Option Pricing under Skewness and Kurtosis using a Cornish Fisher Expansion

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Abstract

This paper revisits the pricing of options, in a context of financial stress, when the underlying asset's returns displays skewness and excess kurtosis. For that purpose, we use a Cornish-Fisher transformation for valuing option contracts with an exact formula allowing for heavy-tails.

An application to the FTSE 100 stock index option contracts during October 2008 provides evidence about the capability of the Cornish-Fisher model to improve calibration and pricing performance during a period of stress.

1 Introduction

Financial markets have been subject to stress periods throughout their history. This raises the question of whether option pricing models have the ability to derive fair contract prices and risk measures in taking such potential stress conditions into account. Although the financial literature on option theory has documented many well-known pervasive features that affect pricing, these are not taken into account in the classic Black-Scholes-Merton framework.

In order to remedy the assumption of a Gaussian marginal distribution for the underlying asset returns in the classical Black-Scholes-Merton model, three principal approaches are proposed in the literature: stochastic volatility models[1], jump-diffusion process for the price dynamics[2], and stochastic volatility with a jump-diffusion process[3]. There are also more general non-Gaussian alternative classes for the underlying asset[4].

Alternatively, others have considered semi-parametric option pricing formulae, particularly when it is not always possible to present the exact distribution in a tractable form; therefore, much effort has been made to approximate the exact distribution. Jarrow and Rudd (1982) model the distribution of stock price with a Edgeworth series expansion. Corrado and Su (1996a) model the distribution of stock log prices with a Gram-Charlier series expansion, while Corrado and Su (1996b) performed the same type of study with an Edgeworth expansion[5]. This method focuses on the skewness and kurtosis deviation from normality for stock

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[1]Scott (1987), Wiggins (1987), Johnson and Shanno (1987), Hull and White (1987, 1988), Stein and Stein (1991), Heston (1993), Nandi (1998), Duan (1995), Heston and Nandi (2000), Hagan, Kumar, Lesniewski and Woodward (2000).

[2]Merton (1976), Bates (1988), Camara and Heston (2008).

[3]Bates (1996), Bakshi, Cao and Chen (1997), Das and Sundaram (1999), Bates (2000), Duffie, Pan and Singleton (2000), Pan (2002), Eraker, Johannes and Polson (2003), Eraker (2004), Duan, Ritchken and Sun (2007).

[4]Madan and Seneta (1990), Madan and Milne (1991), Madan Carr and Chang (1998), Bouchaud, Sornette and Potters (1997), Barndorff-Nielsen (1998), Eberlein, Keller and Prause (1998), Aparicio and Hodges (1998), De Jong and Huisman (2000), Matacz (2000), Corrado (2001), Savickas (2002), Borland (2002), Carr, Geman, Madan and Yor (2002, 2003), Rockinger and Abadir (2003), Kleinert (2004), Dupoyet (2004), Pochard and Bouchaud (2004), Borland and Bouchaud (2004), Carr and Wu (2004), Dutta and Babbel (2005), Sherrick, Garcia and Tirupattur (1996), Albota and Tunaru (2005), Markose and Alentorn (2005), Bakshi, Madan and Panayotov (2008), Aboura, Valeyre and Wagner (2014).

[5]See Barton and Dennis (1952) or Stuart and Ord (1987) for discussion on the distinction between

returns. For clarity, it should be noted that, to correct the bias of the Black-Scholes (1973) model, Corrado and Su (1996a) sum up the Black-Scholes formula with the adjustment terms accounting for non-normal skewness and kurtosis by truncating the expansion after the fourth moment. Under risk-neutral probability, they apply the Gram-Charlier density function to derive European call price formula. The main advantage of Gram-Charlier and Edgeworth expansions is that they allow for additional flexibility over a normal density because they introduce a skewness and kurtosis parameter in the distribution. However, these approaches have noticeable drawbacks. Rockinger and Jondeau (2001) note that, since Gram-Charlier expansions are polynomial approximations, they have the important drawback of yielding negative values for a probability. Actually, it is not guaranteed to be positive, and therefore may violate the domain of validity of the probability distribution. This arises from the fact that the expansions are usually truncated after the fourth power, which may imply negative densities over some interval of their domain of variation (Leon, Mencia and Sentana (2007)), thereby probabilities can be negative for such expansions. This is an undesirable outcome because this situation might occur when the financial markets are in distress, which means that these nearly Gaussian distributions may fail when they are needed most. Therefore, imposing positivity constraints will require the Gram-Charlier approach to be used only for weak departures from normality. Moreover, Blinnikov and Moessner (1998) observe that for strongly non-Gaussian cases, Edgeworth expansion has a small domain of applicability in practical cases, since it diverges like the Gram-Charlier series. Note that Gram-Charlier is itself a particular density expansion of the Edgeworth expansions class.

