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Asset Allocation under (one's own) Sovereign Default Risk

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His main fields are portfolio optimization, asset management, wealth management and tax incidence (in particular on investment return).

Abstract

The Greek drama of the late 2000s has returned sovereign risk awareness to centre stage. The default affected a country with a relatively developed economy. It resulted in huge losses in the value of domestic assets: public debt, but also private debt, equity, real estate and furthermore pension rights and human capital. The burden has, not entirely but importantly, fallen on residents.

Should a sovereign default happen, the consequences are therefore severe for investors, not only on the sovereign's debt, but also on all assets under the sovereign's jurisdiction, which are contaminated by the default.

Investors should take account of sovereign default in their investment plans. The perspective of sovereign default reinforces the case for international diversification and for leaning against home bias.

There are also implications for the asset management industry: it should lean against its own home bias and provide efficient solutions for cross-border investment.

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Keywords : Sovereign risk, Asset Allocation, International diversification, Home bias

1 – Introduction

The Greek drama of the late 2000s has returned sovereign risk awareness to centre stage. Not because there has never been a sovereign default before: they have been occurring for centuries, if not millennia, and Greece has experienced at least a dozen in the last two centuries. But recently, with the “Great Moderation”, we had been in a period of calm since the end of the 1990s. Admittedly, Greece was an old sinner but it is now part of the European Union and, for better or worse, part of the Eurozone: a European democracy that is not rich but actually not poor, and is a member of an exclusive club.

Traditionally, savings were concentrated in developed economies and promising investment opportunities were found in less developed economies. These potential investments were exposed to the risk of sovereign default in the region where they were made: the risk was direct in the case of sovereign funding and indirect in the case of mines, railroads, canals, etc. Although its potential importance was often neglected, sovereign risk was nonetheless viewed as a reality. But, in the minds of savers, sovereign risk was mainly a problem for foreign assets.

Greece, as a developed county, has been faced with the default of its own sovereign. This is a relatively new scenario, which may be relevant to other countries. There are good reasons why sovereign risk has increased and spread.

This paper focuses on the consequences of sovereign default for investors, and how they should modify their asset allocation to cope with it. The results are that they should diversify internationally more and avoid home bias.

2 – Sovereign risk

The increase in sovereign risk is reflected in the numerous downgrades of sovereigns in the rating agencies scales. Though the instrument is not perfect, it gives a quantification of sovereign default risk.

Table 1

S & P Rating	Selected coutries (mid-2015)	Indicative default rate at horizon (%) 1 year	5 years	10 years	15 years
AAA	Australia, Canada, Germany, Norway, Sweden, Switzerland,UK	0.00	0.42	0.87	1.24
AA	Austria, Belgium, Finland, France, Netherlands, US	0.04	0.47	1.11	1.59
A	Ireland, Poland, Greece (Fitch) 2005	0.08	0.77	2.01	3.02
BBB	Brasil, Italy, Mexico, Spain	0.24	2.16	4.84	6.91
BB	Turkey	0.84	8.58	15.43	18.91
B	Albania, Egypt	4.03	19.95	28.06	31.98

Default rates are derived from a study by Standard & Poors (2015) on US corporates and are therefore only indicative.

Why sovereign default has become an actual prospect in the future is tied to the huge built-up in sovereign debt in relation to the potential means for sovereigns to service it (Maillard, 2013a).

Assessing the risk of default by a sovereign borrower is a tricky business and the subject of sizeable literature. First and foremost, of course, risk depends on the size of public debt. This leads to the question of whether there exists a threshold for debt[1], below which risk is likely to be insignificant and above which it would be palpable. However, other factors are involved: the size of the sovereign's off-balance sheet liabilities, in particular retirement benefits; the tax burden level already reached, its concentration or dilution; and, finally, the dynamism of the economy, which provides the tax base.

