m_s^- \sum_{i \in \boldsymbol{\mathcal{S}}ector_j} b_i \leq \sum_{i \in \boldsymbol{\mathcal{S}}ector_j} x_i \leq m_s^+ \sum_{i \in \boldsymbol{\mathcal{S}}ector_j} b_i\right\}$ — or an absolute deviation allowance — $\Omega_2 = \left\{\forall j : \left|\sum_{i \in \boldsymbol{\mathcal{S}}ector_j} (x_i - b_i)\right| \leq \delta_s^+\right\}$. In what follows, we use 4 sets of constraints: $\mathcal{C}_0$ only imposes long-only constraints, $\mathcal{C}_1\left(m_w^-, m_w^+\right)$ adds stock weight constraints, $\mathcal{C}_2(m_s)$ adds sector relative allocation constraints with $m_s^- = 1/m_s$ and $m_s^+ = m_s$, and $\mathcal{C}_3\left(m_w^-, m_w^+, m_s\right) = \mathcal{C}_1\left(m_w^-, m_w^+\right) \cap \mathcal{C}_2(m_s)$ combines $\mathcal{C}_1$ and $\mathcal{C}_2$.

Results We have reported the tracking error volatility (expressed in bps) in Figure 16 when we consider the $\mathcal{C}_0$ constraint. The tracking risk increases when we include scope 2 or upstream scope 3, whereas downstream scope 3 reduces it because of its large dispersion. If we now impose the classical weight constraint $\mathcal{C}_1\left(1/3, 3\right)$, which is very popular in indexing management, we observe a high increase in the tracking error volatility (Figure 17). Moreover, we generally have no solution for $\boldsymbol{\mathcal{R}} > 60\%$. The issue comes from the lower bound, which is way to narrow. Indeed, portfolio decarbonization is, above all, an exclusion process. By imposing a lower bound, we then limit portfolio decarbonization. For instance, we obtain similar results between constraint $\mathcal{C}_1(0,3)$ and constraint $\mathcal{C}_0$. Nevertheless, we must be careful when choosing m_w^+, because a low value can lead to infeasible solutions. For instance, this is the case of constraint $\mathcal{C}_1(0,1.25)$, as shown in Figure 17. If we compare Figures 17 and 18, we notice that the impact of sector constraints is less important than the impact of weight constraints. For instance, constraint $\mathcal{C}_2(1)$ imposes match the benchmark sectoral allocations. For low reduction rates (less than 50%), the increase of tracking risk is lower than 30 bps. The combination of weight and sectoral constraints is a more difficult exercise as shown in the bottom panels in Figure 18.

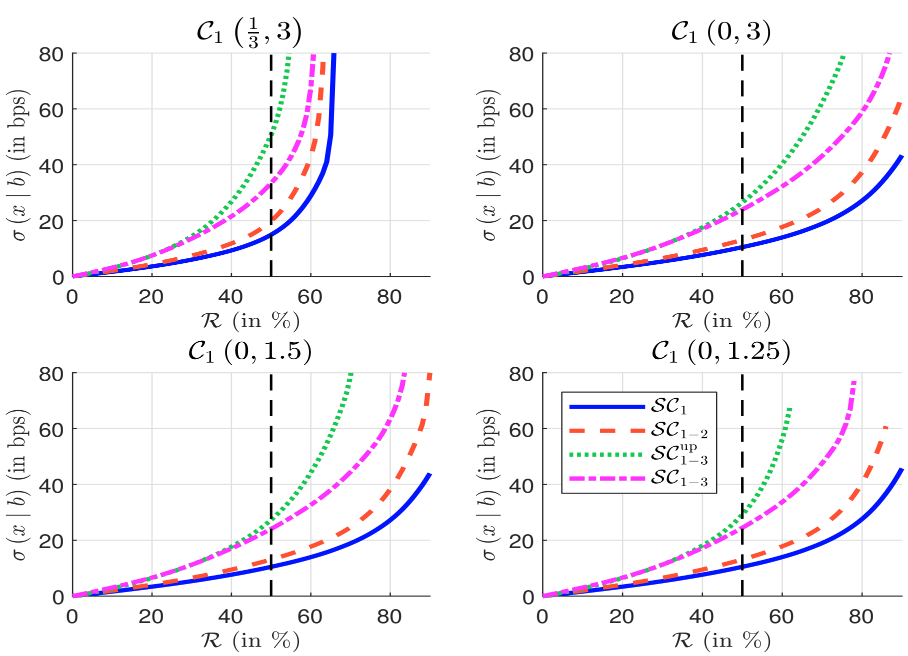
Remark 14. *At first sight, it may be surprising that weight constraints are more binding than sectoral constraints. Indeed, we generally consider that the sector contribution is greater*

Figure 16: Impact of the carbon scope on the tracking error volatility (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint)



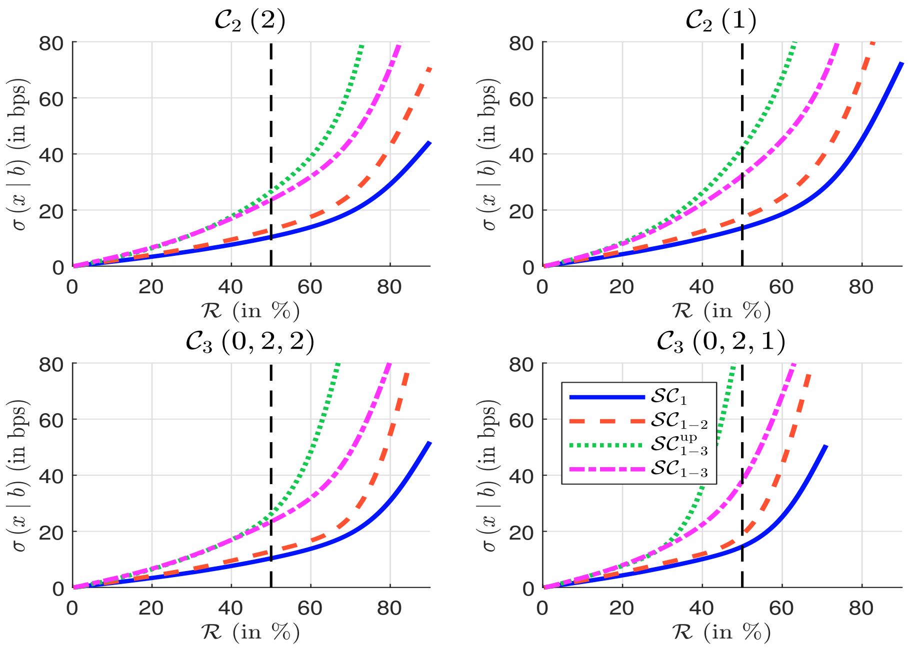
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 17: Impact of the $\mathcal{C}_1$ constraint on the tracking error volatility (MSCI World, Jun. 2022)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 18: Impact of the $\mathcal{C}_2$ and $\mathcal{C}_3$ constraints on the tracking error volatility (MSCI World, Jun. 2022)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

than the idiosyncratic contribution. Therefore, we expect that the inter-class dispersion largely dominates the intra-class variance. Nevertheless, this viewpoint is biased because it considers homogeneous sectors. In our case, we use level one of the GICS classification. The concept of sector is then very heterogeneous. Within a particular sector, we can have low-carbon and high-carbon issuers. For instance, we have reported the boxplots of carbon intensity per sector in Figures 47 and 48 on page 93. We can easily find issuers with low and high carbon footprints for each sector. This is why portfolio decarbonization cannot be reduced to arbitrage between sectors.

Table 10: Sector allocation in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

Sector	Index	Reduction rate $\boldsymbol{\mathcal{R}}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.58	7.95	8.15	8.42	8.78	9.34	10.13	12.27
Consumer Discretionary	10.56	10.69	10.69	10.65	10.52	10.23	9.62	6.74
Consumer Staples	7.80	7.80	7.69	7.48	7.11	6.35	5.03	1.77
Energy	4.99	4.14	3.65	3.10	2.45	1.50	0.49	0.00
Financials	13.56	14.53	15.17	15.94	16.90	18.39	20.55	28.62
Health Care	14.15	14.74	15.09	15.50	16.00	16.78	17.77	17.69
Industrials	9.90	9.28	9.01	8.71	8.36	7.79	7.21	6.03
Information Technology	21.08	21.68	22.03	22.39	22.88	23.51	24.12	24.02
Materials	4.28	3.78	3.46	3.06	2.56	1.85	1.14	0.24
Real Estate	2.90	3.12	3.27	3.41	3.57	3.72	3.71	2.51
Utilities	3.21	2.28	1.79	1.36	0.90	0.54	0.24	0.12

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

In Table 10, we have reported the sectoral allocation considering the $\mathcal{C}_0$ constraint. We observe that portfolio decarbonization is a strategy that is long on Financials and short on Energy, Materials and Utilities, although the extent of reallocation depends on the scope[25]. In particular, we notice that the most favorable case for the Financials sector is when we consider upstream scope 3. Moreover, we observe some strong non-linearities. The allocation in a given sector may increase when the reduction rate is low, but it may also strongly decrease when the reduction rate is very high[26]. These results are obtained with the $\mathcal{C}_0$ constraint, but can be generalized to $\mathcal{C}_1$ or $\mathcal{C}_2$ constraints. Indeed, by imposing sector neutrality for instance, we observe the same phenomenon but at a sub-level category, typically between industries or sub-industries.

Transition dimension As said previously, a decarbonization strategy does not necessarily support a transition to a low-carbon economy for two main reasons. The first one is that the resulting portfolio does not naturally allocate capital toward green activities, as illustrated in Table 11. The green intensity is defined as the green revenue share of the portfolio. We observe a decreasing function between the green intensity and the reduction level. This negative correlation between decarbonization and transition dimensions is particularly problematic from a dynamic perspective. Thus, it is necessary to introduce a green intensity constraint to prevent aligned portfolios from having a lower green intensity.

Table 11: Green intensity in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	5.24	5.21	5.19	5.18	5.16	5.12	5.08	5.01
$\mathcal{SC}_{1-2}$		5.17	5.14	5.09	4.99	4.83	4.64	4.52
$\mathcal{SC}_{1-3}^{\text{up}}$		5.15	5.07	4.89	4.69	4.42	3.90	0.68
$\mathcal{SC}_{1-3}$		5.17	5.12	5.05	4.97	4.80	4.55	3.73

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Similarly, we compute the carbon momentum $\mathcal{CM}^{\mathcal{L}ong}$ of decarbonized portfolios[27]. Most of the time, we observe that the carbon momentum of the decarbonized portfolio is higher than the benchmark. Thus, if all companies pursue their past efforts, the benchmark will decarbonize itself faster than the optimized portfolio. In this scenario, the benchmark's future carbon intensity would be lower than the decarbonized portfolio's future carbon intensity.

Table 12: $\mathcal{SC}_{1-3}^{\text{up}}$ carbon momentum in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	−1.7	−1.5	−1.3	−1.2	−1.2	−1.3	−1.6	−1.8
$\mathcal{SC}_{1-2}$		−1.5	−1.3	−1.3	−1.4	−1.7	−1.9	−2.6
$\mathcal{SC}_{1-3}^{\text{up}}$		−1.7	−1.7	−1.8	−2.1	−2.8	−4.5	−7.7
$\mathcal{SC}_{1-3}$		−1.8	−1.8	−1.7	−1.6	−1.8	−1.8	−1.8

Source: Trucost (2022) & Authors' calculations.

[25] See Tables 37 and 38 on page 79.
[26] For example, this is the case of the Communication Discretionary sector when we consider scope $\mathcal{SC}_{1-3}$.
[27] In the sequel, we use the $\mathcal{SC}_{1-3}^{\text{up}}$ carbon momentum to perform all the analysis.

4.1.2 Bond portfolios

Benchmark analysis We report the carbon intensity of the Global Corp. index[28] and its GICS sectors[29] in Table 13. The index carbon intensity is 249 tCO_2e/$ mn for scope 1, 286 tCO_2e/$ mn if we include scope 2, 435 tCO_2e/$ mn if we add upstream scope 3, and finally 1265 tCO_2e/$ mn if we consider the full scope 3. As in the equity case, we notice a factor of 3 between the full scope 3 and the upstream scope 3. We also observe the same high discrepancy between sectors and hence the same impact on portfolio decarbonization.

Table 13: Carbon intensity in tCO_2e/$ mn per GICS sector (Global Corp., June 2022)

Sector	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	4	28	270	309
Consumer Discretionary	22	73	242	1 011
Consumer Staples	36	65	485	700
Energy	610	698	997	5 694
Financials	1	7	33	590
Health Care	10	21	115	144
Industrials	143	165	318	1 390
Information Technology	11	34	119	254
Materials	655	835	1 167	2 347
Real Estate	25	107	149	904
Utilities	1 666	1 750	2 031	2 957
Global Corp.	249	286	435	1 265

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 14: Sectoral contribution in % (Global Corp., June 2022)

Sector	Index	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	7.34	0.12	0.73	4.55	1.79
Consumer Discretionary	5.97	0.53	1.52	3.32	4.77
Consumer Staples	6.04	0.88	1.38	6.74	3.34
Energy	6.49	15.88	15.82	14.88	29.20
Financials	33.91	0.15	0.84	2.58	15.81
Health Care	7.50	0.30	0.56	1.99	0.85
Industrials	8.92	5.13	5.14	6.52	9.80
Information Technology	5.57	0.23	0.65	1.53	1.12
Materials	3.44	9.04	10.05	9.24	6.39
Real Estate	4.76	0.48	1.78	1.64	3.40
Utilities	10.06	67.25	61.52	47.01	23.52

Source: ICE (2022), Trucost (2022) & Authors' calculations.

In Table 14, we report the contribution of each sector to the portfolio carbon intensity. We notice that with a different sector allocation than the MSCI World, Energy, Materials, and Utilities sectors are still the main contributors to carbon intensity. These sectors also

[28] Only 89% of the index has carbon data since private/unlisted issuers are not covered by Trucost. For these issuers, we associate the average weighted carbon data of their related GICS sector.

[29] These sectors are usually used in the equity space. Therefore, we perform a mapping from the Merrill Lynch sectors to have a comparable sector view.

exhibit the highest ratios of risk contribution in the benchmark, whereas Financials, Health Care, and Information Technology are the sectors with the lowest ratios.

Remark 15. *The corporate bond index structure is significantly different from the equity index structure because of the weight of the Financials sector. Therefore, the results we have obtained for equity portfolios might not be valid for bond portfolios.*

Optimization problem To replicate a market index, fund managers may hold the same securities or a stratified sampling of the securities that comprise the index (Neyman, 1934). Therefore, they track the index portfolio by exhibiting the same risk/return characteristics. In the fixed income space, modified duration (MD) and duration-times-spread (DTS) are the most widely used risk metrics[30]. Indeed, historical volatility, which measures the risk of equity portfolios, is not a reliable predictor of bond volatility since bonds are less frequently traded and mature over time.

In the case of bonds, the objective function is to minimize sectoral active credit risk and the active share (AS) of Portfolio x with respect to Benchmark b subject to a carbon reduction constraint[31]:

$$\begin{aligned} x^{\star}\left(\boldsymbol{\mathcal{R}}\right) &= \arg\min \varphi \underbrace{\sum_{s=1}^{n_{\mathcal{S}ector}} \left| \sum_{i \in s} \left(x_i - b_i\right) \cdot \mathrm{DTS}_i \right|}_{\text{DTS component}} + \underbrace{\frac{1}{2} \sum_{i \in b} \left| x_i - b_i \right|}_{\text{AS component}} \\ & \text{s.t.} \quad \left\{ \begin{array}{l} \boldsymbol{\mathcal{CI}}\left(x\right) \leq \left(1 - \boldsymbol{\mathcal{R}}\right) \cdot \boldsymbol{\mathcal{CI}}\left(b\right) \\ x \in \Omega_1 \cap \Omega_2 \end{array} \right. \end{aligned} \tag{45}$$

where $\boldsymbol{\mathcal{R}}$ is the reduction rate, $\Omega_1 \cap \Omega_2$ is a set of constraints and φ is the trade-off coefficient between DTS and AS components[32]. As in the case of equities, the first set $\Omega_1 = \left\{x : \mathbf{1}_n^\top x = 1, \mathbf{0}_n \leq x \leq \mathbf{1}_n\right\}$ implies that we obtain a long-only portfolio, whereas the second set Ω_2 controls the risk metrics deviation between Portfolio x and Benchmark b. We can use $\Omega_2 = \Omega_{2'} \cap \Omega_{2''} \cap \Omega_{2'''}$ where $\Omega_{2'} = \left\{x : \sum_{i=1}^n \left(x_i - b_i\right) \cdot \mathrm{MD}_i = 0\right\}$, $\Omega_{2''} = \left\{x : \forall j, \sum_{i \in \mathcal{B}ucket(j)} \left(x_i - b_i\right) = 0\right\}$ and $\Omega_{2'''} = \left\{x : \forall j, \sum_{i \in \mathcal{R}ating(j)} \left(x_i - b_i\right) = 0\right\}$. The $\Omega_{2'}$ constraint neutralizes the modified duration at the portfolio level, whereas $\Omega_{2''}$ and $\Omega_{2'''}$ constraint the portfolio to have the same weights as the benchmark per maturity bucket[33] and rating category[34]. We choose not to add further constraints because the current problem is already highly constrained at the sector level, and therefore no sector will vanish when a solution is found.

Results We have reported the duration-times-spread tracking risk $\mathrm{DTS}\left(x \mid b\right) = \sum_{s=1}^{n_{\mathcal{S}ector}} \left| \sum_{i \in s} \left(x_i - b_i\right) \cdot \mathrm{DTS}_i \right|$ and the active share $\mathrm{AS}\left(x \mid b\right) = \frac{1}{2} \sum_{i \in b} \left| x_i - b_i \right|$ in Figures 19 and 20. We observe that the tracking risk is low when we consider the DTS component, whereas it is significant when we focus on the weight component. In particular, $\mathrm{AS}\left(x \mid b\right)$ increases when we include upstream and downstream scope 3. On average, there is a factor of two between $\boldsymbol{\mathcal{SC}}_{1-3}$ and $\boldsymbol{\mathcal{SC}}_{1-2}$. Moreover, we notice that the active share accelerates where the reduction rate $\mathcal{R}$ is above 85% and can reach 50%.

[30] MD is the sensitivity of the bond return to interest risk, and DTS measures the systematic exposure to credit risk by quantifying sensitivity to a shift in the yield spread (Ben Dor *et al.*, 2007).

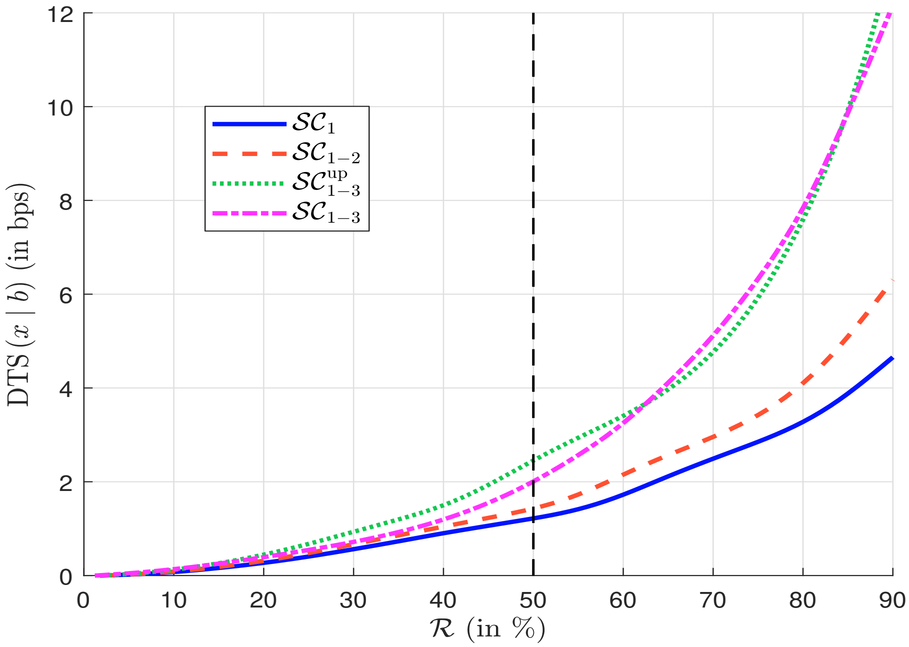
[31] The current exercise does not consider minimum tradable, lot size or the liquidity of bonds. Therefore, solutions may exist theoretically, but their implementation may be challenging.

[32] φ is set to 50, implying that the trade-off is 1% of active share for 2 bps of DTS.

[33] We use the following buckets: 0Y–2Y, 2Y–5Y, 5Y–7Y, 7Y–10Y and 10Y+.

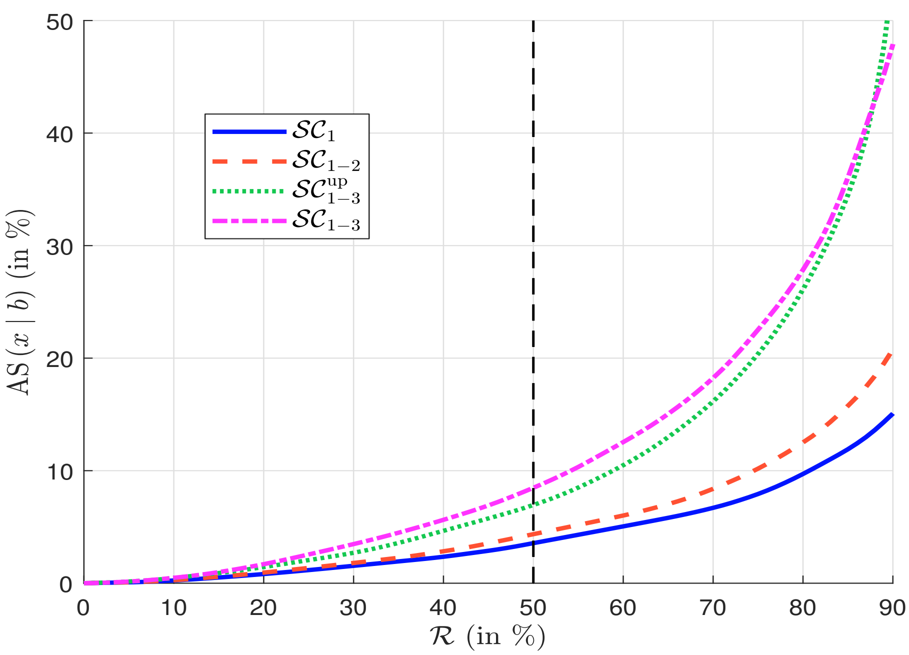
[34] The rating categories are AAA–AA, A and BBB.

Figure 19: Impact of the carbon scope on the duration-times-spread in bps (Global Corp., Jun. 2022)



Source: ICE (2022), Trucost (2022) & Authors' calculations.

Figure 20: Impact of the carbon scope on the active share in % (Global Corp., Jun. 2022)



Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 15: Sector allocation deviation in % (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-3}$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	0.01	0.00	0.03	0.09	0.09	−0.03	−0.04
Consumer Discretionary	5.97	0.00	−0.01	−0.03	−0.04	−0.51	−1.49	−2.42
Consumer Staples	6.04	0.00	0.00	0.00	0.00	−0.02	−0.65	−1.98
Energy	6.49	−1.00	−2.07	−2.65	−2.80	−3.26	−3.91	−3.97
Financials	33.91	0.73	1.75	2.05	2.18	3.45	4.95	5.09
Health Care	7.50	0.00	0.00	0.00	0.00	0.00	0.02	−0.02
Industrials	8.92	0.46	0.70	1.27	2.42	3.15	4.63	9.21
Information Technology	5.57	0.00	0.02	0.02	0.03	0.03	−0.05	−0.30
Materials	3.44	−0.01	−0.13	−0.26	−0.32	−0.80	−1.19	−1.58
Real Estate	4.76	−0.02	−0.02	−0.02	−0.02	−0.10	−0.15	−0.83
Utilities	10.06	−0.17	−0.24	−0.42	−1.54	−2.02	−2.14	−3.18

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 15 shows the deviation of sectoral allocation versus the benchmark when considering the $\mathcal{SC}_{1-3}$ scope. We observe that the decarbonization process is also a strategy that is long on Financials and short on Materials and Utilities. As shown in Tables 39–41 on page 80, reallocation depends on the scope. Health care, Communication Services, Consumer Discretionary, and Information Technology weights are very close to their benchmark's. Regarding the other sectors, the strategy may point in contradictory directions according to the scope. For instance, it is short on Energy with $\mathcal{SC}_{1-3}$ but long on Energy with $\mathcal{SC}_{1-3}^{\text{up}}$. Likewise, it is long on Industrials with scope $\mathcal{SC}_{1-3}$, but no conclusion can be drawn regarding other scopes.

Table 16: Yield variation in bps (Global Corp., Jun. 2022)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	422	−2	−2	−1	−6	−6	−8	−11
$\mathcal{SC}_{1-2}$		−1	−2	−3	−3	−3	−10	−15
$\mathcal{SC}_{1-3}^{\text{up}}$		−3	−3	−4	−10	−16	−23	−57
$\mathcal{SC}_{1-3}$		0	−2	−3	−7	−8	−9	−22

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 16 shows that the yield of the decarbonized portfolio is lower and decreases with the reduction rate. This yield difference in the full scope is due to the lower contribution of the Energy, Materials, and Utilities sectors, partially offset by the higher contribution of Financials and Industrials (see Table 42 on page 81). The breakdown by ratings and durations suggests that BBB-rated bonds and bonds whose duration is between two and five years explain the lower yield.

Remark 16. *In Tables 43 and 44 on page 82, we focus on the two main benchmark sectors: Financials and Utilities. We note that the higher contribution for Financials comes mainly from the short-duration overweighting (0Y–5Y of AAA–AA, 2Y–7Y of A, and the liquidity bucket of BBB). The optimizer also underweights BBB-rated bonds whose duration exceeds two years, resulting in restrained lower yields. In the meantime, regarding Utilities, the optimizer has progressively underweighted BBB-rated bonds and the 0Y–7Y bucket of A-rated bonds. The outcome is partially reallocated to overweight the high-duration of A-rated bonds.*

Transition dimension In Table 17, we see that relative to the benchmark, the decarbonized portfolio has better green intensity[35] that increases with the reduction rate. However, this finding does not apply to scope $\mathcal{SC}_{1-2}$. On the other hand, the green intensity never exceeds twice the benchmark green intensity[36].

Table 17: Green intensity in % (Global Corp., Jun. 2022)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	3.49	3.69	3.84	3.99	4.30	4.70	5.27	5.97
$\mathcal{SC}_{1-2}$		3.44	3.39	3.40	3.42	3.44	3.45	3.06
$\mathcal{SC}_{1-3}^{\text{up}}$		3.55	3.53	3.85	3.95	3.94	3.39	2.00
$\mathcal{SC}_{1-3}$		3.57	3.74	3.97	4.74	5.21	5.84	5.59

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

We illustrate in Table 18 the carbon momentum of the decarbonized portfolio. As its reference, the decarbonized portfolio exhibits negative carbon intensity trends. We note that the carbon momentum of the decarbonized portfolio is generally above the benchmark. Therefore, imposing a constraint on the carbon momentum may help the aligned portfolio to decarbonize faster than the benchmark.

Table 18: Carbon momentum in % (Global Corp., Jun. 2022)

Scope	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	−2.93	−2.29	−1.92	−1.71	−1.26	−1.11	−1.28	−0.93
$\mathcal{SC}_{1-2}$		−2.27	−2.01	−1.89	−1.45	−1.89	−2.30	−2.07
$\mathcal{SC}_{1-3}^{\text{up}}$		−2.27	−2.03	−1.85	−2.26	−2.74	−3.14	−5.27
$\mathcal{SC}_{1-3}$		−3.06	−3.14	−3.12	−1.99	−1.78	−1.97	−0.98

Source: ICE(2022), Trucost (2022) & Authors' calculations.

4.2 Integrated approach

The previous analysis has shown that portfolio decarbonization recovers only one dimension of net zero investing: The reduction of the carbon footprint of asset portfolios. We now consider extending the previous approach by adapting the mathematical optimization problem. This approach is integrated because it tries to solve the problem in one step by integrating the transition dimension, which is multi-faceted.

4.2.1 Equity portfolios

Dynamic decarbonization While the decarbonization problem finds an optimal portfolio $x^\star(\mathcal{R})$ with respect to a given reduction rate $\mathcal{R}$, the alignment problem defines an optimal portfolio $x^\star(t)$ with respect to a given date t. Therefore, this second problem can be seen as a special case of the first problem, where we use the mapping function between the date t and the reduction rate $\mathcal{R}$ (Le Guenedal and Roncalli, 2022). In this case, the

[35] 6.42% of the benchmark has no green data. We apply a zero green intensity for the related issuers.

[36] Table 45 on page 83 displays the results when we apply the average weighted green intensity per sector to issuers with no green data. The results are consistent with the above findings.

decarbonization problem becomes dynamic:

$$\begin{aligned} x^{\star}(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^{\top} \Sigma(t) \left(x - b(t)\right) \\ & \text{s.t.} \quad \left\{ \begin{array}{l} \boldsymbol{\mathcal{CI}}(t, x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) \\ x \in \Omega_1 \cap \Omega_2(t) \end{array} \right. \end{aligned} \tag{46}$$

where t_0 is the base year and $\boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right)$ is the carbon intensity of the benchmark at time t_0. We notice that the benchmark $b(t)$, the covariance matrix $\Sigma(t)$, the carbon intensity $\boldsymbol{\mathcal{CI}}(t, x)$ and the set of additional constraints $\Omega_2(t)$ are functions of time t. This means that the data are updated every time we rebalance the portfolio[37]. In this framework, the constraint $\boldsymbol{\mathcal{CI}}(t, x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right)$ corresponds to the net zero emissions scenario, which is expressed in terms of carbon intensity. We have the following properties:

- The decarbonization of the aligned portfolio becomes easier with time if the benchmark decarbonizes itself:
$$\boldsymbol{\mathcal{CI}}\left(t, b(t)\right) \ll \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) \qquad \text{for } t > t_0 \tag{47}$$
- The decarbonization of the aligned portfolio becomes trickier with the time if the benchmark carbonizes itself:
$$\boldsymbol{\mathcal{CI}}\left(t, b(t)\right) \gg \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) \qquad \text{for } t > t_0 \tag{48}$$
- The aligned portfolio corresponds to the benchmark portfolio if the decarbonization of the benchmark is sufficiently strong:
$$\boldsymbol{\mathcal{CI}}\left(t, b(t)\right) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) \tag{49}$$

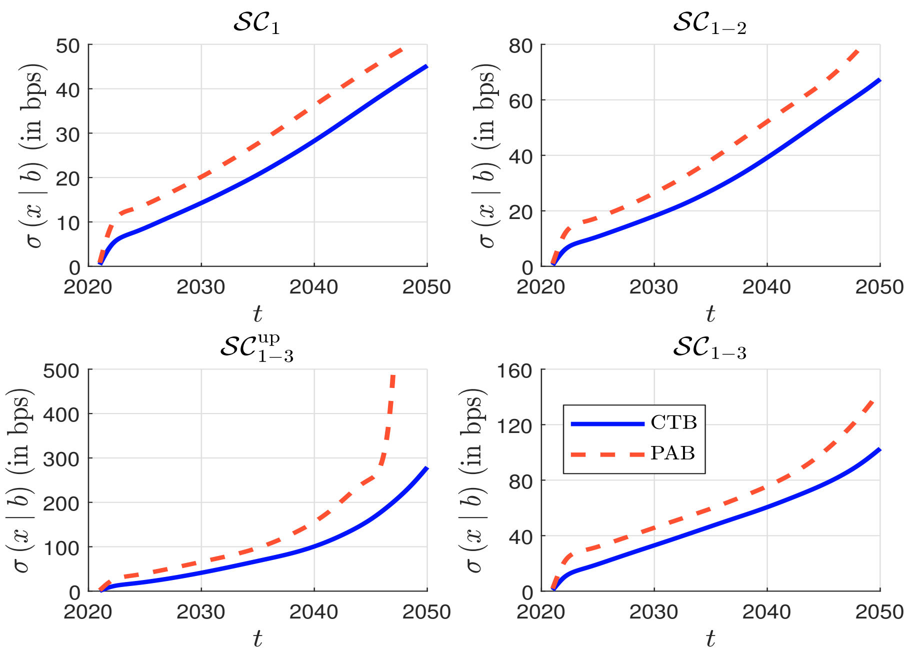
Since we have $\boldsymbol{\mathcal{CI}}\left(t, b(t)\right) = \sum_{i=1}^{n} \boldsymbol{\mathcal{CI}}_i(t) \cdot b_i(t)$, the decarbonization part of a net zero investing process is highly influenced by two pictures: changes in the benchmark weights and the carbon intensity of the assets. Indeed, we can imagine that the decarbonization process becomes easier over time, because the market capitalization of green assets grows faster than the market capitalization of brown assets and/or because the global decarbonization of the world is well established and follows the right way.