Despite the large number of published works on the issue of pricing derivatives during stress periods, the idea to consider a weakly non-Gaussian distributions with an exact formula that allows for skewness and excess kurtosis, to our best knowledge, has been so far ignored. Here we aim to address this issue by deriving a weakly-non Gaussian distribution to price option contracts. We believe that pricing option contracts during financial stress periods induces a trade-off between sophistication (to account for marketplace stylized facts) and simplicity (to avoid any type of risk arising from the model). Most of the aforementioned papers have probably underestimated this trade-off, either because the model is too complex and risky

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Edgeworth and Gram-Charlier expansions.

for the business purpose or it includes approximations that might lead to errors. We argue that the Cornish-Fisher expansion addresses this trade-off by considering risk neutral non-normal third and fourth moments without any approximation. Actually, this expansion is a way to transform a standard Gaussian random variable into a non-Gaussian random variable. The transformation needs to be bijective so that the quantiles of the distribution are conserved. However, computing the moments of the distribution resulting from the Cornish-Fisher transformation, although simple in theory, is difficult in practice.

Fortunately, the use of Cornish-Fisher expansion helps with avoiding two main typical pitfalls in these series expansions. First and most important, the area of the domain of validity, expressed in actual skewness and kurtosis, seems to give sufficient room for manoeuvre in most circumstances, which is not necessarily the case for other expansions. This is a major point for avoiding the case of negative probabilities, typical for higher order expansions, when pricing far-from the money option contracts. What can be shown is that the domain of validity of the Cornish-Fisher is much wider than in the Gram-Charlier case. Second, the transformation gives a simple relation between the skewness and kurtosis parameters and the Value-at-Risk or Expected Shortfall measures.

Finally, our formula provides simple and flexible ways to fit a large variety of skewness and kurtosis data. Indeed, it is written in a simple form, following Corrado and Su (1996a,b), where the third and fourth moments are additional parameters that correct the Black-Scholes-Merton formula. This simplifies the costly numerical analysis, as the model is easy to estimate but also simple to interpret.

This research is relevant for three principal reasons. First, it addresses the issue of option pricing with a weakly non-Gaussian model that accounts for explicit skewness and kurtosis having a large domain of validity. Second, it addresses the issue of option pricing in a context of financial stress. To our best knowledge, no existing studies compare their model's performance during stress periods using intra-day data and short-term out-the-money options, which are the major sources of mispricing. In contrast, our database is mainly composed by short-term out-the-money contracts. Third, our model compares the model's performance to a well-known trading rule, that is used as a benchmark, while the vast majority of the existing studies do comparisons with the Black-Scholes (1973) model or its extensions. This

well known trading rule, called 'sticky strike', was popularized by Derman (1999) to manage the smile dynamics. The sticky strike rule models the volatility as remaining constant to a given strike, whatever the underlying asset moves up or down instantaneously. Ciliberti, Bouchaud and Potters (2008) show that the sticky strike rule is exact for small maturities. Surprisingly, there are few studies using this rule for option pricing (Hagan, et al (2002), Daglish, Hull and Suo (2002) and Ciliberti, Bouchaud and Potters (2008)).

The main contribution of this paper is to derive a weakly non-Gaussian European-style option pricing model, allowing for explicit third and fourth moments estimated implicitly from the derivatives market. To our knowledge, no research paper has addressed the issue of pricing option with a Cornish-Fisher transformation.

The paper is organized as follows: Section 2 displays the characteristics of the novel model; Section 3 exposes the empirical results; and Section 4 summarizes and concludes.

