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Systematic Longevity Risk: To Bear or to Insure?

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Systematic Longevity Risk:

To Bear or to Insure?

Abstract

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We compare two contracts for managing systematic longevity risk in retirement: a collective arrangement that distributes the risk among participants, and a market-provided annuity contract. We evaluate the contracts' appeal with respect to the retiree's welfare, and the viability of the market solution through the financial reward to the annuity provider's equityholders.

We find that individuals prefer to bear the risk under a collective arrangement than to insure it with a life insurers'annuity contract subject to insolvency risk (albeit small). Under realistic capital provision hypotheses, the annuity provider is incapable of adequately compensating its equityholders for bearing systematic longevity risk.

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1 Introduction

Systematic longevity risk is a looming threat to pension systems worldwide. In contrast to idiosyncratic longevity risk, which is the risk surrounding an individual's actual date of death given known survival probabilities, systematic longevity risk concerns the misestimation of future survival probabilities.[1] The persistent trend towards improved life expectancy and the increased uncertainty in mortality developments make it clear that systematic longevity risk can be distressful for retirement financing, especially since longevity-linked assets are not yet commonplace (Tan et al., 2015).

The global transition of funded pensions from Defined Benefit (DB) to Defined Contribution (DC) plans[2] precipitates the need for sustainable means of managing systematic and idiosyncratic longevity risks, which has conventionally been borne by the DB plan sponsor. With the decline of DB pension funds, individuals are given more freedom to manage their retirement capital and rely more on insurers to provide longevity protection. However, insurance companies' ability to fulfill that role on a large scale is questionable. Long-dated guarantees in life annuities are difficult to price and hedge (Koijen and Yogo, 2017). In the 1980s and 1990s, a number of life insurance companies defaulted (e.g., First Executive Corporation in the US, Nissan Mutual Life in Japan). In June 2009, the Hartford Group was bailed out under the US government's Troubled Asset Relief Program after incurring significant losses on life annuity products. For individuals, insurers' insolvency risk has serious consequences. While the optimal, rational individual response to idiosyncratic longevity risk in a frictionless setting is to insure it through pooling (Yaari, 1965; Davidoff et al., 2005; Reichling and Smetters, 2015), the corresponding response to systematic longevity risk is less evident.

We consider two ways to manage systematic longevity risk. The first resembles pension plans in the Netherlands, whereby individuals bear systematic longevity risk under a collective arrangement but pool idiosyncratic longevity risk. This financial contract is similar to Group Self-Annuitization (GSA) introduced by Piggott et al. (2005). The second possibility is for individuals to offload the risk at a cost by purchasing an annuity contract from an equity-backed insurance company. Both options allow individuals to pool idiosyncratic longevity risk, but entail different implications with regard to systematic longevity risk. We compare these arrangements to ascertain the option that maximizes individuals' expected utility. We also investigate the viability of the annuity market by evaluating the risk-return

[1] Systematic and idiosyncratic longevity risks are also referred to as macro- and micro-longevity risks respectively.

[2] In 1975, close to 70% of all US retirement assets were in DB plans. In 2015, DB assets accounted for only 33% of total retirement assets. Over the same period, assets in DC plans and Individual Retirement Accounts (IRAs) grew from 20% to 59% (Investment Company Institute, 2016). In the UK, 98% of the FTSE 350 companies offer a DC pension plan in 2017 (Towers Watson, 2017).

tradeoff with respect to systematic longevity risk for the equityholders of the annuity contract provider.

A number of scholars have examined the appeal to retirees of participating contracts, under which individuals bear systematic risks collectively but pool idiosyncratic ones (see for example Hanewald et al., 2013; Maurer et al., 2013). The main novelty of our work is to concurrently model individual preferences and the business of an equity-backed annuity provider when systematic longevity risk exists. Despite equityholders' critical role in the provision of contracts, comparisons of the GSA and annuity contracts that include systematic longevity risk disregard this aspect, by either exogenously setting a default rate, or putting a loading on the contract that eliminates default risk (e.g., Denuit et al., 2011; Richter and Weber, 2011; Maurer et al., 2013; Qiao and Sherris, 2013).[3] This approach is incompatible with the fact that, in practice, insurers' insolvency risk is non-zero and determines individuals' willingness to pay for insurance contracts (Zimmer et al., 2009).

In our setting, to credibly offer insurance against a systematic risk, the annuity provider requires reserve capital that is constituted either from equity contribution, and/or from contract loading to absorb unexpected shocks.[4] Reserve cushioning has a cost. If the annuity provider solicits capital from equityholders, then it would have to compensate them with a systematic longevity risk premium. If the provider charges too high a loading, then individuals would prefer the GSA over the annuity contract (e.g., Hanewald et al., 2013; Boyle et al., 2015).[5] Therefore, the existence of an annuity market hinges on the provider's ability to set a contract price such that all stakeholders are willing to participate in the market. Previous estimates on individuals' willingness to pay to insure against systematic longevity risk are low. Individuals are willing to offer a premium of between 0.75% (Weale and van de Ven, 2016) to 1% (Maurer et al., 2013) for an annuity contract that insures them against systematic longevity risk, and has no default risk. In contrast, the capital buffer that the annuity provider would have to possess to restrain its default risk is much larger. To limit the default rate to 1%, the necessary buffer is around 18% of the

[3]We differentiate our work from analyses that incorporate only idiosyncratic longevity risk (Stamos, 2008; Donnelly et al., 2013; Milevsky and Salisbury, 2015).

[4]It would be equivalent to consider debt issuance to raise capital, and any dividend policy other than a one-off dividend payment to equityholders (i.e., any gains before the end of the investment horizon are re-invested). This is because the Miller-Modigliani propositions on the irrelevance of capital structure (Modigliani and Miller, 1958) and dividend policy (Miller and Modigliani, 1961) on the market value of firms hold in our setup, which excludes taxes, bankruptcy costs, agency costs, and asymmetric information.

[5]While allocating retirement wealth between the annuity contract and the collective scheme is conceptually appealing, for the feasibility of a collective scheme, individuals can select only an option in our setting (e.g., mandatory participation in a collective scheme averts adverse selection, achieves cost reduction, etc., Bovenberg et al., 2007). Weinert and Gründl (2017) analyze the optimal share of a default-free nominal annuity and a tontine, a type of collective scheme, whereas Zhang and Li (2017) investigate a contract that is partially-indexed to longevity risk, that similarly explores a risk-sharing spectrum between the contract provider and the individual.

contract's best estimate value (Maurer et al., 2013). These figures suggest that the annuity provider has little capacity to compose its reserve capital only from contract loading, as is commonly assumed (Richter and Weber, 2011; Maurer et al., 2013; Boyle et al., 2015). Equity capital is thus necessary. We attempt to reconcile the gap between the maximum loading that individuals are willing to pay, and the minimum capital necessary to provide annuity contracts that individuals are willing to purchase, by introducing equityholders.

While analyses that incorporate both policy and equityholders exist in insurance (e.g., Filipović et al., 2015; Chen and Hieber, 2016), they are unforeseen in the literature on the comparison of the GSA with annuity contracts, which focuses on policyholders only. An exception is Blackburn et al. (2017), who take the equityholders' viewpoint when investigating longevity risk management and the share value of a life annuity provider. Demand for annuities in their model is determined by an exogenous demand function. Instead, we analyze the policyholders and equityholders concurrently when annuity demand is endogenous.

Consistent with the inchoate market for longevity-hedging instruments, we assume that the annuity provider has no particular advantage in bearing systematic longevity risk.[6] Moreover, the annuity provider is required to maintain the value of its assets above the value of its liabilities–a plausible regulatory requirement for such a for-profit entity. In contrast to the literature on collective schemes, which largely focuses on inter-generational risk-sharing (e.g., issues concerning its fairness and stability with respect to the age groups, see Gollier, 2008; Cui et al., 2011; Beetsma et al., 2012; Chen et al., 2017, 2016), we focus instead on risk-sharing between individuals and the annuity provider's equityholders within a generation.

We begin by assuming that the annuity provider composes its buffer entirely from equity capital. In return for their capital contribution, equityholders receive the annuity provider's terminal wealth as a lump sum dividend. Due to equity-capital-cushioning, the annuity contract provides retirement benefits that have a lower standard deviation across scenarios. However, as equity capital is finite, there is a positive (albeit small) probability that the annuity provider defaults. We assess whether individuals are willing to pay for an annuity that adequately compensates equityholders for bearing systematic longevity risk, when individuals have the option to form a collective scheme.

We find that individuals marginally prefer the collective scheme. The Certainty Equivalent Loading (CEL), i.e., the level of loading on the annuity contract

[6]Insurance companies may in practice have a comparative advantage in bearing systematic longevity risk, such as relying on the synergy of product offerings in terms of risk-hedging (Tsai et al., 2010), or the potential of life insurance sales in hedging systematic longevity risk (i.e., natural hedging) (Cox and Lin, 2007; Luciano et al., 2015).

at which individuals would derive the same expected utility under either option, is slightly negative (i.e., -0.35% to -0.052%; Table 3). Furthermore, exposure to systematic longevity risk does not enhance the equityholders' risk-return tradeoff if the annuity provider sells zero-loading contracts, because it yields only half of the Sharpe ratio of an identical investment without exposure to systematic longevity risk, as well as a negative Jensen's alpha (Table 4). Consequently, the annuity contract would not co-exist with the collective scheme. The implication of our results would be even stronger if there were frictional costs, e.g., financial distress, agency, regulatory capital, and double taxation costs, because equityholders would require a higher financial return from the capital they provide.

To further comprehend the tradeoff that an individual faces when selecting a contract, we carry out sensitivity tests with respect to the individual's characteristics, systematic longevity risk, and the annuity provider's default risk. The annuity provider's default risk is the main determinant for the individual's preference for the annuity contract. For features that do not affect default rates materially, such as the deferral period, stock exposure, and parameter uncertainty surrounding the longevity model's time trend, the baseline results stay the same. For features that affect default rates, such as higher standard deviation of the longevity model's time trend, lower equity capital, or an alternate longevity model that presents higher uncertainty of survivorship in the old age, then individuals prefer the collective scheme much more when default risk increases, regardless of the underlying driver of default. Only when we assume substantial uncertainty concerning longevity trends (as in the Cairns et al., 2006 longevity model) and at the same time impose exogenously no default risk, do individuals exhibit a preference for the annuity contract. Yet, the individual's willingness to pay remains insufficient to entice equityholders to contribute the required amount of capital that would enable the provision of the contracts.

We present our model in Section 2 and calibrate it in Section 3. We first discuss the baseline case results from the individual's perspective (Section 4), then from the equityholders' point of view (Section 5). Section 6 is devoted to sensitivity tests on the individual's traits, stock exposure, the annuity provider's leverage ratio, as well as the longevity model's attributes. We conclude in Section 7.

2 Model Presentation

We devise a model to investigate the welfare of individuals under a collective retirement scheme and a market-provided deferred variable annuity contract. The setting comprises a financial market with a constant risk-free rate and stochastic stock index, homogenous individuals with stochastic life expectancies, and two contracts for retirement.[7] We define and discuss these elements in detail in this section.

2.1 Financial Market

In a continuous-time financial market, the investor is assumed to be able to invest in a money market account and a risky stock index. The financial market is incomplete due to the lack of longevity-linked securities. We assume that annual returns to the risk-free asset are constant, r. The money market account is fully invested in the risk-free asset.

The value of the stock index at time t, which is denoted by S_t, follows the diffusion process, $dS_t = S_t\,(r + \lambda_S \sigma_S)\,\mathrm{d}t + S_t \sigma_S\,\mathrm{d}Z_{S,t}$. Z_S is a standard Brownian motion with respect to the physical probability measure, σ_S is the instantaneous stock price volatility, and $\lambda_S \sigma_S$ is the constant stock risk premium.

2.2 Individuals

At time t_0, individuals who are aged $x = 25$ either form a collective scheme or purchase a deferred annuity contract with a lump sum capital that is normalized to one. Both retirement contracts commence retirement benefit payments at age 66, up to the maximum age of 95, conditional on the individual's survival. Individuals' lifespan is determined by survival probabilities that follow the Lee and Carter (1992) model.

We set the maximum age at 95 because there are a small number of survivors beyond that. For example, in 2015, life expectancy for the US population was 78.8 years. A 65-year-old American can expect to live to around age 80 (Xu et al., 2016). The small pool of survivors at high ages amplifies changes in the GSA funding ratio from changes in the number of survivors, subsequently generating extreme benefit adjustments. To enhance the feasibility of a closed GSA that produces no extreme benefits near the end-of-life, in addition to avert the reliability of mortality statistics for high ages that give rise to different risk profiles of deep-deferred and immediate annuity products (Ji and Zhou, 2017), we consider 95 to be the maximum age.[8] The baseline case results do not materially change when the maximum age is extended to 100, but a substantially larger number of replications is necessary to achieve the same accuracy.