The risk of default by a sovereign borrower therefore depends on the amount of its debt. Expressed in monetary units, the amount of debt doesn't mean too much. This is why it is standard practice to compare the value of debt to the size of economies. For households and

[1] A threshold of 90% for the debt-to-GDP ratio suggested by Reinhart and Rogoff and which has been the subject of lively debate and methodological criticisms concerns the impact on GDP growth. As GDP determines the dynamics of the taxable base, it naturally influences a crucial aspect of the sovereign's solvency.

corporations alike, income is frequently measured to assess the debt burden and establish limits. Some hold the view that household debt (in most instances contracted due to a home purchase) should not exceed three or four years of income. For corporations, debt is measured against earnings or cash flow.

For the sovereign, debt is most often measured against GDP and debt-to-GDP ratios are familiar to everyone. GDP corrects monetary and economy-size factors for the purpose of assessment and comparison of sovereign debts throughout time and space. Today, a number of developed countries are hovering around or exceeding a public debt to GDP ratio of 100%. This has been a growing trend for thirty or more years, without these countries having experienced any particular catastrophe, such as a war on their national territory[2]. Under these circumstances, exogenous factors do not properly account for the high ratio and the levels reached are historically unprecedented in certain respects. Several countries have experienced a public expenditure bubble owing to the accumulation of layers of clientelist spending in universal suffrage democracies (Bastiat, 1862, Olson, 1965).

Alongside debt there exists sizeable implicit debt: the commitments of retirement schemes that do not rely on real and financial assets but on the sovereign's power to tax. This is true not only of civil servants' pensions when they are paid directly out of the sovereign's pocket but also of pay-as-you-go schemes that rely on the sovereign's delegation of its power to tax by making contributions to such schemes compulsory. Such benefits, which are quite considerable in France and in other European countries, are equivalent to public debt. But even if this were not so, they restrict the sovereign's power to tax in order to tackle is own debt.

GDP is roughly equivalent to income[3]. But it is national income, that is, the income of the governed and not of the sovereign. The sovereign can levy only a share of national income and the size of national income is not unrelated to the percentage the sovereign seeks to obtain.

[2] An exception to this observation is perhaps Germany, which had to absorb the shock of reunification. Germany nevertheless appears as one of the fiscally healthiest countries.

[3] Not exactly, as GDP is gross of capital consumption, which is basically a loss of wealth during the period; in addition, GDP includes a conventional measure of production by government (so called non-marketable GDP), assessed as its producing cost, which exists only because of the taxes levied on the private sector. GDP therefore overstates the real (taxable) national income.

There are economic limits on the amount of tribute a sovereign can exact from its territory (Dupuit, 1844, Laffer, 1978). There are political limits, too, often referred to as “consent to taxation”.

In France, the tax burden is very heavy – one of the heaviest in the world – and it is increasingly concentrated on a minority. Perhaps we are not far from entering “law of the vital few” territory: 20% of the working and investing population is the source of 60% of all wealth creation and pays 80% of all taxes[4]. On the other hand, expenditure is increasingly concentrated on categories that are not paying the taxes: increased means testing for government benefits and subsidies.

Historically, sovereigns have exacted tribute first from conquered peoples and then from its own subjects. In exchange, sovereigns provided a few services to their subjects, the most important being protection of their person and property. Today sovereigns have a hand in multiple actions and a widening gap is forming between taxpayers and beneficiaries, which is likely to undermine the tax base.

Those considerations point to substantial, and increasing, sovereign default risk.

3 – Modelling the consequences of default

We rely on an as parsimonious as possible modelling of the asset allocation, by assuming that the investor has the choice between three assets:

- A risk-free asset at horizon *T*, if such thing exists, returning annually r_f. A unitary investment in the risk-free asset would thus bring $e^{r_f T}$ at horizon *T*.
- A portfolio of domestic risky assets, comprising sovereign debt, corporate debt, equity, real estate, … This portfolio, which may the market portfolio in broad sense, or may be an actively and dynamically managed portfolio, will return μ yearly on average. If, as we will assume in the illustrations, the risky portfolio return follows a Gaussian random law, a unitary investment will bring $e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}$ at horizon *T*, where ε is a normal standard random variable.

[4] The tax burden on capital in France is especially heavy (Maillard, 2013b).