2 The Cornish-Fisher option pricing model

2.1 The Cornish-Fisher transformation

The Cornish-Fisher expansion, if properly used (Maillard (2012)), allows the generation of distributions with the desired volatility, skewness and kurtosis. The Cornish-Fisher expansion relies on the polynomial transformation of a normal standard distribution z into a distribution Z:

$$Z = z + (z^2 - 1) \times \frac{s}{6} + (z^3 - 3z) \times \frac{k}{24} - (2z^3 - 5z) \times \frac{s}{36} \tag{1}$$

s and k are parameters which determine skewness and kurtosis, but except for very low values, they do not coincide with effective skewness s^* and kurtosis k^*. The parameters will be computed to achieve the effective skewness and kurtosis:

$$s^* = \frac{M_3}{M_2^{1.5}} \tag{2}$$

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$$k^* = \frac{M_4}{M_2^2} - 3 \tag{3}$$

We compute the moments of the Cornish-Fisher distribution:

$$
\begin{aligned}
M_1 &= 0 \\
M_2 &= 1 + \tfrac{1}{96}k^2 + \tfrac{25}{1296}s^4 - \tfrac{1}{36}ks^2 \\
M_3 &= s - \tfrac{76}{216}s^3 + \tfrac{85}{1296}s^5 + \tfrac{1}{4}ks - \tfrac{13}{144}ks^3 + \tfrac{1}{32}k^2 s \\
M_4 &= 3 + k + \tfrac{7}{16}k^2 + \tfrac{3}{32}k^3 + \tfrac{31}{3072}k^4 - \tfrac{7}{216}s^4 - \tfrac{25}{486}s^6 + \tfrac{21665}{559872}s^8 \\
&\quad - \tfrac{7}{12}ks^2 + \tfrac{113}{452}ks^4 - \tfrac{5155}{46656}ks^6 - \tfrac{7}{24}k^2 s^2 + \tfrac{2455}{20736}k^2 s^4 - \tfrac{65}{1152}k^3 s^2
\end{aligned}
\tag{4}
$$

As Z is non-standard (zero mean but variance slightly different from one), we will use the transformation leading to a new definition of Z by retaining:

$$Z = \frac{z + (z-1) \times \tfrac{s}{6} + (z^3 - 3z) \times \tfrac{k}{24} - (2z^3 - 5z) \times \tfrac{s}{36}}{\sqrt{1 + \tfrac{1}{96}k^2 + \tfrac{25}{1296}s^4 - \tfrac{1}{36}ks^2}} \tag{5}$$

Obviously, this new definition has the same higher moments than the initial transformation.

Recall that both the Cornish-Fisher and Gram-Charlier expansions are means of transforming a Gaussian distribution into a non-Gaussian distribution, the skewness and the kurtosis of which can be controlled if the transformations are properly implemented. Their expansions differ in that the Cornish-Fisher is a transformation of quantiles, whereas the Gram-Charlier is a transformation of a probability density. Both transformations must be implemented with care, as their domain of validity does not cover the whole range of possible skewness and kurtosis coefficients. Figure 1 exhibits the domain of validity of both expansions in the skewness-kurtosis plane. What is most appealing, in the Cornish-Fisher expansion, is that its domain of validity is much wider than in the Gram-Charlier case. This characteristic reveals, unambiguously, how much more realistic the Cornish-Fisher expansion is, as compared to the Gram-Charlier expansion; judging by the magnitude of the skewness-kurtosis domain, which potentially encapsulates most of the tradable assets.
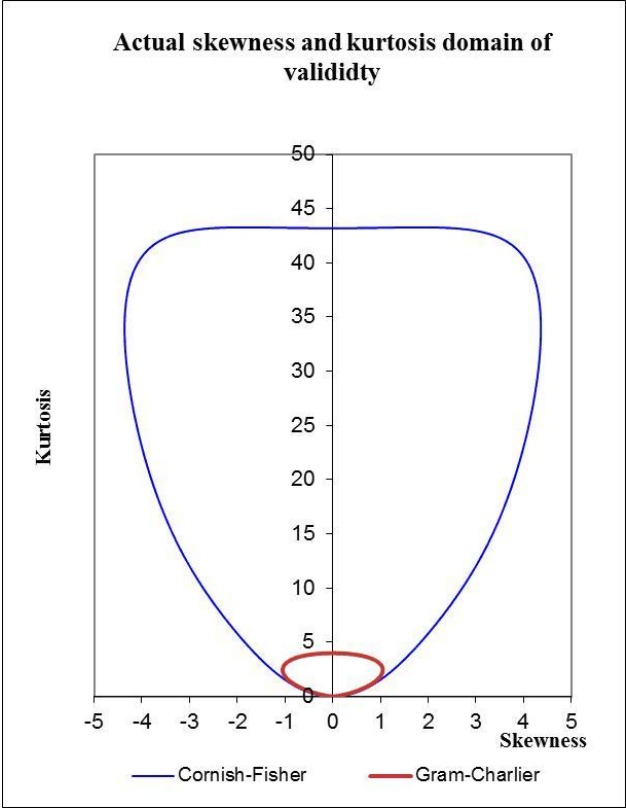
Figure 1: Domain of validity of the Cornish-Fisher and Gram-Charlier models