[7] We abstract from model uncertainty by assuming that the stochastic dynamics underlying the financial assets and life expectancies are known.

[8] Regardless of their choice for a DVA or a GSA, individuals could in practice purchase a DVA contract starting at age 95. Their utility from consumption after 95 would thus be identical.

2.2.1 Longevity Risk Model

We assume that individual mortality rates evolve independently from the financial market. Although productive capital falls as the population ages, empirical evidence on the link between demographic structure and asset prices is mixed.[9]

We adopt the Lee and Carter (1992) model, which is widely used (e.g., by the US Census Bureau and the US Social Security Administration) and studied. This is a one-factor statistical model for long-run forecasts of age-specific mortality rates. It relies on time-series methods and is fitted to historical data. The log central death rate for an individual of age x in year t, $\log(m_{x,t})$[10] is assumed to linearly depend on an age-specific constant, and an unobserved period-specific intensity index, k_t:

$$\log(m_{x,t}) \quad = \quad a_x + b_x k_t + \varepsilon_{x,t} \tag{1}$$

$\exp(a_x)$ is the general shape of the mortality schedule across age; b_x is the rate of change of the log central death rates in response to changes in k_t, whereas the error term, $\varepsilon_{x,t}$, is normally distributed with zero mean and variance σ_x^2.

The Lee and Carter (1992) model is defined for the central death rates, $m_{x,t}$, but we apply it to model the annual rate of mortality, $q_{x,t}$ by the approximation $q_{x,t} \simeq 1 - \exp(-m_{x,t})$. The probability that someone who is aged x at time t_0 is alive in s-year time, ${}_sp_x$, is then ${}_sp_x = \Pi_{l=0}^{s-1}(1 - q_{x+l,t+l})$. We denote the conditional probability in year $t \geq t_0$ that an individual of age x at time t will survive for at least s more years as ${}_sp_x^{(t)}$, ${}_sp_x^{(t)} = \Pi_{l=0}^{s-1}(1 - q_{x+l,t}) = \exp\left(\sum_{l=0}^{s-1} -m_{x+l,t}\right)$.[11]

While many refinements of Lee and Carter (1992) exist (e.g., the two-factor model of Cairns et al., 2006, the addition of cohort effects in Renshaw and Haberman, 2006), the model is not only reasonably robust to the historical data used, but also produces plausible forecasts that are similar to those from extensions of the model (Cairns et al., 2011).

2.2.2 Welfare

Individuals maximize expected utility in retirement.[12] Benefits from the retirement contracts constitute the individual's only source of income. We consider

[9] Erb et al. (1994); Poterba (2001); Ang and Maddaloni (2003); Visco (2006); Schich (2008); Arnott and Chaves (2012)

[10] $m_{x,t}$ is the ratio of $D_{x,t}$, the number of deaths of an individual aged x in year t, over $E_{x,t}$, the exposure, defined as the number of aged x individuals who were living in year t.

[11] This is an exponentiated finite sum of log-normal random variables that has no known analytical distribution function. Therefore, we resort to simulation for our analysis. Alternate ways to proceed include quantile estimation of random survival probabilities in Denuit et al., 2011, or the Taylor series approximation by Dowd et al. (2011).

[12] We can ignore bequest motives as both contracts provide income only when the individual is still alive.

individuals who exhibit Constant Relative Risk Aversion (CRRA), and evaluate their utility in retirement by Equation (2).

$$U(\Xi) \quad = \quad \int_{t_R}^{T} \mathrm{e}^{-\beta(\mathrm{t}-\mathrm{t}_0)} \frac{\Xi_t^{1-\gamma}}{1-\gamma} {}_{t-t_0}p_{25}\,\mathrm{d}t \tag{2}$$

${}_{t-t_0}p_{25}$ is the probability that someone who is 25 years old in year t_0 is alive in year $t \geq t_0$. β is the subjective discount factor $\gamma > 1$ is the risk aversion parameter. Ξ_t is the retirement income in year t, t_R is the retirement year whereas T is the year when the individual attains maximum age.

2.3 Contracts for Retirement

There are two retirement contracts. The first is a collective pension called the Group Self-Annuitization (GSA) scheme. The second is a Deferred Variable Annuity (DVA) contract offered by an annuity provider who is backed by equityholders. We describe both contracts in this section. Appendix A elaborates on the rationale of the definition and provision of the contracts.

The contracts specify the distribution of financial and systematic longevity risks between the stakeholders. As the contracts are intended to portray systematic longevity risk, both treat stock market risk identically–the risk is fully borne by the individuals. The benefits due, henceforth known as entitlements, are fully indexed to the same underlying financial portfolio called the reference portfolio (e.g., a portfolio that is 20% invested in the stock index, and 80% in the money market account). Thus, if the DVA provider adopts the reference portfolio's investment policy, the provider is hedged against financial market risk.

Systematic longevity risk distribution, however, distinguishes the two contracts. Under the GSA, it is shared equally among individuals. Under the DVA, the risk is borne by equityholders up to a limit implied by their equity contribution, beyond which the DVA provider defaults. Both contracts stipulate to distribute mortality credit according to the survival probabilities, conditional on the date of contract sale. The DVA provider's equityholders bear the risk that the survival probability forecast deviates from the realized values. The provider uses its equity capital to finance underestimation of systematic longevity, and disburses any surplus arising from overestimation of longevity to its equityholders as a dividend.

Due to the lack of a liquid market of financial assets associated with systematic longevity risk, the risk cannot be hedged by the DVA provider. Additionally, we assume that the number of individuals who either purchase the DVA or participate in a GSA is large enough such that by the Law of Large Numbers, the proportion of surviving individuals within each pool coincides with that implied by the realized survival probabilities, so we can eliminate idiosyncratic longevity risk.[13] In our setting, mortality credit can be positive or negative depending on systematic longevity evolution.

2.3.1 Deferred Variable Annuity (DVA)

The DVA contract is parametrized by an actuarial construct called the assumed interest rate (AIR), $h = \{h(t)\}_{t=t_0}^{T}$. The AIR is a deterministic rate that determines the cost, A, of a contract sold to an individual who is aged x at time t_0 as follows:

$$A(h, F, t_0, x) = (1+F) \int_{t=t_R}^{T} {}_{t-t_0}p_x^{(t_0)} \exp(-h(t) \times (t - t_R))\, \mathrm{d}t \tag{3}$$

${}_{t-t_0}p_x^{(t_0)}$ is the conditional probability in year t_0 that someone who is x years old lives for at least $t - t_0$ years. h is the AIR, F is the loading factor whereas t_R is the retirement year. The loading factor, F, is a proportional one-off premium that the DVA provider attaches to a contract. A contract that is priced at its best estimate has a loading factor of zero, $F = 0$.

The DVA contract is indexed to a reference investment portfolio that follows a deterministic investment policy, $\theta \equiv \{\theta_t\}_{t=t_0}^{T}$. θ_t is the fraction of portfolio wealth allocated to the risky stock index at time t, while the remaining $1 - \theta_t$ is invested in the money market account. Let $W_t^{Ref}(\theta)$ be the value of the reference portfolio at time t. The dynamics of the reference portfolio are thus $\mathrm{d}W_t^{Ref} = W_t^{Ref}(r + \theta_t \lambda_S \sigma_S)\, \mathrm{d}t + W_t^{Ref} \theta_t \sigma_S\, \mathrm{d}Z_{S,t}$.

Using an annuitization capital that is normalized to one, the individual purchases $A(h, F, t_0, x)^{-1}$ unit(s) of DVA contract(s), and is entitled to Ξ, for every year t in retirement, $t_R \leq t \leq T$.[14]

$$\begin{aligned} \Xi(h, F, t, x) &= \frac{\exp(-h(t) \times (t - t_R))}{A(h, F, t_0, x)} \frac{W_t^{Ref}(\theta)}{W_{t_0}^{Ref}(\theta)} \\ W_t^{Ref}(\theta) &= \text{value of the reference portfolio at time } t \end{aligned} \tag{4}$$

The AIR influences the expectation and dispersion of the benefit payments over time. For instance, the fund units are front- (back-) loaded (i.e., due in the earlier

[13] The GSA in our setting is a specific case of the GSA in Piggott et al. (2005), whereby we omit idiosyncratic longevity risk.

[14] The benefits adjust instantaneously with the value of the portfolio to which the contract is indexed. Maurer et al. (2016) make the case for smoothing of the benefits, which is advantageous to both the policyholder and the contract provider.

(later) years of retirement) under a higher (lower) AIR.[15]

We demonstrate in Appendix A that for any given θ, the AIR that maximizes the individual's expected utility in retirement is Equation (5), which we refer to as the optimal AIR, h^*. h^* depends on the individual's preference and financial market parameters. It serves as the AIR of both the DVA and GSA.

$$h^*(t, \theta_t) \quad = \quad r + \frac{\beta - r}{\gamma} - \frac{1-\gamma}{\gamma}\theta_t \sigma_S \left(\lambda_S - \frac{\gamma \theta_t \sigma_S}{2} \right) \tag{5}$$

The DVA provider merely serves as a distribution platform for annuity contracts. It acts in the best interest of its equityholders, who outlive the individuals. The equityholders provide a lump sum capital that is proportional to the value of its estimated liabilities in the year t_0.[16] At every date $t \geq t_0$, the DVA provider's asset value has to be at least equal to the value of its estimated liabilities.[17] In any year $t_0 \leq t \leq T$, if the DVA provider fails to meet the 100% solvency requirement, then the DVA provider defaults. Regulatory oversight is introduced for the DVA provider, because as a for-profit entity, the DVA provider may have an incentive to take excessive risk at the individuals' expense (Filipović et al., 2015). We impose a solvency constraint as it is not only the norm in regulatory regimes for insurers (e.g., Solvency II in the European Union), but is also shown to be effective in mitigating risk-shifting (Filipović et al., 2015).

In every year of retirement, the individual receives a benefit that is equal to the DVA entitlement,

$$\Xi^{DVA}(h^*, F, t, x) \quad = \quad \Xi(h^*, F, t, x) \tag{6}$$

conditional on the individual's survival and the DVA provider's solvency. $\Xi(.)$ is Equation (4) while h^* is Equation (5).

In the event of default, the residual wealth of the DVA provider is distributed among all living individuals, in proportion to the value of their contracts that remains unfulfilled. Equityholders receive none of the residual wealth. We impose a resolution mechanism that obliges individuals to use the provider's liquidated wealth to purchase an equally-weighted portfolio of zero-coupon bonds, of maturities from the year of default if the individual is already retired, or from the year of

[15]Let $\tilde{r}$ denote the reference portfolio's expected return, and suppose h is time-invariant. Then an annuity contract with $h = \tilde{r}$ has a constant expected benefit payment path. When $h < \tilde{r}$, then the expected benefit stream is upward sloping, with increasing variance as the individual ages. Conversely, when $h > \tilde{r}$, the expected benefit stream is downward sloping, and the variance is higher during the initial payout phase. Horneff et al. (2010) provide an exposition on retirement benefits under numerous *AIR*s and reference portfolios.

[16]The estimation of the value of liabilities is explained in Appendix B.

[17]This is a simplifying assumption. Under Solvency II, for instance, insurance companies face solvency capital requirements calculated on a one-year ruin probability of 0.5%.

retirement, until the year of maximum age. Assuming that the bond issuer poses no default risk, then the individual has a guaranteed income until death, but receives no mortality credit. If the individual dies before the maximum age, the face value of the bonds that mature subsequently is not bequeathed. This resolution to insolvency is harsh on the individuals because it eliminates the mortality credit, but it reflects the empirical evidence that individuals substantially discount the value of an annuity that poses default risk (Wakker et al., 1997; Zimmer et al., 2009).

2.3.2 Group Self-Annuitization (GSA)

Similar to the DVA, the GSA is parameterized by the optimal AIR, h^*, and is indexed to a reference portfolio with the investment policy θ. The aged-x individual receives $A\left(h^*, 0, t, x\right)^{-1}$ contract(s) for every unit of contribution at time t. In any year $t \geq t_R$, the GSA's entitlement depends on the reference portfolio's value at time t, $W_t^{Ref}\left(\theta\right)$.

The description of the GSA thus far is identical to a DVA contract with zero loading, $F = 0$. The GSA's distinctive feature is that the entitlements are adjusted according to its funding status. Let the funding ratio at time t, FR_t, be the ratio of the GSA's value of assets, taking into account the investment return from the preceding year, over the best estimated value of its liabilities.[18] For any year t in retirement, $t_R \leq t \leq T$, the individual is entitled to $\Xi^{GSA}(h^*, 0, t, x)$.

$$\begin{aligned} \Xi^{GSA}(h^*, 0, t, x) &= \Xi(h^*, 0, t, x) \times \frac{FR_t}{1} \\ &= \frac{\exp\left(-h^*\left(t, \theta_t\right) \times \left(t - t_R\right)\right)}{A\left(h^*, 0, t_0, x\right)} \frac{W_t^{Ref}\left(\theta\right)}{W_{t_0}^{Ref}\left(\theta\right)} FR_t \\ FR_t &= \text{Funding Ratio in year } t \end{aligned} \tag{7}$$

The first two terms of Equation (7) are identical to the entitlement for a DVA contract with zero loading, Equation (4). The final term of Equation (7) represents the adjustment. If FR_t is smaller (larger) than 1, then the GSA entitlement, Ξ^{GSA}, is lower (higher) than the DVA entitlement, Ξ^{DVA}, in year t. Equation (7) ensures that the GSA is 100% funded in any year.

