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Incorporating ESG Risk In Fundamental Market Risk Models

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Abstract

This paper investigates the possible impact of ESG Risk when incorporated into front office driven Fundamental Market Risk Measurement approaches. The main principle is, that ESG risk is implicitly embedded in observable market risk factors, like share prices and credit spreads, and interprets the ESG risk of an equity portfolio as an additional jump component to an ordinary GBM process. Thereby, the interdependency of economic entities over different industry sectors is modelled dedicatedly, by using two correlation matrices, for the continuous part and the jump part. These are used by a Gaussian Copula to generate respective correlated equity return movements over time. Further, hazard rates of possible jumps are taken as exogenously given, they are directly derived from Environmental Rating data. Thereby, the hazard rate and the mapped environmental rating carry both the interpretation of the Expected Number of Adverse Jumps during 250 trading days. It is also shown, portfolio risk can be additively decomposed to the single position level, and each position level into the contribution of (1) Ordinary Market Risk, (2) Jump Correlation Risk, and (3) Pure Hazard Rate Jump Risk. In order to calibrate the model, we propose to clearly distinguish between Systematic Environmental risk - which is caused by the product - and Specific Environmental risk - that is caused by the production process. Further, we view a Company as the sum of its Economic Entities, where the activities of each entity belongs to one single economic sector. Each economic entity stands for a jump component in the jump diffusion model. The simulation results show that on a 250 trading day horizon Environmental Risk is on a diversified portfolio level only relevant for longer time horizons (e.g. greater than 50 days), or in case of stressed scenarios. Our simulations indicate, that environmental rating based Exclusion Lists and Exposure Limits on companies with low environmental rating, would already do the job of managing or - more precisely - efficiently restricting current ESG risk. Finally, we derive modification factors based on the Weighted Average Portfolio Hazard Rate and the Weighted Average Portfolio Jump-Correlation, that enable the user for example to adjust a Historical VaR simulation appropriately to consider ESG Risk.

Keywords: ESG Risk, Environmental Risk, Financial Market Risk, Risk Measurement, Stochastic Modelling, Monte Carlo Simulation.

JEL classification: G17, Q5

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Contents

1 Motivation

This working paper is motivated by the current struggle to restrict the climate change by establishing and adhere to criteria that promote primarily **E**nvironmental, but also **S**ocial and **G**overnmental, aspects. These aspects are usually abbreviated by **ESG**. Due to the urgency of the topic, there is currently a large amount of terms and definitions, that partly refer to the same topic, but differ in their meaning and content, e.g. there is still no standard definition of ESG Risk and Sustainable Investment available. Besides these qualitative aspects, there is a need on how to enhance existing quantitative concepts of risk measurement and performance measurement, to account for ESG driven risk and performance. This is the main focus of the paper, while also trying to give some overview of the theme. Thereby, the paper tries to incorporate ESG risk into existing front office driven Fundamental Market Risk Models.

This is done instead of establishing an independent -– rating based -– model, as it was introduced by the banking industry for its dedicated credit risk exposure. This does not mean that **ESG Ratings** are neglected, instead they play an important role to calibrate our models properly. Further, we do not go for accounting figure driven approaches like discounting future EBIT figures or the company value – impacted by CO2 Price increase or CO2 Tax, cf. Friedl et al., 2021 for an overview.

By **Fundamental Market Risk Models**, we refer to models that are front office driven and use a pricing formula of the asset that is based on risk factors that are directly observable at the capital markets, like Share Prices, Interest Rates and Credit Spreads. We also separate our approach from models that use economic indicators or constructed CAPM-like indices to explain observable market risk factors by statistical regression. For example the equity share price or equity return could be explained by several independent economic indicators via regression. Roncalli et al., 2020 and Görgen et al., 2019 provide an extended Capital Asset Pricing Model, where the return over the risk free rate of the classical CAPM is not only explained by the return of the market portfolio, but also by an index they refer to as the Brown-Minus-Green return index (BMG). The risk measurement approach of MSCI Barra is an example for a statistical regression model that is very suitable to incorporate the BMG index as an explaining variable, while MSCI Risk Metrics would be a model we consider of the type fundamental, and this fundamental way we follow in our paper.