2.2 The option pricing model

The Cornish-Fisher expansion is a simple transformation of quantiles, but the corresponding probability density is rather complicated. There is currently no way to compute the integrals analytically, despite numerous approaches. We therefore decided to proceed with necessary numerical computations to compute the quantiles. Our computations give the Cornish-Fisher transformation's density of probability:

$$\Phi(Z) = \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{z^2}{2}}}{z^2\left(\frac{k}{8} - \frac{s^2}{6}\right) + z\frac{s}{3} + 1 - \frac{k}{8} + \frac{5s^2}{36}} \tag{6}$$

with:

$$z = \frac{a}{3} + \sqrt[3]{\frac{-q + \frac{Z}{b} + \sqrt{(q - \frac{Z}{b})^2 + \frac{4}{27}p^3}}{2}} + \sqrt[3]{\frac{-q + \frac{Z}{b} - \sqrt{(q - \frac{Z}{b})^2 + \frac{4}{27}p^3}}{2}}$$

$$a = \frac{-s}{\frac{k}{8} - \frac{s^2}{3}}$$

$$b = \frac{k}{24} - \frac{s^2}{18}$$

$$p = \frac{1 - \frac{k}{8} + \frac{5s^2}{36}}{\frac{k}{24} - \frac{s^2}{18}} - \frac{1}{3}\frac{\frac{s^2}{36}}{(\frac{k}{24} - \frac{s^2}{18})^2}$$

$$q = \frac{-s}{\frac{k}{8} - \frac{s^2}{3}} - \frac{1}{18}\frac{s(1 - \frac{k}{8} + \frac{5s^2}{36})}{(\frac{k}{24} - \frac{s^2}{18})^2} - \frac{2}{27}\frac{\frac{s^3}{216}}{(\frac{k}{24} - \frac{s^2}{18})^3}$$

We use the weakly non-Gaussian probability density $\Phi(.)$, derived from the Cornish-Fisher expansion, to control for skewness and excess kurtosis. $\Phi(Z)$ is the density evaluated on the random variable Z. Assuming risk-neutrality, we derive theoretical European call option formula as the value of the expected payoff, discounted by the risk-free rate r_f at the contract's time to maturity $(T - t)$. For a call option C_{CF} with strike K and underlying asset price S_T at maturity T, the value will be given by:

$$C_{CF} = e^{-r_f(T-t)} \int_{-\infty}^{+\infty} (S_T - K)^+ \Phi(Z) d(Z) \tag{7}$$

If the skewness and the excess kurtosis are zero, this model reduces to the Black-Scholes (1973) model with $C_{CF} = C_{BS}$ when $s^* = 0$ and $k^* = 0$.

To illustrate the model application, we attempt to reproduce smile curves as a function of

kurtosis and skewness. The model allows to compute an option price depending on maturity, risk-free interest rate, volatility and skewness and kurtosis. To do so, we infer the implied volatility for various levels of strike prices. The computations are made for a call option with one-month maturity, a 2% risk-free rate and a 20% volatility.