3 Model Calibration

We consider three groups of individuals, distinguished by their risk aversion levels, $\gamma = 2, 5,$ and 8.[19] Individuals are otherwise homogenous. They have an

[18] Estimation of the GSA liabilities is identical to the estimation of liabilities of the DVA provider. See Appendix B for details.

[19] Using survey responses from the Health and Retirement Study on the US population, Kimball et al. (2008) estimate that the mean risk aversion level among individuals is 8.2, with a standard deviation of 6.8.

annual subjective discount factor of 3%,[20] are aged 25 at time $t_0 = 0$, and use a lump sum that is normalized to one, to either purchase DVA, or join the GSA at time t_0. Both contracts stipulate payment of annual retirement benefits from age 66 until age 95, conditional on the individual's survival in any year, according to the contract specification in Section 2.3.

The portfolio to which the DVA and GSA are indexed is either fully invested in the money market account ($\theta = 0$), or 20% invested in equities and 80% in the money market account ($\theta = 20\%$). These allocations yield the optimal AIR range of 3-4% (Table 1) that is not only observed in the annuity market (Brown et al., 2001), but also typically considered in the related literature (Koijen et al., 2011; Maurer et al., 2013). In Section 6.1.2, we explore alternative investment policies and demonstrate that they uphold the same results as when $\theta = 0, 20\%$.

We assume that the DVA provider's equityholders provide a lump sum capital at date t_0 that is 10% of the contract's best estimate price. The level of equity capital contribution is set such that the annuity provider's leverage ratio is 90%. It reflects the average leverage ratio of US life insurers between 1998-2011.[21]

To provide descriptive calculations on individual welfare under the GSA and the DVA, we calibrate the financial market and life expectancy models to US data. These parameters constitute our baseline case.

3.1 Financial Market

We adopt a constant risk-free rate of $r = 3.6\%$. The stock index has an annualized standard deviation of $\sigma_S = 15.8\%$, and an instantaneous Sharpe ratio of $\lambda_S = 0.467$. This implies that the stock risk premium is $\lambda_S \sigma_S = 7.39\%$. These parameters reflect the performance of the market-capitalization-weighted index of US stocks and the yield on the three-month US Treasury bill over between January 1985 and May 2016.

[20] While field experiments reveal a wide range of implied subjective discount factor (e.g., see Table 1 in Frederick et al., 2002), we choose a value that is commonly adopted in welfare analysis. For example, in similar analyses on retirement income, Feldstein and Ranguelova (2001) and Hanewald et al. (2013) adopt a subjective discount factor of around 2%.

[21] Leverage Ratio $\equiv 1 -$ Value of Equity/Value of Assets. Based on the A.M. Best data used in Koijen and Yogo (2015), the leverage ratio of US life insurers between 1998 to 2011 is 91.36% on average. Assuming that assets are composed of premium and equity capital only, and normalizing Premium $= 1$, we have Leverage Ratio $= 1 - \text{Equity}/(1 + \text{Equity})$, which we use to solve for Equity when the Leverage Ratio $\approx 90\%$.

3.2 Longevity Risk Model

We estimate the Lee and Carter (1992) model using US female death counts, $D_{x,t}$, and the exposure to risk, $E_{x,t}$, both estimated over the full population (Human Mortality Database from 1980 to 2013).[22] The mortality rate for age group x in year t is $D_{x,t}/E_{x,t}$. By relying on population mortality data, we eschew adverse selection that plagues the annuity market, i.e., the individuals who purchase an annuity typically have a longer average lifespan than the general population (Mitchell and McCarthy, 2002; Finkelstein and Poterba, 2004).

Estimation of the Lee and Carter (1992) model proceeds in three steps. First, k_t is estimated using Singular Value Decomposition. In the second step, a_x and b_x are estimated by Ordinary Least Squares on each age group, x. In the third step, k_t is re-estimated by iterative search to ensure that the predicted number of deaths coincides with the data. For identification of the model, we impose the constraints $\sum_x b_x = 1$ and $\sum_t k_t = 0$.

The estimated model is used for forecasting by assuming that the mortality index k_t follows a random walk with drift.

$$\begin{aligned} k_t &= c + k_{t-1} + \delta_t \\ & \delta \sim \mathcal{N}\left(0, \sigma_\delta^2\right) \end{aligned} \tag{8}$$

Forecasts of the log of the central death rates for any year t', $t' \geq t$, are given by $\mathbb{E}_t\left[\log\left(m_{x,t'}\right)\right] = a_x + b_x \hat{k}_{t'}$, with $\hat{k}_{t'} = (t'-t)\,c + k_t$. The realized log of the mortality rate incorporates the independently and identically normally distributed error terms $\varepsilon_x \sim \mathcal{N}\left(0, \sigma_x^2\right)$ and $\delta \sim \mathcal{N}\left(0, \sigma_\delta^2\right)$, where ε_{x,t_1} and δ_{t_2} are uncorrelated for any $t_1, t_2 \in [t_0, T]$ and x. Therefore, the conditional expected forecast error of $\log\left(m_{x,t}\right)$ is zero.

We estimate that $\hat{c} = -1.047$, which implies a downward trend for k_t, while the estimate of σ_δ is $\widehat{\sigma_\delta} = 1.744$. In Figure 1, we present the estimates for a_x, b_x and σ_x. a_x is increasing in age. Estimates for b_x suggest that the change in the sensitivity of age groups to the time trend, k, is not monotone across ages. As for σ_x, it decreases in age non-monotonically until around age 85. With these estimates, 83.8% of the variation in the data is explained.

In Figure 2, we display a fan plot of the fraction of living individuals by age, between 25 and 95, with the population at age 25 normalized to one. The maximum and minimum realizations have a wide range. At its widest at age 88, the difference is as large as 30%.

[22] This fitting period is selected using the method of Booth et al. (2002). It involves determining the longest period when the assumption of linearity of the mortality index k_t (Equation (8)) holds, via a loss of fit ratio.

Figure 1: Lee and Carter (1992) Mortality Model Parameter Estimates

The top panel shows the estimates for a_x, the middle panel displays the estimates for b_x, whereas the bottom panel presents the estimates of σ_x, for the Lee and Carter (1992) model as specified by Equation (1). The calibration sample is the US Female Mortality data from 1980 to 2013, from the Human Mortality Database. The estimate of c is -1.047 and that of σ_δ is 1.744. 83.8% of variation of the sample is explained by these estimates.

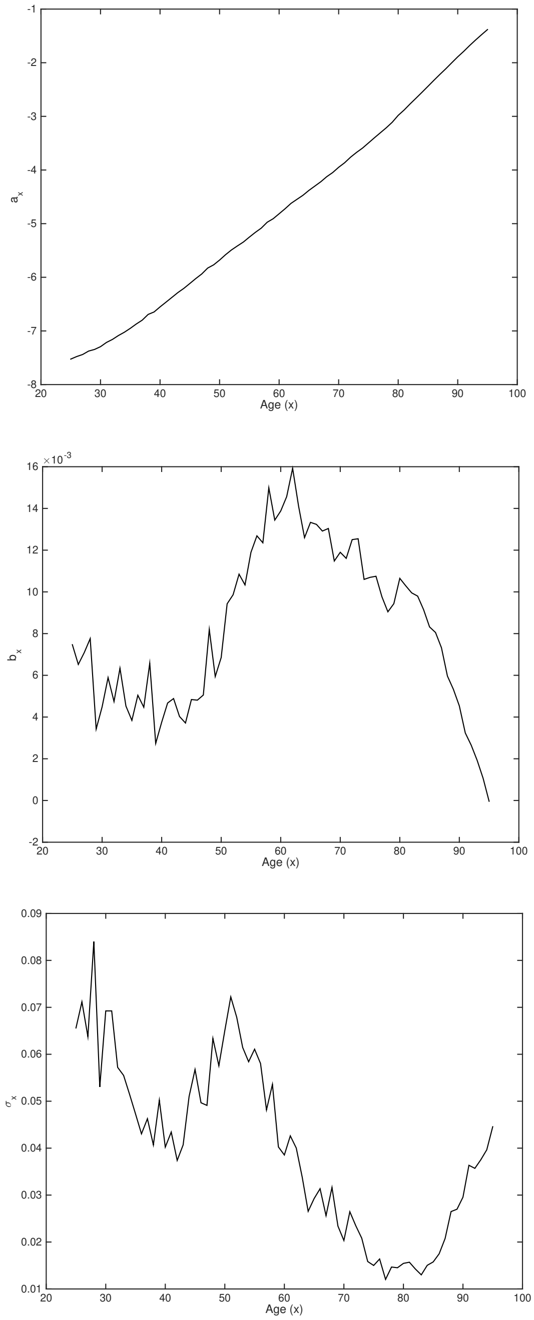
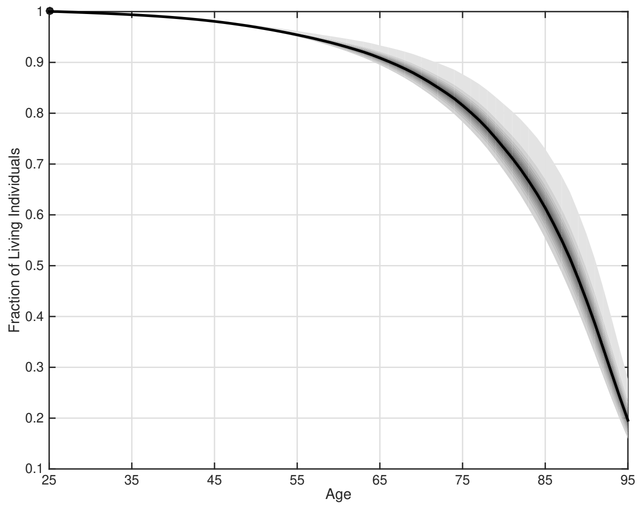


Figure 2: Lee and Carter (1992) Mortality Model: Fan Plot
This figure presents the fan plot of the simulated fraction of living individuals (i.e., the population of 25-year-olds is normalized to one) over 10,000 replications when longevity is modeled according to Lee and Carter (1992), using estimates in Figure 1. Darker areas indicate higher probability mass.



3.3 Contract Characteristics

In order to develop intuition and grasp the contracts' definition, we discuss the characteristics of the GSA and the DVA under the calibrated parameters. Table 1 presents the optimal AIRs as given by Equation (5), and evaluated at the parameters outlined in Sections 3.1 and 3.2.

Table 1: Baseline Case: Optimal AIR, h^* (%)
This table shows the optimal AIR, Equation (5), of the DVA and GSA contracts by the individuals' risk aversion parameter, γ. The underlying portfolio to which the contracts are indexed is either 100% invested in the money market account ($\theta = 0$), or 20% in the risky stock index and 80% in the money market account ($\theta = 20\%$).

θ	γ		
(%)	**2**	**5**	**8**
0	3.31	3.50	3.54
20	4.00	4.48	4.48

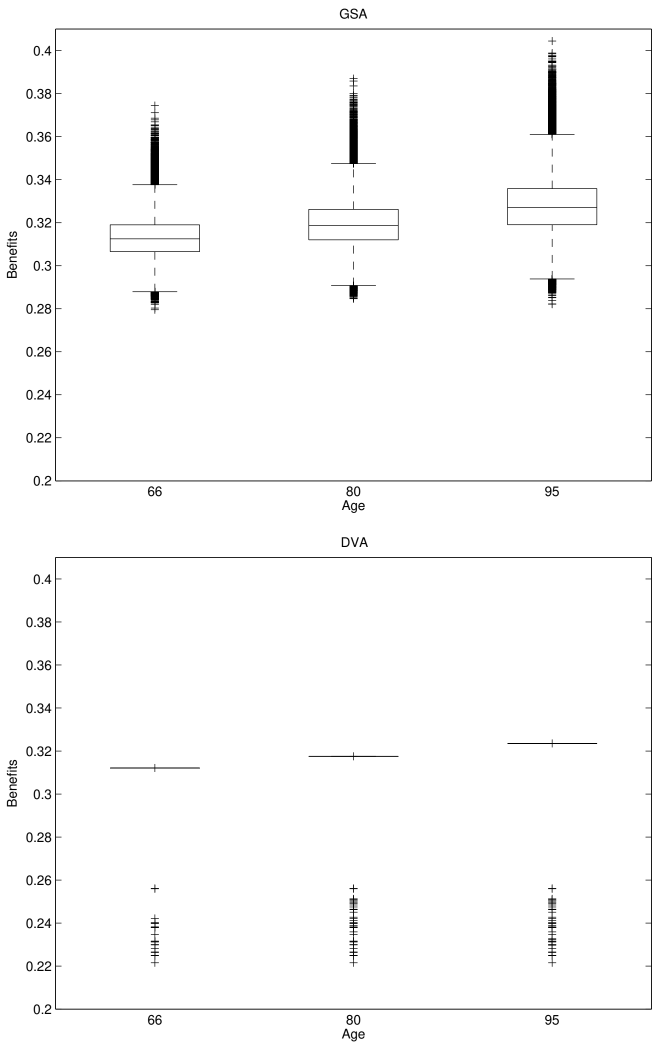
Figure 3 is a box plot of the benefits that individuals receive under the DVA and the GSA. The median benefits of both contracts grow along the retirement horizon because the optimal AIR is lower than the constant financial market return. For the DVA, the median value is also the maximum, because the surplus from life expectancy misestimates belongs to the equityholders.