Remark 17. *In what follows, we consider that the data are not updated since we cannot guess or predict the benchmark composition in the future, the evolution of the covariance matrix, the level of carbon intensity, etc. As in Le Guenedal and Roncalli (2022), we assume that the world does not change. Of course, this is not realistic, but we are more interested in an order of magnitude of the tracking risks and a comparison between the different approaches rather than determining the optimal solutions.*

In Figure 21, we show the relationship between the time and the tracking error volatility with respect to the scope when considering the CTB and PAB decarbonization pathways. As observed by Le Guenedal and Roncalli (2022), including scope 3 has a significant impact on tracking risk, especially when considering the upstream scope 3. On average, including scope 3 results in multiplying the tracking risk by a factor of three. If we include weight and sector constraints, we may face situations where we do not find a solution (Figure 22). This is particularly true when imposing sectoral neutrality. In this case, the solution may not exist even before 2030 for the PAB decarbonization pathway. In order to have acceptable solutions, we relax these constraints and choose the $\mathcal{C}_3(0, 10, 2)$ configuration to challenge the $\mathcal{C}_0$ case (Figure 23).

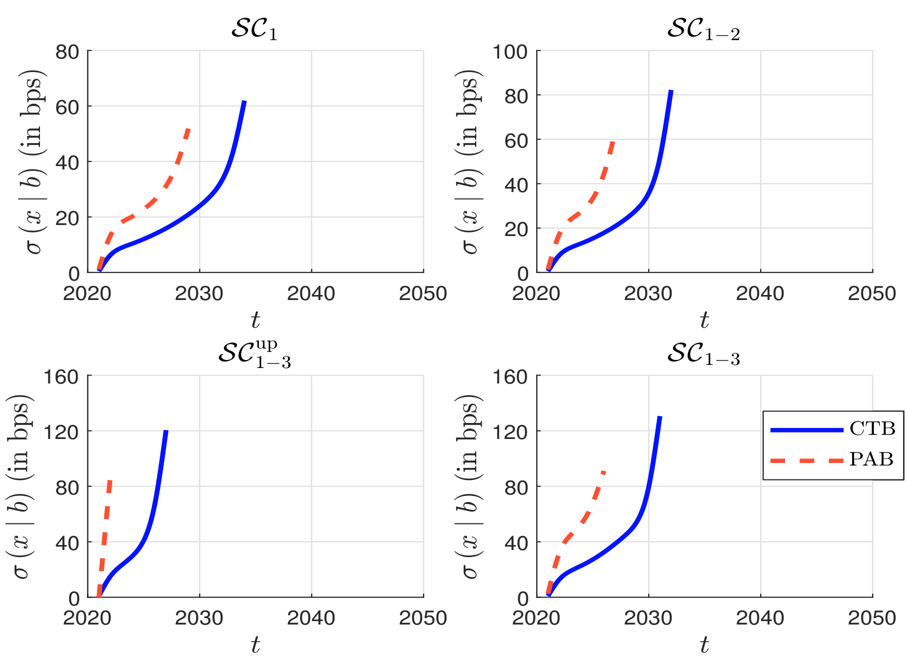
[37] For instance, at time $t+1$, the optimization problem depends on the data available at this current date and not at the past date t.

Figure 21: Tracking error volatility of dynamic decarbonized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint)



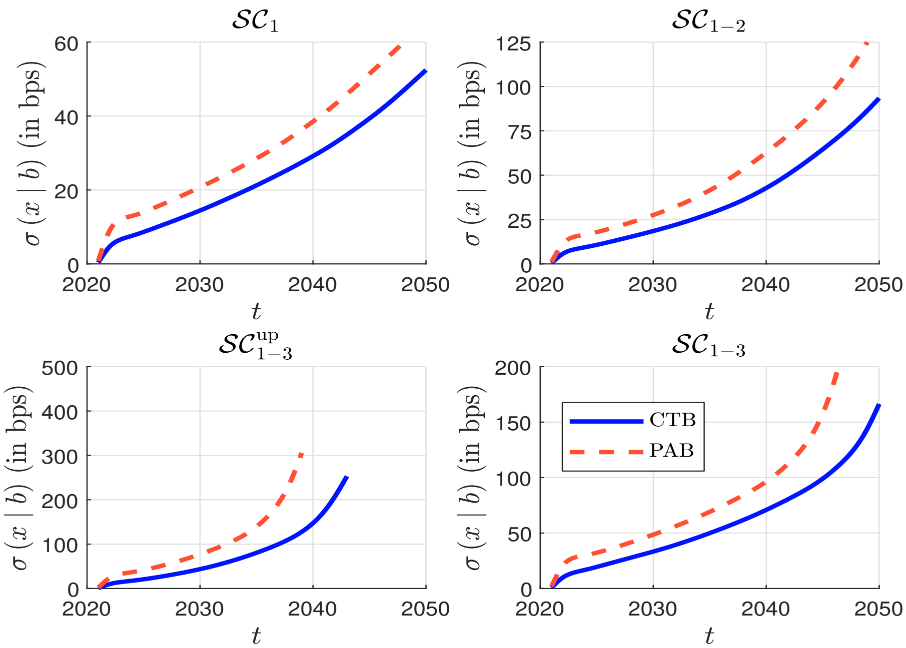
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 22: Tracking error volatility of dynamic decarbonized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3(0,2,1)$ constraint)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 23: Tracking error volatility of dynamic decarbonized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3(0, 10, 2)$ constraint)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Controlling the greenness As explained above, we must introduce the transition dimension. The PAB framework defines the concept of high climate impact sectors (HCIS). It lists several strategic sectors with respect to NACE European classification and imposes the following transition constraint:

$$\mathcal{H}_{CIS}\left(x(t)\right) \geq \mathcal{H}_{CIS}\left(b(t)\right) \tag{50}$$

where $\mathcal{H}_{CIS}(x) = \sum_{i \in \mathcal{H}_{CIS}} x_i$ is the weight of the portfolio that falls into HCIS sectors. As demonstrated by Le Guenedal and Roncalli (2022), this constraint has little impact on the transition dimension. Indeed, it does not help to maintain exposure in key sectors. Moreover, we can show that it does not help finance the transition to a low-carbon economy. This is why it is better to use a green intensity measure instead. We obtain the following optimization problem:

$$\begin{array}{rcl} x^{\star}(t) & = & \arg\min \frac{1}{2}\left(x - b(t)\right)^{\top} \Sigma(t) \left(x - b(t)\right) \\ & & \text{s.t.} \left\{ \begin{array}{ll} \boldsymbol{\mathcal{CI}}(t, x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) & \longleftarrow \text{Decarbonization} \\ \boldsymbol{\mathcal{GI}}(t, x) \geq \left(1 + \boldsymbol{\mathcal{G}}(t)\right) \cdot \boldsymbol{\mathcal{GI}}\left(t_0, b(t_0)\right) & \longleftarrow \text{Transition} \\ x \in \Omega_1 \cap \Omega_2(t) & \end{array} \right. \end{array} \tag{51}$$

Concerning the transition dimension, we can use the current benchmark as the anchor point and define an increasing function for the greenness multiplier $\boldsymbol{\mathcal{G}}(t)$. Another solution is to replace this constraint by the following one:

$$\boldsymbol{\mathcal{GI}}(t, x) \geq (1 + \boldsymbol{\mathcal{G}}) \cdot \boldsymbol{\mathcal{GI}}\left(t, b(t)\right) \tag{52}$$

The underlying idea is to maintain a green intensity for the net zero portfolio that is higher than the green intensity of the benchmark.

Table 19: Additional tracking error cost in bps of the greenness constraint (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\mathcal{G} = 0\%$				
$\mathcal{SC}_1$	0	0	0	0	0	0	0	0	0
$\mathcal{SC}_{1-2}$	0	0	0	0	0	0	0	0	1
$\mathcal{SC}_{1-3}^{\text{up}}$	0	0	0	0	0	1	5	27	
$\mathcal{SC}_{1-3}$	0	0	0	0	0	0	0	2	6
					$\mathcal{G} = 100\%$				
$\mathcal{SC}_1$	22	21	21	20	17	15	13	11	11
$\mathcal{SC}_{1-2}$	21	20	20	19	18	17	16	17	19
$\mathcal{SC}_{1-3}^{\text{up}}$	17	16	15	15	14	19	40	106	
$\mathcal{SC}_{1-3}$	16	15	14	14	12	12	13	22	43
					$\mathcal{G} = 200\%$				
$\mathcal{SC}_1$	51	51	50	49	45	42	38	35	33
$\mathcal{SC}_{1-2}$	50	50	49	48	46	45	43	48	54
$\mathcal{SC}_{1-3}^{\text{up}}$	44	43	42	41	39	50	95	257	
$\mathcal{SC}_{1-3}$	43	42	41	40	36	34	39	57	112

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

We have implemented a fixed greenness multiplier $\mathcal{G}$. In Table 19, we report the additional tracking error cost due to the transition constraint when we consider the PAB decarbonization pathway. We notice that this cost is equal to zero or relatively negligible when the greenness of the benchmark is to be maintained ($\mathcal{G} = 0\%$). Nevertheless, this constraint leads to a portfolio with a green intensity of only 5.24%, which may be weak for a net zero investor who wants to finance the transition. Doubling the green intensity ($\mathcal{G} = 100\%$) implies a marginal tracking error cost between 10 and 20 bps most of the time, except for the scope 3 and long time horizon. We also observe that the relationship between the green intensity and the tracking error cost is highly non-linear. Indeed, if we target a green intensity of 15%, which corresponds to a greenness multiplier $\mathcal{G}$ of about 200%, the additional cost lies between 35 and 100 bps.

Remark 18. *If we consider the $\mathcal{C}_3(0, 10, 2)$ constraint, we observe an increase in the tracking error which is relatively low until 2030 if $\mathcal{G} \leq 100\%$ (see Table 46 on page 84). Moreover, it becomes more and more difficult to find a solution when the greenness multiplier is equal to 200%.*

Managing the carbon momentum In order to manage the carbon momentum, we add a new constraint:

$$\begin{aligned} x^{\star}(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^{\top} \Sigma(t) \left(x - b(t)\right) \\ &\text{s.t.} \left\{ \begin{array}{ll} \mathcal{CI}(t, x) \leq \left(1 - \mathcal{R}(t_0, t)\right) \cdot \mathcal{CI}\left(t_0, b(t_0)\right) & \longleftarrow \text{Decarbonization} \\ x \in \Omega_1 \cap \Omega_2(t) & \\ x \in \Omega_3(t) & \longleftarrow \text{Momentum} \end{array} \right. \end{aligned} \tag{53}$$

For instance, we can impose that the carbon momentum of the portfolio is lower than a global threshold:

$$\Omega_3(t) = \left\{x : \mathcal{CM}^{\mathcal{L}ong}(t, x) \leq \mathcal{CM}^{\star}\right\} \tag{54}$$

In this case, the optimization program will overweight assets with negative momentum. For instance, if $\mathcal{CM}^{\star}$ is set to -7%, we expect the aligned portfolio to decarbonize itself by 7%. However, the previous constraint does not preclude the inclusion, or the overweighting, of companies with rising carbon intensities. Another approach consists in implementing an exclusion process:

$$\Omega_3(t) = \left\{\mathcal{CM}_i^{\mathcal{L}ong}(t) \geq \mathcal{CM}^{+} \Rightarrow x_i = 0\right\} \tag{55}$$

where $\mathcal{CM}^{+}$ is an acceptable upper bound. For example, if $\mathcal{CM}^{+}$ is set to 0, we exclude all the issuers presenting a positive carbon momentum.

Remark 19. *Another approach consists in imposing higher self-decarbonization than the benchmark:*

$$\Omega_3(t) = \left\{x : \mathcal{CM}^{\mathcal{L}ong}(t, x) \leq \mathcal{CM}^{\mathcal{L}ong}\left(t, b(t)\right) - \Delta\mathcal{CM}^{\mathcal{L}ong}\left(x \mid b(t)\right)\right\} \tag{56}$$

This is equivalent to the global threshold approach where:

$$\mathcal{CM}^{\star} = \mathcal{CM}^{\mathcal{L}ong}\left(t, b(t)\right) - \Delta\mathcal{CM}^{\mathcal{L}ong}\left(x \mid b(t)\right) \tag{57}$$

For instance, we saw in Table 12 on page 42 that the carbon momentum of the MSCI World index is estimated at -1.7%. If we would like to improve the carbon momentum of the alignment portfolio, we can set $\mathcal{CM}^{\star} = -5\%$ or $\Delta\mathcal{CM}^{\mathcal{L}ong}\left(x \mid b(t)\right) = 3.3\%$.

Table 20 provides the marginal tracking error cost of adding a global momentum constraint to the $\mathcal{C}_0$ optimization problem. If $\mathcal{CM}^{\star} = -5\%$, the cost is lower than 10 bps, and decreases with the year. If $\mathcal{CM}^{\star} = -7\%$, we can observe a cost greater than 10 bps before 2030. Contrary to the green intensity, the weight constraint $\mathcal{C}_3(0, 10, 2)$ has a significant impact. Indeed, the cost is multiplied by a factor of two at the beginning of the period[38].

Table 20: Additional tracking error cost in bps of a global momentum threshold approach (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\mathcal{CM}^{\star} = -5\%$				
$\mathcal{SC}_1$	9	9	9	9	7	5	3	2	2
$\mathcal{SC}_{1-2}$	8	7	7	7	5	3	1	1	0
$\mathcal{SC}_{1-3}^{\text{up}}$	4	3	2	2	0	0	0	0	
$\mathcal{SC}_{1-3}$	3	3	3	3	2	1	1	1	2
					$\mathcal{CM}^{\star} = -7\%$				
$\mathcal{SC}_1$	17	17	16	16	13	10	8	6	4
$\mathcal{SC}_{1-2}$	15	15	14	13	10	7	4	2	1
$\mathcal{SC}_{1-3}^{\text{up}}$	8	7	6	5	1	0	0	0	
$\mathcal{SC}_{1-3}$	8	8	7	6	4	3	2	2	4

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Let us now consider the exclusion approach. In Table 21, we give some statistics about the distribution of the carbon momentum[39]. It follows that 25% of issuers have a positive carbon momentum. If we consider the case $\mathcal{CM}_i > 5\%$, this figure is equal to 2.3% in terms

[38] See Table 47 on page 84.

[39] We remind that we use $\mathcal{SC}_{1-3}^{\text{up}}$ for estimating the trend and at least 5 years of historical data. This explains that the carbon momentum does not cover 100% of the investment universe.

Table 21: Statistics of the carbon momentum $\mathcal{CM}_i^{\mathcal{L}ong}$ (MSCI World, Jun. 2022)

Statistic	Median	Negative	Positive	$\mathcal{CM}_i <$ -10%	$\mathcal{CM}_i <$ -5%	$\mathcal{CM}_i >$ $+5\%$	$\mathcal{CM}_i >$ $+10\%$
Frequency (in %)	-1.5	75.1	24.9	5.9	14.0	2.3	0.8
Weight (in %)		72.8	24.6	4.3	12.2	1.0	0.5

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

of issuers and 1.0% in terms of allocation. Therefore, we expect that using an upper bound $\mathcal{CM}^+$ greater than 5% has little impact. Let us first consider the case $\mathcal{CM}^+ = 0\%$, implying that we exclude all the issuers with positive carbon momentum. Table 22 shows that the marginal tracking error cost is very high, especially at the beginning of the period. For example, the additional tracking error is greater than 100 bps until 2025. The reason is that a large proportion of issuers in the MSCI World index have a positive trend in their carbon intensity. Nevertheless, if we consider a higher value of $\mathcal{CM}^+$, the cost may be negligible. For instance, this is the case when $\mathcal{CM}^+$ is equal to 10%. Moreover, these different results remain valid with the $\mathcal{C}_3(0,10,2)$ constraint, as shown in Table 48 on page 84.

Table 22: Additional tracking error cost in bps of a momentum exclusion approach (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{CM}^+ = 0\%$								
$\mathcal{SC}_1$	123	122	121	120	114	107	100	93	88
$\mathcal{SC}_{1-2}$	121	119	118	117	109	98	87	78	66
$\mathcal{SC}_{1-3}^{\text{up}}$	109	105	102	98	80	63	37	10	
$\mathcal{SC}_{1-3}$	111	108	106	104	94	85	77	67	50
	$\mathcal{CM}^+ = 5\%$								
$\mathcal{SC}_1$	3	3	3	3	2	1	1	1	1
$\mathcal{SC}_{1-2}$	2	2	2	2	1	1	0	0	0
$\mathcal{SC}_{1-3}^{\text{up}}$	1	1	1	1	0	0	0	0	
$\mathcal{SC}_{1-3}$	2	2	1	1	1	1	1	0	0

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Combining decarbonization and transition Finally, we combine all the constraints to define the final optimization problem. We consider the threshold approach for the carbon momentum and obtain:

$$x^\star(t) = \arg\min \frac{1}{2}\left(x - b(t)\right)^\top \Sigma(t)\left(x - b(t)\right) \quad (58)$$
$$\text{s.t.} \left\{\begin{array}{ll} \mathcal{CI}(t,x) \leq \left(1 - \mathcal{R}(t_0,t)\right)\cdot\mathcal{CI}\left(t_0, b(t_0)\right) & \longleftarrow \text{Decarbonization} \\ x \in \Omega_{\mathcal{T}ransition}(t) & \longleftarrow \text{Transition} \\ x \in \Omega_1 \cap \Omega_2(t) & \end{array}\right.$$

where the decarbonization dimension is defined by using the usual constraint $\mathcal{CI}(t,x) \leq \left(1 - \mathcal{R}(t_0,t)\right)\cdot\mathcal{CI}\left(t_0, b(t_0)\right)$ and the transition dimension is specified by the set of constraints $\Omega_{\mathcal{T}ransition}(t)$. In a first time, we assume that:

$$x \in \Omega_{\mathcal{T}ransition}(t) \Leftrightarrow \left\{ \begin{array}{ll} \mathcal{GI}(t,x) \geq \left(1+\mathcal{G}(t)\right) \cdot \mathcal{GI}\left(t_0, b(t_0)\right) & \longleftarrow \text{ Greenness} \\ \mathcal{CM}^{\mathcal{L}ong}(t,x) \leq \mathcal{CM}^{\star} & \longleftarrow \text{ Momentum} \end{array} \right. \tag{59}$$

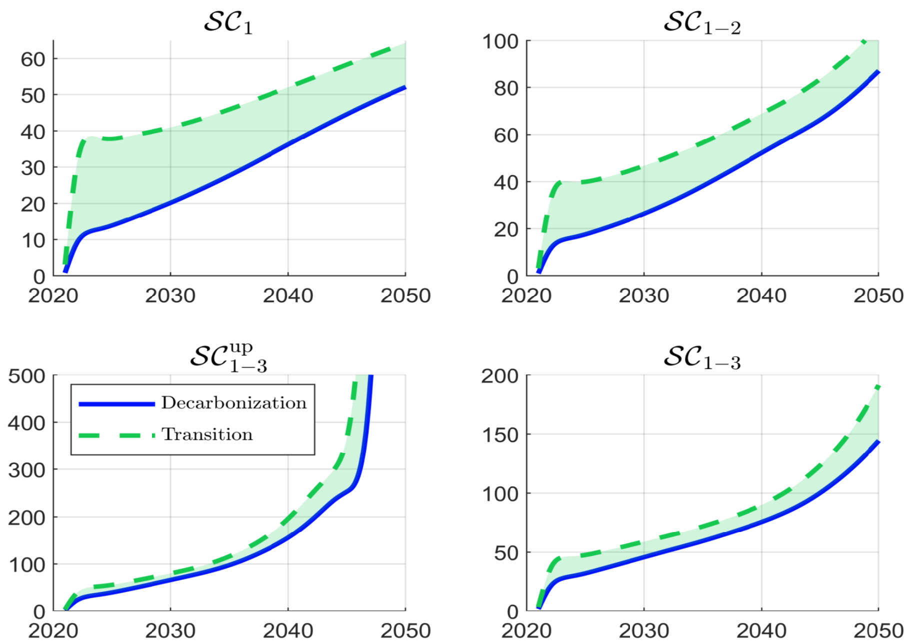
For both CTB and PAB pathways, we consider the previous optimization program (58–59) and compute the solution using several sets of parameters: $\mathcal{C}_0$ vs. $\mathcal{C}_3(0,10,2)$, $\mathcal{G} = 100\%$ vs. $\mathcal{G} = 200\%$ and $\mathcal{CM}^{\star} = -5\%$ vs. $\mathcal{CM}^{\star} = -7\%$. The impact on the tracking error volatility and the decomposition between decarbonization and transition dimensions are reported in Figures 49–56 on pages 94–97. The results of these simulations clearly show that the transition dimension induces a significant cost. On average, if we focus on the case $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$ and the PAB pathway, we observe that the additional tracking error cost for the years 2022–2030 is respectively equal to 23, 22, 16 and 15 bps for scopes $\mathcal{SC}_1$, $\mathcal{SC}_{1-2}$, $\mathcal{SC}_{1-3}^{\text{up}}$ and $\mathcal{SC}_{1-3}$ when we do not consider weight and sector constraints (Figure 24). These figures become 27, 25, 21 and 19 bps if we use the $\mathcal{C}_3(0,10,2)$ constraint (Figure 24). Moreover, there may not be a solution to the optimization problem through 2050, especially when the carbon footprint is based on upstream/downstream scope 3 emissions. Of course, all these results are very sensitive to the choice of the green multiplier $\mathcal{G}$ and the carbon threshold $\mathcal{CM}^{\star}$ as illustrated in Figures 49–56 on pages 94–97.

Remark 20. *The magnitude of the cost of combining green intensity and carbon momentum constraints is significantly higher than the cost of each constraint. This means that the two sub-dimensions of the transition pillar are not currently correlated. For instance, we have reported the scatter plot between the green intensity $\mathcal{GI}_i$ and the carbon momentum $\mathcal{CM}_i^{\mathcal{L}ong}$ in Figure 26 and we do not observe a clear relationship. These two statistical measures are then independent. In practice, there may be a lead-lag effect between these two elements. Indeed, some issuers that are beginning to transform their business model to green activities may have positive carbon momentum because of their old system. For instance, increasing green capex has no direct effect on the current carbon footprint, but it will definitively impact the future carbon footprint. Therefore, we expect that this lead-lag effect will be reduced in some years.*

To measure the discrepancy between the benchmark $b(t_0)$ and the optimized portfolio $x^{\star}(t)$, we compute the active share between the weights of these two portfolios. The results are given in Tables 23 and 24 for $\mathcal{C}_0$ and $\mathcal{C}_3(0,10,2)$ constraints. As expected, we observe that the divergence between the benchmark and the decarbonization portfolio increases with the reduction date. In addition, the active share is far more important when implementing a net zero strategy rather than only a decarbonization pathway. On average, we observe a factor of three. Nevertheless, we observe that both approaches lead to relatively high active shares, meaning that decarbonization and portfolio alignment cannot be achieved without significant active costs. If we now compare the net zero portfolio with the corresponding decarbonized portfolio, we notice that the weights are different (see Tables 49 and 50 on page 49). For constraint $\mathcal{C}_0$, the average active share until 2030 is respectively equal to 11% for the case $\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -5\%$ and 22% for the case $\mathcal{G} = 200\%$ and $\mathcal{CM}^{\star} = -7\%$. These figures become 13% and 35% for constraint $\mathcal{C}_3(0,10,2)$. All these results show that the additional cost of implementing a net zero policy does not only concern the long-term horizon, but they are also important in the short-term horizon. This is a huge difference between the decarbonization dimension and the transition dimension. By construction, this last one implies a spike in the active cost directly at the beginning of the period.

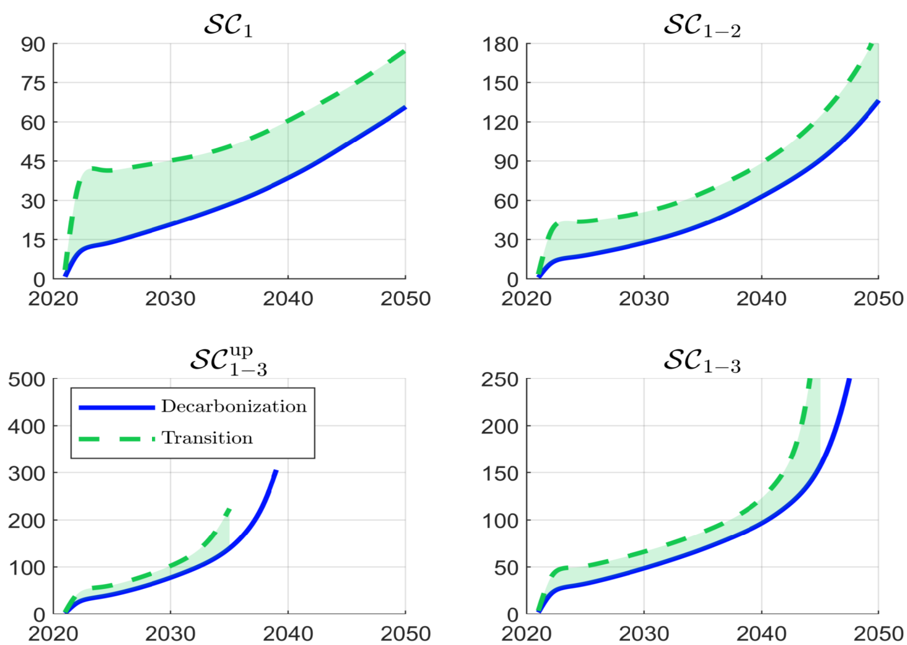
Remark 21. *For the sake of simplicity, we did not impose a constraint on the portfolio turnover and transaction cost, but such optimization problems are specified in Lezmi et al.*

Figure 24: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB)



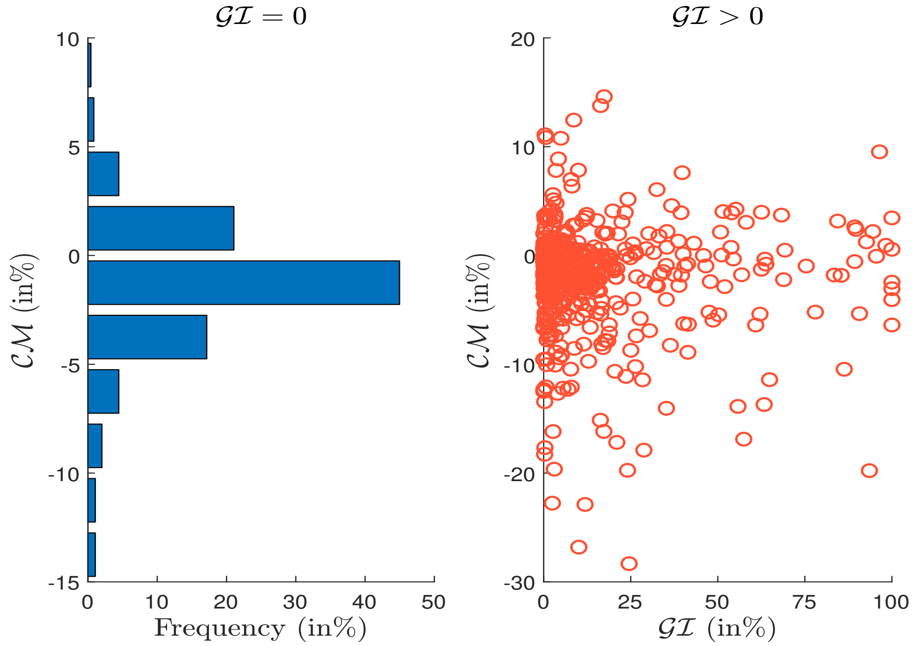
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 25: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 26: Relationship between the green intensity $\mathcal{GI}_i$ and the carbon momentum $\mathcal{CM}_i^{\mathcal{L}ong}$ (MSCI World, Jun. 2022)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Table 23: Active share (in %) between the benchmark and the optimized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	Decarbonized portfolio								
$\mathcal{SC}_1$	3.4	3.7	4.2	4.6	7.2	10.2	13.3	16.0	18.2
$\mathcal{SC}_{1-2}$	4.6	5.2	5.8	6.4	10.4	15.7	21.1	27.8	39.7
$\mathcal{SC}_{1-3}^{\text{up}}$	12.2	14.5	16.8	19.2	30.4	45.1	65.4	82.6	
$\mathcal{SC}_{1-3}$	9.2	10.3	11.3	12.3	18.3	24.1	32.2	45.0	60.7
	Net zero portfolio with $\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	12.1	12.2	12.4	12.6	13.6	15.7	17.7	19.7	21.5
$\mathcal{SC}_{1-2}$	12.4	12.7	12.9	13.3	16.3	20.4	25.2	33.0	44.0
$\mathcal{SC}_{1-3}^{\text{up}}$	16.9	18.9	21.1	23.2	33.9	51.4	71.2	92.7	
$\mathcal{SC}_{1-3}$	14.1	14.8	15.5	16.4	21.4	27.5	37.4	53.0	71.2
	Net zero portfolio with $\mathcal{G} = 200\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	22.5	22.6	22.7	22.7	23.4	24.7	26.0	27.2	28.5
$\mathcal{SC}_{1-2}$	22.7	22.9	23.0	23.3	25.6	28.5	32.5	40.1	49.8
$\mathcal{SC}_{1-3}^{\text{up}}$	25.2	26.7	28.4	30.2	39.9	58.4	75.4	95.7	
$\mathcal{SC}_{1-3}$	23.2	23.5	23.9	24.4	27.5	33.0	45.1	61.7	78.3

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Table 24: Active share (in %) between the benchmark and the optimized portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3(0, 10, 2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	Decarbonized portfolio								
$\mathcal{SC}_1$	3.4	3.7	4.1	4.6	7.2	10.3	14.2	18.9	24.4
$\mathcal{SC}_{1-2}$	4.5	5.1	5.7	6.4	10.6	17.3	27.0	38.8	54.7
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	12.4	14.8	17.2	20.0	36.2	58.7			
$\mathcal{SC}_{1-3}$	9.2	10.2	11.2	12.4	19.8	28.9	40.3	62.2	
	Net zero portfolio with $\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -5\%$								
$\mathcal{SC}_1$	13.8	14.0	14.2	14.4	15.8	18.0	21.9	26.5	32.6
$\mathcal{SC}_{1-2}$	14.2	14.5	14.9	15.2	18.5	25.9	34.9	47.6	65.9
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	18.7	20.9	23.3	26.3	44.4	71.5			
$\mathcal{SC}_{1-3}$	15.2	15.9	16.9	18.0	25.0	34.1	50.3	83.2	
	Net zero portfolio with $\mathcal{G} = 200\%$ and $\mathcal{CM}^{\star} = -7\%$								
$\mathcal{SC}_1$	33.6	33.8	34.1	34.4	36.2	38.3	40.9	46.3	55.3
$\mathcal{SC}_{1-2}$	34.4	35.0	35.5	36.1	38.9	44.8	57.8	73.1	86.9
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	37.4	39.1	40.9	43.5	63.4				
$\mathcal{SC}_{1-3}$	32.4	33.0	33.7	34.7	41.6	54.0	76.0		

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

(2022). Nevertheless, the one-way turnover between dates t and $t+1$ remains low, with an average of 3.2% and 4.5% each year for the decarbonized and net zero portfolios.