Figure 2 reproduces volatility smiles according to various levels of kurtosis and skewness. First, the deeper is the kurtosis, the higher is the volatility curvature. Second, the impact of skewness leads to a rotation in the smile where a negative skewness induces a negative slope.
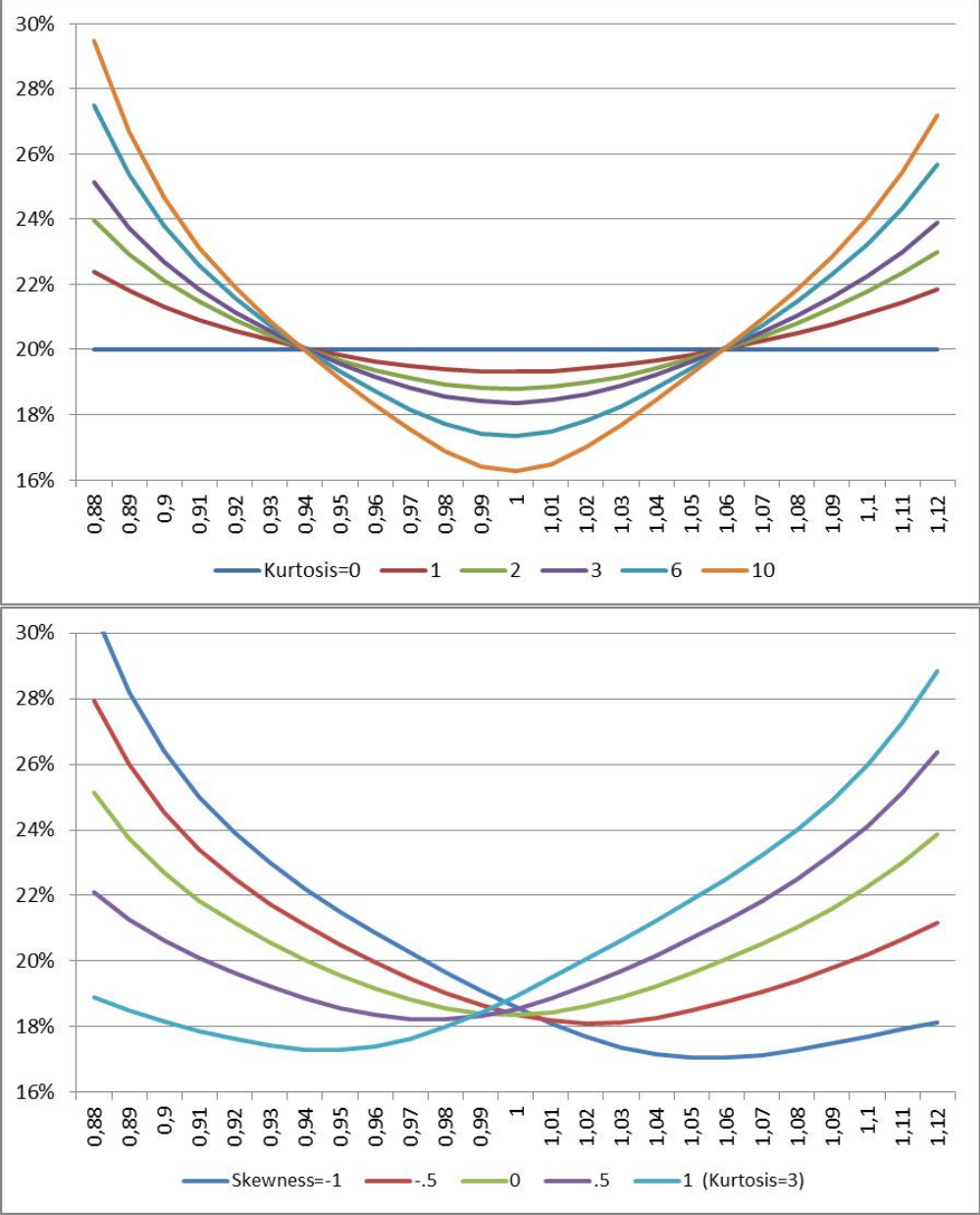
Figure 2: Volatility smiles according to kurtosis and skewness

3 The empirical results

3.1 The dataset

The data set[6] consists of European option contracts on the FTSE 100 stock index during October 2008. The data set contains 364 intra-daily observations from October 1 to October 31. Bid-ask spreads are used in the study for quality measure. This month is critical since it includes the highest volatility peaks of the 2008 financial crisis. It therefore allows for testing our model on one of the most financially distressed periods, contrary to the approach of the vast majority of research papers. Three types of option contracts are discarded. First, only the two shortest maturities are used due to computational time consideration, with a maximum time horizon of three months. The risk-free interest rates are the two week Libor rates. Second, only near-the-money and out-the-money prices are used, since out-the-money contracts are more liquid than in-the-money contracts. Third, in order to have only significant option prices, we retain at each strike, the lowest spread identified from 16:00 to 16:04. Figure 3 plots the volatility smile as a function of strike price K during October 2008.