- A portfolio of foreign risky assets, comprising debt, equity, real estate, … This portfolio, which may the market portfolio in broad sense, or may be an actively and dynamically managed portfolio, will return μ* yearly on average. It will be dynamically managed together with the domestic portfolio

A sovereign default, if it occurs, will provoke a loss on the value of sovereign debt, but also on the other components of the domestic portfolio. There are thus two regimes for the domestic portfolio. Il no sovereign default occurs (probability 1-π), its final value (for a unitary investment) will be $e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}$. Otherwise, if a sovereign default occurs (probability π), its value will be reduced to $(1-d)e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon} = e^{(\mu+\ln(1-d)/T-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}$, where d is the share lost (1-*d* is a sort of recovery rate in case of default).

The consequence of default is thus to decrease the mean return of the risky portfolio by a quantity $\ln(1-d)/T$

4 – The welfare cost of sovereign default

It is interesting at this stage to assess the welfare cost of sovereign default. For an investor who can invest W_0 only in the domestic portfolio, the resulting expected utility will be $E(U(W_T) = E(U(W_0e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})$ in the absence of sovereign default.

If sovereign default is possible with probability π, the expected utility becomes:

$$E(U(W_T) = (1-\pi)E(U(W_0e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}) + \pi E(U((1-d)W_0e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})$$

which is smaller than in the absence of default.

There are two usual was to measure the welfare loss. One would be to ask what additional initial wealth would be necessary to maintain the same level of expected utility. The second one is to ask which additional mean return *s* would be necessary to maintain the same level of expected utility. It is the solution of:

$$(1-\pi)E(U(W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}) + \pi E(U((1-d)^{1-\gamma} W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}) = E(U(W_0 e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})$$

Assuming that the investor's preferences are represented by a Constant Relative Risk Aversion (CRRA) utility function [5] with risk aversion parameter γ, this reduces to (see Appendix):

$$E(U(W_T) = (1-\pi+\pi(1-d)^{1-\gamma})e^{(1-\gamma)sT} E(U(W_0 e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})$$

For the expected utility to reach the same level as with no sovereign risk, we need to have:

$$\ln(1-\pi+\pi(1-d)^{1-\gamma}) + (1-\gamma)sT = 0$$

$$s = \frac{-\ln(1-\pi+\pi(1-d)^{1-\gamma})}{(1-\gamma)T}$$

We present here the value of the welfare s cost for a time horizon of 10 years, a moderate relative risk aversion not too high (γ = 2) and various default probabilities π at this horizon and non-recovery rates (d).

Table 2

Welfare cost of default (T=10, γ=2)

π \ d	0.25	0.5	0.75
1%	0.0333%	0.0995%	0.2956%
2%	0.0664%	0.1980%	0.5827%
3%	0.0995%	0.2956%	0.8618%
4%	0.1325%	0.3922%	1.1333%
5%	0.1653%	0.4879%	1.3976%

The welfare cost exceeds significantly the impact on the mean return of the probability of default times the non-recovery rate (-πln(1-d)/T)

[5] Using a CRRA utility function is convenient. One limitation, which it shares with other utility functions such as a Constant Absolute Risk Aversion (CARA) function is that there is only one parameter to describe risk aversion. An extension to this work could be to use a Bell's utility function, with two parameters (Canevaile, Coën and Hübner, 2009).

Table 3

Welfare cost of default (T=10, γ=2)

π \ *d*	0.25	0.5	0.75
1%	0.0333%	0.0995%	0.2956%
2%	0.0664%	0.1980%	0.5827%
3%	0.0995%	0.2956%	0.8618%
4%	0.1325%	0.3922%	1.1333%
5%	0.1653%	0.4879%	1.3976%

Impact of default on mean return

π \ *d*	0.25	0.5	0.75
1%	0.0288%	0.0693%	0.1386%
2%	0.0575%	0.1386%	0.2773%
3%	0.0863%	0.2079%	0.4159%
4%	0.1151%	0.2773%	0.5545%
5%	0.1438%	0.3466%	0.6931%

That is because sovereign default changes the distribution of returns by introducing negative skewness and excess kurtosis, i.e. tail risk.