The GSA yields more instances of positive than negative adjustments to benefits that are 1.5-time larger than the range between its 75^{th} and 25^{th} percentiles. We infer this from the relative density of "+" symbols above and under the box (Figure 3, top panel). When the individual attains the maximum age of 95, benefits as large as 25% more than the median could occur. In contrast, in the worse scenario at the same age, the reduction in benefits relative to the median is 12.5% at most. This asymmetric effect on benefits arises from the non-linearity of the Lee and Carter (1992) model. For error terms of the same magnitude (i.e., $\{\varepsilon_{x,t}\}_{t=t_0}^{T}$ in Equation (1) and $\{\delta_t\}_{t=t_0}^{T}$ in Equation (8), for any $x \in \mathbb{Z} \cap [25, 95]$), overestimation of the log of the central death rates generates a larger entitlement adjustment than underestimation does. When the DVA provider defaults, the individual is at risk of receiving a much lower benefit. The worst case under the DVA entails up to a 30% lower benefit relative to the median at the maximum age.

The box plots indicate that while both contracts offer comparable benefits at the median, those of the GSA have higher standard deviations across scenarios due to the entitlement adjustments, but upward adjustments are more prevalent than downward ones. The DVA offers less volatile benefits, but is susceptible to severe low benefit outcomes when the provider defaults. These are the main features that the individuals weigh in utility terms.

Figure 3: Baseline Case: Box Plots of GSA and DVA Benefits

The figure presents the box plot of benefits, for the GSA (top panel), and the DVA (bottom panel), for an individual with a risk aversion level of $\gamma = 5$, at ages 66, 80 and 95. The underlying portfolio is invested in the money market account only. The line in the middle of the box is the median, while the edges of the box represent the 25^{th} and 75^{th} percentiles. The height of the box is the interquartile range, i.e., the interval between the 25^{th} and 75^{th} percentiles. The "+" symbols represent data points that are 1.5 times larger than the interquartile range.



4 The Individual's Perspective

We investigate two settings distinguished by the existence of stock market risk. In both, there is systematic longevity risk, but in one instance, there is no investment in the stock market, $\theta = 0$, and so the financial return is constant at r, whereas in the other, $\theta = 20\%$ is invested in the risky stock index while the remaining 80% is allocated to the money market account. All results are based on simulations with 500,000 replications unless specified otherwise. The code that produces all figures and estimates in Sections 4 to 6 are available from the authors upon request.

4.1 Cumulative Default Rate

We measure the DVA provider's default rates with the Cumulative Default Rate, an estimate of the probability that the DVA provider defaults during the individuals' planning horizon.

Let D_t be the indicator function that the DVA provider has defaulted in any year t', $t_0 < t' \leq t \leq T$. For example, if the DVA provider defaults in the year t^*, then $D_t = 1$ for $t \geq t^*$ and $D_t = 0$ for $t < t^*$. Additionally, $D_{t_0} \equiv 0$ because the contracts are sold at their best estimate price, and the equity contribution is non-negative.

The marginal default rate in year t, $d(t)$ is the probability that the annuity provider defaults in year t, conditional on not having defaulted in previous years.

$$\begin{aligned} d(t) &= \text{Marginal Default Rate in year } t \\ &= \frac{\mathbb{E}[D_t]}{1 - \mathbb{E}[D_t]} \end{aligned} \tag{9}$$

We define the Cumulative Default Rate as

$$\begin{aligned} \text{Cumulative Default Rate} &= 1 - \Pi_{t=t_0}^{T} (1 - d(t)) \\ d(t) &= \text{Equation (9)} \end{aligned} \tag{10}$$

The default rates in the baseline case are at most 0.01% (Table 2). As the AIR determines whether the bulk of benefits are due earlier or later in retirement, when combined with the fact that longevity forecast errors are larger at longer horizons, the DVA provider's default rates are inversely related to the AIRs. A higher AIR results in a payment schedule with benefits mostly due earlier in retirement. As such, the longevity estimates are accurate when most of the benefits are paid. Conversely, if the AIR is low, benefit payments are deferred to the end of retirement, when life expectancies are most vulnerable to forecasting errors. Therefore, for a fixed level of equity capital, the DVA provider is less susceptible to defaults when the AIR is higher.[23] For the risk aversion levels $\gamma = 2, 5, 8$, the optimal AIR is

[23]From the regulator's perspective, the notion of an annual probability of default, instead of a

increasing in γ (Table 1), hence the default rates are decreasing in γ (Table 2) for both $\theta = 0, 20\%$. Similarly, the default rates are lower when $\theta = 20\%$ than when $\theta = 0\%$ for all levels of γ because the optimal AIRs are higher under $\theta = 20\%$.

4.2 Individual Preference for Contracts

We quantify the individuals' preference for the contracts via the Certainty Equivalent Loading (CEL). This is the level of loading on the DVA (i.e., F in Equation (3)), that equates an individual's expected utility under the DVA and the GSA. The CEL satisfies Equation (11). A positive (negative) CEL suggests that the individual prefers the DVA (GSA).

$$\mathbb{E}\left[U\left(\frac{\Xi^{DVA}|_{F=0}}{1+CEL}\right)\right] = \mathbb{E}\left[U\left(\Xi^{GSA}\right)\right] \tag{11}$$

$\Xi^{DVA}|_{F=0}$ is the retirement benefits of a DVA with zero loading ($F = 0$ for Equation (6)). Ξ^{GSA} is the retirement benefits of a GSA, (7). $U(.)$ is the utility function, Equation (2). Confidence intervals for the CELs are estimated via the Delta Method, for which more details are in Appendix C.

Table 3 presents the CEL in the baseline case. The CELs are negative for all risk aversion levels. This implies that individuals prefer the GSA over the DVA, but only marginally. If the DVA contracts were to be sold at a discount of between 0.052% and 0.350%, then individuals would be indifferent between the two contracts. The CEL is increasing in the risk aversion level, γ. This is because more risk-averse individuals have greater preference for the DVA benefits' lower standard deviation across scenarios.

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cumulative one may be more salient. We explore the "Maximum Annual Conditional Probability of Default", defined as $\max_{\{t=t_0,\dots,T\}} d(t)$, and find that the maximum annual default rate in the baseline case is 0.0008%. This suggests that the 10% buffer capital is sufficient to restrict default rates of DVA providers who are exposed to only systematic longevity risk to existing regulatory limits (e.g., Solvency II for insurers in Europe).

Table 2: Baseline Case: Cumulative Default Rates (%)

This table displays the Cumulative Default Rates, Equation (10), of the DVA provider who sells zero-loading variable annuity contracts with a 40-year deferral period, and has equity capital valued at 10% of the liabilities in the year that the contract was sold. The underlying portfolio to which the DVA and GSA are indexed is either fully invested in the money market account ($\theta = 0$), or 20% in the stock index, and 80% in the money market account ($\theta = 20\%$).

θ	γ		
(%)	**2**	**5**	**8**
0	0.0102	0.0084	0.0082
20	0.0070	0.0038	0.0038

Table 3: Baseline Case: Certainty Equivalent Loading (CEL) (%)

This table presents the CEL, Equation (11), by the risk aversion levels (γ). Individuals aged 25 purchase either the DVA or join the GSA with a lump sum capital normalized to one. The reference portfolio is either fully invested in the money market account ($\theta = 0$), or is $\theta = 20\%$ invested in the stock index and 80% in the money market account. The expected utilities to which the CELs are associated are computed over individuals' retirement between ages 66 and 95. The equityholder's capital is 10% of the present value of liabilities at the date when the contract is sold. The default rates that ensue at this level of equity capitalization are shown in Table 2. The 99% confidence intervals estimated by the Delta Method are in parentheses.

θ	γ		
(%)	**2**	**5**	**8**
0	-0.350 [-0.362, -0.339]	-0.200 [-0.211, -0.188]	-0.055 [-0.067, -0.044]
20	-0.349 [-0.361, -0.338]	-0.200 [-0.216, -0.184]	-0.052 [-0.088, -0.016]

5 The Equityholders' Perspective

To evaluate the equityholders' risk-return tradeoff on systematic longevity risk exposure, we consider the Sharpe ratio and the Jensen's alpha of providing capital to the annuity provider, against those of investing the same amount of capital in the reference portfolio over the same time period.[24] As in Section 4, the annuity

[24]The stochastic discount factor, $\{M_t\}_{t=t_0}^{T}$, that follows $\mathrm{d}M_t/M_t = -r\,\mathrm{d}t - \lambda_S\,\mathrm{d}Z_{S,t}$, allows us to price any contingent claim exposed to stock market risk only: If X_t is a (random) cash flow generated

provider offers contracts at zero loading.

Equityholders contribute 10% of the DVA provider's best estimate value of liabilities at time t_0, and receive the terminal wealth of the annuity provider, $W_T^{(A)}$, as a dividend. When the value of liabilities is normalized to one, the continuously compounded annualized return of capital provision in excess of the risk-free rate of return is $R^{(A_{exs})} = \log\left(W_T^{(A)}/0.1\right)/(T-t_0) - r$. We evaluate the equityholders' profitability via the Sharpe ratio, $SR = \mathbb{E}\left[R^{(A_{exs})}\right]/\sigma^{(A_{exs})}$, and we compute the Sharpe ratio's confidence intervals in accordance with Mertens (2002).

The Jensen's alpha, α, is given by Equation (12) (Jensen, 1968).

$$R^{(A_{exs})} = \alpha + \beta R^{(S_{exs})} + u \tag{12}$$

$R^{(S_{exs})}$ is the annualized return of the stock index in excess of the return on the money market account, and u is the error term. We estimate Equation (12) by Ordinary Least Squares. α assesses the investment performance of providing capital to the annuity provider, relative to that of the market portfolio, on a risk-adjusted basis. A positive α suggests that systematic longevity risk exposure enhances the equityholders' risk-return tradeoff. When $\theta = 0$, $\beta = 0$ due to the assumption that the mortality evolution is uncorrelated with the financial market dynamics. If in Equation (12), $R^{(A_{exs})}$ is replaced by the annualized return in excess of the risk-free rate of return for the reference portfolio, then $\alpha = 0$ and $\beta = \theta$. This is because the reference portfolio has identical financial market risk exposure as capital provision, but is not exposed to systematic longevity risk.

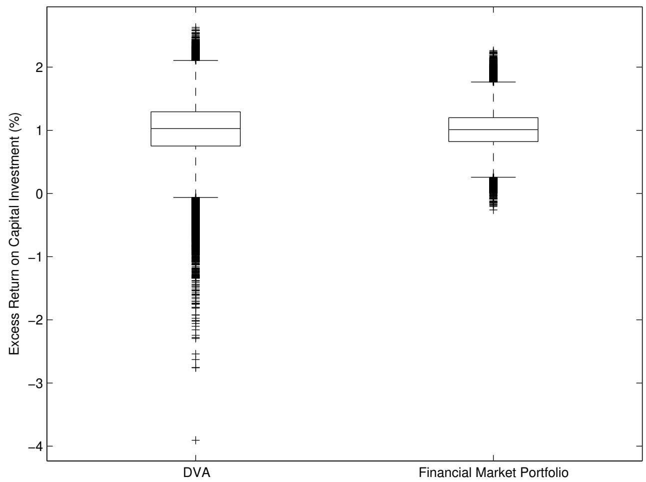
When $\theta = 0$, the annualized excess return of capital provision is between -0.008 and -0.007%, and the standard deviation is 3.9% (Table 4, top panel). Relative to the zero excess return from investing in the money market account, equity capital provision is inferior, but the difference is economically insignificant. When $\theta = 20\%$, investing in the DVA provider yields an expected excess return of 1.44% (Table 4, bottom panel). This is of no material difference with the expected excess return on the identical financial market portfolio, i.e., $\theta\lambda_S\sigma_S - \theta^2\sigma_S^2/2 = 1.43\%$ when $\theta = 20\%$. However, the standard deviation of excess returns is considerably higher when equityholders are exposed to systematic longevity risk (i.e., $\approx 5\%$, Table 4, bottom panel), than when their investment is subject to stock market risk only (i.e., $\theta\sigma_S = 3.17\%$ with $\theta = 20\%$). Consequently, investing in the financial market only is associated with a Sharpe ratio that is around 50% higher than the

by a contingent claim at time t, then its price at time t_0 is $\mathbb{E}_{t_0}\left[\int_{t=t_0}^{T} (M_t/M_{t_0}) X_t \, \mathrm{d}t\right]$. However, when such pricing is carried out for claims due on a long horizon, and the market price of stock risk (i.e., the Sharpe ratio) exceeds its volatility, the price depends on extreme sample paths along which the claim's return explodes (Martin, 2012). As the claims are susceptible to severe underpricing when the Monte Carlo replication sample size is small, we refrain from valuing contingent claims when comparing the equityholders' investment opportunities.