Remark 22. *The previous results are valid for the MSCI World index, a large investment universe. Let us focus on smaller investment universes by considering the MSCI EMU and USA indexes. The results are reported in Figures 57–60 on page 98. Tracking errors for smaller universes become greater in fewer years than for the MSCI World index and we also fail to find solutions sooner. We could separate these results by putting scope 1 and scope 2 alignment on one side and scope 3 on the other. Considering scopes 1 and 2, we observe that, for both universes, our aligned portfolio breaks earlier than for the MSCI World. However, even though the MSCI EMU universe is smaller than the USA one, we can find solutions for a longer period. The reason lies in the distribution of green revenue and carbon momentum, which can be more easily conciliated with the intensity reduction constraint for the EMU. Including scope 3 intensities paints another picture. Although the EMU portfolios have lower tracking errors than those from the USA, larger universes tend to give solutions longer. The fact that we are not able to align our EMU portfolio after 2040 in terms of scope 3 carbon intensities therefore highlights the difficulty of portfolio alignment for a relatively small investment universe.*

Preventing greenwashing In finance, greenwashing is the action of making people think an investment is not harmful to the environment while this is not really the case. Intentional or not, greenwashing is a reputational risk for financial institutions. Providing full transparency about a financial process helps to reduce this risk. Therefore, a quantitative top-down approach is useful because the different steps of the process are fully described, in particular the objective function and the different constraints. Nevertheless, a top-down approach is not sufficient because some issuers may be selected or overweighted compared to the benchmark, albeit, they do not meet all the conditions of a net zero investment policy. Of course, we can always define an optimization problem by increasing the number of constraints. However, too many of them may produce no solution. This is why we think that a top-down approach must be based on a few intelligible constraints and it must be completed by an ex-post analysis to avoid greenwashing risks.

Various KPIs of a company should be considered when aligning a portfolio to a net zero trajectory. For instance, tracking error minimization can lead to the inclusion of companies that do not actually meet the emission reduction objective. For example, a company with a positive carbon emission momentum can be overweighted compared to the benchmark, as only the carbon intensity momentum is taken into account in the constraints. Similarly, if the optimization is based on scope $\boldsymbol{\mathcal{SC}}_{1-2}$, it can favour companies that better manage this scope than scope $\boldsymbol{\mathcal{SC}}_{1-3}$. Moreover, it seems important to perform a bottom-up analysis of the aligned portfolio to make sure that the selected companies are not subject to climate (or ESG) controversies. The ex-post analysis consists then in analyzing the optimized portfolio and defining a new set of exclusions $x \in \Omega^{\text{ex-post}}_{\boldsymbol{\mathcal{E}}xclusion}(t)$, which generally complete a set of ex-ante or pre-defined exclusions[40] $x \in \Omega^{\text{ex-ante}}_{\boldsymbol{\mathcal{E}}xclusion}$. Therefore, the global optimization problem becomes:

$$
\begin{aligned}
x^{\star}(t) &= \arg\min \frac{1}{2}\left(x - b(t)\right)^{\top} \Sigma(t) \left(x - b(t)\right) \\
&\text{s.t.} \left\{
\begin{array}{lll}
x \in \Omega_{\boldsymbol{\mathcal{E}}xclusion}(t) = \Omega^{\text{ex-ante}}_{\boldsymbol{\mathcal{E}}xclusion} \cap \Omega^{\text{ex-post}}_{\boldsymbol{\mathcal{E}}xclusion}(t) & \longleftarrow & \text{Exclusion} \\
\boldsymbol{\mathcal{CI}}(t, x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(t_0, b(t_0)\right) & \longleftarrow & \text{Decarbonization} \\
x \in \Omega_{\boldsymbol{\mathcal{T}}ransition}(t) & \longleftarrow & \text{Transition} \\
x \in \Omega_1 \cap \Omega_2(t) & &
\end{array}
\right.
\end{aligned}
\tag{60}
$$

4.2.2 Bond portfolios

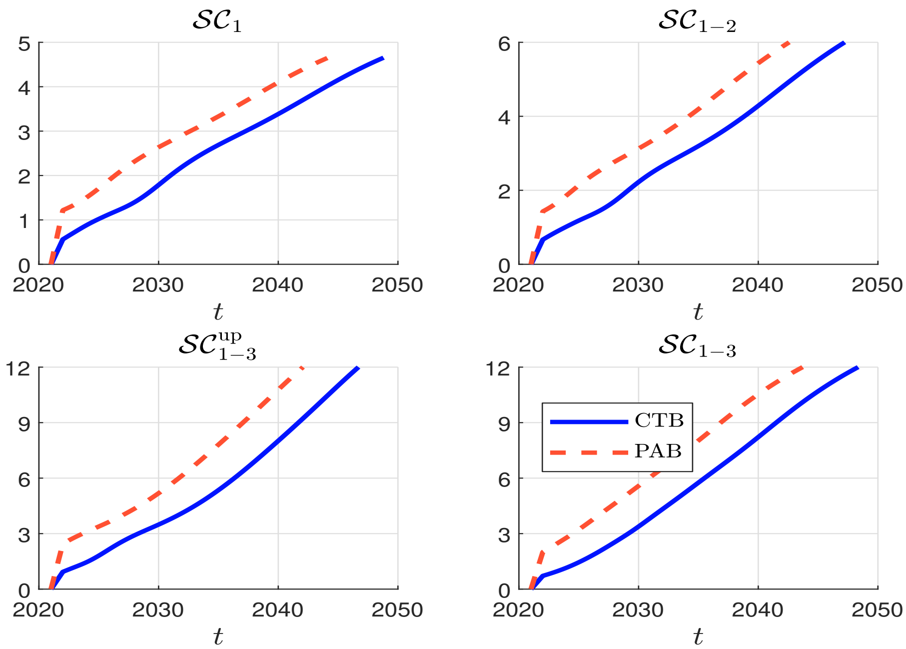
Dynamic decarbonization We adapt the equity dynamic decarbonization problem to bonds. The solution $x^{\star}(t)$ at time t requires to know the investment universe $b(t)$, the bond risk metrics $\text{DTS}_i(t)$ and $\text{MD}_i(t)$, and the carbon intensity $\boldsymbol{\mathcal{CI}}_i(t)$. In what follows, we perform the exercise assuming that the world does not change[41]. We perform the optimization by considering only the decarbonization pathways of CTB and PAB labels. The results are given in Figures 27 and 28. The DTS tracking risk is not significant and is lower than 6 bps until 2030. This is not the case of the active share risk, since it can reach 20% for the PAB decarbonization pathway in 2030. We also notice that the active share risk is an increasing function of the year and the scope until 2040. After this year, scope $\boldsymbol{\mathcal{SC}}^{\text{up}}_{1-3}$ takes the lead on scope $\boldsymbol{\mathcal{SC}}_{1-3}$. Nevertheless, we do not have the significant gap observed in the case of equities between upstream scope 3 and the other scopes.

Controlling the greenness We apply the transition constraint for different values of $\boldsymbol{\mathcal{G}}$: 0%, 100% and 200%. In Table 25, we present PAB results, but CTB results are comparable and available in the appendix (Table 51 on page 85). We do not report the DTS tracking risk since it is negligible (less than 1 bp for $\boldsymbol{\mathcal{G}} = 100\%$). The active share cost is low and close to zero when the goal is to maintain the greenness of the benchmark. The reason is that most decarbonized portfolios already have a green intensity greater than or equal to that of the benchmark (see Table 17 on page 47). When $\boldsymbol{\mathcal{G}} = 100\%$, the additional cost is between 0.2% and 0.9% until 2030. This cost becomes high when we want to triple the green intensity, and can reach 4.2%.

[40] Generally, asset managers exclude worst-in-class ESG issuers, companies with a large business on thermal coal and oil, etc.

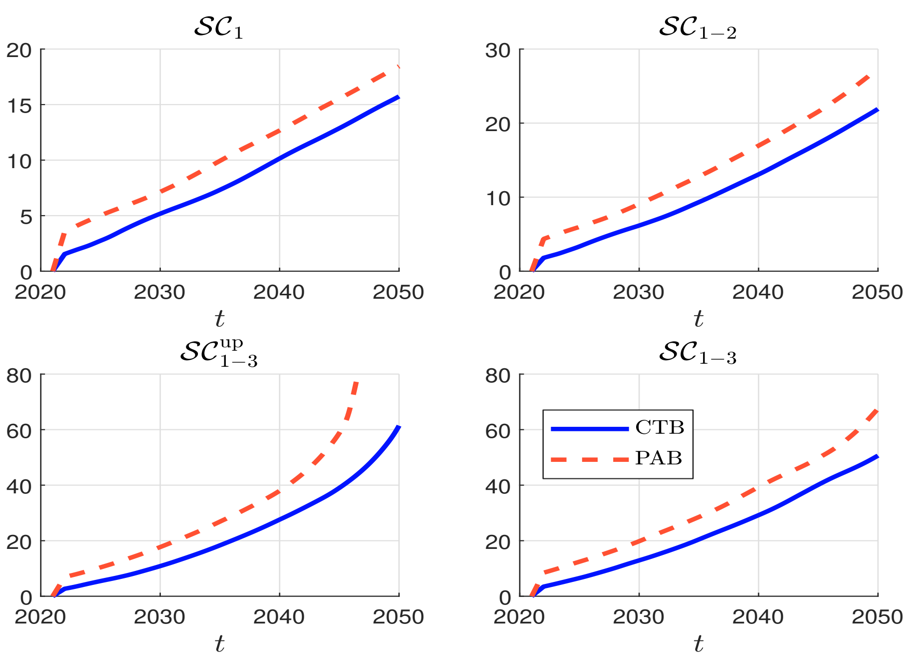
[41] This implies that $\boldsymbol{\mathcal{CI}}_i(t) = \boldsymbol{\mathcal{CI}}_i(t_0)$, $b(t) = b(t_0)$, $\text{DTS}_i(t) = \text{DTS}_i(t_0)$ and $\text{MD}_i(t) = \text{MD}_i(t_0)$.

Figure 27: Duration-times-spread of dynamic decarbonized portfolios in bps (Global Corp., Jun. 2022)



Source: ICE (2022), Trucost (2022) & Authors' calculations.

Figure 28: Active share of dynamic decarbonized portfolios in % (Global Corp., Jun. 2022)



Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 25: Additional active share cost in % when we control the green intensity (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\mathcal{G} = 0\%$				
$\mathcal{SC}_1$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\mathcal{SC}_{1-2}$	0.1	0.1	0.1	0.1	0.2	0.1	-0.2	-0.3	-0.2
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.1	0.1	0.2	0.2	0.0	-0.2	-0.1	0.2	0.3
$\mathcal{SC}_{1-3}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
					$\mathcal{G} = 100\%$				
$\mathcal{SC}_1$	0.9	0.6	0.4	0.3	0.2	0.2	0.0	-0.2	-0.4
$\mathcal{SC}_{1-2}$	0.6	0.4	0.4	0.4	0.4	0.2	0.0	-0.1	0.1
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.3	0.4	0.4	0.5	0.5	0.5	0.4	1.8	0.3
$\mathcal{SC}_{1-3}$	0.4	0.3	0.3	0.3	0.4	0.5	0.3	-0.6	-3.6
					$\mathcal{G} = 200\%$				
$\mathcal{SC}_1$	4.2	3.8	3.5	3.2	2.0	0.8	0.2	0.0	-0.3
$\mathcal{SC}_{1-2}$	3.7	3.3	3.0	2.7	1.5	1.0	0.4	0.3	0.6
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	2.3	1.8	1.4	1.3	1.3	1.6	1.6	3.8	0.3
$\mathcal{SC}_{1-3}$	1.6	1.2	1.0	0.8	0.7	0.9	0.7	0.0	-2.2

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Integrating the carbon momentum constraint In Table 26, we report some statistics about the carbon momentum. Obviously, the higher the upper bound $\mathcal{CM}^+$, the lower the number of excluded issuers. Removing all issuers with positive carbon momentum represents 542 out of 2362 issuers and 23.5% of the benchmark, while only 51 issuers (and 1.5% of the benchmark) are discarded when $\mathcal{CM}^+$ is equal to 5%.

Table 26: Statistics of the carbon momentum $\mathcal{CM}_i^{\mathcal{L}ong}$ (Global Corp., Jun. 2022)

Statistic	Median	Negative	Positive	$\mathcal{CM}_i <$		$\mathcal{CM}_i >$	
				-10%	-5%	$+5\%$	$+10\%$
Frequency (in %)	-1.3	77.1	22.9	3.3	14.9	2.2	0.8
Weight (in %)		76.5	23.5	4.2	13.4	1.5	0.9

Source: ICE (2022), Trucost (2022) & Authors' calculations.

We suppose that $\mathcal{G}$ is equal to 100%. Since we have seen that the additional tracking cost (DTS and AS) is small when we control the green intensity, we add the momentum exclusion constraint to the previous optimization problem. Table 27 shows the additional active share cost after the greenness control. When $\mathcal{CM}^+$ is equal to 0%, this cost until 2030 is above 20% for the scope $\mathcal{SC}_1$ and 15% for the scope $\mathcal{SC}_{1-3}$. As expected, excluding issuers with positive carbon momentum has a substantial cost compared to the cost of doubling the green intensity. When we set $\mathcal{CM}^+ = 5\%$, the cost is negligible since these issuers represent about 1.5% of the benchmark.

Remark 23. *In Table 53 on page 86, we report additional DTS cost because it is the only case where it is significant. Indeed, when $\mathcal{G} = 100\%$ and $\mathcal{CM}^+ = 0\%$, we can observe an additional DTS cost that is close to 5 bps.*

In Table 55 on page 87, we have reported the carbon momentum difference $\Delta\mathcal{CM}(t) = \mathcal{CM}(t, x^\star(t)) - \mathcal{CM}(t, b(t))$. We note that the difference is negative with a null $\mathcal{CM}^+$ and frequently decreases with the years. When we set $\mathcal{CM}^+$ to 5%, the story is different.

Table 27: Additional active share cost in % when we implement a momentum exclusion approach (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	20.5	20.6	20.5	20.5	20.1	18.6	17.5	16.7	16.1
$\mathcal{SC}_{1-2}$	20.1	20.0	19.8	19.7	18.5	16.5	15.1	13.6	11.4
$\mathcal{SC}_{1-3}^{\text{up}}$	18.6	18.1	17.6	17.0	13.8	11.6	9.7	4.1	0.3
$\mathcal{SC}_{1-3}$	17.2	16.8	16.5	16.3	14.6	13.2	12.5	12.3	13.9
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	0.6	0.7	0.9	1.0	1.0	0.9	0.9	0.8	0.7
$\mathcal{SC}_{1-2}$	0.7	0.8	0.9	0.9	0.6	0.6	0.5	0.3	0.1
$\mathcal{SC}_{1-3}^{\text{up}}$	0.9	0.8	0.7	0.7	0.5	0.3	0.2	0.1	0.0
$\mathcal{SC}_{1-3}$	1.1	1.1	1.1	1.0	0.8	0.6	0.5	0.3	0.0

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

The decarbonized portfolio often shows a worse carbon reduction trend than the benchmark reference, and the difference may even increase with the years. To ensure a better trajectory for the decarbonized portfolio, we change the momentum approach and use the global momentum constraint $\mathcal{CM}^{\mathcal{L}ong}(t, x) \leq \mathcal{CM}^{\star}$. This second strategy is less harmful in active share, especially compared to $\mathcal{CM}^{+} = 0\%$. Indeed, if we apply $\mathcal{CM}^{\star} = -5\%$ and $\mathcal{CM}^{\star} = -7\%$, the difference in active share remains below 1.4% (Table 28).

Table 28: Additional Active share cost in % of a global momentum threshold approach (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -5\%$								
$\mathcal{SC}_1$	0.6	0.7	0.7	0.7	0.7	0.2	0.2	0.2	0.2
$\mathcal{SC}_{1-2}$	0.6	0.6	0.6	0.6	0.3	0.1	0.1	0.2	0.1
$\mathcal{SC}_{1-3}^{\text{up}}$	0.5	0.5	0.5	0.4	0.2	0.2	0.1	0.4	−0.2
$\mathcal{SC}_{1-3}$	0.3	0.3	0.4	0.4	0.4	0.4	0.7	0.9	1.9
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -7\%$								
$\mathcal{SC}_1$	1.4	1.4	1.3	1.3	1.4	0.8	0.4	0.4	0.4
$\mathcal{SC}_{1-2}$	1.2	1.1	1.1	1.1	1.0	0.4	0.4	0.4	0.3
$\mathcal{SC}_{1-3}^{\text{up}}$	0.9	0.9	0.9	0.8	0.5	0.5	0.4	0.0	−0.1
$\mathcal{SC}_{1-3}$	0.6	0.7	0.7	0.7	0.7	0.8	1.2	1.7	3.5

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Remark 24. *Results for the CTB pathway are shown in Tables 52, 54 and 56 on page 86.*

Preventing greenwashing We have already presented the ex-post exclusion approach on page 57. In this paragraph, we explore other approaches. For instance, we can impose that the weight in the aligned portfolio can not exceed the weight in the benchmark for issuers with a positive carbon momentum. For each issuer j, we note $\mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CE}, \mathcal{SC})$ and $\mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CI}, \mathcal{SC})$ the carbon emission and intensity momentum measures for the corresponding scope $\mathcal{SC}$. Let $\mathcal{NCM}_j$ be the total number of positive carbon momentum:

$$\mathcal{NCM}_j = \sum_{\mathcal{SC}=\mathcal{SC}_1, \mathcal{SC}_{1-2}, \mathcal{SC}_{1-3}} \left\{ \mathbb{1}\left\{\mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CE}, \mathcal{SC})\right\} + \mathbb{1}\left\{\mathcal{CM}_j^{\mathcal{L}ong}(\mathcal{CI}, \mathcal{SC})\right\} \right\} \quad (61)$$

$\mathcal{NCM}_j$ takes its values between 0 and 6. We also define $\mathcal{NCM}'_j$ when we only consider the carbon intensity momentum:

$$\mathcal{NCM}'_j = \mathbb{1}\left\{\mathcal{CM}_j^{\mathcal{L}ong}\left(\mathcal{CI},\mathcal{SC}_1\right)\right\} + \mathbb{1}\left\{\mathcal{CM}_j^{\mathcal{L}ong}\left(\mathcal{CI},\mathcal{SC}_{1-2}\right)\right\} + \mathbb{1}\left\{\mathcal{CM}_j^{\mathcal{L}ong}\left(\mathcal{CI},\mathcal{SC}_{1-3}\right)\right\} \tag{62}$$

In this case, $\mathcal{NCM}'_j$ takes its values between 0 and 3.

Table 29: Frequency and weight of positive carbon momentum (Global Corp., Jun. 2022)

$\mathcal{NCM}_j$	$\mathcal{NCM}'_j$ Frequency in (%) 0	1	2	3	Weight in (%) 0	1	2	3
0	32.26				32.84			
1	10.75	0.76			8.67	1.53		
2	12.49	1.14	0.59		8.93	3.64	0.85	
3	14.27	1.61	0.47	2.54	12.51	4.53	0.66	3.38
4		2.84	2.20	0.47		3.01	3.01	0.56
5			4.57	0.47			5.32	1.04
6				12.57				9.51
Total	69.77	6.35	7.83	16.05	62.94	12.72	9.85	14.49

Source: ICE (2022), Trucost (2022) & Authors' calculations.

In Table 29, we report the frequencies of $\left(\mathcal{NCM}_j, \mathcal{NCM}'_j\right)$. We notice that less than one-third of issuers have six negative carbon trends, implying that the matrix of carbon trends is exactly this one:

	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}$
$\mathcal{CE}$	$\searrow$	$\searrow$	$\searrow$
$\mathcal{CI}$	$\searrow$	$\searrow$	$\searrow$

12.57% of issuers have six positive carbon trends:

	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}$
$\mathcal{CE}$	$\nearrow$	$\nearrow$	$\nearrow$
$\mathcal{CI}$	$\nearrow$	$\nearrow$	$\nearrow$

This implies that about 55% of issuers have both positive and negative trends. Among them, 14.27% of issuers are in the following configuration:

	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}$
$\mathcal{CE}$	$\nearrow$	$\nearrow$	$\nearrow$
$\mathcal{CI}$	$\searrow$	$\searrow$	$\searrow$

whereas 2.54% of issuers are in the opposite configuration:

	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}$
$\mathcal{CE}$	$\searrow$	$\searrow$	$\searrow$
$\mathcal{CI}$	$\nearrow$	$\nearrow$	$\nearrow$

Finally, about 28% of issuers are in the other configurations.

Let us assume that we use the carbon intensity trend based on the scope $\mathcal{SC}_{1-3}$ to define the self-decarbonization constraint in the optimization problem, the bad case is the following configuration:

	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}$
$\mathcal{CE}$	$\nearrow$	$\nearrow$	$\nearrow$
$\mathcal{CI}$	$\nearrow$	$\nearrow$	$\searrow$

For these issuers, we want to underweight their allocation relative to the benchmark. More generally, we can define the following constraint[42]:

$$\Omega_{\mathcal{G}reen\mathcal{W}ash} = \left\{ \mathcal{NCM}_j > 0 \implies \sum\nolimits_{i \in \mathcal{I}ssuer(j)} x_i \leq \sum\nolimits_{i \in \mathcal{I}ssuer(j)} b_i \right\} \tag{64}$$

Table 30 shows the impact of $\Omega_{\mathcal{G}reen\mathcal{W}ash}$ on active share cost. Just as it is below 0.7% for $\mathcal{SC}_1$, it remains below 1% until 2030 for the other scopes. Applying the greenwashing constraint $\Omega_{\mathcal{G}reen\mathcal{W}ash}$ on $\mathcal{NCM}'_j$ yields lower additional costs due to a lower frequency of constrained issuers. For instance, these costs would remain below 0.4% for the three scopes.

Table 30: Additional Active share cost in % of the constraint $\Omega_{\mathcal{G}reen\mathcal{W}ash}$ (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4	0.7
$\mathcal{SC}_{1-2}$	0.2	0.2	0.2	0.2	0.2	0.5	1.0	2.5	6.4
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.2	0.2	0.2	0.3	0.9	2.2	8.6	2.2	−2.1
$\mathcal{SC}_{1-3}$	0.4	0.5	0.5	0.5	1.0	1.7	3.1	6.6	20.4
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.3	0.5
$\mathcal{SC}_{1-2}$	0.2	0.2	0.2	0.1	0.1	0.4	1.0	2.3	6.3
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.2	0.2	0.2	0.2	0.9	2.2	8.0	4.2	0.1
$\mathcal{SC}_{1-3}$	0.5	0.5	0.5	0.5	1.0	1.7	3.0	6.5	20.5

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

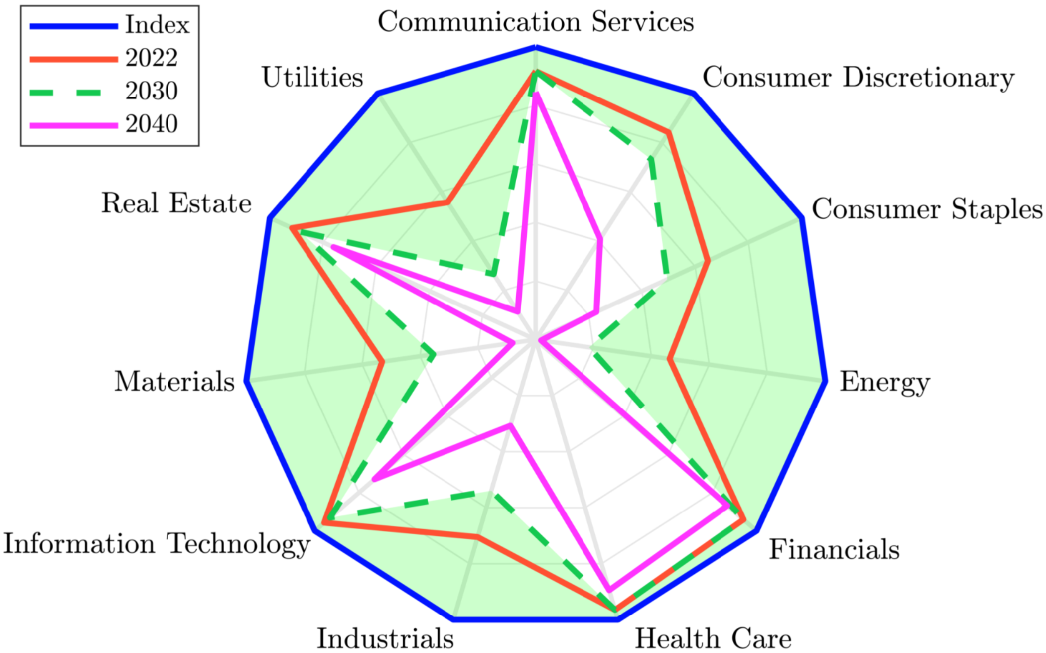
4.2.3 Diversification and liquidity risk

In practice, a lot of constraints can be used in the construction of aligned portfolios. We have previously seen that the tracking error cost can be significant and that the solution may also not exist for long time horizons. Since some assets are excluded from the net zero portfolio, this one may be more concentrated than the benchmark. Therefore, we might face not only a diversification risk, but also a liquidity risk. These risks will be reduced if the economy decarbonizes itself in the coming years. Nevertheless, we are not immune that carbon emissions keep increasing in the short term. In this case, the solutions will be very sensitive to the gap between the carbon objective of net zero portfolios and the carbon footprint of the economy.

[42] An alternative approach is to constraint each bond of these issuers:

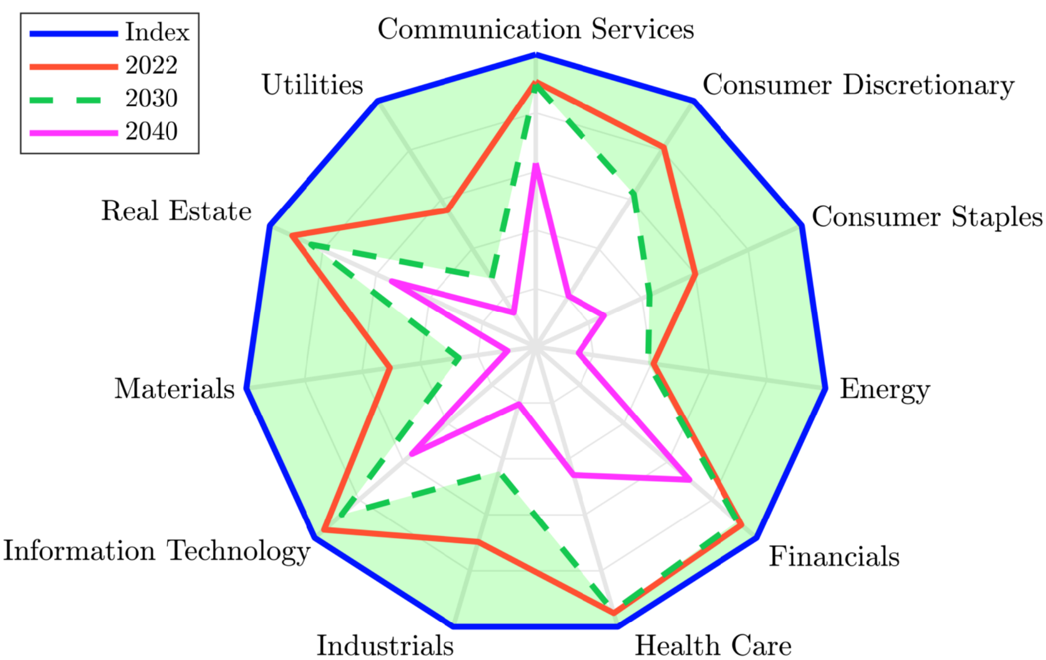
$$\Omega_{\mathcal{G}reen\mathcal{W}ash} = \left\{ \mathcal{NCM}_j > 0 \implies \forall i \in \mathcal{I}ssuer(j) : x_i \leq b_i \right\} \tag{63}$$

Figure 29: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 30: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^\star = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

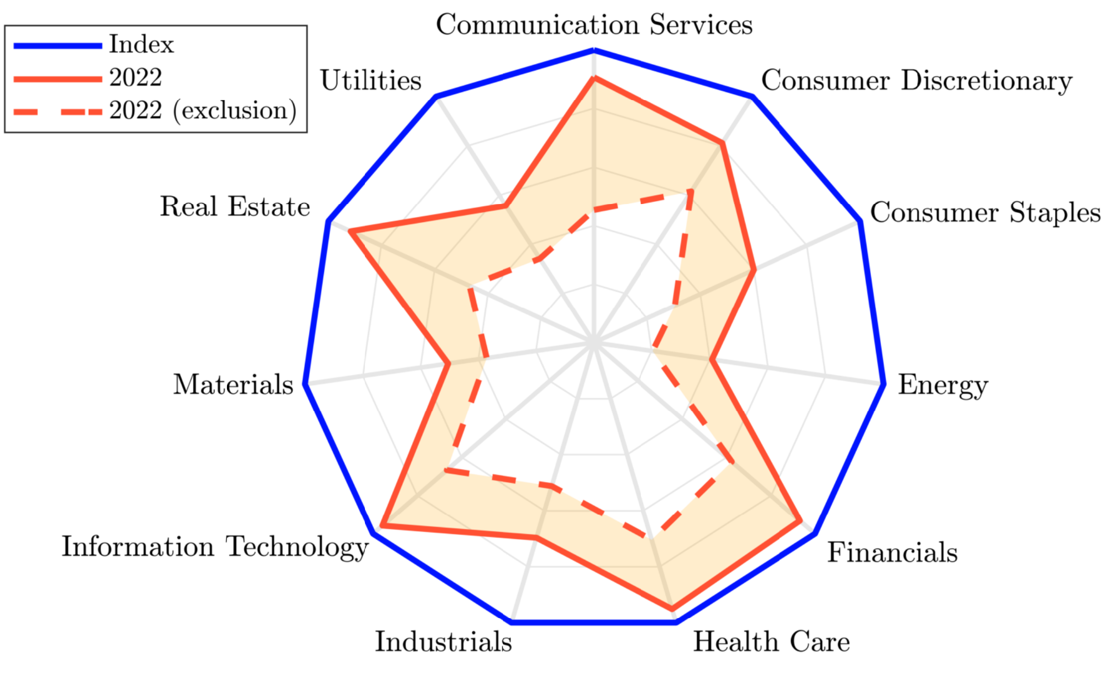


Source: MSCI (2022), Trucost (2022) & Authors' calculations.