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[6]We would like to thank Sebastien Valeyre for having provided us with this data set that comes from Liffe-Nyse-Euronext.
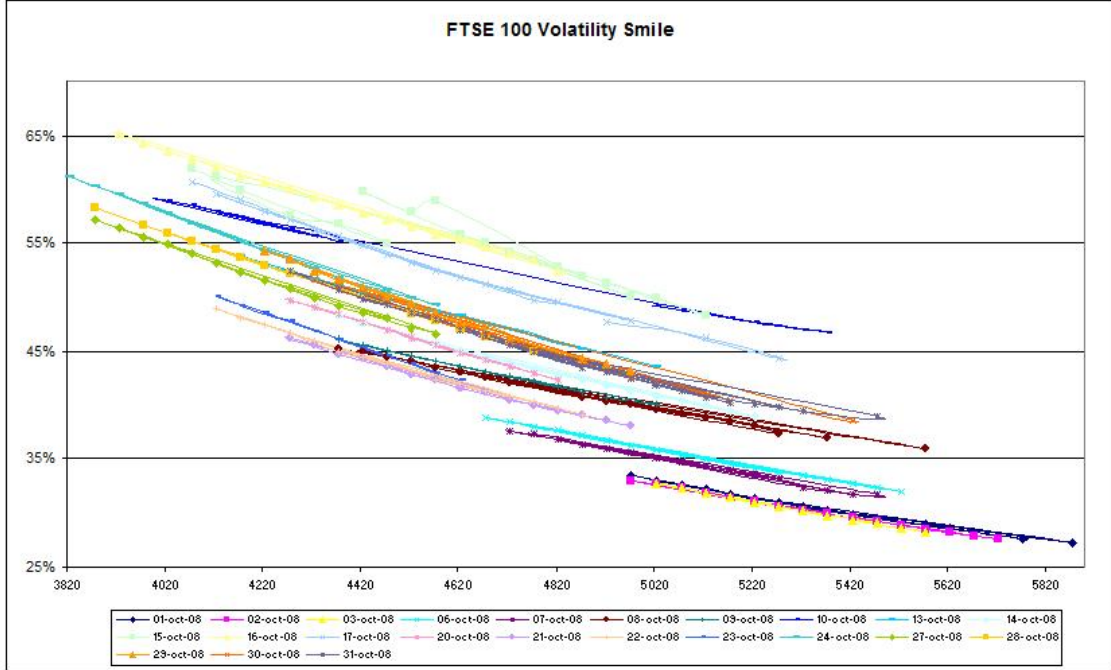
Figure 3: Volatility smiles in October 2008

3.2 The model estimation

We consider a minimizing procedure for the model estimation. The first step corresponds to the in-the-sample model calibration, while the second step corresponds to the out-the-sample valuation. For the in-the-sample procedure, the model is calibrated on the intra-daily data by minimizing the following sum of squared errors:

$$\min_{(\sigma, s^*, k^*)} \left(\sum_{K=1}^{N_t} e_{K,t}^2 \right) \tag{8}$$

With $e_{K,t}$ representing the normalized difference $\left(\frac{C_{Market} - C_{CF}}{S_t} \right)$ between the market call option prices C_{Market} and the theoretical Cornish-Fisher call option prices C_{CF} for the Kth strike price with $K = 1, ..., N_T$ and the underlying stock index price S_t at date t. The estimation procedure is implemented on the set of parameters (σ, s^*, k^*) that is estimated implicitly by the quadratic minimization procedure, yielding the implied volatility, the implied skewness, and the implied kurtosis, respectively.

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To assess the differences between market prices and model prices for an out-the-sample fit, we compute the mean price forecast error to quantify the error magnitude.

$$\sum_{K=1}^{N_t} e_{K,t}^2 \tag{9}$$

With $e_{K,t}$ again representing the normalized difference between the market call option prices and the call option prices computed by the Cornish-Fisher model for the Kth strike price with $K = 1, ..., N_T$ and the underlying stock index price S_t at date t. The set of estimated parameters (σ, s^*, k^*) are computed from the in-the-sample calibration and remain constant for the one-day ahead out-the-sample pricing; this assumption is reasonable since these parameters are relatively stable for short horizons. Hence, during the out-the-sample procedure, we re-compute option prices of the current day using the previous day's implied volatility, implied skewness, and implied kurtosis; the interest rate r is set constant for the period. Therefore, only the underlying stock index price S_t and the time to maturity $(T-t)$ change.

3.3 The empirical findings

Table 1 summarizes the model calibration and pricing on FTSE 100 call options for the month of October 2008. The choice of this month is motivated by the highest concentration of volatility peaks during the 2008 financial crisis. The option mispricing overall affects short-term contracts and out-the-money contracts. For that reason, it is relevant to test the Cornish-Fisher model on the October 2008 month options, which consist of short-term (less then 3 months) and generally deep-out-the money contracts (average moneyness defined by (S/K) is 0.90). Therefore, we posit that the Cornish-Fisher model is able to fairly price the option contracts characterized by underlying asset prices that are weakly non-Gaussian.