Sharpe ratio of providing capital to the DVA provider (i.e., 0.29 in Table 4, bottom panel, as compared to $\lambda_S - \theta\sigma_S/2 = 0.45$[25] when $\theta = 20\%$). Thus, if equityholders were risk-neutral, then the excess returns imply that they would be indifferent between either investment opportunity. If equityholders were risk averse, then by the Sharpe ratio, investing in systematic longevity risk worsens the equityholders' risk-return tradeoff when the annuity provider sells the contracts at zero loading. The negative Jensen's alpha of -0.0001 corroborates this inference. Any positive loading is infeasible, because it intensifies individuals' preference for the GSA. Therefore, the annuity provider is incapable of adequately compensating its equityholders for exposure to systematic longevity risk.

The box plot in Figure 4 indicates that the medians of the excess returns from either investing in the DVA provider, or in the portfolio having the same investment policy as the DVA contract reference portfolio are comparable. While excess returns on the financial market only are less volatile across scenarios, their maximum is lower than the best excess returns attainable via capital provision. Therefore, systematic longevity risk exposure allows the equityholders to achieve higher excess returns in the best scenario, but entails greater downside risk due to the possible default of the DVA provider.

Figure 4: Box Plot of Equityholders' Annualized Excess Return (%): $\theta = 20\%$
This figure presents the box plot of the equityholders' annualized return in excess of the risk-free rate (%), from either capital provision to the DVA provider (left), or investing in the reference portfolio (right). The reference portfolio is 20% invested in the risky stock index and 80% in the money market account.



[25]This is the discrete Sharpe ratio, which is the parameter we estimate using simulation replications, as opposed to the instantaneous Shape ratio, λ_S (Nielsen and Vassalou, 2004).

Table 4: Baseline Case: Equityholders' Investment Performance Statistics
This table displays the equityholders' mean annualized return in excess of the risk-free rate of return ($\mathbb{E}\left[R^{(A_{exs})}\right]$, %), standard deviation of annualized excess return ($\sigma^{(A_{exs})}$, %), the Sharpe ratio (SR) and Jensen's alpha ($\mathbb{E}[\alpha]$, %), Equation (12), of capital provision to the DVA provider. The underlying portfolio is either invested in the money market account only ($\theta = 0$, top panel), or is 20% invested in the risky stock index, and 80% invested in the money market account ($\theta = 20\%$, bottom panel). The 99% confidence intervals are in parentheses.

$\theta = 0$

Statistic	γ		
	2	**5**	**8**
$\mathbb{E}\left[R^{(A_{exs})}\right]$ (%)	-0.008 [-0.010, -0.006]	-0.007 [-0.009, -0.005]	-0.007 [-0.008, -0.005]
$\sigma^{(A_{exs})}$ (%)	3.96 [3.95, 3.40]	3.91 [3.90, 3.91]	3.89 [3.88, 3.90]
SR	-0.002 [-0.0056, 0.0016]	-0.0017 [-0.0054, 0.0019]	-0.0017 [-0.0053, 0.0020]
$\mathbb{E}[\alpha]$ (%)	-0.0001 [-0.0001, -0.0001]	-0.0001 [-0.0001, -0.0001]	-0.0001 [-0.0001, -0.0001]

$\theta = 20\%$

Statistic	γ		
	2	**5**	**8**
$\mathbb{E}\left[R^{(A_{exs})}\right]$ (%)	1.44 [1.44, 1.44]	1.44 [1.44, 1.45]	1.44 [1.44, 1.45]
$\sigma^{(A_{exs})}$ (%)	5.04 [5.03, 5.06]	4.95 [4.94, 4.96]	4.95 [4.94, 4.96]
SR	0.29 [0.29, 0.29]	0.29 [0.29, 0.29]	0.29 [0.29, 0.29]
$\mathbb{E}[\alpha]$ (%)	-0.0001 [-0.0001, -0.0001]	-0.0001 [-0.0001, -0.0001]	-0.0001 [-0.0001, -0.0001]

6 Sensitivity Analysis

6.1 Features that indirectly affect default rates

For model features that only indirectly affect the DVA provider's default rates via the optimal AIR,[26] such as the contract's length of the deferral period and stock market risk exposure, the baseline case results hold.

[26]Refer to footnote 15.

6.1.1 Deferral Period

As the accuracy of longevity forecast depends on its horizon, the preference for either contract may be sensitive to the age when the individual annuitizes. In the baseline case, individuals are aged 25 when purchasing a DVA contract or participating in the GSA. As retirement benefit payments commence at age 66, the deferral period is 40 years. When the deferral period is shorter, survival probability forecasts are more accurate. Thus, we expect smaller differences in the average level and standard deviation of benefits between contracts. However, this does not necessarily imply that the CEL estimates would be closer to zero, because the time-preference discounting, as governed by the subjective discount factor, β in Equation (2), plays a larger role when retirement is imminent. Thus, while the difference between the benefits would be smaller, the effect in terms of utility would be greater. By shortening the deferral period to 20 years, we find that the effect due to shorter time-discounting dominates the more accurate survival probability forecast; the CEL estimates are negative across all γs.

6.1.2 Stock Exposure

As long as the allocation to the stock index corresponds to an optimal AIR with similar default rates as those in the baseline case.

We consider four alternative exposures to the stock index. The first three are constant allocations over the planning horizon: $\theta_1 = 40\%$, $\theta_2 = 60\%$, $\theta_3 = \frac{\lambda_S}{\gamma\sigma_S}$. θ_3 corresponds to the individual's optimal exposure to stocks (See Appendix A). For the least risk-averse individual ($\gamma = 2$), θ_3 is 147.2%. The moderately risk-averse individual ($\gamma = 5$) optimally invests 58.9% in the stock index whereas the most risk-averse individual ($\gamma = 8$) optimally invests 36.8% in stocks. The fourth exposure that we consider is an age-dependent allocation that begins with around 90% allocation to stocks at age 25, and gradually diminishes to a minimum of about 30% post-retirement until the maximum age, $\theta_4 = \{\theta_{4,x}\}_{x=25}^{95}$.[27]

For all θs, the optimal AIRs that are set according to Equation (5) are higher than those in the baseline case. Due to the inverse relationship between the default rate and the AIR, the default rates are marginally smaller than those in the baseline case. Consequently, individuals prefer the GSA to a similar extent as in the baseline case.

[27] This glidepath allocation is based on the 2014 Target-Date Fund industry average (Yang et al., 2016). A decreasing exposure to stocks as the individual grows older is consistent with popular financial advice (Viceira, 2001). In theory, when the investment opportunity set is constant, horizon-dependent investment strategies are optimal in situations where, for instance, the individual receives labor income (Viceira, 2001; Cocco et al., 2005), or where the individual's risk aversion parameter is time dependent (Steffensen, 2011).

6.2 Features that directly affect default rates

The DVA provider's default rates are directly affected by systematic longevity risk when equity capital is kept constant, or alternately, the level of equity capital when systematic longevity risk is held constant. Systematic longevity risk manifests via the standard deviation of the error term governing the longevity model's time trend (σ_δ), the standard deviation of the age-dependent errors on the mortality matrix (σ_x), and parameter uncertainty of the time trend's drift term, c .

6.2.1 Consequential

The level of equity capital and σ_δ have consequential effects on the default rates. A lower level of equity capital or a higher σ_δ increases the CDR, hence individuals prefer the GSA more, as indicated by the CELs that are decreasing in CDRs (Figures 5 and 6).[28] Moreover, the extent to which individuals prefer the GSA relative to the DVA with respect to their risk aversion levels varies with the CDR, i.e., the curves intersect. At low levels of CDR, risk averse individuals have a weaker preference for the GSA relative to a more risk averse individual. This is due to the appeal of more stable benefits under a DVA from a sound provider. When there is a small amount of default risk of about 0.5%, more risk averse individuals soon find the DVA to be less attractive than the GSA. Individuals' preference for the DVA diminishes in the CDR regardless of whether the high default rates are induced by heightened systematic longevity risk or less equity capital. Additionally, individuals have a slight preference for the GSA over a zero-default-risk DVA, as seen from a negative CEL when CDR is zero. This is due to the asymmetric effect that longevity time-trend shocks in the Lee and Carter (1992) model have on the benefit adjustments. A negative shock which causes life expectancy decline results in larger upward benefit adjustment than the downward benefit adjustment that a positive shock of the same magnitude entails.

[28] Figure 6 contains only σ_δ that generates positive CDRs. Situations when σ_δ is small such that 10% equity capital is sufficient to ensure no default are discussed in Appendix E.

Figure 5: CEL and CDR: By Changing Level of Equity Capital

This figure presents the CEL and CDR for the base case's risk aversion levels, γ. The CDR is altered by changing the level of equity capital from 0 to 12.5% of the contract's best estimate value. The top panel is for $\theta = 0\%$, whereas the bottom panel is for $\theta = 20\%$. The data points are based on simulations with 100,000 replications.

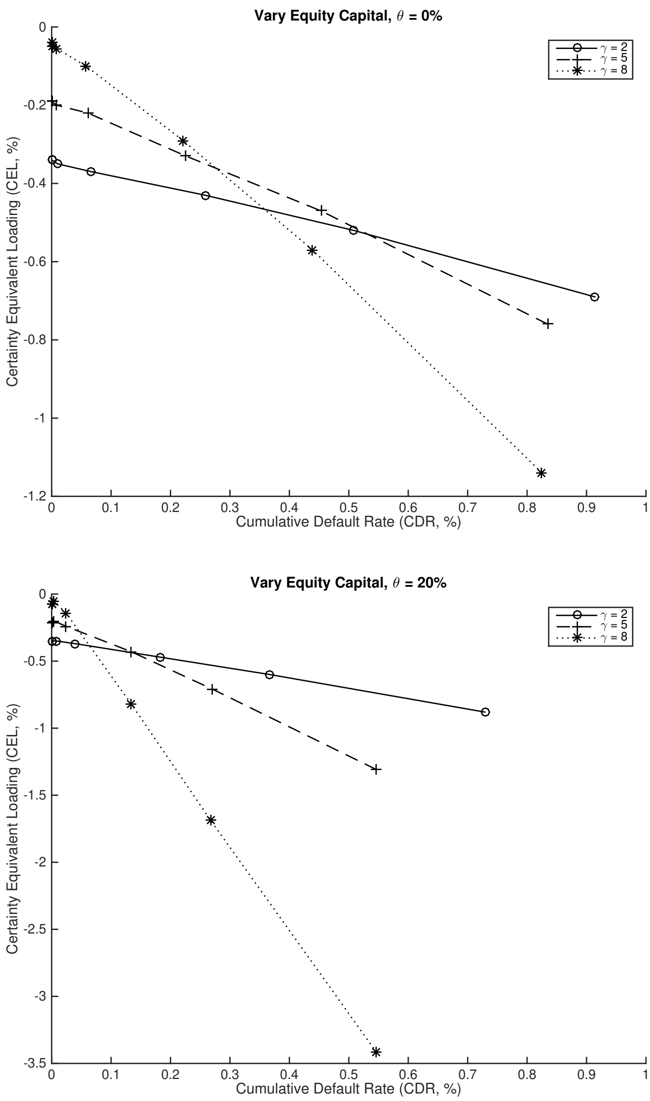
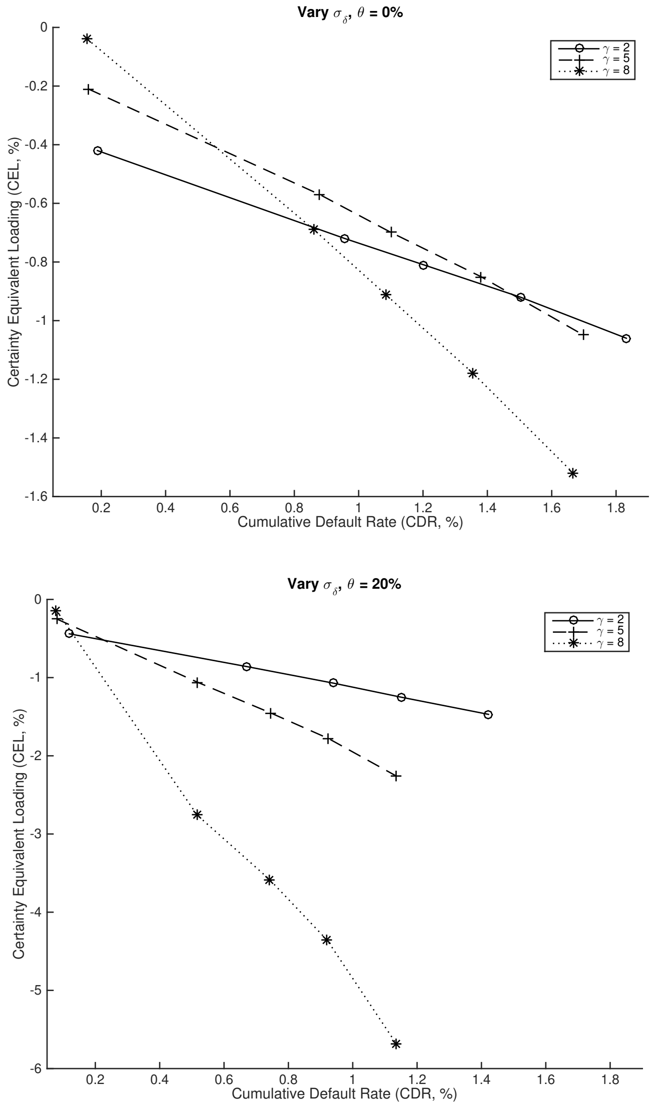


Figure 6: CEL and CDR: By Changing σ_δ, higher CDRs
This figure presents the CEL and CDR for the base case's risk aversion levels, γ. The CDR is altered by changing σ_δ from 0 to 2 times the calibrated value in the base case. The top panel is for $\theta = 0\%$, whereas the bottom panel is for $\theta = 20\%$. The data points are based on simulations with 100,000 replications.