To illustrate the shrinkage risk of the investment universe, we compute the number of selected stocks per sector for each optimized portfolio and divide these figures by the corresponding stocks number in the index[43]. In the case of the scope $\mathcal{SC}_{1-3}$, the radar charts of these frequencies are reported[44] in Figures 29 and 30. We observe that the investment universe is shrunk at the first date. The green area represents the removed part by 2030. With the exception of the Communication Services, Financials, Health Care, Information Technology and Real Estate sectors, the investment in the other sectors is concentrated on few stocks. This shrinkage effect can also be observed for small investment universes[45].

By construction, the shrinkage of the investment universe worsens if we add other constraints. For instance, the impact of the momentum exclusion constraint is illustrated in Figure 31. In this case, we complete the set of constraints by the threshold constraint $\left\{\mathcal{CM}_i^{\mathcal{L}ong}(t) \geq 0 \Rightarrow x_i = 0\right\}$, meaning that we exclude issuers with a positive carbon trend. We notice that the investment universe is highly reduced even from the first year. This type of high impact is also observed when we compare Case #1: $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$ and Case #2: $\mathcal{G} = 200\%$, $\mathcal{CM}^\star = -7\%$ (see Figures 73 and 74 on page 106).

Figure 31: Impact of momentum exclusion on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

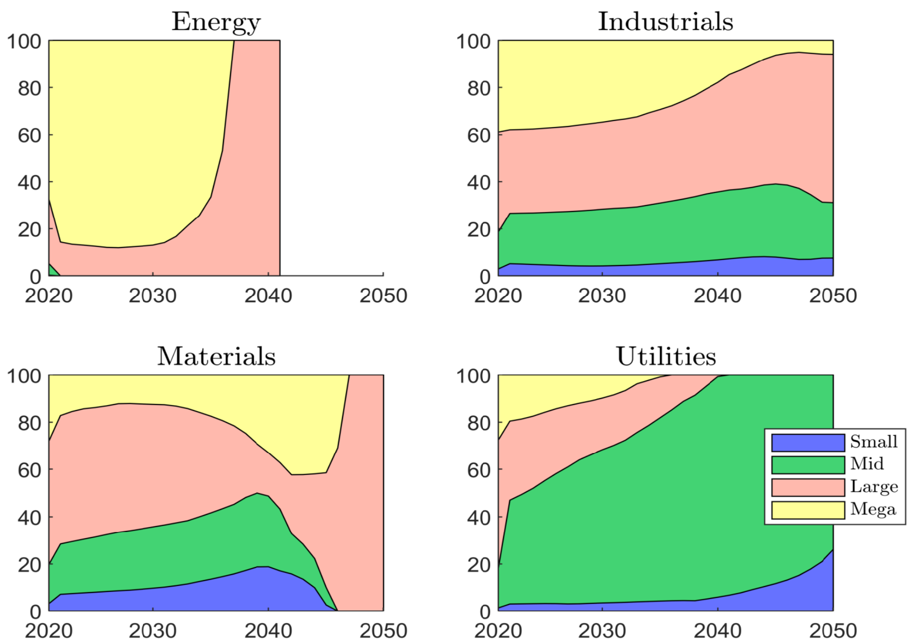
Remark 25. *These results show that we cannot reduce the cost of net zero investing to the cost of tracking risk. As seen above, there is also a cost of diversification risk. In this paper, we do not consider the cost of liquidity risk, but it does not mean that it is negligible. To give an idea, we have calculated the breakdown of the allocation with respect to the market*

[43] For instance, if the frequency is equal to 25% for the Energy sector, this means that the optimized portfolio has selected 25% of Energy stocks and removed 75% of the Energy investment universe.

[44] The results for the different scopes are shown on pages 100–103.

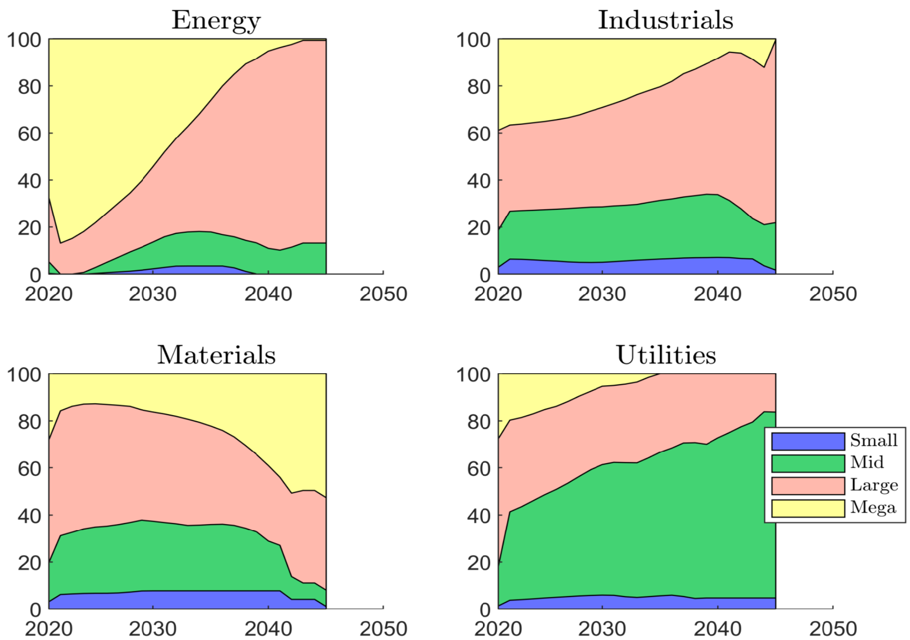
[45] See Figures 69 and 70 on page 104 for the MSCI EMU index.

Figure 32: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 33: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)

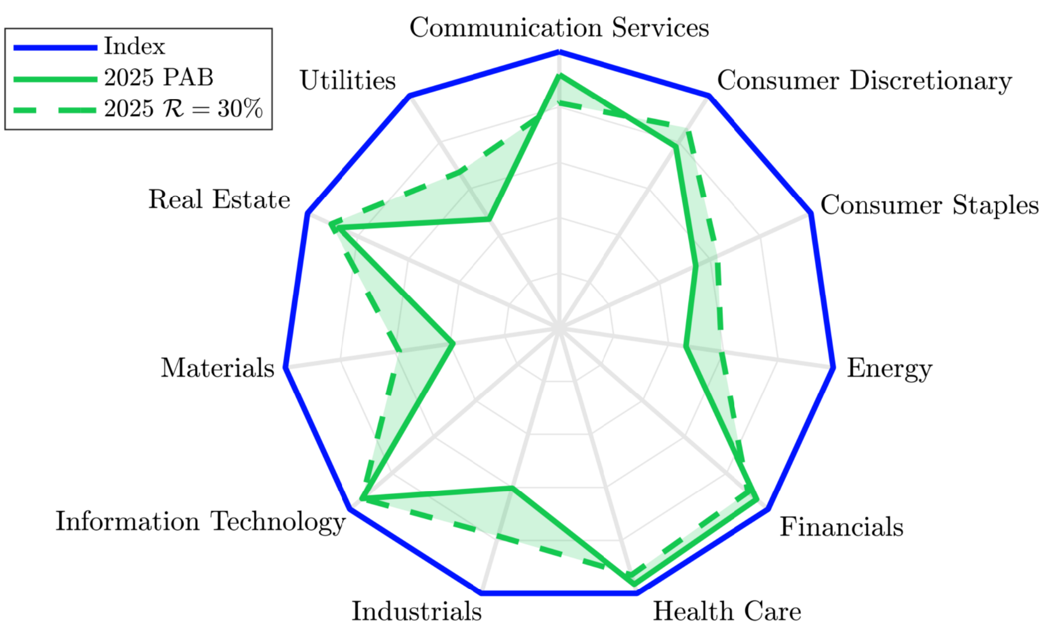


Source: MSCI (2022), Trucost (2022) & Authors' calculations.

capitalization. We consider four buckets: small-cap (below \$4.5 bn, mid-cap (between \$4.5 and \$12.5 bn), large-cap (between \$12.5 and \$50 bn and big-cap (above \$50 bn). The results for the four strategic sectors (Energy, Industrials, Materials and Utilities) are reported in Figures 32 and 33 (See Figures 75–78 on page 107 for the other sectors). We notice that the allocation to large- and mid-cap buckets is reduced while the allocation to small- and micro-cap buckets increases over time.

Of course, the previous results depend on the parameter values $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{G}}$ and $\boldsymbol{\mathcal{CM}}^{\star}$, because they have an important impact on the investment universe shrinkage. For instance, Figure 34 shows the investment universe shrinkage if we target (a) $\boldsymbol{\mathcal{R}} = 30\%$ in 2025 instead of using the PAB decarbonization pathway. In Figures 35–38, we consider the impact of other parameters: (b) $\boldsymbol{\mathcal{G}} = 0\%$; (c) $\Delta\boldsymbol{\mathcal{CM}}^{\star} = 0\%$; (d) $\boldsymbol{\mathcal{G}} = 0\%$ and $\Delta\boldsymbol{\mathcal{CM}}^{\star} = 0\%$; (e) $\boldsymbol{\mathcal{R}} = 30\%$, $\boldsymbol{\mathcal{G}} = 0\%$ and $\Delta\boldsymbol{\mathcal{CM}}^{\star} = 0\%$. The choice of the parameter values $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{G}}$ and $\boldsymbol{\mathcal{CM}}^{\star}$ is then crucial to define a net zero investment policy. Today, no consensus exists on the right solution.

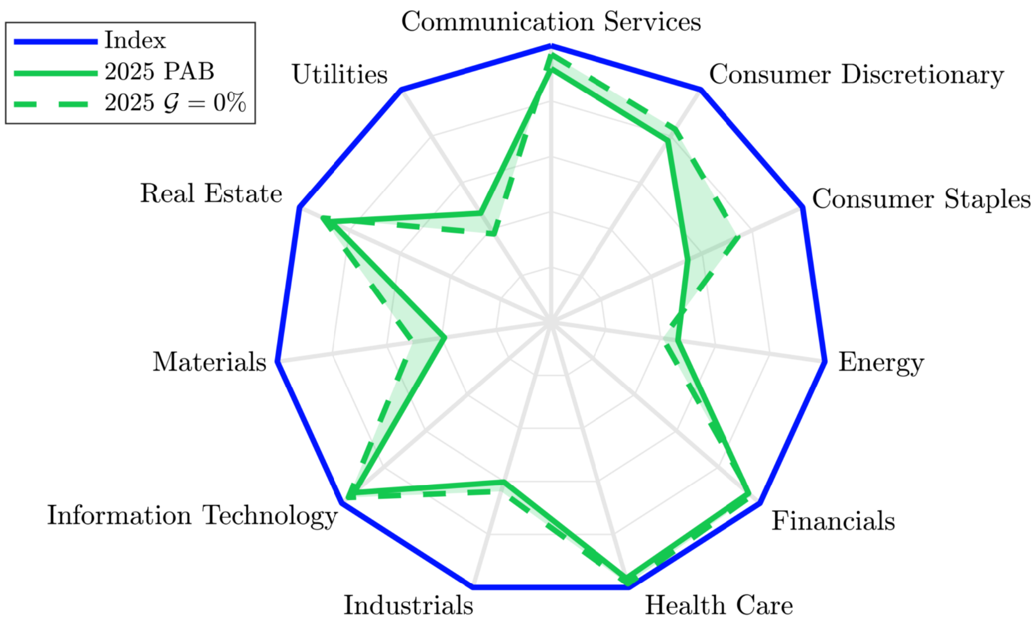
Figure 34: (a) Impact of the reduction rate on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3(0,10,2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

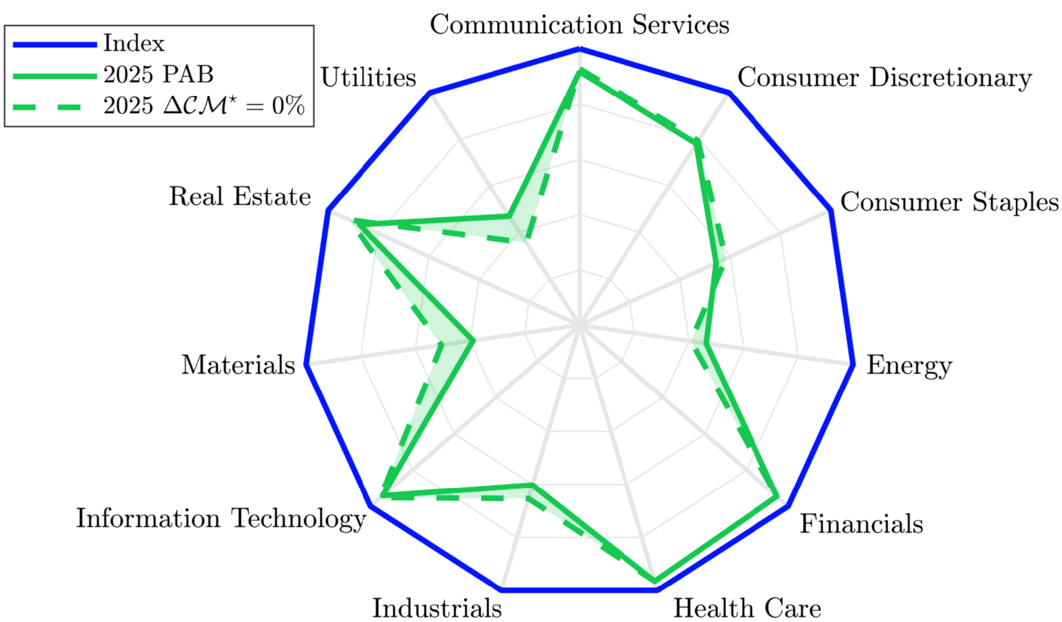
Regarding bond portfolios, we measure the issuer concentration by the inverse of the Herfindahl index. This indicator defines the number of bets, or the degrees of freedom of portfolio weights. It is equal to one if the portfolio is concentrated on one asset. Conversely, it is equal to the number of assets for an equally-weighted portfolio, which is the least concentrated portfolio in terms of weights. The current benchmark is comprised of 2 362 companies corresponding to 342 equally-weighted issuers. The benchmark is far from being highly diversified as the first quintile of issuers represents 77.2% of the benchmark weights while the last quintile corresponds to 1.2%. Table 31 displays the number of bets of optimized portfolios. This number decreases with the year, indicating more and more concentrated portfolios. This is especially true for scope $\boldsymbol{\mathcal{SC}}_{1-3}$, where the number of bets is divided by

Figure 35: (b) Impact of the greenness intensity on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, scope $\mathcal{SC}_{1-3}$)



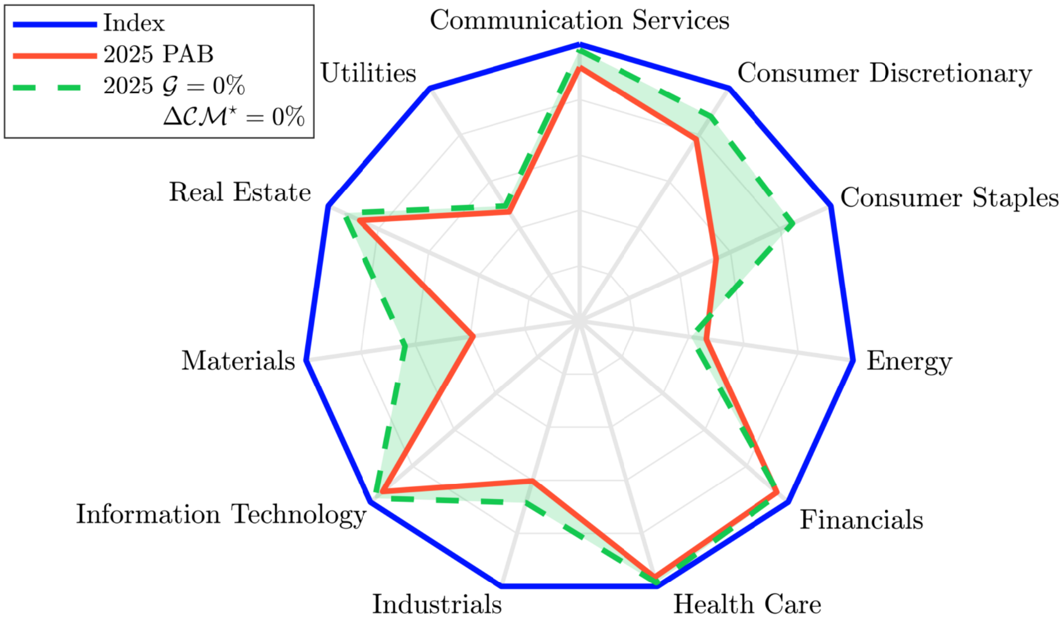
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 36: (c) Impact of the carbon momentum constraint on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, scope $\mathcal{SC}_{1-3}$)



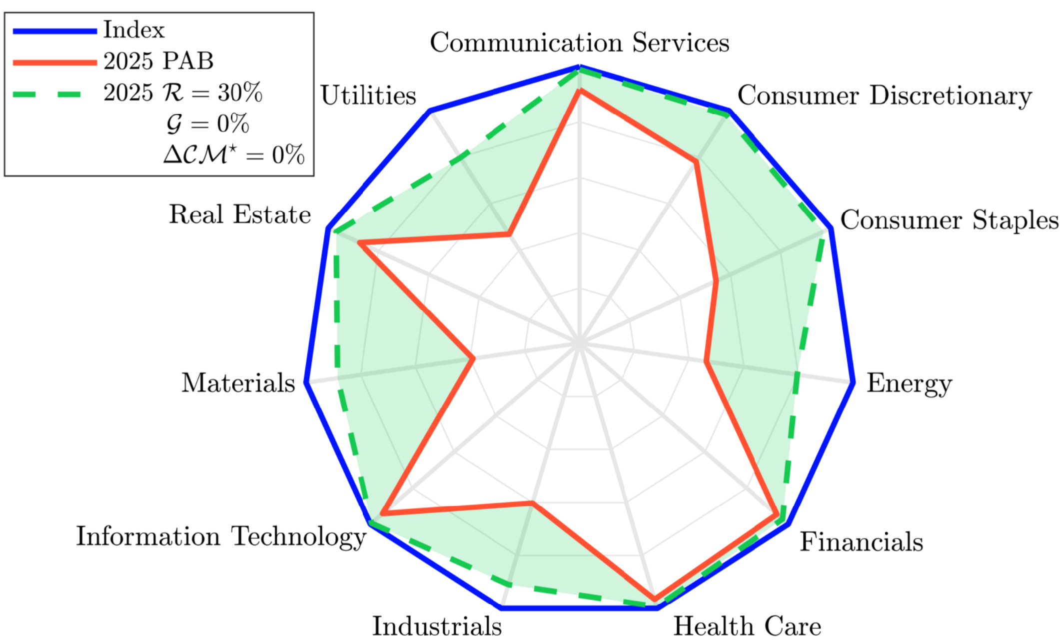
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 37: (d) Impact of combined constraints on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 38: (e) Impact of combined constraints on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, scope $\boldsymbol{\mathcal{SC}}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

a factor of 2.5 by 2030 and 5 by 2035. The evolution of the top 10 issuers' weights gives another picture of the extent of the diversification (Table 32). On average, we observe that the concentration in the top 10 issuers is multiplied by a factor of 2 in 2030 and 5 in 2045 if we focus on scope $\mathcal{SC}_{1-3}$.

Table 31: Number of bets (Global Corp., Jun. 2022, PAB)

Scope	Index	2022	2023	2024	2025	2030	2035	2040	2045	2050
		$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	342	265	265	255	256	249	213	200	161	130
$\mathcal{SC}_{1-2}$		259	257	255	254	205	187	156	122	66
$\mathcal{SC}_{1-3}^{\text{up}}$		246	229	202	191	121	75	34	19	8
$\mathcal{SC}_{1-3}$		227	216	208	198	131	69	34	22	9
		$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	342	225	228	228	222	226	180	167	148	121
$\mathcal{SC}_{1-2}$		229	222	217	222	205	169	133	110	71
$\mathcal{SC}_{1-3}^{\text{up}}$		229	216	194	181	118	74	47	23	6
$\mathcal{SC}_{1-3}$		197	190	191	180	124	65	43	24	11

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Table 32: Top 10 issuers' weight in % (Global Corp., Jun. 2022, PAB)

Scope	Index	2022	2023	2024	2025	2030	2035	2040	2045	2050
		$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	10.9	13.4	13.4	13.7	13.5	13.6	15.3	16.4	19.5	21.9
$\mathcal{SC}_{1-2}$		13.6	13.6	13.7	13.6	15.8	17.5	19.5	22.6	30.8
$\mathcal{SC}_{1-3}^{\text{up}}$		13.8	14.2	15.8	16.3	21.9	29.6	42.2	56.6	88.3
$\mathcal{SC}_{1-3}$		15.3	15.6	16.4	16.8	21.7	29.9	42.2	55.0	80.8
		$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	10.9	14.9	14.7	14.2	14.5	14.8	17.0	18.0	19.7	22.4
$\mathcal{SC}_{1-2}$		14.5	14.6	14.9	15.0	16.0	18.1	20.9	24.3	31.1
$\mathcal{SC}_{1-3}^{\text{up}}$		14.4	15.4	16.3	16.7	23.1	29.7	40.4	57.2	90.9
$\mathcal{SC}_{1-3}$		16.8	17.4	17.2	18.1	22.2	30.8	41.4	53.6	79.4

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

4.3 Core-satellite approach

Another solution to build a net zero portfolio is to implement a core-satellite approach. Indeed, a net zero investment strategy implies two building blocks. The first building block concerns the decarbonization of the portfolio while the objective of the second building block is to finance the transition to a low-carbon economy. In this context, the decarbonization portfolio plays the role of a core investment, whereas the transition portfolio is like a thematic portfolio or a satellite basket. Typically, the underlying idea of a core-satellite strategy is to boost a passive portfolio with actively managed strategies or 'exotic' asset classes that have the potential to enhance risk-adjusted returns. In our case, the purpose of the core-satellite strategy is to boost the greenness or the alignment of a decarbonized portfolio with respect to net zero objectives.

The portfolio construction is defined in Table 33. As we have already seen, decarbonization is typically a top-down approach, whereas transition is more a bottom-up approach, or

Table 33: The core-satellite approach

Decarbonized portfolio		Transition portfolio
• Low-carbon portfolio		• Climate/green solutions
• Decarbonization pathway(s)		• Financing the transition
• Top-down approach	+	• Bottom-up approach
• Portfolio optimization		• Security selection
• Carbon measures		• ≠ Carbon measures
• Net zero **carbon** metrics		• Net zero **transition** metrics
$\mathbf{1-\alpha(t)}$		$\boldsymbol{\alpha}(\mathbf{t})$

a security selection process. The core-satellite approach circumvents the problem of the negative correlation between decarbonization and transition in the short-term. It also reduces the complexity of dealing with many constraints and many climate risk measures that are not always compatible. Moreover, portfolio managers have extensive experience in portfolio decarbonization and its associated metrics. They don't need to have a strong background about climate investing. Therefore, portfolio decarbonization can be implemented on a massive scale. This is not the case with the transition basket, which requires specialized portfolio managers. These last ones must understand net zero challenges, metrics and concepts such as self-decarbonization, green capex or climate taxonomy. In this case, it is obvious that traditional carbon metrics are not adapted to the transition dimension. For instance, if we consider investment in hydrogen solutions, it may have a high carbon footprint. This is not incompatible with the transition dimension if this investment is helpful in building a low-carbon economy in the future. Therefore, the reporting of the transition basket must be based more on impact investing and net zero transition measures than on traditional carbon footprints.

We may wonder why the transition portfolio corresponds to the satellite portfolio. Mainly because we have seen that transition and green activities are today a small portion of the investment universe. From a strategic asset allocation viewpoint, allocating 10% of a global portfolio to green solutions is already a big progress. But it is important to notice that the proportion $\alpha(t)$ allocated to the transition dimension is time-varying and must increase with the enlargement of the green investment universe in the future.

Remark 26. *The comprehensive integrated approach is widely used when building equity and fixed-income funds and ETFs. Nevertheless, it is not suitable when managing a large diversified portfolio. On the contrary, the core-satellite approach is more appropriate when considering multi-asset portfolios. Moreover, it is the relevant approach when asset owners would like to implement a net zero strategic asset allocation. The second part of this article will be dedicated to the core-satellite approach in these two contexts (*Roncalli et al., *2023).*

5 Conclusion

This article is part of a comprehensive research project on net zero investing. In this paper dedicated to the comprehensive integrated approach, we break down a net zero investment policy into two dimensions: decarbonization and transition. First, we present the two families of metrics needed to implement such a policy. While we assess the first dimension through traditional carbon footprint measures and a decarbonization pathway, we suggest some metrics to evaluate the ability to finance the transition to a low-carbon economy and the willingness of issuers to participate in the net zero journey. In particular, the green revenue share is an interesting proxy for assessing this second dimension because it grants high data coverage. Since portfolio alignment is a dynamic process, we also highlight the need to consider static and forward-looking metrics for decarbonization and transition. To this extent, we use carbon trends for portfolio decarbonization and emphasize the lack of forward-impactful transition data. For example, green capex — in addition to being seldom disclosed — does not always lead to patent filing and even less to commercialization. Beyond these metrics, we introduced key concepts to better understand net zero investment portfolios. These concepts mainly encompass the $\mathcal{PAC}$ framework, and in particular the participation pillar. Indeed, net zero investing implies a dynamic view of portfolio decarbonization. Therefore, we propose using carbon momentum measures to gauge the self-decarbonization ability of issuers. A portfolio could only be labeled net zero if it reaches some minimum requirements of self-decarbonization. Indeed, if the decarbonization pathway is achieved only because the fund manager rebalances the portfolio at a given frequency to obtain a higher reduction rate, the decarbonization pathway followed by the portfolio is purely exogenous and is explained by the rebalancing process. In the case of a net zero portfolio, a part of the decarbonization pathway must become endogenous and explained by self-decarbonization. In this approach, decarbonization is not due to external factors (e.g., the rebalancing scheme), but internal factors also participate. This is one of the two main differences between a net zero investment policy and a low-carbon strategy, the former being to focus on the transition pillar, as seen previously. Alongside our suggestions, we implement an optimization-based approach for aligning a portfolio by integrating various constraints based on the previous metrics. Generally, we use three constraints. The first one targets the time-varying decarbonization rate, the second imposes a minimum green revenue share, and the last one uses carbon momentum metrics to forecast the self-decarbonization rate. If we consider the classical framework that consists in replicating a benchmark and controlling the tracking risk, our empirical results show the following lessons.

The first lesson concerns the sensitivity of the solution to parameters and data. In particular, fund managers need to be careful when they select the carbon scope metric to assess the decarbonization rate. Net zero only makes sense if it concerns a closed system. Therefore, scope 3 emissions must be considered to align a portfolio with respect to a net zero scenario. The issue is that we observe a lack of data reliability on scope 3 emissions today. Nevertheless, it is important that asset owners and managers begin to use scope 3 in order to create incentives to improve data quality. These incentives concern several actors: regulators, issuers, and data providers. However, including scope 3 increases the tracking error risk, particularly with the upstream emissions. Similarly, the solution is highly dependent on the figure we target for the green revenue share and the carbon momentum rate we would like to achieve for the self-decarbonization level. Fund managers must then be careful because too much ambition in the short term implies that there may be no solution in the medium term to the optimization problem. The no-solution issue depends on the relative speed of the portfolio's decarbonization pathway with respect to the economy's decarbonization pathway and the initial starting point.

The second main result is that portfolio decarbonization and portfolio alignment give different solutions. In particular, decarbonizing a portfolio is easier than aligning a portfolio. We show that decarbonizing along CTB or PAB pathways never leads to exploding tracking errors until 2030. In fact, the real issue of the decarbonization exercise lies in the diversification and liquidity risk an investor might face. These results are amplified when we add the transition dimension into the optimization program. Along with a higher tracking error cost, there is no guarantee that a solution always exists. Besides, introducing the transition pillar emphasizes the difficulty of choosing a proper set of constraints for net zero portfolios, because some metrics can be negatively correlated with others. Portfolio decarbonization is systematically a strategy that is long on Financial issuers and short on Energy, Materials and Utilities issuers. Therefore, we have a situation where the transition dimension of a decarbonized portfolio is weaker than that of the benchmark portfolio as green solutions are also located in carbon-intensive sectors. Thus, it is crucial to distinguish between issuers with a high carbon footprint that will not participate in the transition and those that will reduce their carbon emissions and find low-carbon solutions. Since the transition dimension is multi-faceted, professionals are tempted to multiply the transition metrics. This is not always a good idea because these metrics may be independent in the short run. For example, we observe no current relationship between carbon momentum and green revenue share. However, we can assume that these two metrics will be correlated in the long term when the economy will be on the right track to reach net zero. Since many independent metrics do not ensure the existence of a solution, it is better to concentrate on a small number of transition constraints and to understand the objective of each one. True to the saying that "*less is more*", a concise problem for defining net zero is more useful than a complex patchwork and a diffuse stack of criteria. In this last case, the balance is always difficult to find.