Naturally, the welfare cost increases with risk aversion.

Table 4

Welfare cost of default

(T=10, γ=2)

π \ *d*	0.25	0.5	0.75
1%	0.0333%	0.0995%	0.2956%
2%	0.0664%	0.1980%	0.5827%
3%	0.0995%	0.2956%	0.8618%
4%	0.1325%	0.3922%	1.1333%
5%	0.1653%	0.4879%	1.3976%

(T=10, γ=5)

π \ *d*	0.25	0.5	0.75
1%	0.0534%	0.3494%	3.1674%
2%	0.1058%	0.6559%	4.5207%
3%	0.1570%	0.9289%	5.3939%
4%	0.2072%	1.1750%	6.0398%
5%	0.2564%	1.3990%	6.5526%

Note that for moderate risk aversions, the welfare cost does not depend a lot on the time horizon, provided the default probabilities are adjusted to be equal in annualized terms, to make things comparable.

Table 5a

Welfare cost of default

(T=10, γ=2)

π \ *d*	0.25	0.5	0.75
1%	0.0333%	0.0995%	0.2956%
2%	0.0664%	0.1980%	0.5827%
3%	0.0995%	0.2956%	0.8618%
4%	0.1325%	0.3922%	1.1333%
5%	0.1653%	0.4879%	1.3976%

(T=5, γ=2)

π \ *d*	0.25	0.5	0.75
0.50%	0.0334%	0.1000%	0.2985%
1.01%	0.0669%	0.2000%	0.5941%
1.51%	0.1005%	0.3000%	0.8869%
2.02%	0.1342%	0.4001%	1.1769%
2.53%	0.1681%	0.5001%	1.4643%

This is less true for higher risk aversions.

Table 5b

Welfare cost of default

(T=10, γ=5)

π \ *d*	0.25	0.5	0.75
1%	0.0534%	0.3494%	3.1674%
2%	0.1058%	0.6559%	4.5207%
3%	0.1570%	0.9289%	5.3939%
4%	0.2072%	1.1750%	6.0398%
5%	0.2564%	1.3990%	6.5526%

(T=5, γ=5)

π \ *d*	0.25	0.5	0.75
0.50%	0.0539%	0.3625%	4.1169%
1.01%	0.1074%	0.7021%	6.3528%
1.51%	0.1607%	1.0217%	7.8991%
2.02%	0.2136%	1.3236%	9.0839%
2.53%	0.2663%	1.6097%	10.0456%

It appears from the previous tables that the welfare cost of default is nearly proportionate to the probability of default. That is what the first-order development gives:

$$s \approx \frac{\pi(1-(1-d)^{1-\gamma})}{(1-\gamma)T}$$

On the other hand, it increases more than proportionately with loss given default parameter *d*.

5 – Asset allocation model

5.1 – The investor's objectives

Following a long trail in the economic and financial literature, we assume that a representative investor has a (long run) horizon T and is a Neuman-Morgenstern expected utility maximizer. Her objective is thus to maximize:

$$E[U(W_T)]$$

U is the utility function depending of the investor's wealth W at time T. For the applications, we will conveniently use a constant relative risk aversion (CRRA) utility function with risk aversion parameter γ.

The investor's choice is to allocate a share α or her wealth into the domestic risky portfolio, a share α^* into the foreign risky portfolio, the rest 1-α-α^* being invested into the risk-free asset if it exists.

5.2 – International diversification without sovereign risk

With a CRRA utility function, the optimal asset allocation may be derived exactly (see Appendix).

$$\alpha = \frac{1}{\gamma}\frac{t-\rho t^*}{\sigma(1-\rho)}$$
$$\alpha^* = \frac{1}{\gamma}\frac{t^*-\rho t}{\sigma^*(1-\rho)}$$

The relative shares allocated to the domestic and foreign risky assets do not depend on risk aversion. They depend on the Sharpe ratios of the portfolios t and t^*, on the volatility of their returns and on the correlation between them.