Date	σ	s*	k*	(T-t)	Average Call Price	Average Stock Index Price	Number of Calls	Calibration	Pricing
01/10/2008	42.47%	-2.38	12.55	0.22	142.17	4905.38	16	2.6E-05	
02/10/2008	32.75%	-0.37	1.23	0.22	139.36	4899.96	16	1.1E-04	1.5E-05
03/10/2008	37.61%	-1.74	7.34	0.22	179.15	4991.02	12	2.7E-05	5.9E-05
06/10/2008	42.68%	-1.18	7.68	0.21	159.01	4634.92	18	1.8E-04	2.0E-03
07/10/2008	46.63%	-2.06	10.03	0.21	148.27	4636.69	16	2.3E-05	2.2E-04
08/10/2008	55.85%	-2.42	11.07	0.20	161.23	4339.39	21	2.2E-04	1.9E-03
09/10/2008	52.60%	-1.96	8.48	0.20	212.62	4320.57	13	4.5E-05	1.3E-04
10/10/2008	66.66%	-2.22	9.07	0.20	213.75	3966.98	13	2.2E-04	4.9E-03
13/10/2008	58.48%	-1.84	7.27	0.19	260.10	4176.22	12	9.3E-06	1.3E-03
14/10/2008	54.39%	-2.03	7.69	0.19	164.34	4327.30	16	1.2E-04	7.5E-04
15/10/2008	69.73%	-1.72	6.61	0.18	170.29	3910.38	19	4.1E-04	9.3E-03
16/10/2008	66.81%	-1.31	7.24	0.18	208.00	3824.33	19	3.4E-04	1.0E-05
17/10/2008	71.89%	-2.40	16.64	0.18	198.64	4040.86	20	5.0E-04	2.4E-03
20/10/2008	52.92%	-1.60	7.34	0.17	202.35	4204.29	12	9.6E-05	3.2E-03
21/10/2008	53.86%	-1.75	5.93	0.17	172.00	4222.33	15	1.8E-04	1.5E-04
22/10/2008	53.64%	-1.63	4.92	0.16	150.70	4059.54	16	1.4E-05	1.2E-05
23/10/2008	58.71%	-2.00	10.77	0.16	195.09	4069.80	11	3.4E-05	3.4E-06
24/10/2008	68.33%	-1.99	8.43	0.16	197.75	3795.29	16	5.6E-05	2.6E-03
27/10/2008	59.18%	-1.60	7.37	0.15	185.38	3844.28	15	1.4E-04	7.2E-04
28/10/2008	59.28%	-1.24	6.50	0.15	165.62	3834.02	13	2.2E-04	2.8E-04
29/10/2008	59.15%	-1.56	7.63	0.15	173.95	4152.81	14	9.7E-05	4.2E-04
30/10/2008	54.09%	-1.28	4.78	0.14	140.59	4270.70	19	9.6E-05	5.1E-04
31/10/2008	50.40%	-1.31	2.75	0.14	128.68	4295.62	22	2.9E-04	1.5E-04
Average CF	**55.14%**	**-1.72**	**7.80**	**0.18**	**176.91**	**4248.81**	**15.83**	**1.5E-04**	**1.4E-03**
Average CF/SS	**120.85%**							**49.42%**	**76.59%**

Table 1: Cornish-Fischer Model Calibration and Pricing

The in-the-sample fit (see column "Calibration") shows that the sum of squared errors made by the Cornish-Fisher model (49.42%) is only half of the errors made by the sticky strike model. This signifies that the in-the-sample fit brought by the weakly non-Gaussian model adheres fairly well to the market data during this turbulent period. The empirical results reveal that the Cornish-Fisher average implied volatility is 55.14%, which is 20% higher than the sticky strike implied volatility for the same period. The Cornish-Fisher implied volatility ranges from 32.75% to 71.89%, which represents a variation of as much as 100% in only eleven trading days. The volatility peak of 71.89% occurred on October 17, after a stock index decline from 4,327.30 (October 14) to 3,824.33 (October 16), followed by an increase to 4,204.29 (October 20).