The third main result is that portfolio decarbonization and alignment are two exclusion processes. This means it is quite impossible to achieve net zero alignment without allowing the algorithm to exclude companies from the benchmark. For instance, the optimization program does not generally find a solution when imposing lower bounds other than zero. Therefore, some key actors of the transition such as Energy and Utilities companies unfortunately disappear. Moreover, imposing sector neutrality may lead to similar problems finding a solution. The exclusion process that we observe at both issuer and industry levels raises the question of benchmarking. Indeed, if portfolio decarbonization can be viewed as a tilt of the benchmark portfolio, portfolio alignment may imply a significant shrinkage of the investment universe. As such, defining the net zero investing benchmark is complex because it is too far from business-as-usual investing. Of course, in the long run, we will observe a convergence between net zero and market portfolios when the world economy reaches net zero emissions. But, in the short term, the gap remains wide, and an alternative benchmark choice is an important issue for all net zero investors.

Another lesson concerns the question of greenwashing, which is a key challenge of net zero investment. Here, we are referring to explicit and deliberate greenwashing, which is a mis-selling risk from a regulatory viewpoint, but rather unintentional greenwashing, which is more of a mis-interpretation risk. This risk occurs when (1) the practices and definitions are not unique and (2) the practices and definitions change over time. Regulators have not yet defined a normative and comprehensive framework for net zero investment policies. As a result, two investors may have two different visions about net zero, implying that they do not use the same criteria. Moreover, as we said previously, it is really difficult to manage all aspects of a top-down optimization process. Therefore, it is always easy to analyze the net zero portfolio of an investor and to find some issuers that are not net zero using other criteria. For example, our optimization model uses intensity-based carbon momentum including scope 3, because the decarbonization pathway is expressed with the carbon intensity measure and scope 3. We could also use emission-based carbon momentum or another scope. We can multiply the criteria but as we explained before, the no-solution risk increases. Moreover, another dimension that is difficult to integrate in a top-down approach is the engagement and ESG stewardship of asset owners and managers. Therefore, we need a bottom-up analysis of the issuers that make up the net zero portfolio. The fund manager must validate each constituent. In a sense, building a net zero portfolio is an active management strategy, and the fund manager must be convinced that each exposure is justified. Applying a bottom-up check will then reduce the risk of greenwashing controversies.

Contrary to some academic publications dedicated to the integrated approach, we find that the tracking error cost may be significant even in the short term. This is particularly true for equity portfolios and small investment universes, but much less for bond portfolios. At first sight, this result may be surprising because there is no reason that net zero impacts equity and fixed-income markets differently. In fact, there are two explanations. First, the structure of equity and bond indices are different, with a more balanced allocation across sectors and a high exposure to Financial issuers for the latter. Second, bond indices are highly affected by new fresh capital, whereas equity indices are sticky to the stock of existing capital (or old capital). This is because the primary bond market is very active, implying a significant impact on the secondary market. This is not the case in the equity market, where IPOs and capital increases only represent a small fraction of the secondary market. This implies that portfolio holdings change faster for bond indices than equity indices. Therefore, the greenness of bond indices increases more quickly than the greenness of equity indices. All these factors show that the cost of implementing net zero investing with respect to traditional investing will be higher for equity portfolios than bond portfolios and the fixed-income market will benefit more quickly from the transition to a low-carbon economy.

In this research paper, the cost is measured with respect to three risk dimensions: tracking risk, diversification risk and liquidity risk. We have put aside the question of financial performance, which we discussed in a previous publication (Laugel and Roncalli, 2022). The idea is not to reiterate what we have said. As shown by Pastor *et al.* (2021), the risk premium of green assets must be lower. Nevertheless, expected returns are different from actual returns, which depend on the investment flows and the supply/demand imbalance. Since we do not have a crystal ball, net zero portfolios may outperform or underperform business-as-usual portfolios. For instance, it is very interesting to notice that the investment universe of the greenest stocks from the transition viewpoint has behaved like a growth strategy in recent years. Indeed, we observe that these assets have been systematically overvalued, except during the Covid-19 crisis. Of course, this may change in the future. Investing in green assets could also be a low-risk or a quality strategy, or it could be correlated to momentum and value risk factors. In this case, predicting whether net zero investing has a positive or negative alpha is a pipe dream.

The final remark concerns the implementation of net zero investment policies. In this research, we have focused on the traditional top-down approach because we can easily obtain quantitative results. This approach is suitable when we build an equity or bond investment fund or ETF vehicle. However, this is not the only solution. In particular, active management makes a lot of sense if we want to implement net zero investing. Moreover, the comprehensive integrated approach is not relevant when we consider multi-asset portfolios or strategic asset allocation of asset owners. In this case, it is better to implement the core-satellite approach[46], which consists of the decarbonization dimension for the core investment and the transition dimension for the satellite strategy. This framework is easier to implement than the integrated optimization approach. Moreover, it allows control over the breakdown between the two dimensions, and the weight of the transition bucket to be progressively changed based on the greenness of the economy. Currently, net zero could be viewed as thematic investing because the universe of transition assets is small. But in the future, there will be no difference between net zero and core investing. If there is, that would mean that we have collectively failed to limit global warming.

[46] This approach will be extensively developed in Part 2 of the net zero research project (Roncalli *et al.*, 2023).

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A Technical appendix

A.1 Notations

Table 34: Carbon and transition risk measures

	Symbol	Description
Carbon	$\mathcal{CB}$	Carbon budget
	$\mathcal{CE}$	Carbon emission
	$\mathcal{CI}$	Carbon intensity
	$\mathcal{CM}$	Carbon momentum
	$\mathcal{CM}^{\mathcal{L}ong}$	Long-term carbon momentum
	$\mathcal{CM}^{\mathcal{S}hort}$	Short-term carbon momentum
	$\mathcal{R}$	Carbon reduction
	v	Carbon velocity
	$\mathcal{SC}_1$	Scope 1
	$\mathcal{SC}_2$	Scope 2
	$\mathcal{SC}_3^{\mathrm{up}}$	Upstream scope 3
	$\mathcal{SC}_3^{\mathrm{down}}$	Downstream scope 3
	$\mathcal{SC}_3$	Scope 3 ($= \mathcal{SC}_3^{\mathrm{up}} + \mathcal{SC}_3^{\mathrm{down}}$)
	$\mathcal{SC}_{1-2}$	Scope $1+2$
	$\mathcal{SC}_{1-3}^{\mathrm{up}}$	Upstream scope $1+2+3$ ($= \mathcal{SC}_1 + \mathcal{SC}_2 + \mathcal{SC}_3^{\mathrm{up}}$)
	$\mathcal{SC}_{1-3}$	Scope $1+2+3$
	$\mathcal{SR}$	Self-decarbonization ratio
Transition	$\mathcal{BI}$	Brown intensity
	$\mathcal{GI}$	Green intensity
	$\mathcal{GC}$	Green capex
	$\mathcal{GM}$	Green momentum
	$\mathcal{GRS}$	Green revenue share

A.2 Mathematical results

A.2.1 Carbon momentum aggregation at the portfolio level

If we consider carbon momentums built on intensities, we recall that we have:

$$\mathcal{CM}_i^{\mathcal{L}ong}(t) = \frac{\hat{\beta}_{i,1}(t)}{\mathcal{CI}_i(t)} \tag{65}$$

where i is the issuer, $\mathcal{CI}_i(t)$ is the carbon intensity and $\hat{\beta}_{i,1}(t)$ is the slope of the trend model:

$$\widehat{\mathcal{CI}}_i(t) = \mathcal{CI}_i(t) + \hat{\beta}_{i,1}(t) \cdot (t - t_0)$$

Let $x = (x_1, \ldots, x_n)$ be the weights of the stocks that belong to the portfolio. Its carbon intensity is given by its weighted average:

$$\mathcal{CI}_x(t) = \sum_{i=1}^{n} x_i \cdot \mathcal{CI}_i(t) \tag{66}$$

It follows that:

$$\begin{aligned}
\widehat{\mathcal{CI}}_x(t) &= \sum_{i=1}^{n} x_i \cdot \widehat{\mathcal{CI}}_i(t) \\
&= \sum_{i=1}^{n} x_i \cdot \mathcal{CI}_i(t_0) + (t - t_0) \sum_{i=1}^{n} x_i \cdot \hat{\beta}_{i,1}(t) \\
&= \mathcal{CI}_x(t_0) + \hat{\beta}_{x,1}(t) \cdot (t - t_0)
\end{aligned}$$

where $\hat{\beta}_{x,1}(t) = \sum_{i=1}^{n} x_i \cdot \hat{\beta}_{i,1}(t)$. We deduce that:

$$\begin{aligned}
\mathcal{CM}_x^{\mathcal{L}ong}(t) &= \frac{\hat{\beta}_{x,1}(t)}{\mathcal{CI}_x(t)} && (67) \\
&= \frac{\sum_{i=1}^{n} x_i \cdot \hat{\beta}_{i,1}(t)}{\sum_{i=1}^{n} x_i \cdot \mathcal{CI}_i(t)} && (68) \\
&= \frac{\sum_{i=1}^{n} x_i \cdot \mathcal{CI}_i(t) \cdot \mathcal{CM}_i^{\mathcal{L}ong}(t)}{\sum_{i=1}^{n} x_i \cdot \mathcal{CI}_i(t)} && (69) \\
&= \sum_{i=1}^{n} \tilde{x}_i \cdot \mathcal{CM}_i^{\mathcal{L}ong}(t) && (70)
\end{aligned}$$

where the adjusted weight $\tilde{x}_i$ is equal to:

$$\tilde{x}_i = \frac{x_i \cdot \mathcal{CI}_i(t)}{\sum_{j=1}^{n} x_j \cdot \mathcal{CI}_j(t)} \tag{71}$$

Remark 27. *In particular, we see that* $\mathcal{CM}_x^{\mathcal{L}ong}(t) \neq \sum_{i=1}^{n} x_i \cdot \mathcal{CM}_i^{\mathcal{L}ong}(t)$. *At the sector level, we aggregate the carbon momentum following the same method with the weights of each issuer in its respective sector.*

B Additional results

B.1 Tables

Table 35: Breakdwon (in %) of carbon emissions in 2019

Sector	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_3^{\mathrm{up}}$	$\mathcal{SC}_3^{\mathrm{down}}$	$\mathcal{SC}_3$	$\mathcal{SC}_{1-3}$
Communication Services	0.1	5.1	0.8	1.5	0.2	0.4	0.5
Consumer Discretionary	1.7	9.7	2.9	14.1	10.2	10.8	9.1
Consumer Staples	2.3	6.7	2.9	18.6	1.6	4.4	4.1
Energy	15.0	8.5	14.0	14.1	40.1	36.0	31.2
Financials	0.7	1.8	0.9	2.6	1.8	2.0	1.7
Health Care	0.3	1.7	0.5	2.6	0.2	0.6	0.6
Industrials	10.2	8.9	10.0	15.6	24.2	22.8	20.0
Information Technology	0.6	6.8	1.5	4.9	2.3	2.7	2.5
Materials	29.8	40.7	31.4	20.2	13.5	14.6	18.2
Real Estate	0.3	2.8	0.6	1.1	1.0	1.0	0.9
Utilities	39.0	7.3	34.4	4.7	4.8	4.8	11.2
Total (in $GtCO_2e$)	15.1	2.6	17.6	10.3	53.7	64.0	81.6

Source: Trucost (2022) & Authors' calculations.

Table 36: Distribution of carbon emissions in 2019

Sector	$\frac{\mathcal{SC}_2}{\mathcal{SC}_1}$	$\frac{\mathcal{SC}_3^{\mathrm{up}}}{\mathcal{SC}_{1-2}}$	$\frac{\mathcal{SC}_3^{\mathrm{down}}}{\mathcal{SC}_{1-2}}$	$\frac{\mathcal{SC}_3}{\mathcal{SC}_{1-2}}$	$\frac{\mathcal{SC}_{1-2}}{\mathcal{SC}_{1-3}}$	$\frac{\mathcal{SC}_3}{\mathcal{SC}_{1-3}}$
Communication Services	7.9	1.1	0.8	1.8	0.35	0.65
Consumer Discretionary	0.9	2.8	10.7	13.6	0.07	0.93
Consumer Staples	0.5	3.7	1.7	5.4	0.16	0.84
Energy	0.1	0.6	8.7	9.3	0.10	0.90
Financials	0.4	1.8	6.5	8.2	0.11	0.89
Health Care	1.1	3.2	1.3	4.5	0.18	0.82
Industrials	0.1	0.9	7.4	8.3	0.11	0.89
Information Technology	1.8	1.9	4.6	6.5	0.13	0.87
Materials	0.2	0.4	1.3	1.7	0.37	0.63
Real Estate	1.8	1.0	4.7	5.8	0.15	0.85
Utilities	0.0	0.1	0.4	0.5	0.67	0.33
Total	0.2	0.6	3.0	3.6	0.22	0.78

Source: Trucost (2022) & Authors' calculations.

Table 37: Sector allocation in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, scope $\mathcal{SC}_{1-2}$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.58	7.74	7.83	7.93	8.08	8.31	8.68	9.02
Consumer Discretionary	10.56	10.71	10.78	10.84	10.91	11.08	11.18	10.69
Consumer Staples	7.80	7.98	8.05	8.12	8.17	8.08	7.55	5.89
Energy	4.99	4.80	4.66	4.40	3.99	3.30	2.00	0.14
Financials	13.56	14.05	14.32	14.67	15.20	16.19	18.30	23.11
Health Care	14.15	14.40	14.53	14.68	14.90	15.21	15.73	16.02
Industrials	9.90	10.04	10.07	10.13	10.19	10.12	9.83	8.86
Information Technology	21.08	21.38	21.54	21.74	22.02	22.44	23.09	23.93
Materials	4.28	3.80	3.54	3.20	2.69	2.04	1.20	0.59
Real Estate	2.90	2.98	3.00	3.01	2.97	2.77	2.27	1.50
Utilities	3.21	2.11	1.70	1.28	0.88	0.45	0.19	0.24

Source: Trucost (2022) & Authors' calculations.

Table 38: Sector allocation in % (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, scope $\mathcal{SC}_{1-3}^{\text{up}}$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.58	7.95	8.28	8.81	9.61	10.50	10.88	0.18
Consumer Discretionary	10.56	10.76	10.77	10.67	10.35	9.44	6.86	0.00
Consumer Staples	7.80	7.44	6.99	6.17	4.94	3.12	0.93	0.24
Energy	4.99	4.55	4.06	3.36	2.28	1.00	0.00	0.00
Financials	13.56	14.90	15.99	17.86	21.09	26.04	37.75	81.71
Health Care	14.15	14.69	15.02	15.39	15.61	14.87	10.74	3.98
Industrials	9.90	9.94	9.76	9.07	7.55	6.30	4.74	2.85
Information Technology	21.08	21.73	22.18	22.78	23.48	24.07	24.41	9.55
Materials	4.28	3.16	2.43	1.58	0.86	0.43	0.22	0.17
Real Estate	2.90	3.21	3.39	3.60	3.80	3.83	3.02	0.94
Utilities	3.21	1.67	1.12	0.71	0.43	0.41	0.45	0.38

Source: Trucost (2022) & Authors' calculations.

Table 39: Sector allocation deviation in % (Global Corp., Jun. 2022, scope $\mathcal{SC}_1$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Consumer Discretionary	5.97	0.00	0.00	0.09	0.09	0.09	0.00	0.02
Consumer Staples	6.04	0.00	0.00	0.00	0.00	0.00	0.00	−0.01
Energy	6.49	0.00	0.00	0.02	0.02	0.38	0.70	−0.12
Financials	33.91	0.45	0.65	1.13	1.45	1.15	1.85	3.02
Health Care	7.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industrials	8.92	0.00	0.00	0.00	0.00	−0.02	−0.22	−0.42
Information Technology	5.57	0.11	0.11	0.00	0.32	0.02	0.00	0.00
Materials	3.44	−0.10	−0.12	−0.13	−0.16	−0.20	−0.68	−0.92
Real Estate	4.76	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01
Utilities	10.06	−0.47	−0.64	−1.11	−1.72	−1.43	−1.63	−1.58

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 40: Sector allocation deviation in % (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-2}$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Consumer Discretionary	5.97	0.00	−0.04	−0.04	−0.04	−0.06	−0.08	−0.08
Consumer Staples	6.04	0.00	0.00	0.00	0.00	0.00	−0.01	−0.07
Energy	6.49	0.00	0.00	−0.10	−0.12	−0.06	−0.21	−2.88
Financials	33.91	0.72	1.14	1.84	2.22	2.35	3.06	5.28
Health Care	7.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industrials	8.92	0.00	0.00	0.00	0.00	0.03	0.02	0.73
Information Technology	5.57	0.00	0.00	0.00	0.00	0.00	0.00	−0.05
Materials	3.44	−0.11	−0.16	−0.17	−0.19	−0.43	−0.85	−1.40
Real Estate	4.76	0.00	0.00	0.00	0.00	0.00	−0.06	−0.12
Utilities	10.06	−0.61	−0.95	−1.53	−1.87	−1.83	−1.87	−1.41

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 41: Sector allocation deviation in % (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-3}^{\text{up}}$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	−0.04	−0.03	0.02	−0.05	0.00	−0.17	−1.29
Consumer Discretionary	5.97	0.00	−0.02	−0.03	−0.04	−0.06	−0.18	−2.81
Consumer Staples	6.04	0.00	−0.01	−0.08	−0.31	−0.81	−2.41	−3.72
Energy	6.49	0.00	−0.07	0.21	0.53	1.02	1.85	2.21
Financials	33.91	1.43	2.72	2.97	4.39	5.38	8.10	14.88
Health Care	7.50	0.00	0.00	0.00	0.00	0.00	−0.07	−1.96
Industrials	8.92	0.00	0.00	0.01	−0.19	−0.29	−0.76	4.01
Information Technology	5.57	0.00	0.00	0.00	0.00	0.00	−0.17	−2.17
Materials	3.44	−0.09	−0.14	−0.17	−0.59	−0.87	−1.04	−1.22
Real Estate	4.76	0.00	0.00	0.00	−0.01	−0.06	−0.13	−2.56
Utilities	10.06	−1.30	−2.46	−2.92	−3.74	−4.31	−5.03	−5.39

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 42: Contribution to yield variation in bps (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-3}$)

	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Rating								
AAA–AA	33	1	0	0	0	0	0	0
A	160	0	−1	0	1	0	3	4
BBB	229	−1	−1	−2	−7	−8	−12	−26
Duration								
0Y–2Y	41	1	1	1	0	2	3	1
2Y–5Y	148	−1	−2	−3	−5	−8	−11	−23
5Y–7Y	67	0	−1	−1	−1	−1	1	5
7Y–10Y	60	0	0	−1	−1	−2	−1	−2
10Y+	107	1	1	1	0	0	−1	−3
Sector								
Communication Services	32	0	0	0	0	0	0	0
Consumer Discretionary	24	0	0	0	0	−1	−4	−6
Consumer Staples	25	0	0	0	0	0	−1	−4
Energy	30	−4	−7	−9	−9	−11	−14	−14
Financials	138	4	6	8	9	15	22	19
Health Care	31	0	0	0	0	0	0	0
Industrials	37	1	1	2	3	3	3	9
Information Technology	24	0	0	0	0	0	0	−1
Materials	16	0	−1	−1	−1	−3	−5	−7
Real Estate	24	0	0	0	0	0	0	−3
Utilities	43	−1	−1	−2	−8	−10	−11	−15

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 43: Weight (in %) and yield (in bps) variations of the Financials sector (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-3}$)

		Index	Reduction rate $\mathcal{R}$						
			30%	40%	50%	60%	70%	80%	90%
Weight									
AAA–AA	0Y–2Y	0.8	0.0	0.2	0.7	1.3	1.4	2.5	4.2
	2Y–5Y	1.6	0.4	0.8	0.6	0.1	0.3	−0.2	−0.9
	5Y–7Y	0.4	0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.2
	7Y–10Y	0.2	0.0	0.0	0.0	0.0	0.0	0.0	−0.1
	10Y+	0.4	0.0	0.0	0.0	0.0	0.0	0.0	−0.3
A	0Y–2Y	3.6	0.0	0.0	−0.1	−0.1	−0.1	−0.1	0.4
	2Y–5Y	9.7	0.0	0.1	0.2	1.0	1.5	2.9	0.8
	5Y–7Y	2.8	0.5	0.9	1.0	0.6	0.6	0.0	2.4
	7Y–10Y	2.4	0.0	0.0	0.0	0.0	0.0	−0.1	3.0
	10Y+	2.0	0.0	0.0	0.0	0.0	0.0	0.0	−0.9
BBB	0Y–2Y	1.7	0.4	0.4	0.2	−0.1	0.6	0.6	−0.6
	2Y–5Y	4.9	−0.3	−0.3	−0.4	−0.4	−0.4	−0.5	−1.8
	5Y–7Y	1.3	−0.1	−0.1	−0.1	−0.1	−0.1	0.2	−0.1
	7Y–10Y	1.1	0.0	0.0	−0.1	−0.1	−0.1	−0.2	−0.4
	10Y+	0.9	0.0	0.0	0.0	0.0	0.0	−0.1	−0.4
Yield									
AAA–AA	0Y–2Y	2	0	1	3	5	5	9	15
	2Y–5Y	5	2	3	3	1	1	0	−3
	5Y–7Y	1	0	0	0	0	0	0	−1
	7Y–10Y	1	0	0	0	0	0	0	0
	10Y+	2	0	0	0	0	0	0	−1
A	0Y–2Y	11	0	0	0	0	0	0	2
	2Y–5Y	38	0	0	1	4	7	13	6
	5Y–7Y	12	2	2	4	2	2	0	12
	7Y–10Y	11	0	0	0	0	0	0	9
	10Y+	10	0	0	0	0	0	0	−4
BBB	0Y–2Y	6	2	2	1	0	4	4	−2
	2Y–5Y	22	−1	−1	−2	−2	−2	−2	−8
	5Y–7Y	7	0	0	−1	−1	−1	2	1
	7Y–10Y	6	0	0	0	0	0	−1	−2
	10Y+	5	0	0	0	0	0	0	−2

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 44: Weight (in %) and yield (in bps) variations of the Utilities sector (Global Corp., Jun. 2022, scope $\mathcal{SC}_{1-3}$)

		Index	Reduction rate $\mathcal{R}$ 30%	40%	50%	60%	70%	80%	90%
Weight									
AAA–AA	0Y–2Y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2Y–5Y	0.1	0.0	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1
	5Y–7Y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	7Y–10Y	0.1	0.0	0.0	0.0	0.0	−0.1	−0.1	−0.1
	10Y+	0.1	0.0	0.0	0.0	0.0	−0.1	−0.1	−0.1
A	0Y–2Y	0.3	0.0	0.0	0.0	−0.2	−0.2	−0.2	−0.2
	2Y–5Y	0.7	0.0	0.0	−0.1	−0.5	−0.5	−0.6	−0.6
	5Y–7Y	0.4	0.0	0.0	−0.1	−0.2	−0.3	−0.3	−0.3
	7Y–10Y	0.6	0.3	0.6	0.8	1.2	1.9	2.4	−0.6
	10Y+	1.7	0.0	−0.1	0.0	0.5	0.2	1.9	4.8
BBB	0Y–2Y	0.6	0.0	−0.1	−0.1	−0.2	−0.3	−0.5	−0.5
	2Y–5Y	2.0	−0.1	−0.2	−0.3	−0.9	−1.2	−1.7	−1.9
	5Y–7Y	1.1	0.0	−0.1	−0.2	−0.4	−0.7	−1.0	−1.1
	7Y–10Y	0.9	0.0	0.0	−0.1	−0.2	−0.5	−0.7	−0.9
	10Y+	1.3	0.0	0.0	−0.2	−0.4	−0.1	−1.1	−1.3
Yield									
AAA–AA	0Y–2Y	0	0	0	0	0	0	0	0
	2Y–5Y	0	0	0	0	0	0	0	0
	5Y–7Y	0	0	0	0	0	0	0	0
	7Y–10Y	0	0	0	0	0	0	0	0
	10Y+	1	0	0	0	0	−1	−1	−1
A	0Y–2Y	1	0	0	0	−1	−1	−1	−1
	2Y–5Y	2	0	0	0	−2	−2	−2	−2
	5Y–7Y	1	0	0	0	−1	−1	−1	−1
	7Y–10Y	3	1	2	3	5	7	10	−2
	10Y+	8	0	−1	0	1	0	7	18
BBB	0Y–2Y	2	0	0	0	−1	−1	−2	−2
	2Y–5Y	8	0	−1	−1	−4	−5	−7	−8
	5Y–7Y	5	0	0	−1	−2	−3	−4	−5
	7Y–10Y	4	0	0	0	−1	−2	−3	−4
	10Y+	7	0	0	−1	−2	0	−5	−7

Source: ICE (2022), Trucost (2022) & Authors' calculations.

Table 45: Green intensity in % (Global Corp., Jun. 2022), average sector data applied for missing green data

Scope	Index	Reduction rate $\mathcal{R}$ 30%	40%	50%	60%	70%	80%	90%
$\mathcal{SC}_1$	3.82	4.03	4.18	4.30	4.53	4.90	5.45	6.14
$\mathcal{SC}_{1-2}$		3.77	3.72	3.69	3.62	3.64	3.62	3.23
$\mathcal{SC}_{1-3}^{\text{up}}$		3.89	3.81	4.09	4.13	4.12	3.55	2.01
$\mathcal{SC}_{1-3}$		3.90	4.06	4.29	4.98	5.38	5.95	5.61

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Table 46: Additional tracking error cost in bps of the greenness constraint (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\boldsymbol{\mathcal{G}} = 0\%$				
$\boldsymbol{\mathcal{SC}}_1$	0	0	0	0	0	0	0	0	0
$\boldsymbol{\mathcal{SC}}_{1-2}$	0	0	0	0	0	0	0	0	1
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	0	0	0	0	0	4			
$\boldsymbol{\mathcal{SC}}_{1-3}$	0	0	0	0	0	0	1	6	
					$\boldsymbol{\mathcal{G}} = 100\%$				
$\boldsymbol{\mathcal{SC}}_1$	24	23	22	22	19	16	14	13	13
$\boldsymbol{\mathcal{SC}}_{1-2}$	23	22	22	21	19	20	21	30	51
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	18	18	18	18	23	83			
$\boldsymbol{\mathcal{SC}}_{1-3}$	18	17	16	16	15	16	24	133	
					$\boldsymbol{\mathcal{G}} = 200\%$				
$\boldsymbol{\mathcal{SC}}_1$	69	69	68	67	64	61	59	58	62
$\boldsymbol{\mathcal{SC}}_{1-2}$	69	69	68	68	71	78	93	135	233
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	67	68	70	72	112				
$\boldsymbol{\mathcal{SC}}_{1-3}$	62	62	61	61	64	73	137		

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Table 47: Additional tracking error cost in bps of a global momentum threshold approach (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\boldsymbol{\mathcal{CM}}^{\star} = -5\%$				
$\boldsymbol{\mathcal{SC}}_1$	11	11	11	11	9	8	8	7	6
$\boldsymbol{\mathcal{SC}}_{1-2}$	10	9	9	9	6	6	4	2	1
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	5	4	3	3	2	1			
$\boldsymbol{\mathcal{SC}}_{1-3}$	4	4	3	3	3	2	3	7	
					$\boldsymbol{\mathcal{CM}}^{\star} = -7\%$				
$\boldsymbol{\mathcal{SC}}_1$	23	23	23	23	22	21	19	19	20
$\boldsymbol{\mathcal{SC}}_{1-2}$	21	21	20	20	19	16	14	16	13
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	14	13	12	10	6	8			
$\boldsymbol{\mathcal{SC}}_{1-3}$	11	10	10	9	9	9	8	18	

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Table 48: Additional tracking error cost in bps of a momentum-based exclusion approach (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
					$\boldsymbol{\mathcal{CM}}^{+} = 0\%$				
$\boldsymbol{\mathcal{SC}}_1$	124	122	122	120	114	107	99	89	81
$\boldsymbol{\mathcal{SC}}_{1-2}$	121	120	118	117	108	96	80	64	41
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	109	105	101	98	74	44			
$\boldsymbol{\mathcal{SC}}_{1-3}$	112	109	107	105	94	84	76	80	
					$\boldsymbol{\mathcal{CM}}^{+} = 10\%$				
$\boldsymbol{\mathcal{SC}}_1$	2	2	2	1	1	1	0	0	0
$\boldsymbol{\mathcal{SC}}_{1-2}$	2	1	1	1	1	0	0	0	0
$\boldsymbol{\mathcal{SC}}_{1-3}^{\text{up}}$	1	1	1	1	0	0			
$\boldsymbol{\mathcal{SC}}_{1-3}$	1	1	1	1	1	0	0	0	0

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Table 49: Active share (in %) between the decarbonized and net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	Net zero portfolio with $\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	11.6	11.6	11.6	11.5	11.4	11.5	11.6	11.5	11.3
$\mathcal{SC}_{1-2}$	11.6	11.6	11.6	11.6	12.0	12.2	12.1	12.1	12.3
$\mathcal{SC}_{1-3}^{\text{up}}$	11.3	11.4	11.4	11.4	11.4	12.6	17.8	37.6	
$\mathcal{SC}_{1-3}$	11.4	11.3	11.3	11.3	11.1	11.3	12.9	16.2	21.6
	Net zero portfolio with $\mathcal{G} = 200\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	22.1	22.0	22.0	21.9	21.8	21.8	21.9	21.9	21.6
$\mathcal{SC}_{1-2}$	22.0	22.0	22.0	22.0	22.4	22.7	22.4	22.5	22.9
$\mathcal{SC}_{1-3}^{\text{up}}$	21.5	21.4	21.3	21.3	21.3	23.4	29.8	56.0	
$\mathcal{SC}_{1-3}$	21.7	21.6	21.5	21.4	21.0	21.0	24.8	30.3	36.0

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Table 50: Active share (in %) between the decarbonized and net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	Net zero portfolio with $\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$								
$\mathcal{SC}_1$	13.3	13.3	13.2	13.2	13.2	13.1	14.1	15.4	17.7
$\mathcal{SC}_{1-2}$	13.3	13.3	13.4	13.4	13.6	15.2	16.6	19.7	23.2
$\mathcal{SC}_{1-3}^{\text{up}}$	12.9	12.7	12.8	13.2	16.0	30.5			
$\mathcal{SC}_{1-3}$	12.8	12.7	12.7	12.8	13.2	14.6	19.2	46.0	
	Net zero portfolio with $\mathcal{G} = 200\%$ and $\mathcal{CM}^\star = -7\%$								
$\mathcal{SC}_1$	33.1	33.2	33.3	33.5	34.6	35.9	36.5	39.8	45.5
$\mathcal{SC}_{1-2}$	33.7	34.1	34.5	34.9	36.7	38.0	44.1	53.5	57.9
$\mathcal{SC}_{1-3}^{\text{up}}$	33.7	34.3	35.0	35.3	44.1				
$\mathcal{SC}_{1-3}$	31.6	31.7	31.9	32.1	33.6	39.1	56.2		

Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Table 51: Additional active share cost in % when we control the green intensity (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 0\%$								
$\mathcal{SC}_1$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\mathcal{SC}_{1-2}$	0.0	0.0	0.0	0.1	0.1	0.2	0.1	−0.2	−0.3
$\mathcal{SC}_{1-3}^{\text{up}}$	0.0	0.1	0.1	0.1	0.2	0.0	−0.2	−0.1	0.4
$\mathcal{SC}_{1-3}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	$\mathcal{G} = 100\%$								
$\mathcal{SC}_1$	2.2	1.9	1.7	1.4	0.2	0.2	0.2	0.0	−0.2
$\mathcal{SC}_{1-2}$	2.0	1.7	1.4	1.1	0.4	0.4	0.2	−0.0	−0.1
$\mathcal{SC}_{1-3}^{\text{up}}$	1.5	1.0	0.6	0.4	0.5	0.5	0.5	0.4	2.6
$\mathcal{SC}_{1-3}$	0.9	0.6	0.4	0.4	0.3	0.4	0.5	0.3	−0.7
	$\mathcal{G} = 200\%$								
$\mathcal{SC}_1$	5.6	5.3	5.1	4.8	3.1	1.9	0.7	0.2	0.0
$\mathcal{SC}_{1-2}$	5.4	5.2	4.8	4.4	2.6	1.5	0.9	0.4	0.3
$\mathcal{SC}_{1-3}^{\text{up}}$	4.9	4.3	3.6	3.0	1.3	1.3	1.6	1.6	6.5
$\mathcal{SC}_{1-3}$	4.2	3.5	2.9	2.4	0.8	0.7	0.9	0.7	−0.1

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Table 52: Additional active share cost in % of a momentum exclusion approach (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	20.0	20.1	20.3	20.4	20.5	20.1	18.5	17.5	16.6
$\mathcal{SC}_{1-2}$	19.8	19.9	20.0	20.1	19.6	18.4	16.4	15.0	13.4
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	19.4	19.4	19.4	19.2	16.8	13.6	11.4	9.5	3.0
$\mathcal{SC}_{1-3}$	19.1	18.6	18.2	17.8	16.2	14.4	13.1	12.5	12.2
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	0.4	0.4	0.4	0.5	1.0	1.0	0.9	0.9	0.8
$\mathcal{SC}_{1-2}$	0.3	0.3	0.4	0.5	0.8	0.6	0.6	0.4	0.2
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.4	0.5	0.7	0.9	0.6	0.4	0.2	0.2	0.0
$\mathcal{SC}_{1-3}$	0.7	1.0	1.1	1.1	1.0	0.8	0.6	0.5	0.3

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Table 53: Additional DTS cost in bps of a momentum exclusion approach (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	4.62	4.66	4.67	4.70	4.05	4.13	3.72	3.59	3.52
$\mathcal{SC}_{1-2}$	4.51	4.39	4.39	4.14	3.65	3.58	2.93	2.61	1.54
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	3.81	3.71	3.55	3.46	2.89	2.24	1.67	0.38	0.00
$\mathcal{SC}_{1-3}$	3.84	3.70	3.72	3.65	3.05	2.80	2.47	2.13	2.35
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	0.20	0.30	0.29	0.39	0.35	0.34	0.27	0.27	0.27
$\mathcal{SC}_{1-2}$	0.28	0.22	0.28	0.29	0.21	0.23	0.18	0.05	0.05
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	0.35	0.26	0.32	0.32	0.16	0.05	0.03	0.00	0.00
$\mathcal{SC}_{1-3}$	0.36	0.38	0.46	0.32	0.16	0.24	0.23	0.12	0.04

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Table 54: Carbon momentum difference $\Delta\mathcal{CM}(t)$ in % of a momentum exclusion approach (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	−3.3	−3.3	−3.3	−3.2	−2.9	−2.8	−2.2	−2.4	−1.8
$\mathcal{SC}_{1-2}$	−2.3	−2.3	−2.2	−2.0	−1.6	−1.4	−1.2	−0.9	−0.6
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	−2.3	−2.3	−2.0	−1.8	−1.5	−2.1	−1.8	−2.8	−4.8
$\mathcal{SC}_{1-3}$	−2.1	−2.1	−2.2	−2.2	−1.6	−1.3	−1.4	−0.6	−0.8
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	−0.1	0.1	0.3	0.5	1.3	1.5	1.3	1.7	1.9
$\mathcal{SC}_{1-2}$	−0.1	0.1	0.5	0.6	1.2	0.8	0.3	0.6	0.8
$\mathcal{SC}_{1-3}^{\mathrm{up}}$	−0.1	0.3	0.4	0.6	0.3	−0.0	−0.4	−1.7	−4.4
$\mathcal{SC}_{1-3}$	−0.5	−0.5	−0.7	−1.0	0.3	0.3	0.2	1.1	1.0

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Table 55: Carbon momentum difference $\Delta\mathcal{CM}(t)$ in % of a momentum exclusion approach (Global Corp., Jun. 2022, PAB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 0\%$								
$\mathcal{SC}_1$	-2.9	-2.9	-2.8	-2.8	-2.8	-2.3	-2.4	-1.9	-1.6
$\mathcal{SC}_{1-2}$	-1.7	-1.7	-1.6	-1.6	-1.4	-1.3	-0.9	-0.6	-0.5
$\mathcal{SC}_{1-3}^{\text{up}}$	-1.6	-1.7	-1.6	-1.6	-2.0	-1.8	-2.7	-4.6	-5.2
$\mathcal{SC}_{1-3}$	-2.2	-2.3	-1.9	-1.6	-1.3	-1.5	-0.6	-0.7	-1.0
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{+} = 5\%$								
$\mathcal{SC}_1$	0.8	0.9	1.0	1.2	1.5	1.3	1.6	1.9	1.9
$\mathcal{SC}_{1-2}$	0.8	0.8	1.0	1.1	0.8	0.3	0.6	0.8	0.5
$\mathcal{SC}_{1-3}^{\text{up}}$	0.8	0.6	0.5	0.4	-0.1	-0.2	-1.7	-4.0	-5.1
$\mathcal{SC}_{1-3}$	-0.7	-0.2	-0.0	0.2	0.3	0.2	1.1	1.1	1.0

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Table 56: Additional Active share cost in % of a global momentum threshold approach (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -5\%$								
$\mathcal{SC}_1$	0.7	0.6	0.6	0.6	0.7	0.7	0.2	0.2	0.2
$\mathcal{SC}_{1-2}$	0.6	0.5	0.5	0.5	0.6	0.3	0.1	0.2	0.2
$\mathcal{SC}_{1-3}^{\text{up}}$	0.4	0.4	0.4	0.4	0.3	0.2	0.2	0.1	1.0
$\mathcal{SC}_{1-3}$	0.5	0.4	0.3	0.2	0.4	0.4	0.4	0.7	1.0
	$\mathcal{G} = 100\%$ and $\mathcal{CM}^{\star} = -7\%$								
$\mathcal{SC}_1$	1.7	1.6	1.6	1.5	1.2	1.4	0.7	0.4	0.5
$\mathcal{SC}_{1-2}$	1.6	1.5	1.4	1.3	1.1	1.0	0.4	0.4	0.4
$\mathcal{SC}_{1-3}^{\text{up}}$	1.3	1.1	1.0	0.9	0.8	0.5	0.5	0.4	0.0
$\mathcal{SC}_{1-3}$	1.4	1.2	0.9	0.7	0.7	0.7	0.9	1.2	1.7

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

Table 57: Additional Active share cost in % of the constraint $\Omega_{\mathcal{G}reen\mathcal{W}ash}$ (Global Corp., Jun. 2022, CTB)

Scope	2022	2023	2024	2025	2030	2035	2040	2045	2050
				$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -5\%$					
$\mathcal{SC}_1$	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4
$\mathcal{SC}_{1-2}$	0.2	0.2	0.2	0.2	0.2	0.2	0.5	1.1	2.6
$\mathcal{SC}_{1-3}^{\text{up}}$	0.3	0.3	0.2	0.3	0.3	1.0	2.3	9.6	−1.4
$\mathcal{SC}_{1-3}$	0.4	0.6	0.7	0.6	0.6	1.1	1.8	3.1	7.1
				$\mathcal{G} = 100\%$ and $\mathcal{CM}^\star = -7\%$					
$\mathcal{SC}_1$	0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.3
$\mathcal{SC}_{1-2}$	0.2	0.2	0.2	0.2	0.1	0.1	0.5	1.0	2.4
$\mathcal{SC}_{1-3}^{\text{up}}$	0.2	0.2	0.2	0.2	0.3	1.0	2.3	8.8	1.8
$\mathcal{SC}_{1-3}$	0.4	0.5	0.6	0.6	0.5	1.0	1.8	3.0	7.0

Source: ICE (2022), MSCI (2022), Trucost (2022) & Authors' calculations.

B.2 Figures

Figure 39: Estimated value $\Delta\boldsymbol{\mathcal{R}}^{\star}(2020,t)$ (in %) from the IEA NZE scenario — $g_Y = 6\%$

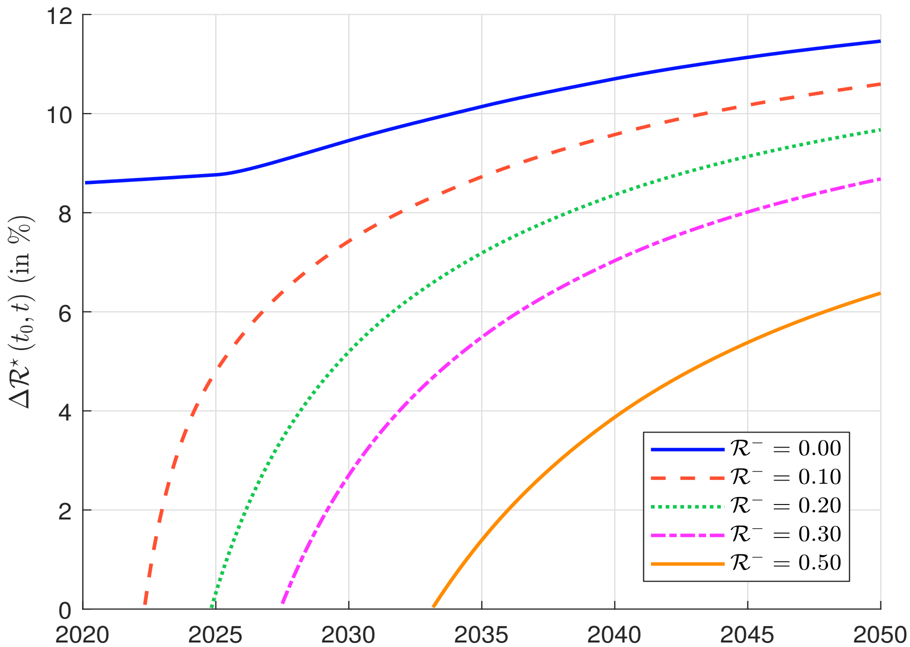


Figure 40: Estimated value $\boldsymbol{\mathcal{R}}^{\star}(t)$ (in %) from the IEA NZE scenario — $g_Y = 6\%$

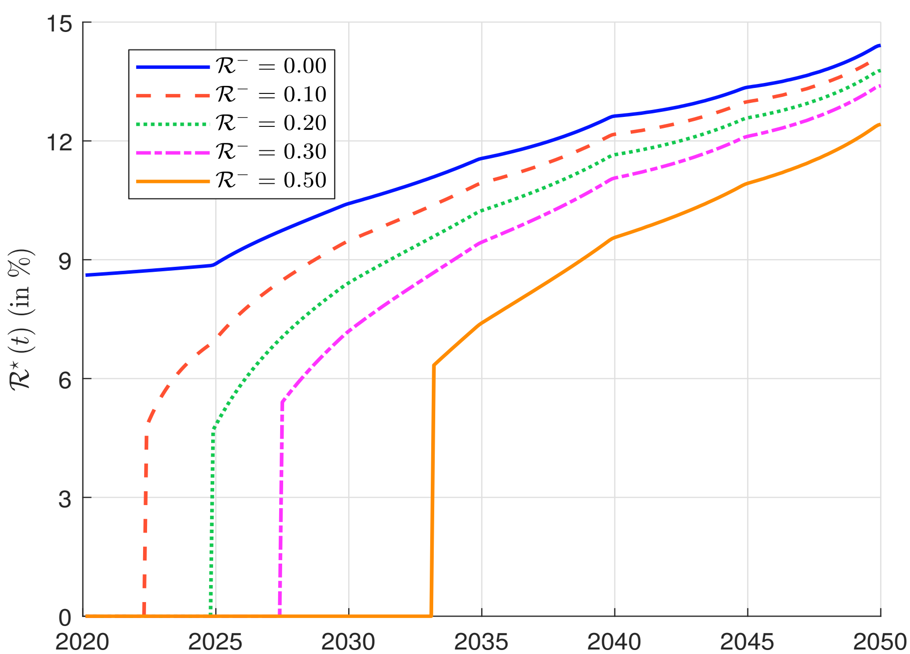


Figure 41: Relationship between $\Delta\boldsymbol{\mathcal{R}}^{\star}(2020, t)$ and $\boldsymbol{\mathcal{R}}^{\star}(t)$ (in %) — $g_Y = 3\%$, $\boldsymbol{\mathcal{R}}^{-}_{\mathcal{CI}} = 0$

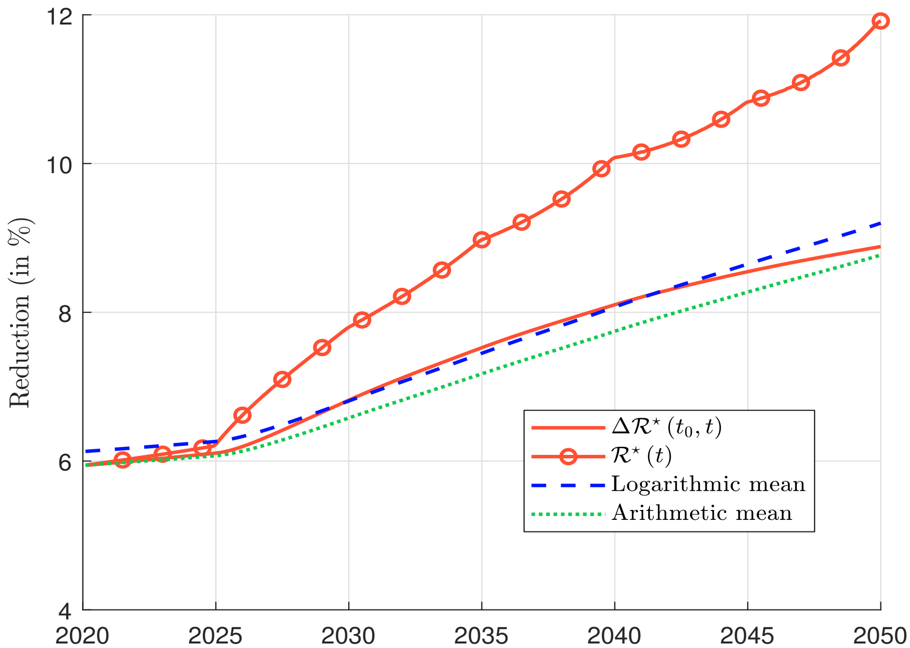


Figure 42: Relationship between $\Delta\boldsymbol{\mathcal{R}}^{\star}(2020, t)$ and $\boldsymbol{\mathcal{R}}^{\star}(t)$ (in %) — $g_Y = 3\%$, $\boldsymbol{\mathcal{R}}^{-}_{\mathcal{CI}} = 0$

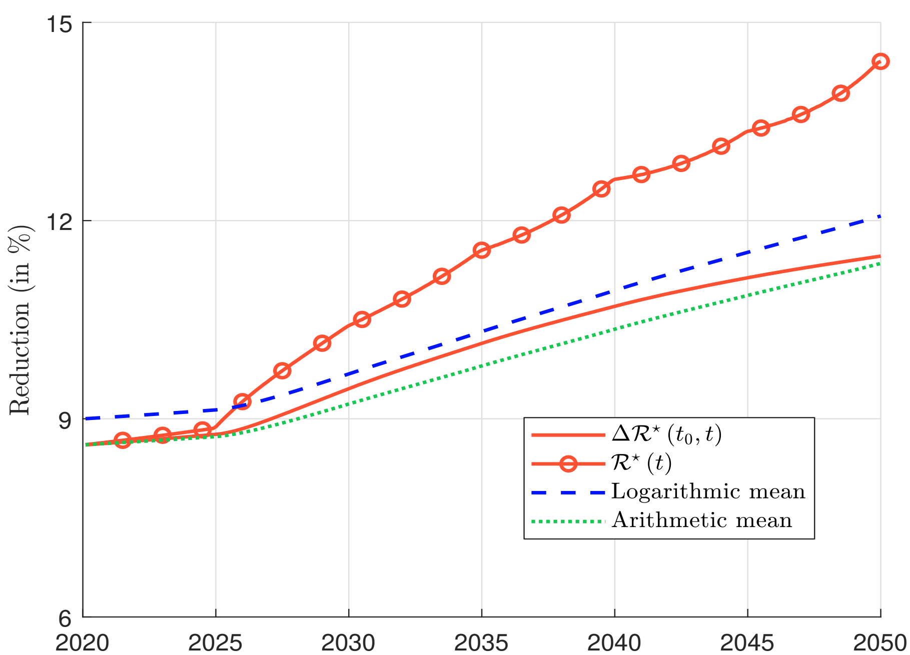


Figure 43: Fitted CTB and PAB decarbonization pathways

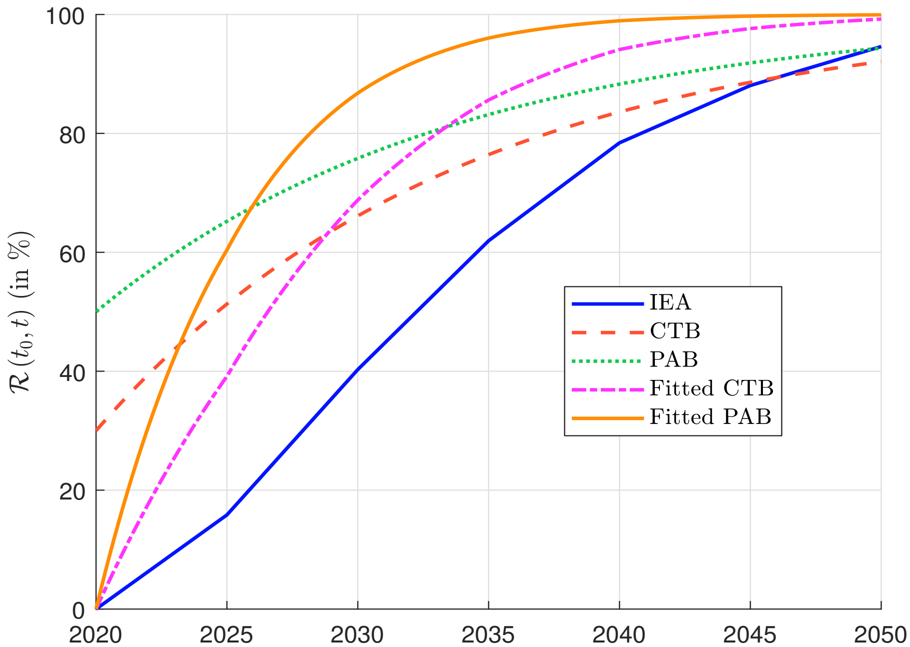


Figure 44: Calibrated growth rate $g_Y^{\star}(t_0, t)$

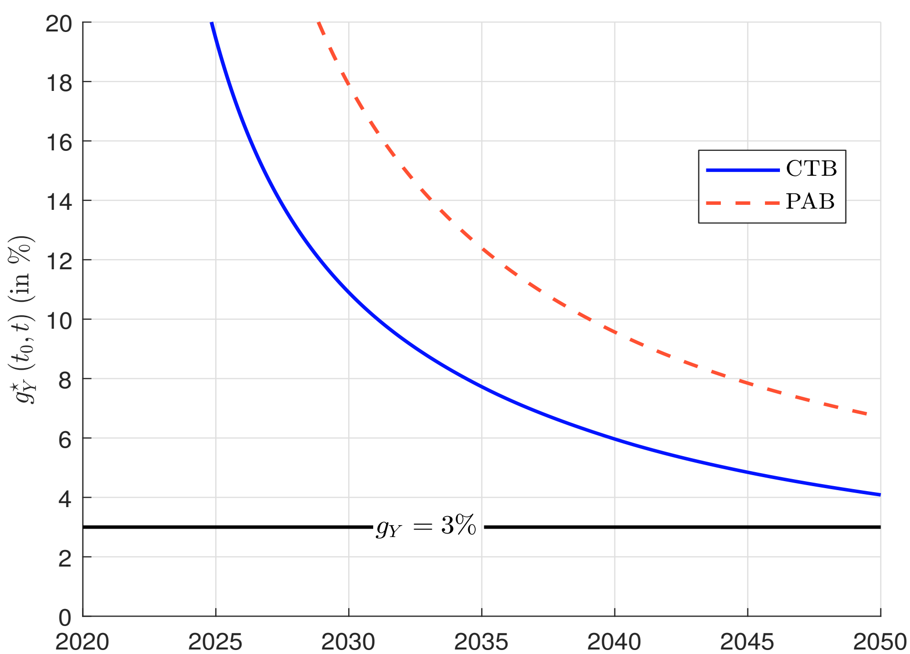


Figure 45: Sequential versus self-decarbonization (Case #1)

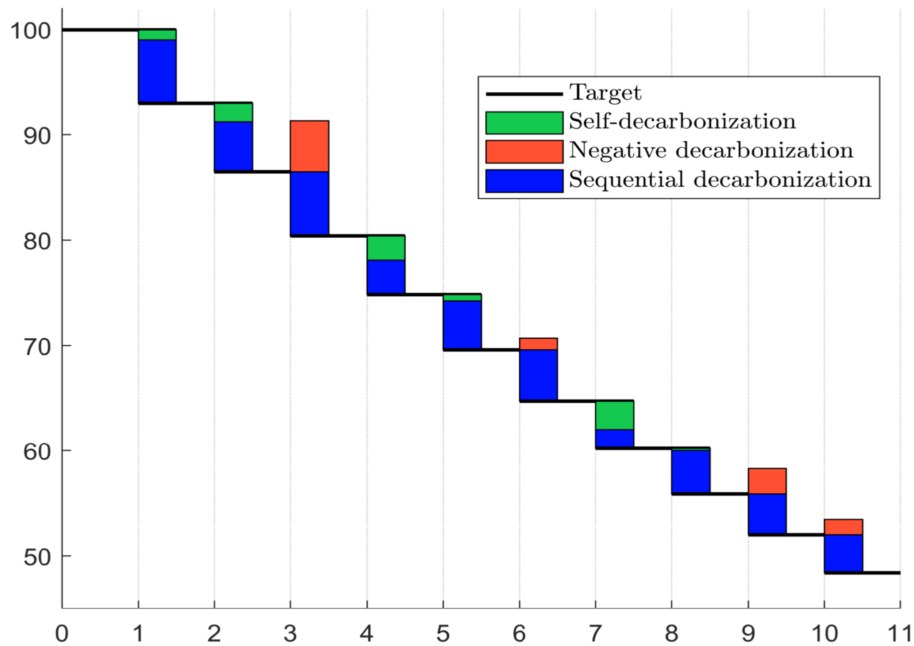


Figure 46: Sequential versus self-decarbonization (Case #2)

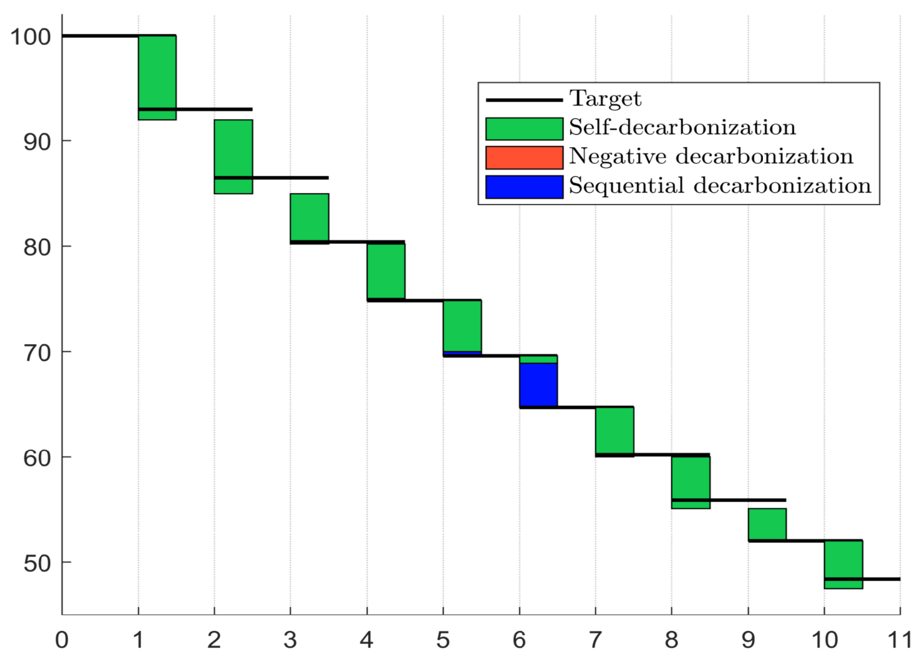
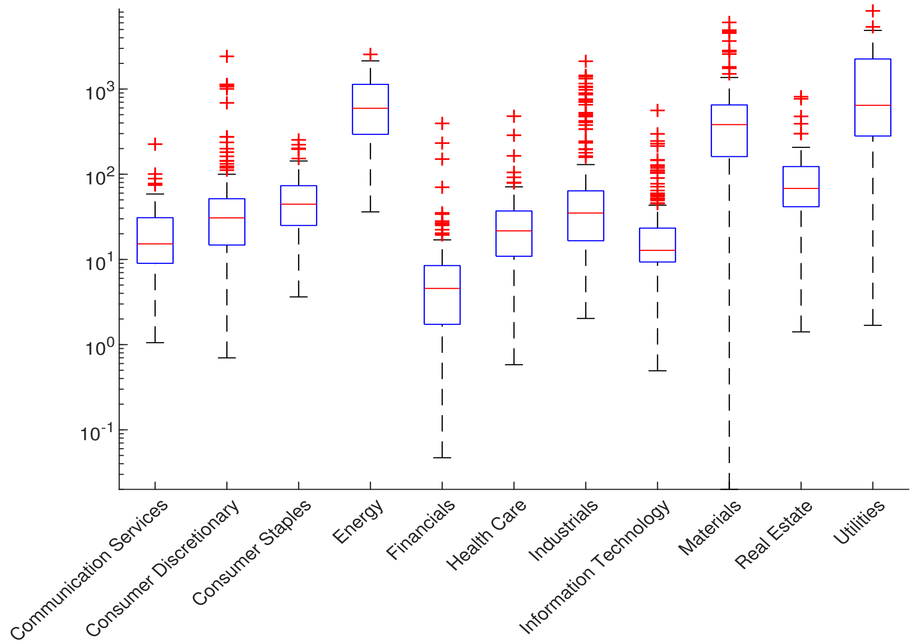
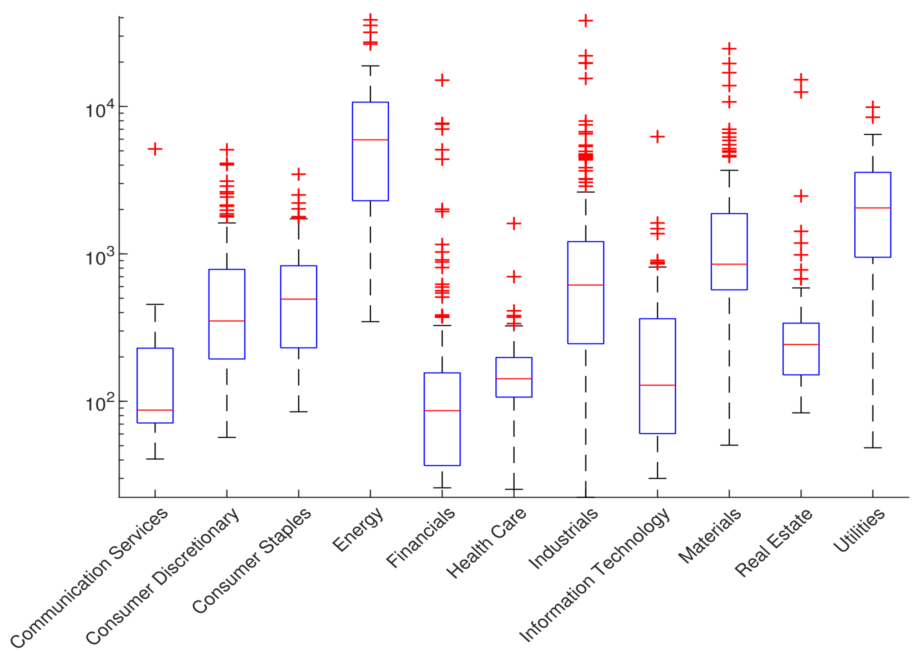


Figure 47: Boxplot of carbon intensity per sector (MSCI World, Jun. 2022, scope $\mathcal{SC}_{1-2}$)



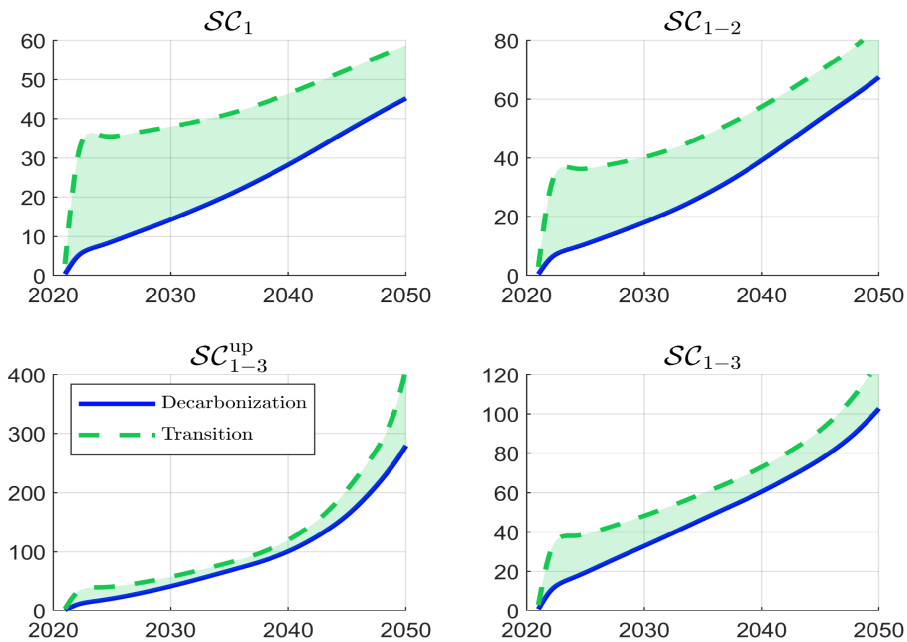
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 48: Boxplot of carbon intensity per sector (MSCI World, Jun. 2022, scope $\mathcal{SC}_{1-3}$)