The paper starts to review the framework in which our approach is embedded, and then briefly touches how to incorporate ESG risk into a Normal Distribution. The main part of the paper is concerned with our view of how ESG risk is reflected by existing market risk factors like equity share prices and credit spreads, whereby we will concentrate in this paper on equity portfolios. Finally, a simulation study compares the developed concept of Jump Diffusion for different ESG rating and cross company correlation scenarios with ordinary Geometric Brownian Motion modelling. This is done for the risk factor "equity price" of an equity portfolio, covering 49 members of the EuroStoxx50[1]. While our approach differs remarkably from those of Agliardi and Karydas, we have the same starting point which is the jump diffusion model for equity or asset prices, which enables in principle to reconcile between the different concepts.

Finally, based on the investigation of the simulation results, we provide some analytical properties of the model like additive risk decomposition and delta-sensitivity, as well as a rule of thumb that tries to avoid a regular full Monte Carlo simulation as much as possible, but just to modify ordinary VaR figures, dependent on the Weighted Average Portfolio Hazard Rate, and the Weighted Average

[1] Agliardi and Agliardi, 2021 use a model very similar to our jump diffusion, in order to consider carbon emission risk for the pricing of defaultable bonds. For the calibration of the needed hazard rate they refer to Karydas and Xepapadeas, 2019. While for our paper exogenous calibration of the model - based on given ESG rating data – is a main topic. Further, we want to extend the risk scope from carbon risk to environmental risk and even ESG risk. The idea to go for a jump diffusion process for equity returns was actually already proposed by Merton, 1976.

Portfolio Correlation. The result should serve to enable the user to clearly distinguish how ESG risk will impact market risk. This impact may vary, depending on whether we are in a **Transition Phase** to more sustainable investment driven markets, or whether such a transition is finalized.

2 Framework of the Model

2.1 Definition of a Company

What is produced by **Company** $c \in \{1, ..., n\}$, its products/services or **Activities**, distribute to the economic sectors $s \in \{1, ..., k\}$[2], that form some kind of equivalent classes insofar the companies therein produce the same product. More precisely, we introduce the term **Economic Entity**, which is part of company c, and whose economic activities belong to exactly one industry sector s. A company c is thus made up by its economic entities $\{c_1, \ldots, c_k\}$ with respective economic activities and company sales share $Q(c_s)$ each. Each economic entity operates in exactly one economic sector $s \in \{1, \ldots, k\}$. If we refer to all $n \cdot k$ many economic entities that exist in the considered market, then we use the notation $e \in \{1, ..., n \cdot k\}$. In case we want to identify to which sector and company an economic entity e belongs, we can write $e(c, s)$ and then we have the identity $e(c, s) = c_s$. In case there is only one sector in which company c acts, then we have even the identity $e(c, s) = c_1 = c$. For the aggregate sales share $Q(c)$ of the company over all sectors it operates in via its economic entities, we have:

$$Q(c) = \sum_{s=1}^{k} Q(c_s) = 1 \quad \text{with} \quad Q(c_s) \geq 0 \tag{2.1}$$

Hence, for some economic sectors there may be no activity of the company, and the number of possible sectors is equal for all companies. Further, this is the **Definition of a Company** as the **sum of its activities/products** they sell to clients. This definition follows the idea given by the EU Taxonomy concept, where a company is analyzed according to its economic activities[3]. We will need this concept, for the model we use for Monte Carlo simulation, cf. Section 3.3

We assume within each (equivalent) class of economic entities (which is the industry sector) the same impact on ESG by the activities/products of the economic entities therein. Note, from a theoretical point of view, we find it useful to distinguish between this primary environmental impact caused by the product, and the secondary environmental impact caused by the production process[4]. It leads us to the notion of **Systematic** and **Specific Environmental Risk**. This is the environmental part of ESG risk, where we are mainly interested in.