6.2.2 Inconsequential

The standard deviation of errors on the mortality matrix (σ_x) and drift parameter uncertainty affect the default rates only marginally. Hence, the baseline results are unchanged.

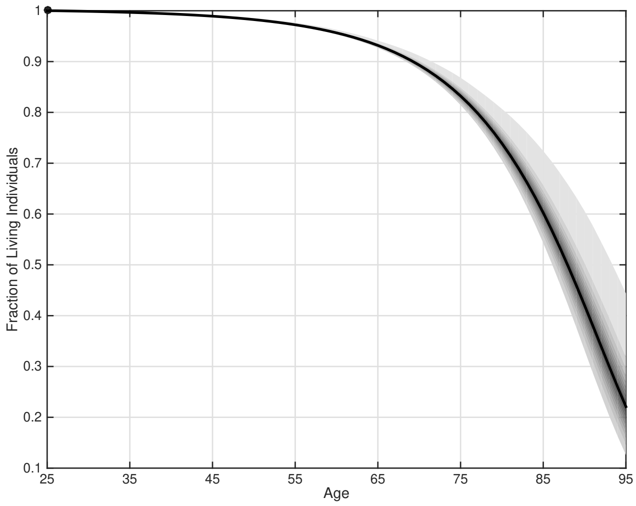
The time trend in the Lee and Carter (1992) model accounts for the bulk of the uncertainty surrounding systematic longevity evolution. Thus, even when σ_x is five times as large as the calibrated values, CDRs and CELs deviate little from those in the baseline case.

When there is uncertainty around the drift parameter, the DVA is disadvantaged by a higher default probability. However, the GSA's appeal also diminishes as entitlement adjustments have a wider variation, especially near the maximum age. Neither of these drawbacks is sufficiently decisive to sway individual preferences. Therefore, the CELs differ only marginally from those in the baseline case. The incorporation of parameter uncertainty is described in Appendix D.

6.3 Alternate Longevity Model

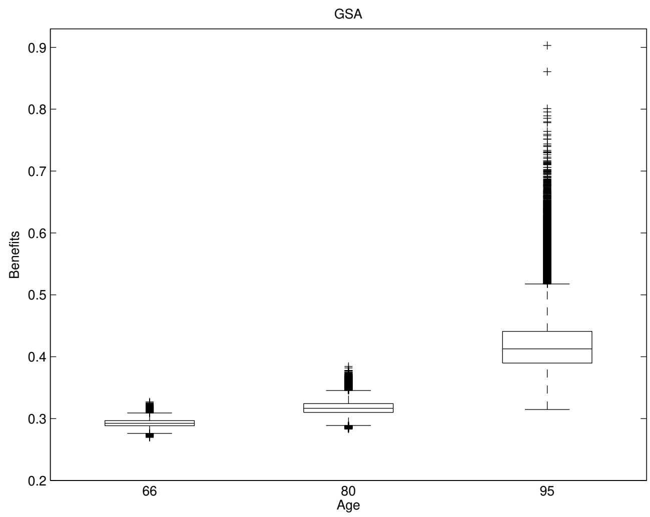
We next explore the choice of the longevity model by replacing the Lee and Carter (1992) model with the Cairns et al. (2006) model, which produces a wider range of survival probabilities at old age. We calibrate the Cairns et al. (2006) model over the same sample of mortality data as that in Section 3.2. Figure 7 presents the fan plot of the simulated fraction of living individuals under the Cairns et al. (2006) model. The maximum range of the fraction of 25-year-olds still alive at older ages is 45% (i.e., at age 91), 50% more than the maximum range under the Lee and Carter (1992) model (i.e., 30% interval at age 88; Figure 2). This wider range translates into greater variability in benefits for the GSA, and higher default rates for the DVA provider.

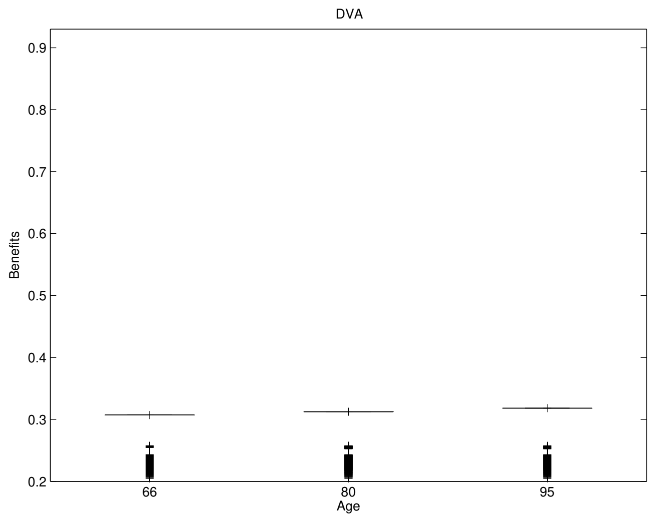
Figure 7: Cairns et al. (2006) Mortality Model: Fan Plot
This figure presents the fan plot of the simulated fraction of living individuals (i.e., the population of 25-year-olds is normalized to one) over 10,000 replications when longevity is modeled according to the Cairns et al. (2006) model. The model is calibrated on US female death counts from 1980 to 2013 taken from the Human Mortality Database. Darker areas indicate higher probability mass.



With either the Lee and Carter (1992) or the Cairns et al. (2006) model, the rise in GSA benefits with age is accompanied by more uncertainty surrounding the benefits. However, the Cairns et al. (2006) model produces greater uncertainty as the individual ages, as seen by comparing the top panels in Figures 3 and 8. This generates greater individual preference for the DVA under the Cairns et al. (2006) model.

Figure 8: Cairns et al. (2006) Mortality Model: Box Plots of GSA and DVA Benefits
The figure presents the box plots of benefits for the GSA (top panel) and the DVA (bottom panel), for an individual with a risk aversion level of $\gamma = 5$, at ages 66, 80 and 95. The underlying portfolio is invested in the money market account only. The line in the middle of the box is the median, while the edges of the box represent the 25^{th} and 75^{th} percentiles. The height of the box is the interquartile range, i.e., the interval between the 25^{th} and 75^{th} percentiles. The "+" symbols represent data points 1.5 times larger than the interquartile range.





For a fixed level of equity capital, the Cairns et al. (2006) model yields higher default rates because of the heightened uncertainty surrounding old age survival. If we maintain the baseline case's 90% leverage ratio, the default rates under the

Cairns et al. (2006) model are between 0.48% to 2.21% (Table 5), substantially higher than the at-most 0.01% default rates when the Lee and Carter (1992) model is adopted (Table 2). Consequent to more defaults, individuals have a lower preference for the DVA (Table 5, bottom panel), as the CEL estimates are more negative than those in the baseline case (Table 3). Therefore, individuals prefer the DVA contract under the Cairns et al. (2006) model only if the associated default risk is curtailed. Regardless of whether equityholders provide enough capital to eliminate default risk, the Sharpe ratio of equity provision is lower than the ratio of abstaining from systematic longevity risk exposure. The Jensen's alpha of equity provision is positive but economically insignificant.

Table 5: Cairns et al. (2006) Mortality Model with Default: Cumulative Default Rates (%) and CEL (%)
The top panel presents the Cumulative Default Rates, Equation (10), whereas the bottom panel displays the CEL, Equation (11), when life expectancy follows the Cairns et al. (2006) model, calibrated to the same sample as the Lee and Carter (1992) model. All other parameters are identical to those in the baseline case. The 99% confidence intervals are in parentheses.

Cumulative Default Rates (%)

θ	γ		
(%)	**2**	**5**	**8**
0	2.2120	1.8082	1.7120
20	0.9676	0.4808	0.4756

CEL (%)

θ	γ		
(%)	**2**	**5**	**8**
0	-0.950 [-0.970, -0.930]	-0.660 [-0.690, -0.630]	-0.975 [-1.025, -0.924]
20	-0.877 [-0.906, -0.847]	-0.503 [-0.571, -0.436]	-1.515 [-1.763, -1.268]

Additionally, the choice of the longevity model underlies the inference of Maurer et al. (2013). While we find that individuals marginally prefer the GSA, Maurer et al. (2013) observe the opposite (positive CEL for the contract indexed to systematic longevity; Table 7 of Maurer et al., 2013). When we assume that no default occurs, as do Maurer et al. (2013), we are able to reconcile our results to theirs. For instance, individuals who are moderately risk-averse to risk-averse, $\gamma = 5$ and 8, prefer the DVA; Table 6, top panel. The most risk-averse individual is willing to pay as much as 1% in loading to shed systematic longevity risk. Despite that, when the annuity provider sets the loading to be equal to the CEL, the accompanying Sharpe ratio remains inferior to the Sharpe ratio of investing in only the financial market, i.e., 0.45 when $\theta = 20\%$, whereas the Jensen's alpha is positive but economically insignificant (Table 6, bottom panel). Therefore, while individual preference is sensitive to the choice of the longevity model, the extent that individuals are willing to pay to insure against systematic longevity risk is insufficient to entice equityholders to gain exposure to that risk.

Table 6: Cairns et al. (2006) Mortality Model with No Default: Certainty Equivalent Loading (CEL) (%) and Investment Performance Statistics
The top panel presents the CEL, Equation (11), when life expectancy follows the Cairns et al. (2006) model, calibrated to the same sample as the Lee and Carter (1992) model. The bottom panel shows the Sharpe ratio (SR) and Jensen's alpha (α), Equation (12), when the loading is set at the CEL estimates in the top panel. Equity capital is sufficiently high such that no default occurs. All other parameters are identical to those in the baseline case. The 99% confidence intervals are in parentheses.

CEL (%)

θ	γ		
(%)	**2**	**5**	**8**
0	-0.089 [-0.099, -0.079]	0.528 [0.519, 0.537]	1.019 [1.011, 1.028]
20	-0.092 [-0.101, -0.082]	0.461 [0.448, 0.475]	0.874 [0.835, 0.913]

Sharpe Ratio and Jensen's Alpha: No Default Risk, Loading = CEL

θ	**Statistic**	γ		
(%)		**2**	**5**	**8**
0	*SR*	0.0206 [0.0170, 0.0242]	0.0481 [0.0444, 0.0517]	0.0701 [0.0665, 0.0738]
	$\mathbb{E}[\alpha]$ (%)	0.0001 [0.0001, 0.0001]	0.0002 [0.0002, 0.0002]	0.0002 [0.0002, 0.0002]
20	SR	0.4337 [0.4337, 0.4337]	0.4362 [0.4362, 0.4362]	0.4379 [0.4379, 0.4379]
	$\mathbb{E}[\alpha]$ (%)	0.0001 [0.0001, 0.0001]	0.0001 [0.0001, 0.0001]	0.0002 [0.0002, 0.0002]

7 Conclusion

We investigate systematic longevity risk management in retirement planning in the presence of two alternatives: individuals participate in a collective scheme that adjusts retirement income according to longevity evolution, or purchase a variable annuity contract offered by an equityholder-backed annuity provider. Our model features the perspective of not only the individuals, who evaluate their welfare in retirement, but also of the equityholders, who weigh their risk-return tradeoff from systematic longevity risk exposure.