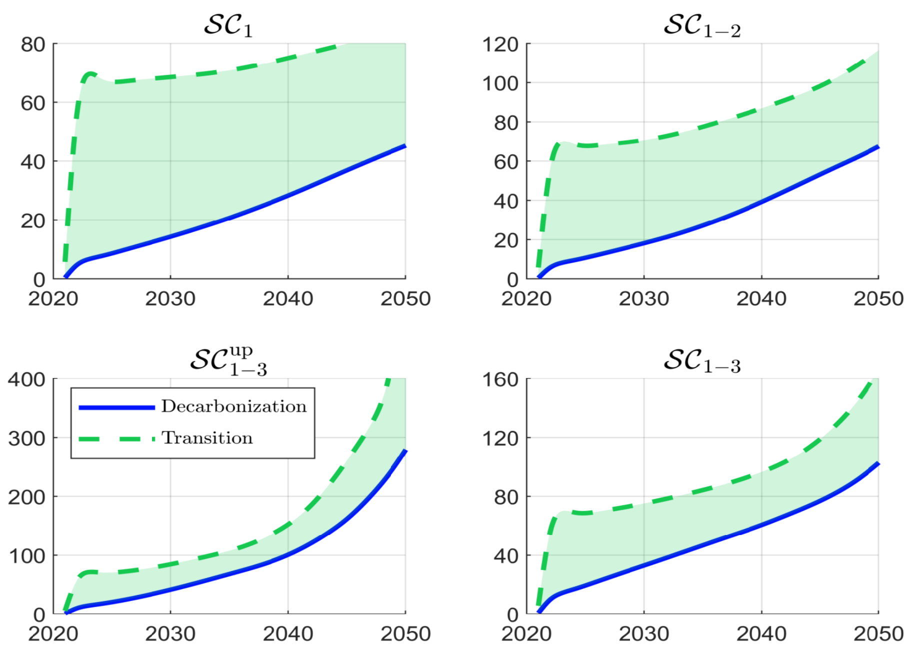
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 49: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, CTB)



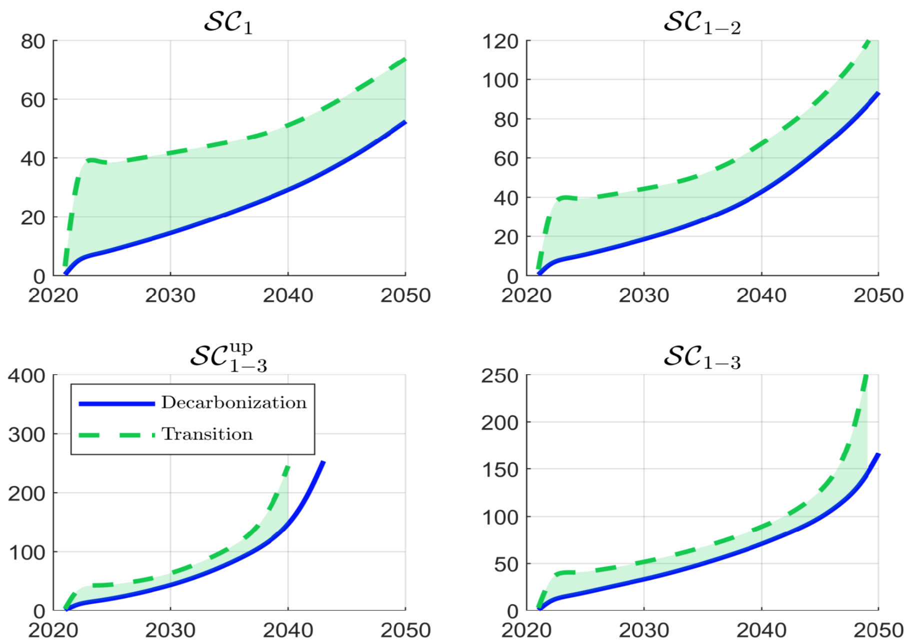
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 50: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 200\%$, $\mathcal{CM}^\star = -7\%$, CTB)



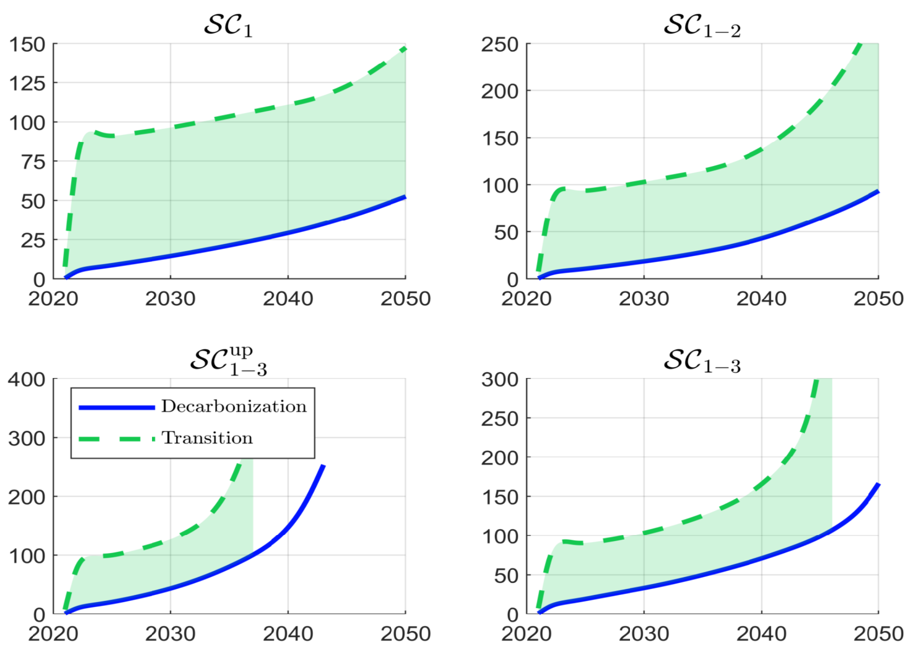
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 51: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, CTB)



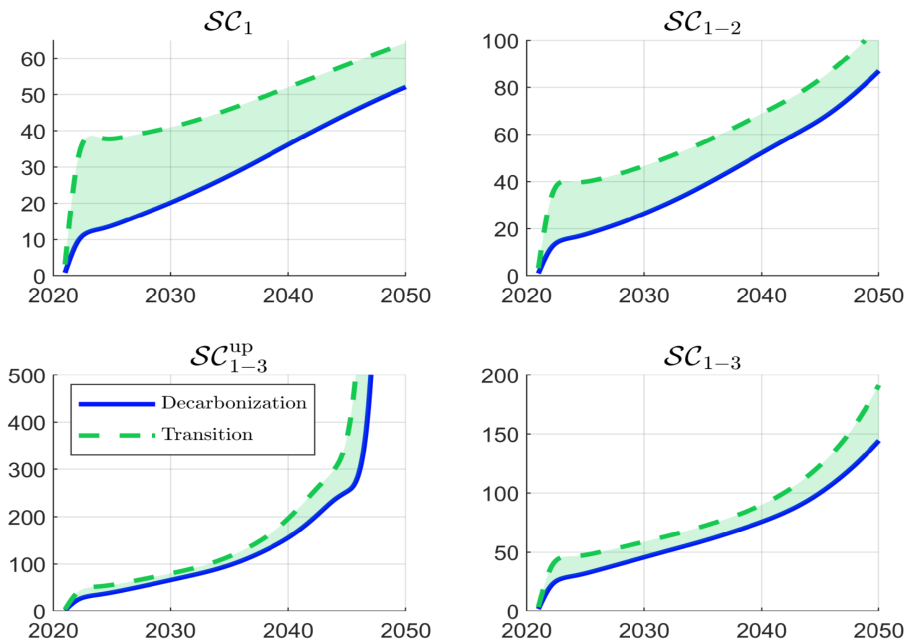
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 52: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\mathcal{G} = 200\%$, $\mathcal{CM}^{\star} = -7\%$, CTB)



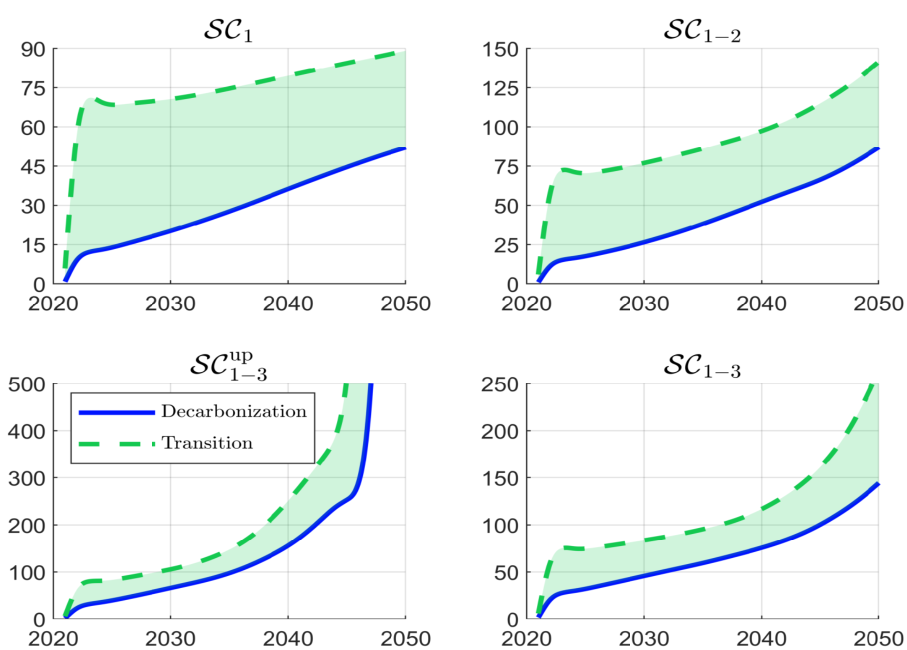
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 53: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB)



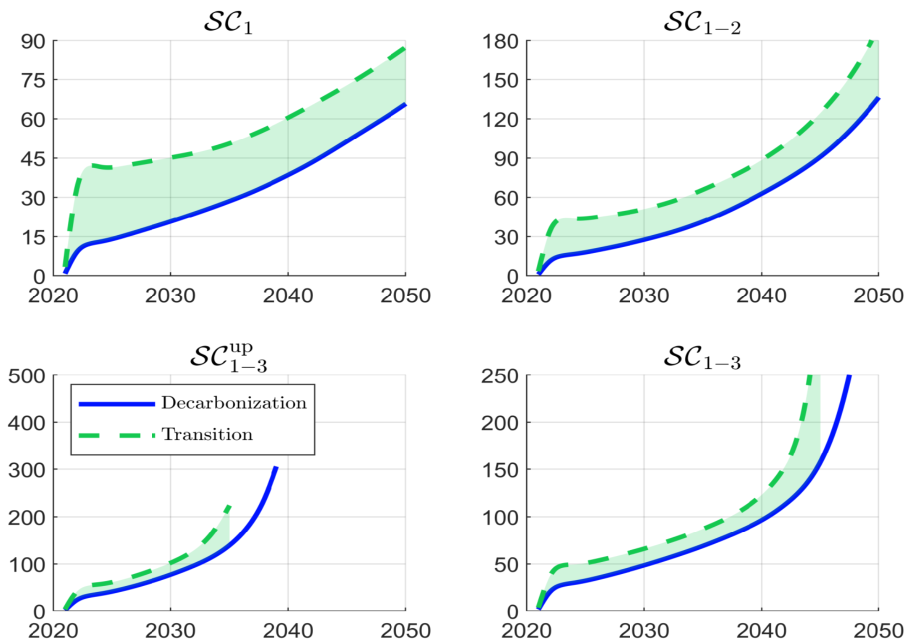
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 54: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 200\%$, $\mathcal{CM}^\star = -7\%$, PAB)



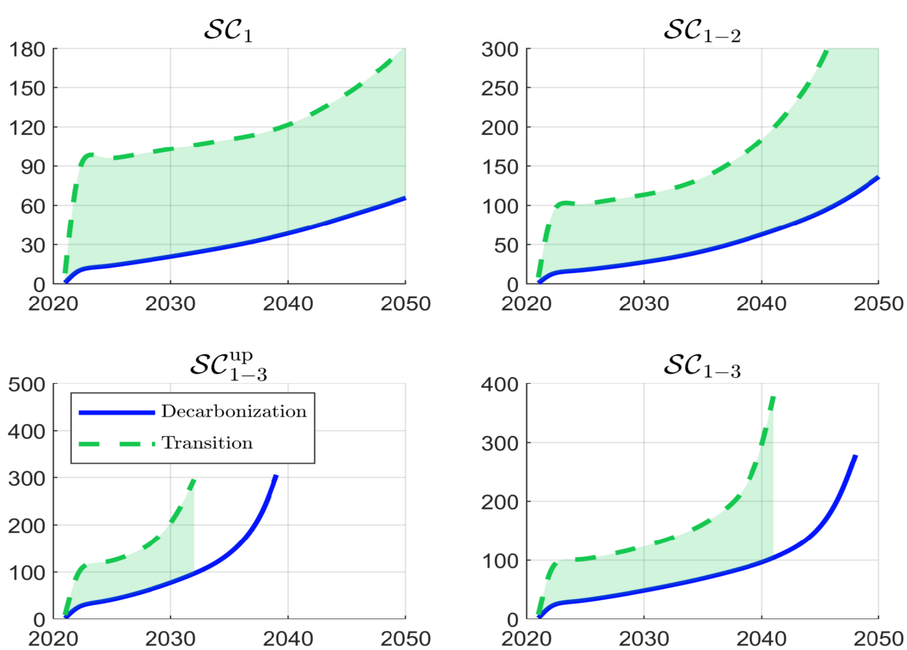
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 55: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB)



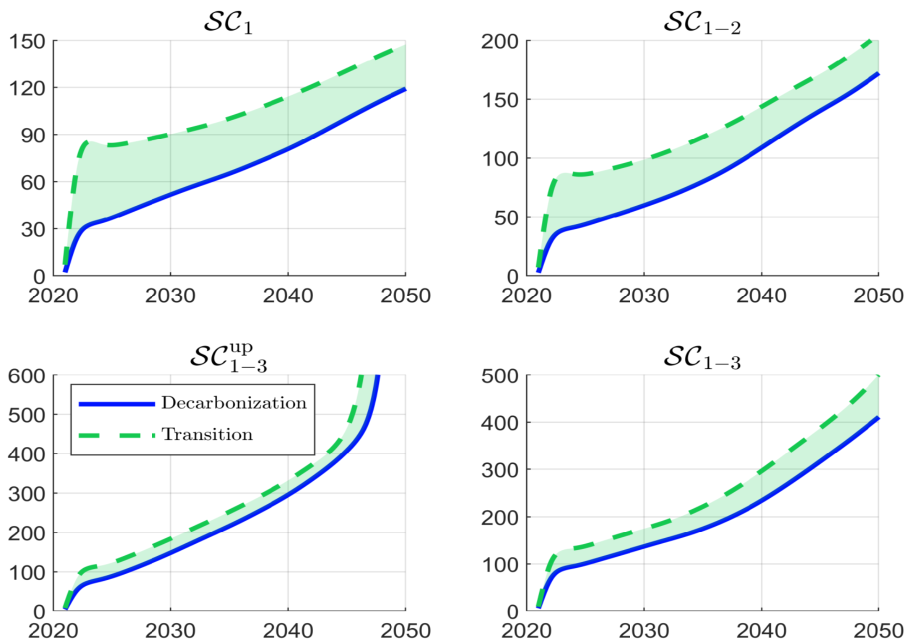
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 56: Tracking error volatility of net zero portfolios (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\boldsymbol{\mathcal{G}} = 200\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -7\%$, PAB)



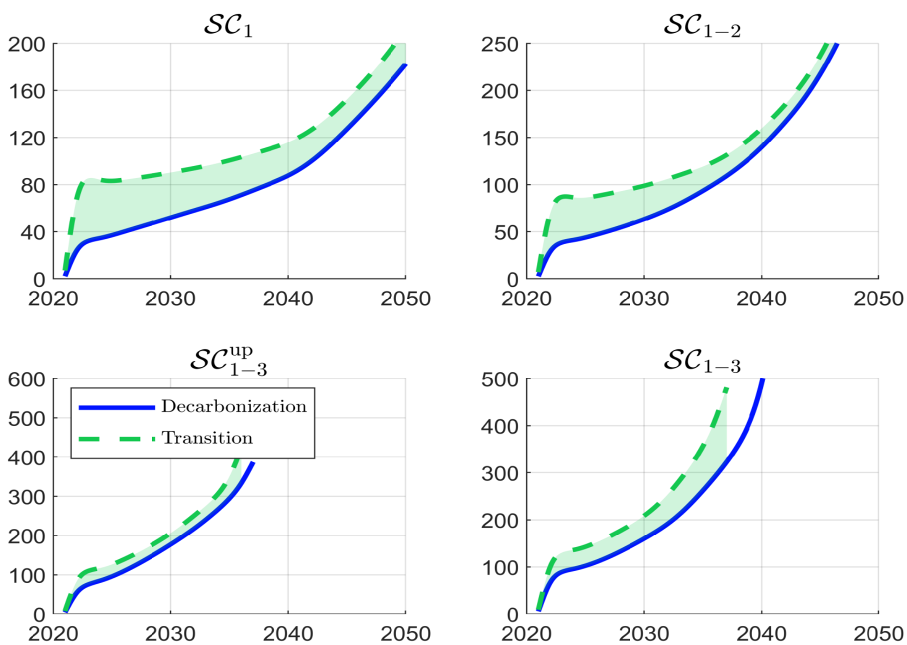
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 57: Tracking error volatility of net zero portfolios (MSCI EMU, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB)



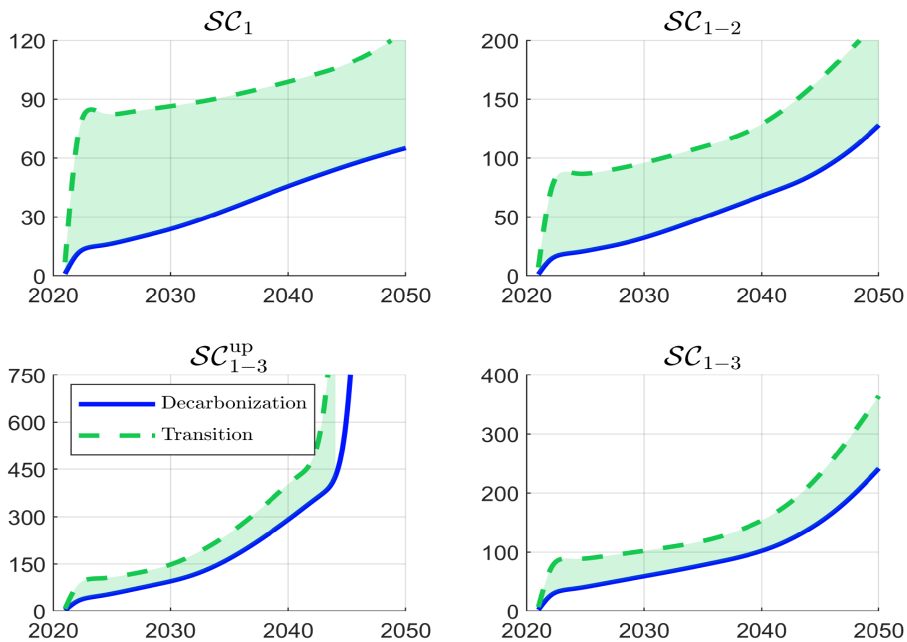
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 58: Tracking error volatility of net zero portfolios (MSCI EMU, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB)



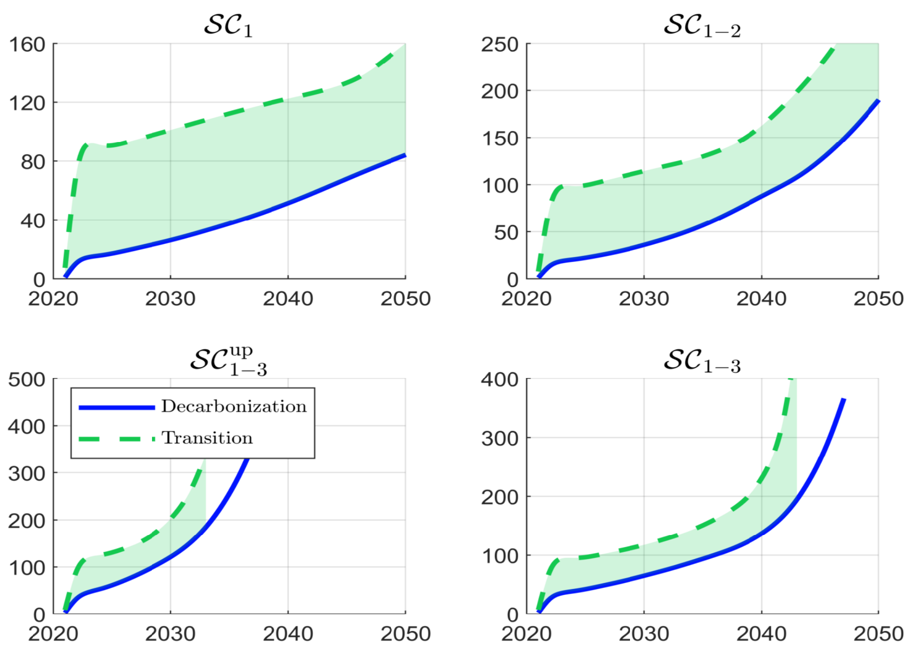
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 59: Tracking error volatility of net zero portfolios (MSCI USA, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB)



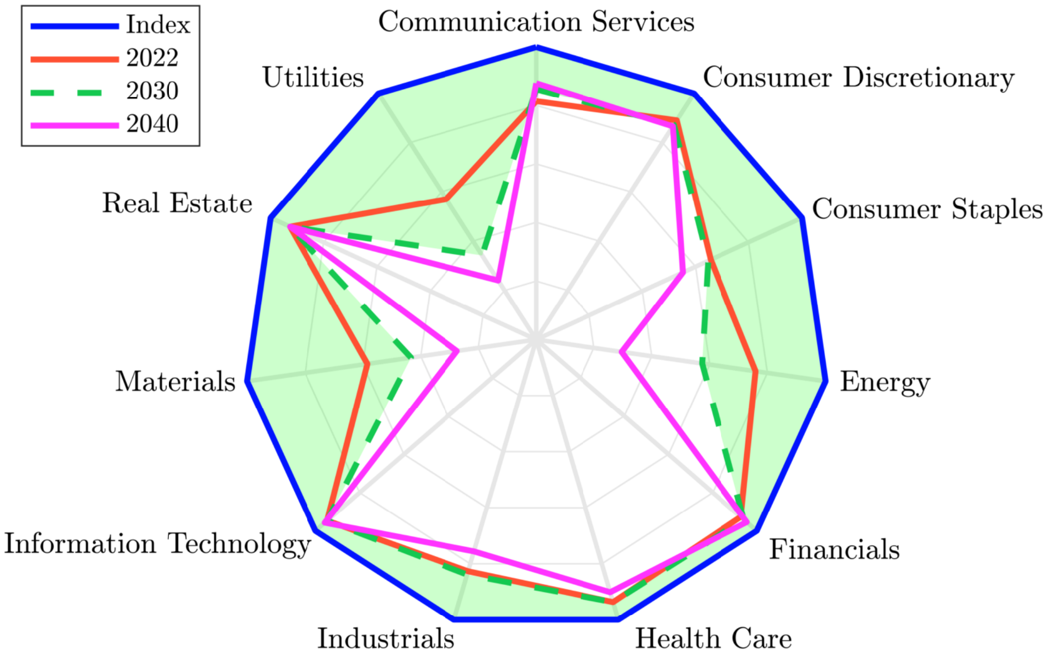
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 60: Tracking error volatility of net zero portfolios (MSCI USA, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB)



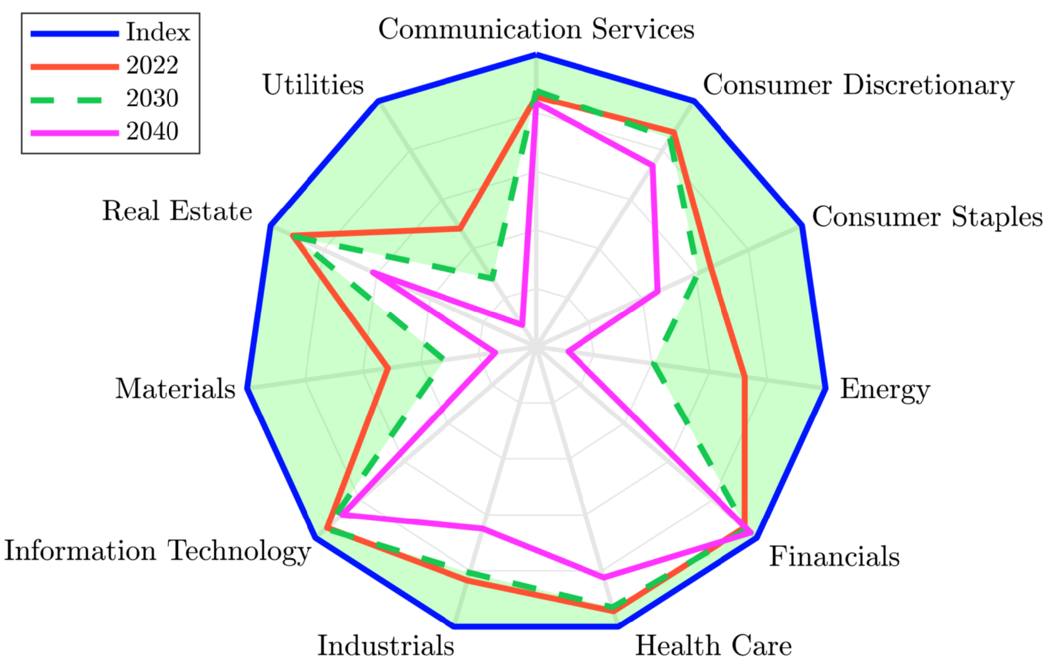
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 61: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_1$)



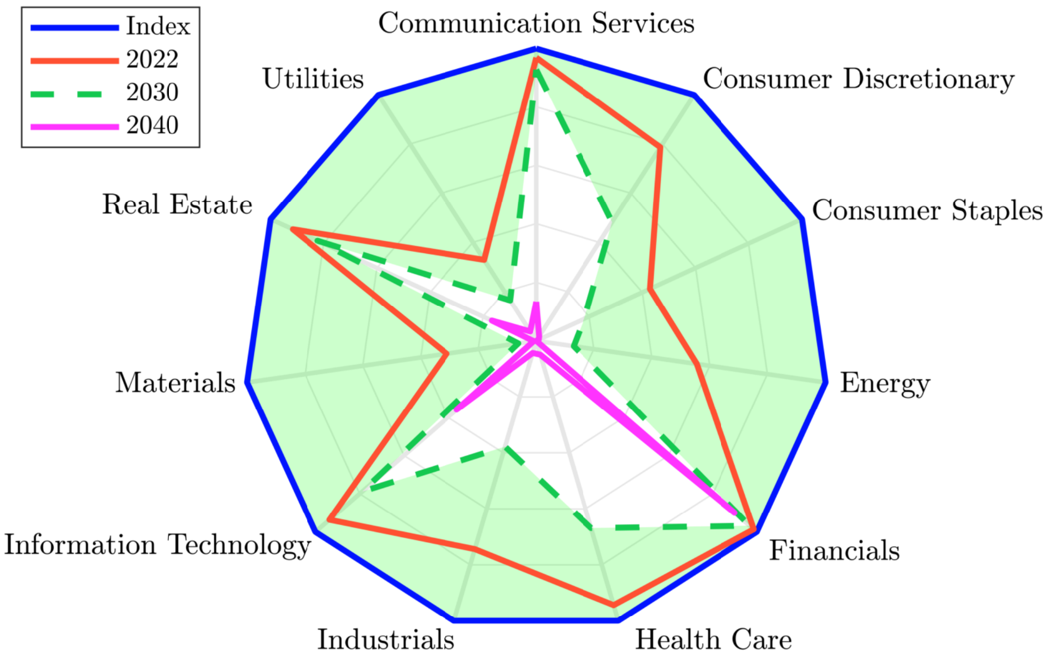
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 62: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-2}$)



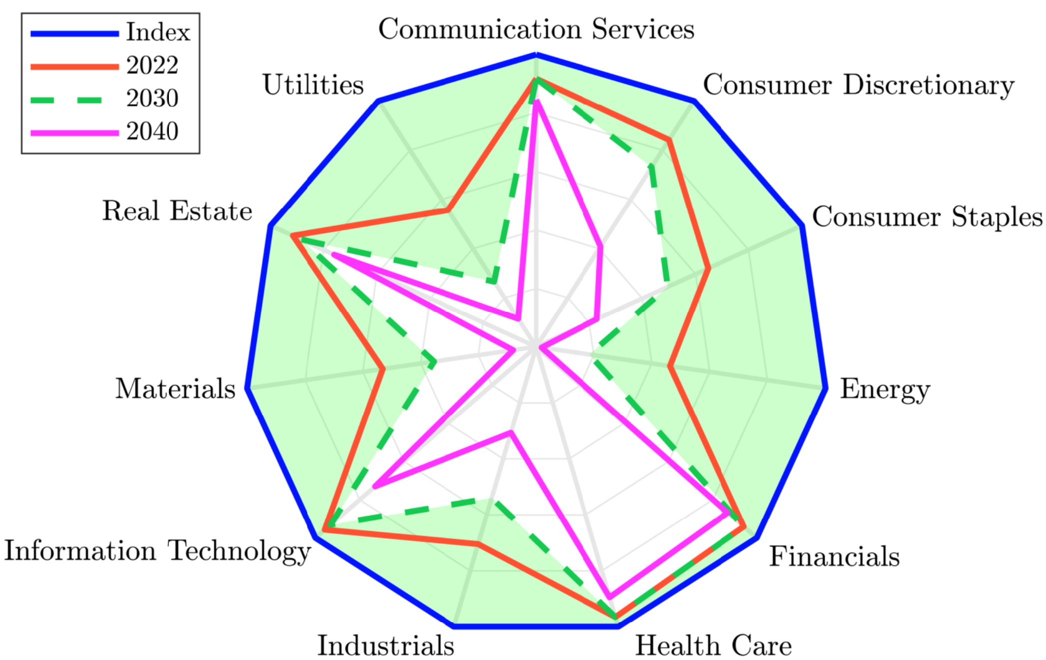
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 63: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB, scope $\mathcal{SC}_{1-3}^{\text{up}}$)