We focus on the share of foreign risky assets in the total of risky assets. This is

$$s^* = \frac{\alpha^*}{\alpha+\alpha^*} = \frac{(t^*-\rho t)/\sigma^*}{(t^*-\rho t)/\sigma^*+(t-\rho t^*)/\sigma}$$
$$s^* = \frac{\alpha^*}{\alpha+\alpha^*} = \frac{1}{1+\dfrac{(t-\rho t^*)\sigma^*}{(t^*-\rho t)\sigma}}$$

As a base case, we will choose excess (above risk-free rates) returns of 4% for domestic risky assets (the domestic "risk premium"), 20% volatility (which is common for equity portfolios, maybe a little high for diversified risky portfolios) and a correlation of .5.

Table 6

Share of risky foreign assets in total risky assets

$\sigma^* \backslash \mu^* - r_f$	2%	3%	4%	5%	6%
15%	25.0%	57.1%	76.9%	90.3%	100.0%
20%	0.0%	28.6%	50.0%	66.7%	80.0%
25%	-11.1%	10.3%	28.6%	44.4%	58.3%
30%	-15.4%	0.0%	14.3%	27.6%	40.0%
35%	-16.7%	-5.5%	5.4%	16.0%	26.3%

Note that except in the case of very poor foreign expected return or very high volatility for those returns, the optimal share of foreign assets is substantial, and much higher than usually recorded. It does depend on the risk aversion parameter.

Why would foreign expected returns be very low compared to domestic expected returns ? The most often put forward reason is related to transaction fees in a broad sense : acquiring and managing foreign assets induce additional fees. There are also possibly higher information costs, and foreigners are at higher risk of acquiring lemons.

As for volatility, it is basically lower for foreign assets, as the markets are wider and more diversified (even for a US investor). One has to rely on exchange rate risk, if this is not negatively correlated with returns and is difficult to hedge, to make foreign assets looking as more risky than domestic ones..

We will go back to the home bias issue on which there is a consequent literature (Coeurdacier, Nicolas & Hélène Rey, 2011). We will examine how the optimal share of foreign assets is affected by the possibility of domestic sovereign default .

5.3 – International Diversification with sovereign risk

Our investor will still be maximizing her expected utility by investing a share α in the domestic risky portfolio and a share α^* in the foreign risky portfolio.

If there is no domestic sovereign default, the final value of the investment will be:

$$W_T = W_0 e^{(M-\Sigma^2/2)T+\Sigma\sqrt{T}\varepsilon}$$

With:

$$M = r_f + \alpha(\mu - r_f) + \alpha^*(\mu^* - r_f)$$
$$\Sigma^2 = \alpha^2\sigma^2 + \alpha^{*2}\sigma^{*2} + 2\alpha\alpha^*\rho\sigma\sigma^*$$

If there is a domestic sovereign default, only the domestic part of the risky portfolio will be impacted, which means that the final value of the whole portfolio will reach:

$$W_T = (1-d)^\alpha W_0 e^{(M-\Sigma^2/2)T+\Sigma\sqrt{T}\varepsilon}$$

The expected utility will therefore write:

$$E(U(W_T) = (1-\pi)E(W_0 e^{(M-\Sigma^2/2)T+\Sigma\sqrt{T}\varepsilon}) + \pi E\left((1-d)^\alpha W_0 e^{(M-\Sigma^2/2)T+\Sigma\sqrt{T}\varepsilon}\right)(1-d)^\alpha W_0 e^{(M-\Sigma^2/2)T+\Sigma\sqrt{T}\varepsilon}$$

With a CRRA utility function, the first order conditions for maximization still give, for the derivative relating to the share of foreign assets (see Appendix):

$$\alpha^* = -\alpha\rho\sigma/\sigma^* + \frac{t^*}{\gamma\sigma^*}$$

The other derivative relating to the share of domestic assets gives a more convoluted expression:

$$\left[\pi(1-d)^{\alpha(1-\gamma)}\ln(1-d)\right] + \left[1-\pi+\pi(1-d)^{\alpha(1-\gamma)}\right]T\left[-\gamma\sigma^2(1-\rho^2)\alpha + \sigma(t-\rho t^*)\right] = 0$$

which can be solved only numerically.