Due to the entitlement adjustments arising from errors in survival probability forecasts, the collective scheme provides more volatile benefits than those of an annuity contract. However, the collective scheme also offers a slightly higher average level of benefits, because for errors of the same magnitude, over- and underestimating the log central death rates produce asymmetric effects.

The annuity contract provider relies on limited equity capital to subsume forecasting errors, and so is subject to default risk. Although the annuity contract shields individuals from downward entitlement adjustments up to a limit, it deprives individuals of any upward adjustments, as these gains belong to the equityholders.

We find that individuals marginally prefer the collective scheme over the annuity contract priced at its best estimate. This implies that the annuity provider is unable to charge a positive loading on the contract, subsequently failing to compensate its equityholders who bear longevity risk. Therefore, when individuals have the choice to form a collective scheme, the annuity provider who has no advantage at managing systematic longevity risk, and who has to fully hedge financial market risk would not exist in equilibrium. Our finding is robust to individuals' risk aversion level, the contract deferral period, and stock market risk exposure. Individuals' desire for an annuity contract diminishes in the provider's default risk regardless of whether the underlying determinant of default risk is the magnitude of systematic longevity risk or the level of equity capital. When faced with higher systematic longevity risk, individuals are willing to purchase an annuity contract with no default risk, but the loading that individuals offer remains insufficient to compensate equity holders for taking systematic longevity risk.

The results advocate for collective mechanisms in pension provision, which exist in a handful of countries (e.g., Collective Defined Contribution in the Netherlands, Target Benefit Plans in Canada). The pressing issue of population aging, and the gradual maturation of the longevity risk derivatives market, is likely to spur reform. For example, the US Chamber of Commerce (2016) recommends new plan designs to enhance the private retirement system. Our results also highlight the difficulty for insurers to offer long-term guarantees on non-diversifiable risks such as systematic longevity, while offering an attractive remuneration to their shareholders.

A limitation of our work is the exclusion of channels that may reduce the insurer's effective longevity exposure, such as synergies in product offering (e.g., natural hedging of systematic longevity risk via the sale of annuities and life insurance contracts; Gatzert and Wesker, 2012; Wong et al., 2017), access to reinsurance (Baione et al., 2017) and shadow insurance (Koijen and Yogo, 2016). There are also alternative resolutions in the case of default, and other factors that may influence annuitization decisions, such as bequest motives, medical expenses, social security, uninsurable income, etc. (Lockwood, 2012; Pashchenko, 2013; Peijnenburg et al., 2017; Ai et al., 2017; Yogo, 2016). Additionally, when the number of individuals is not large enough to assume away idiosyncratic longevity risk, it may become a larger risk driver of benefits relative to the systematic component of longevity risk. Examining these features in future research would enrich our knowledge of retirement planning.

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Appendices

A Rationale of the Contract Definition

The DVA and GSA contracts are not only modeled along the variable annuity contracts studied in the literature (Koijen et al., 2011; Maurer et al., 2013), but are also relatable to an individual's optimal consumption and investment.

The problem of optimal consumption and investment is composed of two separate parts: the allocation of initial wealth over each retirement year, and the investment strategy. Aase (2015) shows that for an expected-CRRA-utility-maximizing individual facing idiosyncratic longevity and stock market risks, the optimal allocation of initial wealth decays geometrically in the retirement horizon. The AIR in our setting represents precisely this rate of decay.

When individuals are subject to longevity risk, its existence would not change the optimal wealth and asset allocation; complication to the solution arises from the inability to react to longevity evolution (Huang et al., 2012). We, however, assume that the contract's parameters are deterministic (i.e., fixed in the year when it is sold, and the incorporation of new information thereafter is prohibited). Therefore, by an appropriate choice of the AIR, h^*, the contract described by Equations (3) and (4) coincides with the optimal decumulation path of the individual.

We next solve the utility maximization problem, (13), to obtain the optimal AIR and investment strategy for a contract defined by Equations (3) and (4).

At time t_0, the individual purchases the maximum number of variable annuity contracts affordable with a lump sum capital normalized to one. The annuity contract commences benefit payment in year t_R, until the year T, conditional on the individual's survival. In the financial market setting as described in Section

2.1, with a deterministic fraction of wealth $\theta = \{\theta_t\}_{t=t_0}^{T}$ invested in the risky stock index, and $1-\theta$ invested in the money market account, the value of the reference portfolio evolves according to $dW_t/W_t = (r + \theta_t \lambda \sigma_S)\,\mathrm{d}t + \theta_t \sigma_S \,\mathrm{d}Z_{S,t}$.

$$
\begin{aligned}
\max_{\{\theta_t,\, h(t,\theta_t)\}_{t=t_R}^{T}} \quad & \mathbb{E}_{t_0}\left[U(\Xi)\right] \qquad (13)\\
= \quad & \mathbb{E}_{t_0}\left[\int_{t_R}^{T} \mathrm{e}^{-\beta(t-t_0)} \frac{\Xi_t^{1-\gamma}}{1-\gamma} \left(\Pi_{s=t_0}^{t}\, {}_1p_{x+(s-t_0)}^{(s)}\right) \mathrm{d}t\right]\\
\Xi_t \quad = \quad & \begin{cases} \frac{1}{A(h)} \exp\left(-h(t,\theta_t)(t-t_R)\right) \frac{W_t}{W_{t_0}} & \text{if alive in year } t \\ 0 & \text{otherwise} \end{cases}\\
A(h) \quad = \quad & \int_{t_R}^{T} \exp\left(-h(t,\theta_t)(t-t_R)\right) \times \\
& \mathbb{E}_{t_0}\left[\left(\Pi_{s=t_0}^{t}\, {}_1p_{x+(s-t_0)}^{(s)}\right)\right] \mathrm{d}t\\
h(t,\theta_t) \quad = \quad & \text{AIR}\\
\beta \quad = \quad & \text{subjective discount factor}\\
\gamma \quad = \quad & \text{risk aversion parameter}\\
W_t \quad = \quad & \text{value of the reference portfolio with}\\
& \text{the investment policy } \theta\\
\mathbb{E}_{t_0}\left[\Pi_{s=t_0}^{t}\, {}_1p_{x+(s-t_0)}^{(s)}\right] \quad = \quad & {}_{t-t_0}p_x^{(t_0)}
\end{aligned}
$$

$A(h)$ is the cost per unit of a zero-loading contract. It is straightforward to verify that the contract has a present expected value of one for any $h \in \mathbb{R}^{T-t_R}$, and thus satisfies the budget constraint. Given any θ, the first order condition, $\partial \mathbb{E}_{t_0}\left[U(\Xi)\right]/\partial h = 0$ yields the optimal AIR, Equation (14).

$$
\begin{aligned}
h^*(t,\theta_t) \quad = \quad & r + \frac{\beta - r}{\gamma} - \frac{1-\gamma}{\gamma}\theta_t \sigma_S \left(\lambda_S - \frac{\gamma \theta_t \sigma_S}{2}\right) \qquad (14)\\
r \quad = \quad & \text{constant short rate}\\
\beta \quad = \quad & \text{subjective discount factor}\\
\gamma \quad = \quad & \text{risk aversion parameter}\\
\theta_t \quad = \quad & \text{fraction of wealth allocated to the stock index}\\
& \text{at time } t,\ t_R \le t \le T\\
\sigma_S \quad = \quad & \text{standard deviation governing the stock index's dynamics}\\
\lambda_S \quad = \quad & \text{instantaneous Sharpe ratio of the stock index}
\end{aligned}
$$

Equation (14) is composed of the risk-free rate, the difference between the subjective discount factor and the risk-free rate, adjusted by the risk aversion parameter, and a term concerning the exposure to the stock index, weighted by the risk aversion level.

If the returns on the investment were constant at r (e.g., either $\theta = 0$ or $\sigma_S = 0$), for any given level of risk aversion, γ, the shape of the optimal consumption path depends on the relative magnitude of β and r. An individual who discounts future consumption at a higher rate than the constant interest rate (i.e., $\beta > r$, an impatient individual) prefers a downward sloping consumption path whereas a more patient person (i.e., $\beta < r$) optimally chooses an upward sloping path. When $\theta \neq 0$ and $\sigma_S \neq 0$, then the risk aversion level, the standard deviation and the market price of stocks also have a role in determining the optimal consumption path.

The first-order condition corresponding to the allocation to the stock index, $\partial \mathbb{E}_{t_0}[U(\Xi)]/\partial\theta = 0$, implies the optimal allocation to the risky asset:

$$\theta^* = \frac{\lambda_S}{\gamma\sigma_S} \tag{15}$$

The optimal allocation to the stock index, θ^*, is independent of time and wealth, and is identical to the optimal investment policy of Merton (1969).

The variable annuity contract provides the optimal decumulation path when the AIR is set to $h^*(t, \theta_t^*)$. By prohibiting the incorporation of new information into the contract definition after its date of sale (i.e., enforcing deterministic, but possibly time-varying contract parameters), longevity risk does not influence the optimal AIR and the optimal portfolio choice.

The conception of the GSA as a collective justifies the assumption that it prioritizes individual welfare (i.e., maximizes individuals' expected utility in retirement). Therefore, the GSA offers an AIR that is in the best interest of the individuals, without conflict among its stakeholders. As for the annuity provider, such contracts are also conceivable. For instance, Froot (2007) suggests that insurers should shed all liquid risks for which they have no comparative advantage to outperform (e.g., financial market risk), and devote their entire risk budget to insurance risks (e.g., longevity risk). The selling of variable annuities without any financial guarantee achieves precisely this goal. Besides, Gatzert et al. (2012) demonstrate that if an insurance company sets contract parameters for a participating life insurance contract such that they maximize the contract's value (e.g., expected utility) to the individual, the individual may be willing to pay more for the contract. Therefore, the provision of contracts defined according to Equations (3) and (4) under either a cooperative setup or by a for-profit entity is plausible.

B Definition of the Book Value of Liabilities

Suppose that the DVA provider or the GSA administrator issues contract(s) to a cohort who is aged x at time t_0, promising entitlements of $\Xi^K(h^*, F, t, x)$, $K \in$

$\{DVA, GSA\}$, in every year t, $t_R \leq t \leq T$, conditional on the individual's survival. The estimate of the entity's book value of liabilities at time t, $t_0 \leq t \leq T$, is:

$$L_t \equiv \Xi^K(h^*, F, t, x) \int_{s=\max\{t_R, t\}}^{T} \exp\left(-h^*(s, \theta)(s-t)\right) \times {}_{s-t}p^{(t)}_{x+t-t_0} \mathrm{ds} \tag{16}$$

${}_{s-t}p^{(t)}_{x+t-t_0}$	$=$	conditional probability in year t that a living individual of age $x+t$ lives for at least $s-t$ more years
$h^*(t, \theta)$	$=$	Optimal AIR, Equation (14)
$\Xi^K(h^*, F, t, x)$	$=$	benefit at time t for contract $K \in \{GSA, DVA\}$

B.1 Illustration of the Case with No Risk

To motivate the definition of Equation (16), let us consider a three-period case ($t = t_0, t_1, t_2$) in the absence of stock market and longevity risks. Assume that the individual buys exactly one unit of the retirement contract at retirement in year t_0, lives with certainty to collect the benefits in year $t_1 = t_0 + 1$, and dies with certainty before the year $t_2 = t_1 + 1$. Suppose that the reference portfolio is fully invested in the money market account, earning an interest rate that is constant at 2%. Furthermore, we adopt a constant AIR, $h = 3\%$, and zero contract loading, $F = 0$. As there is no uncertainty in this example, Equation (16) should yield precisely the value of liabilities at time t.

By definition of the DVA contract, there are two payments to be made: one in the year t_0 and another in the year t_1. The individual receives a payment immediately, in t_0, that is valued at:

$$\begin{aligned}\Xi(h, 0, t_0, x) &= 1 \times \frac{W^{Ref}_{t_0}}{W^{Ref}_{t_0}} e^{-h \times (t_0 - t_0)} \\ &= 1\end{aligned}$$

The second payment, in present value at time t_1 is:

$$\begin{aligned}\Xi(h, 0, t_1, x) &= 1 \times \frac{W^{Ref}_{t_1}}{W^{Ref}_{t_0}} e^{-h \times (t_1 - t_0)} \\ &= \frac{W^{Ref}_{t_0} e^{0.02}}{W^{Ref}_{t_0}} e^{-h} \\ &= e^{-h+0.02} \\ &= e^{-0.01}\end{aligned} \tag{17}$$

Discounting Equation (17) by the constant interest rate, we obtain the present value at time t_0, of the payment due at time t_1:

$$\begin{aligned} PV_{t_0}\left[\Xi(h,0,t_1,x)\right] &= \Xi(h,0,t_1,x)e^{-0.02\times(t_1-t_0)} \\ &= e^{-0.01-0.02} \\ &= e^{-0.03} \end{aligned}$$

The present value of liabilities at time t_0 is

$$\Xi\left(h,0,t_0,x\right)+PV_{t_0}\left[\Xi(h,0,t_1,x)\right] = 1+e^{-0.03} \tag{18}$$

It remains to show that Equation (16) yields Equation (18):

$$\begin{aligned} L_t &= \Xi(h,0,t_1,x)\times\left(e^{-h\times 0}\,{}_0p_t^{(t)}+e^{-h\times 1}\,{}_1p_t^{(t)}\right) \\ &= 1\times\left(1+e^{-h}\right) \\ &= 1+e^{-0.03} \end{aligned}$$

B.2 Illustration of the General Case

We price the liabilities of the pension provision entity by constructing a replicating portfolio for its contractual obligation. We demonstrate that the price of the portfolio that replicates all the cash flows of an annuity contract is Equation (16).