Let us consider the example of Crude Oil production, while this is very problematic in view of the environment and climate, a difference within this industry sector can be made, by how efficient and ESG compliant an economic entity uses the necessary resources to create this product. This leads to the term **Best-In-Class**, and the respective approaches to measure this feature: like minimized Power Consumption, Greenhouse Gas Emission, minimal use of Water and Waste Recycling – during the production process. We will often abbreviate Best-In-Class by **BiC** in the sequel.

Thereby, one needs to be aware that BiC should ideally be a relative measure within the considered class. That is to say, it is[5] "[...] reflecting the ESG performance of a company compared to the average performance of its industry.". The impact of the products of the industry class on the environment is absolute and systematic, as far as the respective industry sector is concerned, as

[2]Sometimes we will also use the index variables i and j to denote the economic sector, instead of s. But this will be mentioned explicitly.

[3]The European Commission states: "The EU taxonomy is a classification system, establishing a list of environmentally sustainable economic activities. [...] The EU taxonomy would provide companies, investors and policymakers with appropriate definitions for which economic activities can be considered environmentally sustainable.", cf. EU, n.d.-a

[4]Product and Production Process environmental impact is related to the notion of Downstream and Upstream impact on environment. Upstream is everything that is needed before the company starts its core production process, and Downstream is what comes after the company has completed the product, cf. Funk C M, n.d. for a good and structured explanation.

[5]Amundi ESG Rating Methodology, July 2020, p. 8.

it is equal for all companies that produce this same product/service.

In this sense an electric car is not the same product as a car that uses gasoline, even though both are cars. As concerns ESG rating approaches, that are mainly based on the Best-In-Class concept, we observed that even though they mainly focus on the production process, there is no strict separation as we propose it above, instead they bring product information into the rating, in case the product is supposed to be a Sustainable Activity[6]. A good example is the Automobile Industry, where "Green Car" is an additional 5-th rating criteria, besides Energy & Emissions, Water Management, BioDiversity & Pollution, Supply Chain. In this respect, they consider this product information a company specific feature. As a possible example consider a car manufacturer, whose production process alone (specific risk) would lead to a bad rating, while the product is completely Green - finally yielding a rating that is some notches better. Another interesting example is the industry sector of Transportation: the product/service is just movement, while the production process is using for example train or ship, and the main topic is how the movement is achieved, that is the production process.

A possible interpretation of this notion of systematic and specific ESG risk is, that the impact of the economic activity/product on environment is **Systematic ESG risk**, while being Best-In-Class minimizes the company's **Specific ESG risk**[7]. On the other hand, there are industry sectors where the main ESG risk is in the production process, instead of the product. Electricity and Transport are examples, one can produce electricity with wind, water or by burning coal, to cite the most controversial productions processes in this sector. These specifics are to be taken into account in course of the risk model calibration.

We are of the opinion that the ESG vulnerability of a product and its impact on company returns and environment, that is systematic ESG risk for the industry sector, will play a more important role in future as concerns possible adverse impacts on the company value. This should be reflected in a clear separation between systematic and specific ESG risk, and lead to a separate rating for product related (systematic) ESG risk.

Even though we are of the opinion that a strict separation between systematic ESG risk and specific ESG risk in the rating process is desirable, we observe that this is not perfectly the case in reality, but ratings cover primarily specific indicators that are mixed up to some extend by systematic components. We will come back to this issue in course of our model calibration.

2.2 Definition of ESG Risk

Based on the outline of the last section we explain two possible approaches to define ESG risk, and a possible synthesis of these two definitions. First is to say that ESG risk separates into the components

1. **Systematic ESG Risk**, that is related to the product or economic activity and
2. The **Company Specific ESG Risk**, that is related to the production process.