This allows us to compute how the optimal share of foreign assets in the risky portfolio varies with the domestic sovereign default rate and loss in case of default. The illustration below is constructed in the base case when risk excess returns are equal to 4%, domestically and abroad, and volatilities are equal to 20%, with the correlation coefficient equal to 0.5, and for a ten-year horizon.

Table 7

Share of risky foreign assets in total risky assets (T=10, γ=2)

π \ d	0.25	0.5	0.75
0%	50.00%	50.00%	50.00%
1%	50.59%	51.63%	54.02%
2%	51.19%	53.27%	57.85%
3%	51.79%	54.89%	61.55%
4%	52.39%	56.52%	65.13%
5%	52.99%	58.15%	68.63%

In the absence of sovereign default risk, the optimal share of foreign assets is 50%. This share increases significantly in the presence of domestic sovereign default risk.

Note that, rather counterintuitively, the results are not much more pronounced with higher risk aversion, as appears from Table 8

Table 8

Share of risky foreign assets in total risky assets (T=10, γ=5)

π \ d	0.25	0.5	0.75
0%	50.00%	50.00%	50.00%
1%	50.63%	51.86%	55.06%
2%	51.26%	53.69%	59.56%
3%	51.89%	55.49%	63.66%
4%	52.52%	57.27%	67.48%
5%	53.15%	59.02%	71.09%

This may be because diversification within the risky portfolio has a benefit for risk-adverse investors, even if one the diversifier asset is of low quality.

Finally, to test the robustness of the impact sovereign default on the asset allocation, we consider a situation that is less favourable to foreign assets, by degrading by one percent their expected return.

Table 9

Share of risky foreign assets in total risky assets (T=10, γ=2, μ* /-1%)

π \ d	0.25	0.5	0.75
0%	28.57%	28.57%	28.57%
1%	29.17%	30.27%	32.99%
2%	29.77%	31.97%	37.21%
3%	30.37%	33.67%	41.30%
4%	30.97%	35.37%	45.28%
5%	31.58%	37.07%	49.18%

The impact of sovereign risk is to add around 2 percentage points to the share of foreign asset per one per cent of probability default at a ten year horizon with a recovery rate of 50%, and more than 4 percentage points if the recovery rate is just 25%.

6 – Sovereign risk and home bias

In practice, people tend to own much less foreign assets than optimality would suggest. Typically, domestic equity represents the major share of equity portfolios, even for relatively small countries (51 % in Switzerland, 76 % in Australia, 77 % in the United States, 99 % in Brazil and China[6]).

Home bias is even more pronounced in real estate. In France for example, foreigners just own 4 % of residential real estate[7]. There are several explanations to this bias.

One is that the expected return on foreign assets may be lowered by higher costs : higher management fees, higher taxes (some countries have tax-favoured saving plans where only domestic assets can be hold, there may be double taxation of the returns of foreign assets), or higher informational costs. In selecting the assets, foreigners may be more prone to end up with the ‘lemons”.

A portfolio of foreign assets should not be riskier than a portfolio of domestic assets, as there is more opportunity to diversify. There is a good economic case for national economies to

[6] Source : Coeurdacier & Rey, 2011
[7] Source : File FILOCOM, French official statistic

specialize and concentrate on their comparative advantages, à la Pareto. The result is a concentration of domestic economies on particular sectors.

However, perceived risk may be higher due to exchange rate risk. At short horizon, exchange risk is normally easy to hedge at relatively low cost. At a long horizon, what counts is the real exchange rates. If long term misalignments cannot be dismissed, there are good reasons for real long term exchange rates to be negatively correlated with the return differential, reducing the risk of holding foreign assets.

Finally, there are good reasons to have a home bias as far as homes are concerned. Self-occupied real estate has many advantages. It is generally tax-favoured, compared to renting and leasing, and it may be considered as a sort of hedge for an important part of the consumption basket. However, not everyone is a home-owner and there are other forms of real estate (leasing, second homes, non-residential), for which a home bias seems to exist.