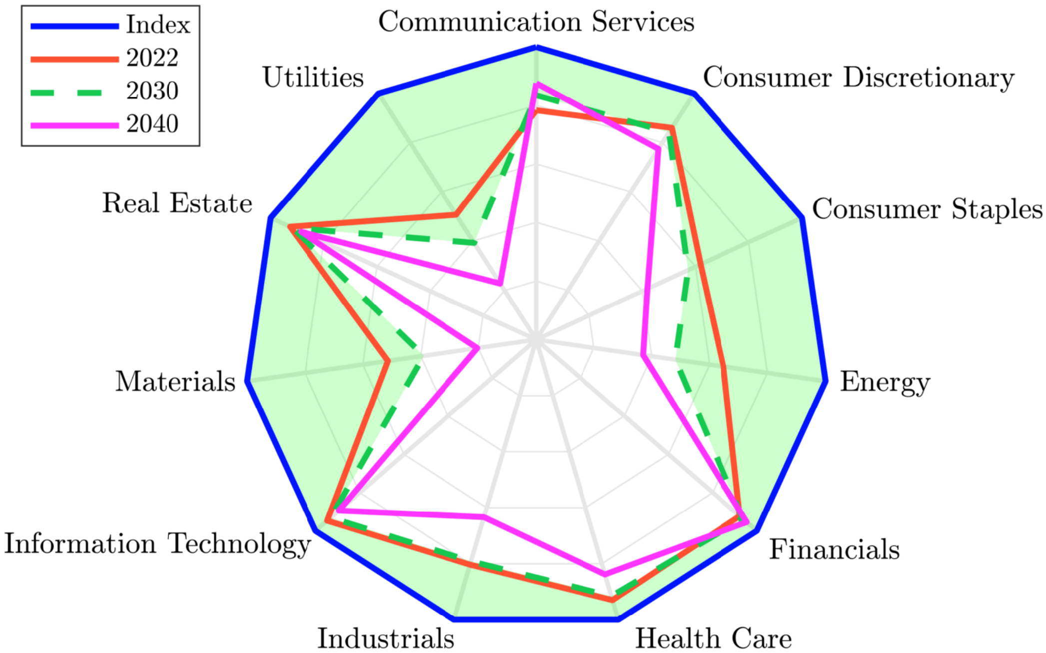
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 64: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



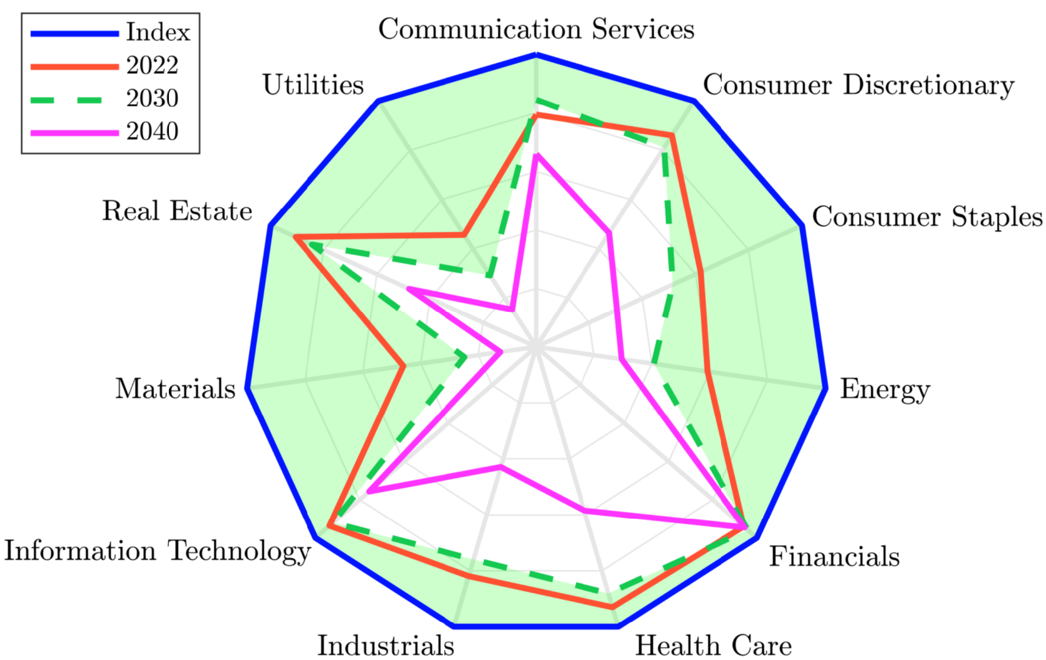
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 65: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_1$)



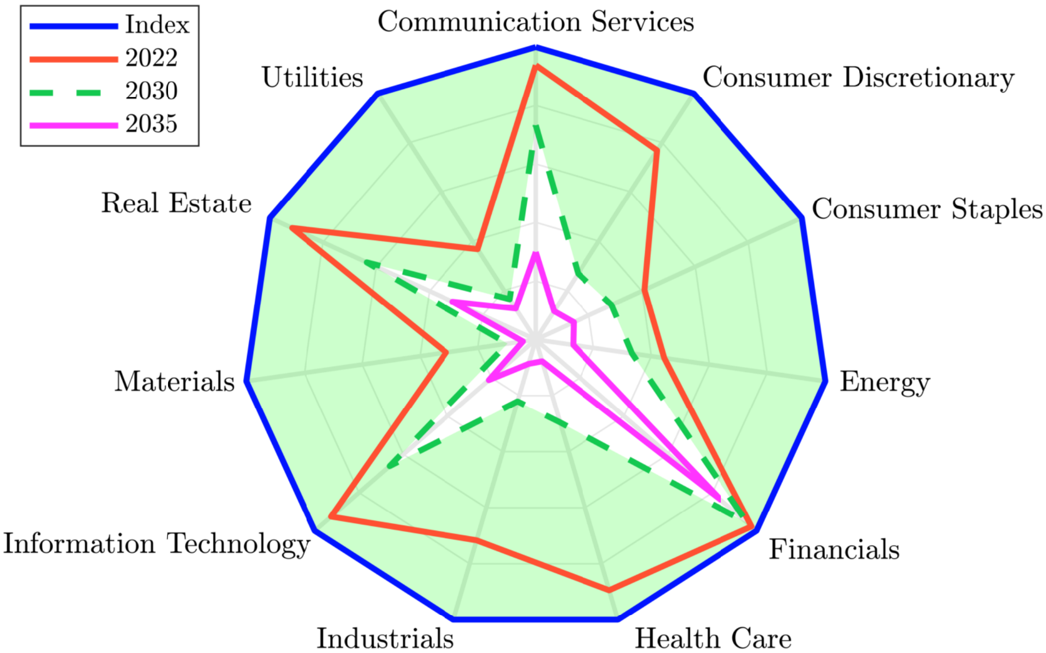
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 66: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0,10,2)$ constraint, $\boldsymbol{\mathcal{G}} = 100\%$, $\boldsymbol{\mathcal{CM}}^{\star} = -5\%$, PAB, scope $\boldsymbol{\mathcal{SC}}_{1-2}$)



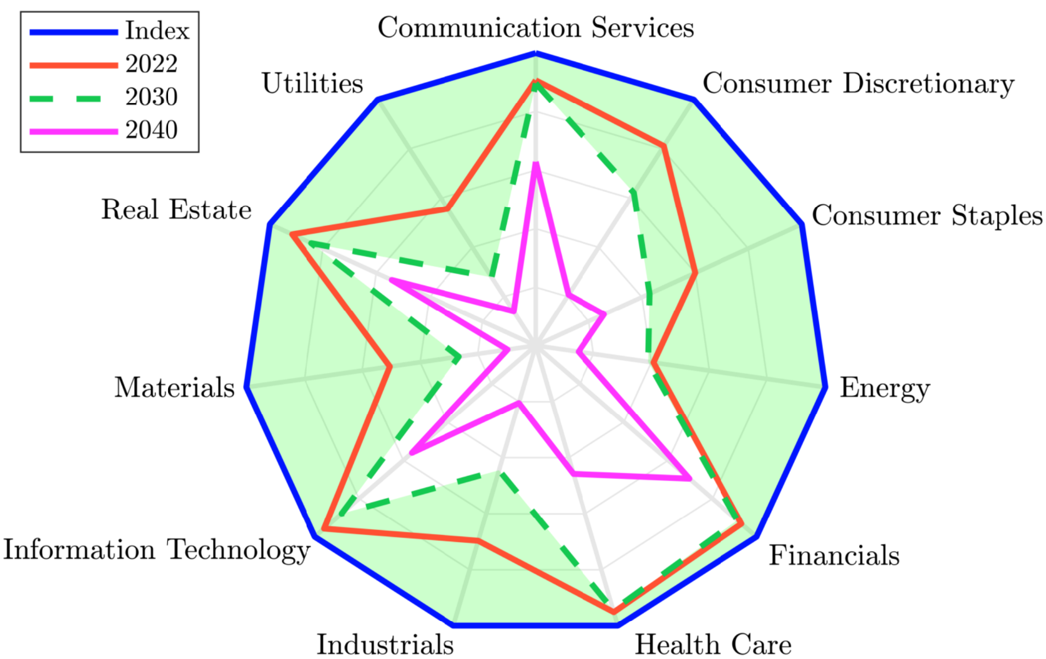
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 67: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}^{\mathrm{up}}_{1-3}$)



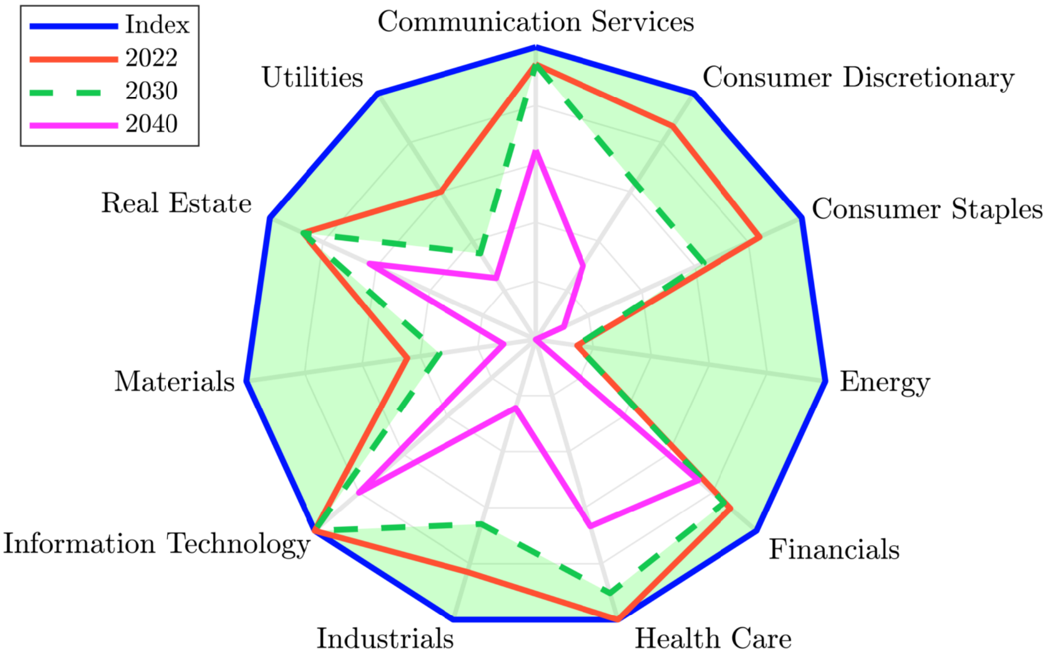
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 68: Radar chart representation of investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



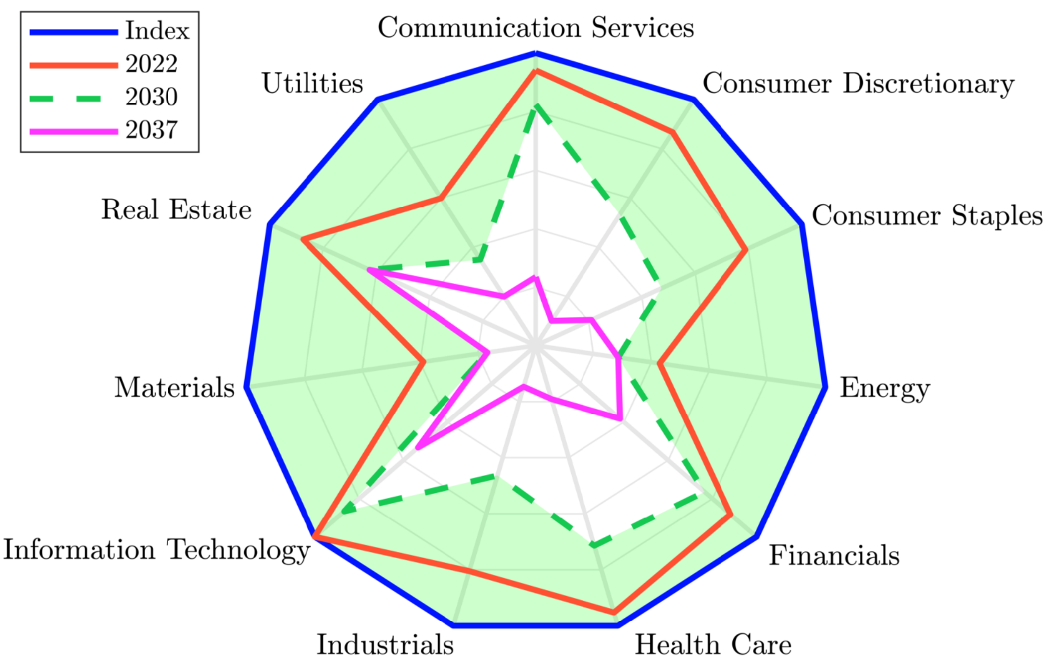
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 69: Radar chart representation of investment universe shrinkage (MSCI EMU, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



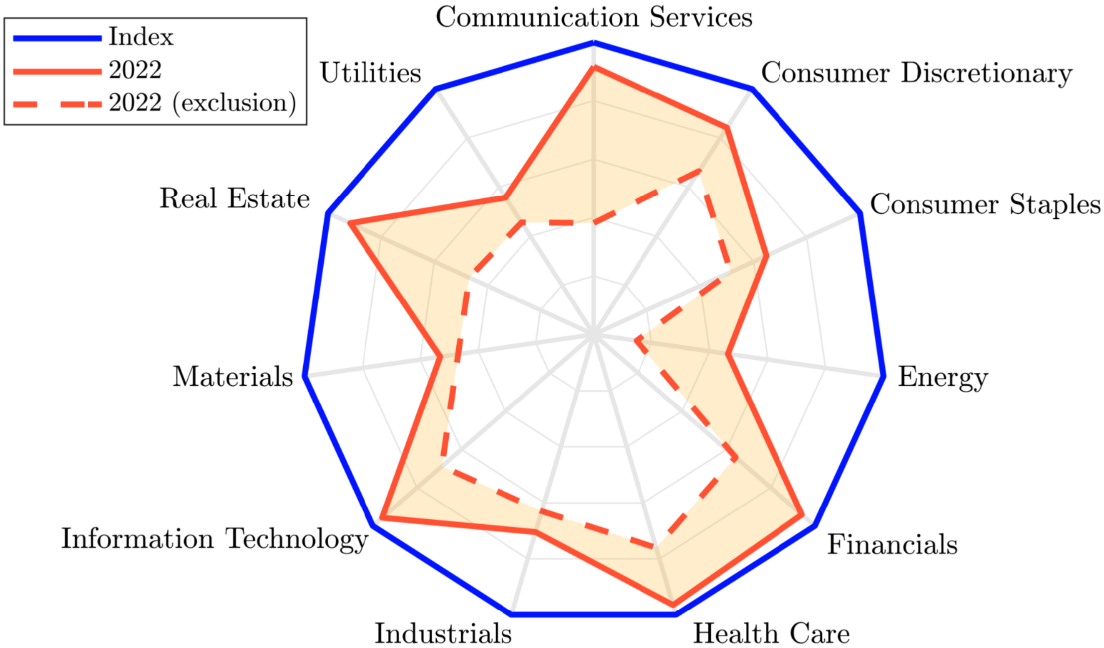
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 70: Radar chart representation of investment universe shrinkage (MSCI EMU, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^\star = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



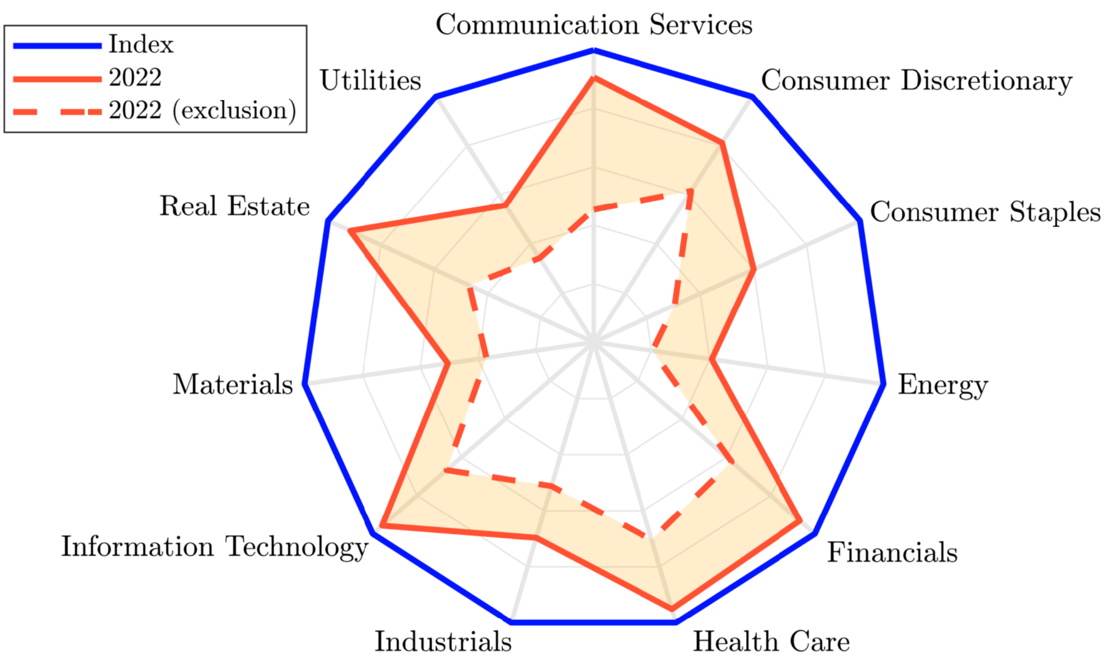
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 71: Impact of the momentum exclusion constraint on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



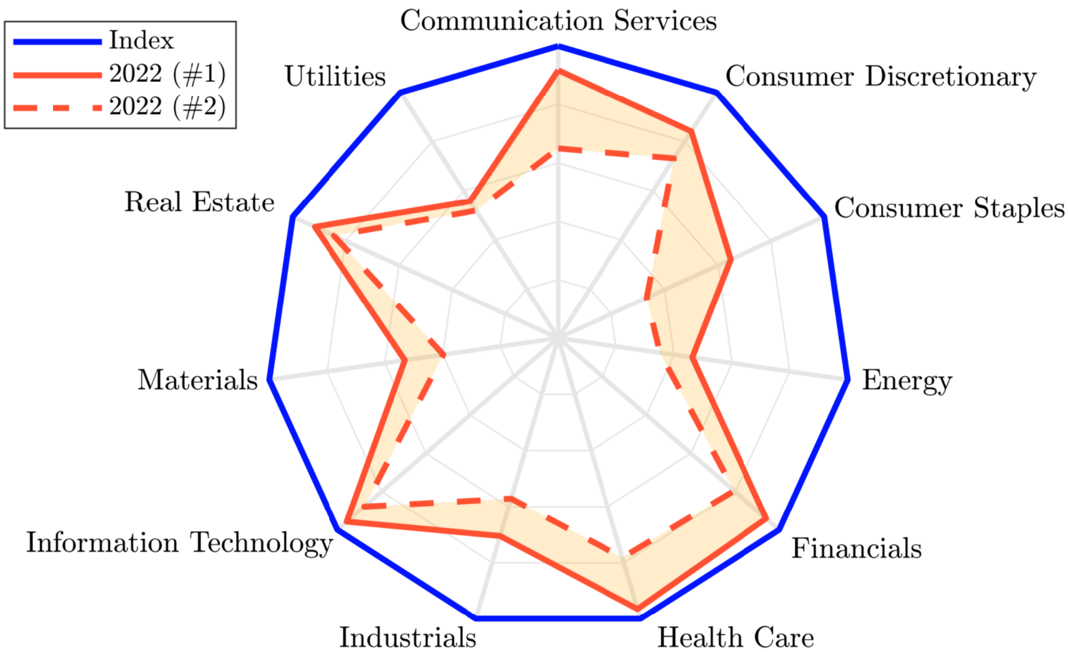
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 72: Impact of the momentum exclusion constraint on the investment universe shrinkage (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



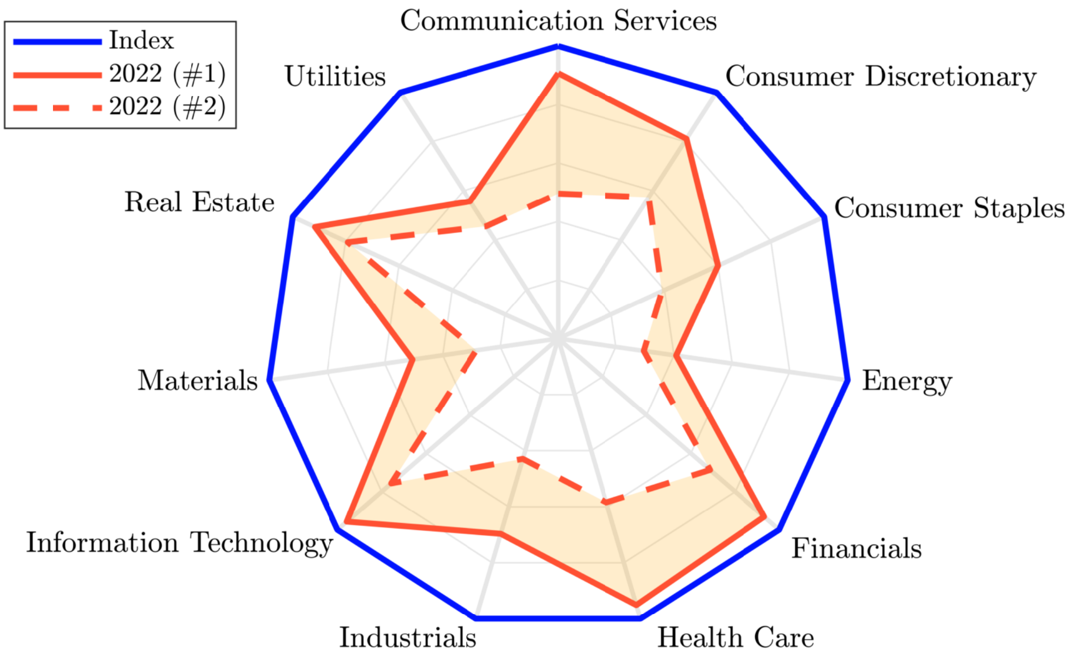
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 73: Case #1: $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$ vs. Case #2: $\mathcal{G} = 200\%$, $\mathcal{CM}^{\star} = -7\%$ (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, PAB, scope $\mathcal{SC}_{1-3}$)



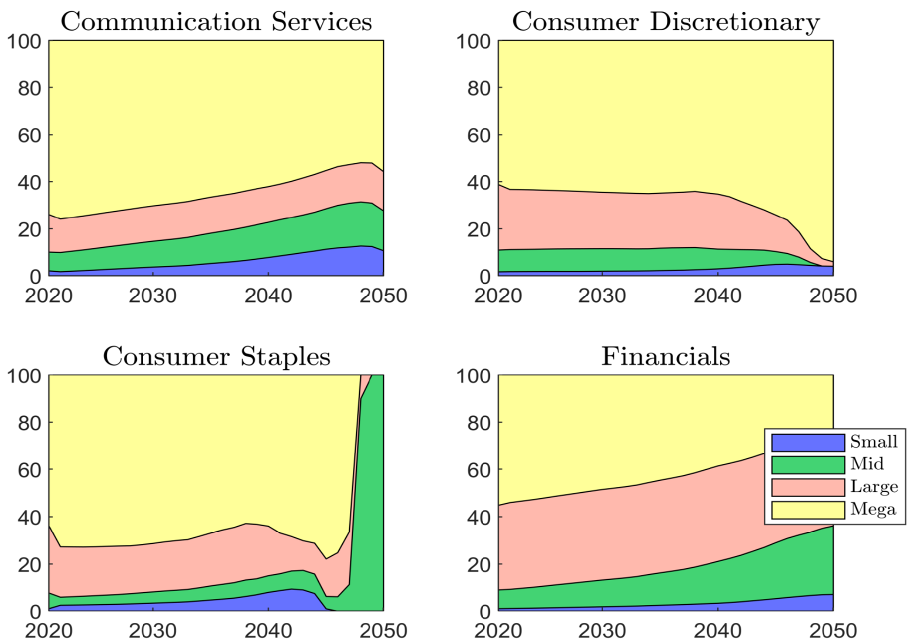
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 74: Case #1: $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$ vs. Case #2: $\mathcal{G} = 200\%$, $\mathcal{CM}^{\star} = -7\%$ (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, PAB, scope $\mathcal{SC}_{1-3}$)



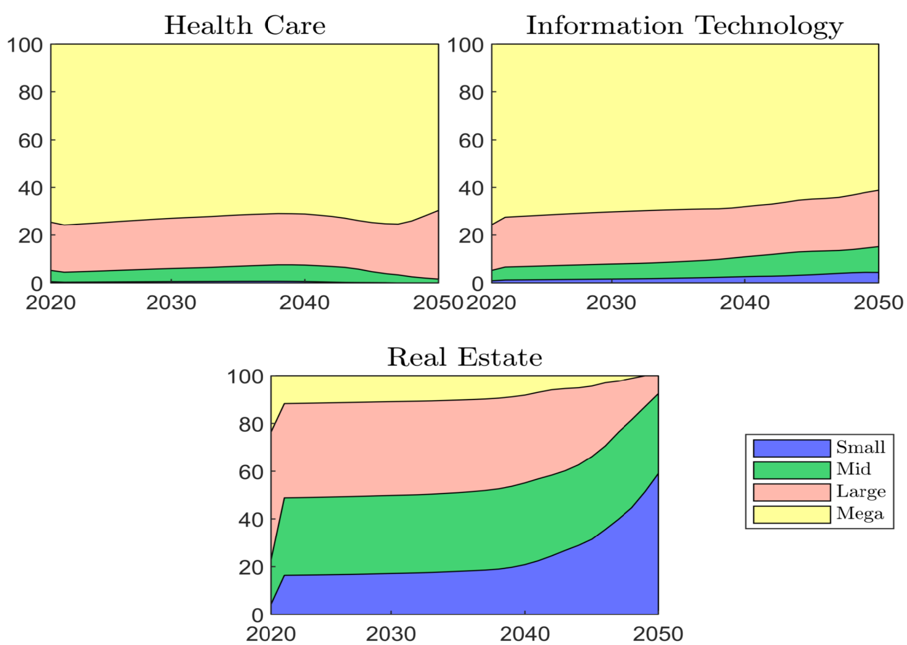
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 75: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



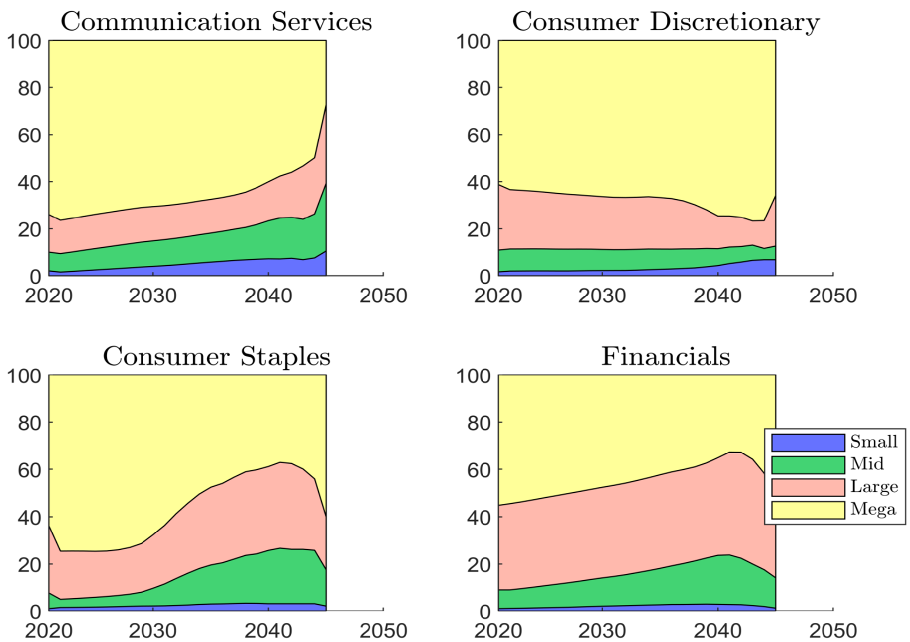
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 76: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_0$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



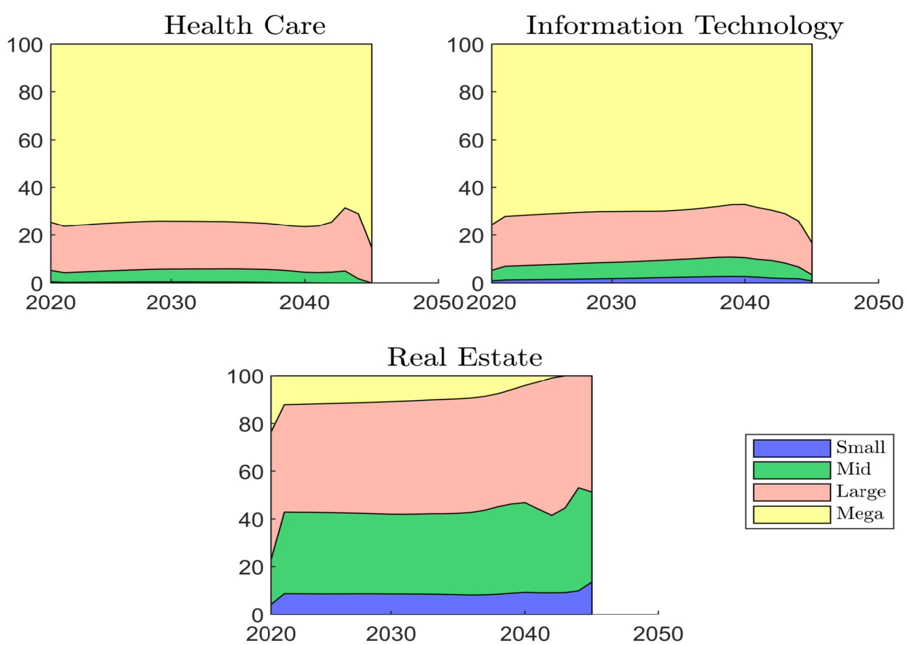
Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 77: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

Figure 78: Breakdown of net zero allocation with respect to the market capitalization (MSCI World, Jun. 2022, $\mathcal{C}_3\,(0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $\mathcal{CM}^{\star} = -5\%$, PAB, scope $\mathcal{SC}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Authors' calculations.

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