Instead of sticking to systematic and specific risk, one can view ESG risk from a completely different angle, driven by what is the **Source of ESG risk**. This leads to the notion of a possible negative impact of

1. the **Transition** to an ESG compliant state of the world, on the return of financial investments, mainly caused by political decisions (e.g. introduction of carbon tax), and the

[6] In addition to the hybrid Best-In-Class approach, the Asset Management industry applies exclusion lists, in order to restrict the systematic ESG risk impact of their investments. Examples are the Tobacco industry or the Military industry.

[7] MSCI, n.d. states: "Companies are rated on a AAA-CCC scale relative to the standards and performance of their industry peers".

2. negative impact of the change of **ESG Factors** on financial investments (e.g. climate catastrophe or water pollution).

If we compare the notion of systematic and specific ESG risk with ESG transition risk and physical risk, we observe that transition risk is made up by systematic (product/activity) and specific (process) risk, while physical risk is independent from transition risk and an additional impact, which depends on the geographic location of the company and its industry sector, as some industry sectors are more exposed to physical risk (e.g. Reinsurance Companies). We further note that current rating approaches are more in line with this second notion of ESG risk above.

One can bring the two notions of ESG risk together by separating the sources of risk into a systematic and specific source, leading to

1. **Transition Risk**, caused by political decision, is related to
   (a) the product (systematic risk), and to
   (b) the production process (specific risk)
2. Similarly, **Physical Risk** can be caused by
   (a) the product or by
   (b) the production process

The separation of transition risk into a product driven and a production process driven part can serve as a starting point to refine industry sector specific stress scenarios. Industry sector specific scenarios are considered in course of the second part of our simulation study, that is based on the notion of transition- and physical risk.

Within this paper, we focus on the environmental part of ESG risk, but the principle of our approach extends easily to the whole ESG scope. We model the stochastic return with a continuous and a discontinuous jump component. The jump component focuses on substantial changes of observable risk factors that drive the portfolio return, while gradual changes are covered by the continuous part. The hazard rate of the jump is directly linked to the observed E- or ESG rating of the considered company. Similar to a Credit Rating there are ESG risk rating agencies like MSCI and Systainalytics that provide **ESG Rating Levels**. In this paper we heavily rely on the existence of such exogenously given rating data, to calibrate the jump part of our stochastic model. For example the 7 possible MSCI ESG rating levels are $\{AAA, AA, A, BBB, BB, B, CCC\}$[8]. Following this principle, we assume for this paper the availability of rating data of 7 rating classes $\{A, B, C, D, E, F, G\}$ for the rating types/themes listed below:

1. Aggregated Environmental Rating
2. Environmental Rating related to the Production Process (specific risk)
3. Environmental Rating related to the Product (systematic risk)
4. Transition Risk Rating
5. Physical Risk Rating

As far as systematic environmental risk is concerned, we use the CO2 Intensity Industry Sector Average to derive a systematic environmental rating and subsequently we link the respective industry sector hazard rate to this rating. This will be explained in detail by Section 3.3.1. As regards rating data for transition risk, physical risk and company specific Best-In-Class rating, these kind of data are readily provided by external ESG rating agencies. Some market participants

[8] Cf. MSCI, n.d.

also maintain respective inhouse rating research[9].

Karydas and Xepapadeas, 2019 calculate the Environmental Risk Premium for equity required by the markets within a macroeconomic portfolio optimization environment, where their model portfolio consists of General Equity Assets and Brown Equity Assets and a Risk Free investment. The needed equity valuation formula is the present value of all future dividends, and the discounting is based on a stochastic pricing kernel, which is derived from the utility function of the representative investor. For this they model the environmental hazard rate endogenously by a mean reverting SDE, with an average hazard rate parameter $\bar{\lambda}^E$- the respective stochastic process swings around. Thereby, they combine physical risk and transition risk into one expected hazard rate parameter of the SDE. More precisely, they derive the following linear relationship, they use to calibrate the hazard rate parameter:

$$\bar{\lambda}^E = \tilde{\lambda}^E + \xi \cdot \tfrac{\Lambda}{\delta} \cdot \tfrac{\phi}{\nu} \cdot n_B$$

With ξ and $\tilde{\lambda}^E$ the slope and constant of the linear relation, Λ the climate sensitivity, δ the possible outgoing radiation from earth, ϕ the emission intensity, ν the expenditure share for environment, and finally $n_B = \frac{K_B}{K_B+K_G}$ the ratio of brown risky assets in percent of total risky assets. Further, $\frac{\phi}{\nu} \cdot n_B = E_t$ are the carbon emissions, cf. Karydas and Xepapadeas, 2019, Section 3.4 and 4.2.2 for details.