The Cornish-Fisher average implied skewness s^* is -1.72 (ranging from -2.42 to -0.37), while the average implied kurtosis k^* is 7.80 (ranging from 1.23 to 16.64). Under these circumstances, a larger domain of validity is paramount since it allows the model to capture strong market swings that affect these three structural parameters (σ, s, k) within a short interval of time.
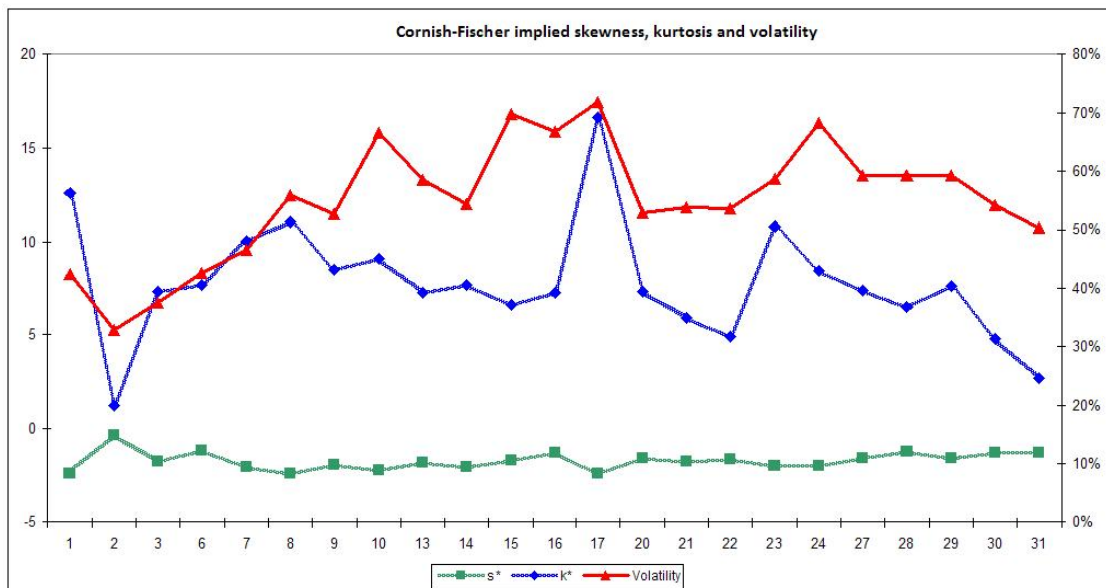
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Figure 4: Cornish-Fischer model calibration

Figure 4 illustrates the ranges of values for the three parameters estimated during October 2008. Given the volatile nature of this month, the implied skewness and kurtosis may appear relatively stable in general.

To assess the out-of-sample forecasting (see column "Pricing") performance of the Cornish-Fisher model, we set constant the implied volatility σ_{t-1}, implied skewness s_{t-1}, and implied kurtosis k_{t-1} calibrated from the previous trading day $t-1$, in order to price option contracts for the day t. It appears that the sum of squared errors made by the Cornish-Fisher model represent two thirds (76.59%) of the error made by the sticky strike model.

Overall, we conclude that the Cornish-Fisher model has improved calibration and pricing accuracy during the most volatile month of the 2008 financial crisis. This can be explained that this weakly non-Gaussian model has a larger domain of validity than comparable models (Gram-Charlier, Edgeworth etc.) and is easier to estimate and more stable than the stochastic volatility models. The day of October 17 represents the biggest stock index price swing of the period, characterized by the highest volatility (71.89%), the highest kurtosis (16.64), the

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second highest negative skewness (-2.40), and the highest in-the-sample fit error (5.0e-04); the highest out-the-sample fit error (9.3e-03) occurred immediately prior, on October 15. The model's capability to adhere to the data is fairly good, although it has been disturbed by the sudden regime change of the underlying stock index from October 15 to October 20. Note that a possible improvement would have been to extend the empirical test to a larger set of data.

4   Conclusion

This paper derives a new option pricing model based on a weakly non-Gaussian risk-neutral probability density. This density relies on a Cornish-Fisher transformation with an exact formula allowing for heavy-tails in the presence of non-normal skewness and kurtosis. An in and out-the-sample analysis is carried out on intra-day data from the FTSE 100 stock index options during October 2008, which was the most volatile month of the 2008 financial crisis. We conclude that the Cornish-Fisher model has improved calibration and pricing performance, as comparison to the sticky strike model. This improvement is due to its larger domain of validity. It is worth emphasizing that this weakly non-Gaussian model is easier to implement than the stochastic volatility models or other alternative classes of stable non-Gaussian models. This work can be applied to large data sets or extended to risk management measures, such as VaR and Expected Shortfall.

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