In the setting with systematic but no idiosyncratic longevity risk, we consider the liability associated with a contractholder who purchased $1/A$ unit(s) of contracts when aged x in the year $t_0=0$, retired in the year $t=t_R$, while being subject to unknown survival probabilities throughout the horizon, until the maximum age in the year $t=T$, when death is certain.

The pension provision entity is contractually obliged to make annual benefit payments from the individual's retirement in the year $t=t_R$ until he or she attains maximum age in the year $t=T$, conditional on her survival. Let W_t^{Ref} be the price at time t of the reference portfolio to which the benefits are indexed, $t\in[t_0,T]$.

Absent longevity risk, by purchasing the sum of all the units of the reference portfolio in Column (2) of Table 7 at time t, the annuity provider would be able to fulfill its contractual obligation with certainty. For instance, to meet the payment at time t_R, the annuity provider purchases $1/\left(AW_{t_0}^{Ref}\right)e^{-h\times 0}\,{}_{t_R-t}p_x^{(t_0)}$ units of the reference portfolio at time t_0. When longevity risk is absent, the conditional expectation, made at time t_0, of the individual's survival in year t_R coincides with the realized survival probability, i.e., ${}_{t_R-t}p_x^{(t_0)}={}_{t_R-t}p_x$. The value

Table 7: Future Cash Flow and the Best Replicating Portfolio of the Pension Provision Entity

This table shows the value of entitlements due in each year of retirement until maximum age (column (1)), and the corresponding Best Replicating Portfolio in units of the reference portfolio (column (2)). The Best Replicating Portfolio is the conditional expectation of the benefits in future value.

Time	**Benefits in Future Value**	**Best Replicating Portfolio (constructed at time t)** Units of the Reference Portfolio to purchase at time t
	(1)	(2)
t_R	$\frac{1}{A}\frac{W^{Ref}_{t_R}}{W^{Ref}_{t_0}} e^{-h(t_R-t_R)} \times \Pi_{l=t_0}^{t_R-1}\ {}_1p_{x+l-t_0}$	$\frac{1}{A}\frac{1}{W^{Ref}_{t_0}} e^{-h(t_R-t_R)}\ {}_{t_R-t}p^{(t)}_{x+t-t_0}$
t_R+1	$\frac{1}{A}\frac{W^{Ref}_{t_R+1}}{W^{Ref}_{t_0}} e^{-h(t_R+1-t_R)} \times \Pi_{l=t_0}^{t_R}\ {}_1p_{x+l-t_0}$	$\frac{1}{A}\frac{1}{W^{Ref}_{t_0}} e^{-h(t_R+1-t_R)}\ {}_{t_R+1-t}p^{(t)}_{x+t-t_0}$
t_R+2	$\frac{1}{A}\frac{W^{Ref}_{t_R+2}}{W^{Ref}_{t_0}} e^{-h(t_R+2-t_R)} \times \Pi_{l=t_0}^{t_R+1}\ {}_1p_{x+l-t_0}$	$\frac{1}{A}\frac{1}{W^{Ref}_{t_0}} e^{-h(t_R+2-t_R)}\ {}_{t_R+2-t}p^{(t)}_{x+t-t_0}$
$\vdots$	$\vdots$	$\vdots$
T	$\frac{1}{A}\frac{W^{Ref}_{T}}{W^{Ref}_{t_0}} e^{-h\times(T-t_R)} \times \Pi_{l=t_0}^{T-1}\ {}_1p_{x+l-t_0}$	$\frac{1}{A}\frac{1}{W^{Ref}_{t_0}} e^{-h(T-t_R)}\ {}_{T-t}p^{(t)}_{x+t-t_0}$

of this portfolio will evolve along with the financial market, to be worth exactly $\frac{1}{A}\frac{W_{t_R}^{Ref}}{W_{t_0}^{Ref}} \times \Pi_{l=t_0}^{t_R-1}\,{}_{l-t_0}p_{x+l-t_0}^{(l)}$, the payment due at time t_R. By the same reasoning for the rest of the entries in Column (2), Equation (19) is thus the total units of the reference portfolio to be held at any time t, such that the pension provision entity fully hedges financial market risk.

$$\int_{s=\max\{t_R,t\}}^{T} \frac{1}{A}\frac{1}{W_{t_0}^{Ref}} e^{-h(s-t_R)}\,{}_{s-t}p_{x+t-t_0}^{(t)}\,\mathrm{ds} \tag{19}$$

Equation (19) is an estimate of the liabilities at time t, in terms of the *units* of reference portfolio. Each unit is worth W_t^{Ref} at time t. To obtain the *value* of liabilities, we take the portfolio's corresponding value:

$$W_t^{Ref} \quad \times \quad \int_{s=\max\{t_R,t\}}^{T} \frac{1}{A}\frac{1}{W_{t_0}^{Ref}} e^{-h(s-t_R)}\,{}_{s-t}p_{x+t-t_0}^{(t)}\,\mathrm{ds} \tag{20}$$

As $\Xi(h,F,t,x) = W_t^{Ref}/\left(AW_{t_0}^{Ref}\right)e^{-h(t-t_R)}$ by definition, we can substitute it into Equation (20) to get

$$\begin{aligned} L_t \quad := \quad & \Xi(h^*,F,t,x)\int_{s=\max\{t_R,t\}}^{T} \exp\left(-h^*(s,\theta)(s-t)\right)\times \\ & {}_{s-t}p_{x+t-t_0}^{(t)}\mathrm{ds} \end{aligned} \tag{21}$$

Equation (21) is identical to Equation (16).

When there is systematic longevity risk, the Best Replicating Portfolio is identical to column (2) of Table 7, but this best estimate may not necessarily provide the exact cash flow to meet the annuity provider's contractual obligations because the realized survival probability may deviate from its conditional expectation made at time t, which then triggers the provider's default.

C Delta Method

We apply the Delta Method (Theorem 5.5.4 of Casella and Berger, 2002) to estimate the variance of the CELs, which is used to compute their confidence intervals.

Consider the function $g(x,y) = (x/y)^{1/(\gamma-1)} - 1$. By the definition of Equation (11), $CEL = g\left(U\left(\Xi^{GSA}\right), U\left(\Xi^{DVA}\right)\right)$. We estimate the CEL by plugging the expected utility into $g(.)$, $g\left(\mathbb{E}_0\left[U\left(\Xi^{GSA}\right)\right], \mathbb{E}_0\left[U\left(\Xi^{DVA}\right)\right]\right)$. Theorem 5.5.24 of Casella and Berger (2002) suggests the following estimate for its variance:

$$
\begin{aligned}
\operatorname{Var}\left\{g\left(\mathbb{E}_0\left[U\left(\Xi^{GSA}\right)\right],\right.\right. &= g_x^2\operatorname{Var}\left(U\left(\Xi^{GSA}\right)\right)+g_y^2\operatorname{Var}\left(U\left(\Xi^{DVA}\right)\right)+ \\
\left.\left.\mathbb{E}_0\left[U\left(\Xi^{DVA}\right)\right]\right)\right\} & \quad 2g_xg_y\operatorname{cov}\left(U\left(\Xi^{GSA}\right),U\left(\Xi^{DVA}\right)\right) \qquad (22)\\
g_x &= g_x\left(\mathbb{E}_0\left[U\left(\Xi^{GSA}\right)\right],\mathbb{E}_0\left[U\left(\Xi^{DVA}\right)\right]\right) \\
g_y &= g_y\left(\mathbb{E}_0\left[U\left(\Xi^{GSA}\right)\right],\mathbb{E}_0\left[U\left(\Xi^{DVA}\right)\right]\right)
\end{aligned}
$$

g_x and g_y denote the first partial derivative of $g(.)$ with respect to x and to y respectively. $\operatorname{Var}\left(U\left(\Xi^{K}\right)\right)$ for $K\in\{GSA,DVA\}$ and $\operatorname{cov}\left(U\left(\Xi^{GSA}\right),U\left(\Xi^{DVA}\right)\right)$ are estimated by the sample variance and sample covariance.

D Uncertainty of the Drift Parameter

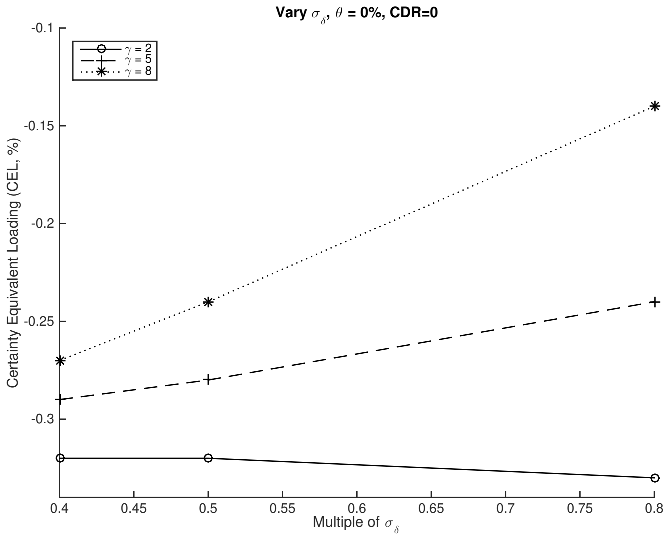
The maximum likelihood estimate for the drift term of the longevity model is normally distributed, $\hat{c}\sim\mathcal{N}\left(c,\sigma_c^2\right)$. Based on the sample used for the model calibration, we obtain $\hat{c}=-1.0689$ and $\widehat{\sigma_c}$=0.0521. Without parameter uncertainty, the best m-year-ahead forecast at time t is $\widehat{k_{t+m}}=m\hat{c}+k_t$. To incorporate parameter uncertainty, we draw c_l from the distribution $\mathcal{N}\left(\hat{c},\widehat{\sigma_c^2}\right)$ for the l^{th} simulation replication. The time trend governing longevity is thus $k_{t+m,l}=mc_l+k_{t,l}+\sum_{i=1}^{m}\varepsilon_{\delta,l}$, $\varepsilon_{\delta,l}\sim\mathcal{N}\left(0,\widehat{\sigma_\delta^2}\right)$, while the best m-year-ahead forecast relies on $\hat{c}$ as c_l is unobserved, i.e., $\widehat{k_{t+m,l}}=m\hat{c}+k_{t,l}$.

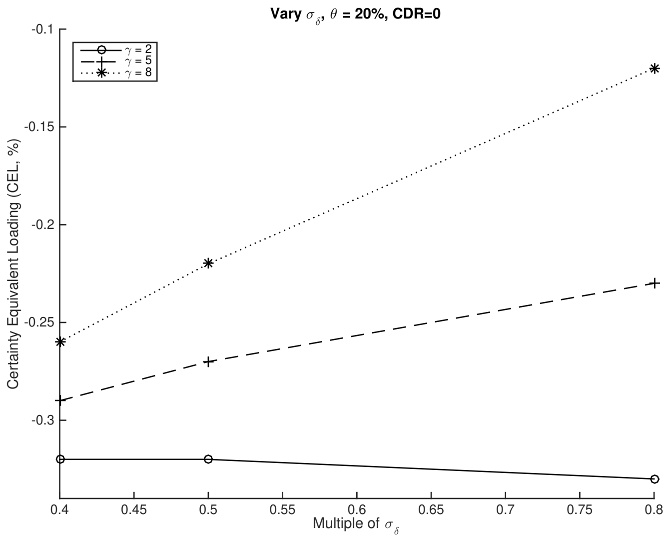
E σ_δ Generating Zero Default

When σ_δ is very small, the baseline 10% equity capital ensures that the probability of default is zero. The CEL for $\gamma=5,8$ increases in the multiple of σ_δ, conditional on zero default probability. When systematic longevity risk is small, risk averse individuals find the welfare improvement due to upward adjustments of GSA benefits outweigh the welfare cost of the scheme's more volatile payments, but this effect reverses when systematic longevity risk rises (Figure 6).

Figure 9: CEL and Multiple of σ_δ: CDR=0

This figure presents the CEL and the multiple of σ_δ for the base case's risk aversion levels, γ. The top panel is for $\theta = 0\%$, whereas the bottom panel is for $\theta = 20\%$.





Cross asset

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