The possibility of sovereign default increases the potential cost of home bias.

7 – Conclusions

Taking into account one's own sovereign default risk should therefore lead to take more advantage of international diversification in portfolio choices. Sovereign default risk enhances the cost of home bias for investors.

The wealth and asset management industries should help to reduce the practical and informational costs of investing abroad.

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Appendix

Let's consider a portfolio whose mean return is M and volatility is Σ. The final value of wealth will be:

$$W_T = W_0 e^{(M-\Sigma^2/2)T+\Sigma\sqrt{T}\varepsilon}$$

With a CRRA utility function,

$$U(W_T) = (1-\gamma)^{-1} W_0^{1-\gamma} e^{(1-\gamma)(M-\Sigma^2/2)T+(1-\gamma)\Sigma\sqrt{T}\varepsilon}$$

As $E(e^{b\varepsilon}) = e^{b^2/2}$ for a normal standard random variable,

$$E(U(W_T)) = (1-\gamma)^{-1} W_0^{1-\gamma} e^{(1-\gamma)(M-\Sigma^2/2)T} e^{(1-\gamma)^2\Sigma^2 T/2} = (1-\gamma)^{-1} W_0^{1-\gamma} e^{(1-\gamma)RT}$$

with $R = (M - \Sigma^2/2) + (1-\gamma)\Sigma^2/2 = M - \gamma\Sigma^2/2$

The expected utility is thus equivalent to the utility provided by a risk-free return (certainty equivalent return) equal to R.

Welfare loss of sovereign default

We compute the expected utility of a portfolio entirely composed of domestic assets, whose mean return is $\mu+s$

$$U((1-d)W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}) = (1-d)^{1-\gamma} U(W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})$$
$$E(U((1-d)W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})) = (1-d)^{1-\gamma} E(U(W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}))$$
$$E(U(W_T) = (1-\pi)E(U(W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon}) + \pi(1-d)^{1-\gamma} E(U(W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})$$
$$E(U(W_T) = (1-\pi+\pi(1-d)^{1-\gamma})E(U(W_0 e^{(\mu+s-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})$$
$$E(U(W_T) = (1-\pi+\pi(1-d)^{1-\gamma})e^{(1-\gamma)sT} E(U(W_0 e^{(\mu-\sigma^2/2)T+\sigma\sqrt{T}\varepsilon})$$

Optimization with a domestic risky portfolio

If the portfolio is composed of a share α in the domestic risky portfolio (volatility σ), and the rest in the risk-free asset, then:

$$R = M - \gamma\Sigma^2/2 = r_f + \alpha(\mu - r_f) - \gamma\alpha^2\sigma^2/2$$

R, and therefore expected utility, is maximum for

$$(\mu - r_f) - \gamma\alpha\sigma^2 = 0$$

$$\alpha = \frac{\mu - r_f}{\gamma\sigma^2} = \frac{t}{\gamma\sigma}$$

where t is the Sharpe ratio of the domestic portfolio.

Optimization with a domestic and a foreign risky portfolio

If the portfolio is composed of a share α in the domestic risky portfolio, a share α^* in the foreign risky portfolio, and the rest in the risk-free asset, then

$$R = M - \gamma\Sigma^2 / 2 = r_f + \alpha(\mu - r_f) + \alpha^*(\mu^* - r_f) - \gamma\left[\alpha^2\sigma^2 + \alpha^{*2}\sigma^{*2} + 2\alpha\alpha^*\rho\sigma\sigma^*\right]/2$$

where ρ is the correlations between domestic and foreign returns.

The first-order conditions for maximizing R, and thus expected utility, are

$$(\mu - r_f) - \gamma\left[2\alpha\sigma^2 + 2\alpha^*\rho\sigma\sigma^*\right]/2 = 0 \quad \mu - r_f = \gamma(\alpha\sigma^2 + \alpha^*\rho\sigma\sigma^*)$$

$$\mu^* - r_f = \gamma(\alpha^*\sigma^{*2} + \alpha\rho\sigma\sigma^*)$$

$$\sigma\alpha + \rho\sigma^*\alpha^* = \frac{\mu - r_f}{\gamma\sigma} = \frac{t}{\gamma}$$

$$\rho\sigma\alpha + \sigma^*\alpha^* = \frac{\mu^* - r_f}{\gamma\sigma} = \frac{t^*}{\gamma}$$

This solves easily into:

$$\alpha = \frac{1}{\gamma}\frac{t - \rho t^*}{\sigma(1-\rho^2)}$$

$$\alpha^* = \frac{1}{\gamma}\frac{t^* - \rho t}{\sigma^*(1-\rho^2)}$$

Optimization with a domestic sovereign default risk

In case of no default (probability 1-π), the final value of wealth will be:

$$W_T = W_0 e^{(M - \Sigma^2/2)T + \Sigma\sqrt{T}\varepsilon}$$

In case of default (probability 1-π), the domestic part of the risky portfolio will lose d of its value.

Therefore, the final value of wealth will be:

$$W_T = (1-d)^\alpha W_0 e^{(M - \Sigma^2/2)T + \Sigma\sqrt{T}\varepsilon}$$

The expected utility will reach:

$$E(U(W_T)) = (1-\pi)(1-\gamma)^{-1} W_0^{1-\gamma} e^{(1-\gamma)RT} + \pi(1-\gamma)^{-1} W_0^{1-\gamma} (1-d)^{\alpha(1-\gamma)} e^{(1-\gamma)RT}$$
$$E(U(W_T)) = (1-\gamma)^{-1} W_0^{1-\gamma} \left[1-\pi+\pi(1-d)^{\alpha(1-\gamma)}\right] e^{(1-\gamma)RT}$$

The partial derivative with respect to the share of foreign risky assets α^* is still equal to zero when:

$$\frac{\partial R}{\partial \alpha^*} = (\mu^* - r_f) - \gamma\left[2\alpha^* \sigma^{*2} + 2\alpha\rho\sigma\sigma^*\right]/2 = 0 \quad \alpha^* = -\alpha\rho\sigma/\sigma^* + \frac{1}{\gamma}\frac{\mu^* - r_f}{\sigma^{*2}} = -\alpha\rho\sigma/\sigma^* + \frac{t^*}{\gamma\sigma^*}$$

As for the partial derivative with respect to the share of foreign assets α, it is equal to:

$$(1-\gamma)^{-1} W_0^{1-\gamma} \left\{ \left[\pi(1-d)^{\alpha(1-\gamma)}(1-\gamma)\ln(1-d)\right] e^{(1-\gamma)RT} + \left[1-\pi+\pi(1-d)^{\alpha(1-\gamma)}\right](1-\gamma)T \frac{\partial R}{\partial \alpha} e^{(1-\gamma)RT} \right\}$$

It is equal to zero when:

$$\left[\pi(1-d)^{\alpha(1-\gamma)}\ln(1-d)\right] + \left[1-\pi+\pi(1-d)^{\alpha(1-\gamma)}\right] T \frac{\partial R}{\partial \alpha} = 0$$
$$\frac{\partial R}{\partial \alpha} = \mu - r_f - \gamma\left(\alpha\sigma^2 + \alpha^* \rho\sigma\sigma^*\right) = \mu - r_f - \gamma\left(\alpha\sigma^2 - \alpha\rho^2\sigma^2 + \rho\sigma\sigma^* \frac{t^*}{\gamma\sigma^*}\right)$$
$$\frac{\partial R}{\partial \alpha} = \mu - r_f - \gamma\sigma^2(1-\rho^2)\alpha - \rho\sigma t^* = -\gamma\sigma^2(1-\rho^2)\alpha + \sigma(t - \rho t^*)$$

Finally, α solves:

$$\left[\pi(1-d)^{\alpha(1-\gamma)}\ln(1-d)\right] + \left[1-\pi+\pi(1-d)^{\alpha(1-\gamma)}\right] T \left[-\gamma\sigma^2(1-\rho^2)\alpha + \sigma(t-\rho t^*)\right] = 0$$

This equation may only be solved numerically.

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