Compared to this approach, our model takes the hazard rate as exogenously given or directly linked to the ESG- or E rating of the company, and we dedicatedly model the correlation structure between companies and industry sectors in course of our Monte Carlo simulation. As concerns the needed correlation structure, the work of Adenot et al., 2022 gives useful insight on how to analyze and visualize such dependencies based on **Upstream Cascading Effects**. We will come back to this topic in course of our simulation framework in Section 3.3.1.

The asset management industry and regulators suggest to measure **ESG Factors** (and in particular the E factors) by key ratios, like **Carbon Footprint**. For a pure equity portfolio, the Equity Ownership Approach of carbon footprint is a good measure, where the "Company Value" is the market capitalization of the company. If we allow for multi asset portfolios the company value would comprise of equity, debt and cash, and is called the company's enterprise value[10]. This more general carbon footprint of a portfolio ptf with portfolio market value $f(ptf)$ is defined by the Sustainable Financial Disclosure Regulation (SFDR) Level 2 as given by Equation 2.2. Thereby, the portfolio consists of n-many financial assets a_c with market value $f(a_c)$ each, and portfolio weights w_{a_c}. These are related to companies $c \in \{1, ..., n\}$, with company enterprise value $f(c)$ and company carbon emissions in tCO_2 denoted by $CrbEm(c)$[11]:

$$CrbFtp(ptf) = \frac{\sum_{c=1}^{n} f(a_c) \cdot \frac{CrbEm(c)}{f(c)}}{f(ptf)} \quad with \quad f(a_c) \leq f(c) \tag{2.2}$$

$$\iff CrbFtp(ptf) = \sum_{c=1}^{n} w_{a_c} \cdot \frac{CrbEm(c)}{f(c)}$$

This definition measures the share of carbon emissions that can be attributed to our portfolio investments, relative to the portfolio value. The second formula gives the interpretation of Weighted Average Portfolio Carbon Emissions in $tCO_2/M.EUR$. Definition 2.2 restricts the scope of assets for which $CrbFtp$ can be measured: Assets that are in scope, need to reflect the equity capital or

[9]For our paper we used the Amundi ESG rating.

[10]Cf. EU, n.d.-c Appendix 1. The level 2 regulation of SFDR defines "enterprise value means the sum, at fiscal year-end, of the market capitalisation of ordinary shares, the market capitalisation of preferred shares, and the book value of total debt and non-controlling interests, without the deduction of cash or cash equivalents".

[11]Cf. Funk C M, n.d., p. 9-10.

the debt capital, that is, equity and bonds, and direct derivatives on them via the delta equivalent[12].

Different from the ESG risk definition above, but closely related and important, in order to prepare the way to sustainable investments is the notion of **Principal Adverse Impact (PAI)**. This is the negative impact of an investment on ESG factors. Limits on PAI are one means to restrict future ESG risk. For example, one can limit the investment in tobacco-related companies, or the investment in crude-oil related companies, or may limit the carbon footprint of the portfolio. Thus, systematic ESG risk and exclusion lists are related to PAI, cf. Section 2.1[13].

2.3 Definition of Sustainable Investment Strategy

Apart from the definition of ESG risk in the finance industry, there is also a discussion of what is a **Sustainable Investment Strategy**. Some argue this is already the case if PAI is considered, by setting up rules that only allow for a certain percentage share of non ESG compliant investments (e.g. <5% tobacco, or <10% military weapons). Normally, this is made sure by **Exclusion Rules** that relate to concerned companies. A more direct definition would be to set up limits on -– for example – carbon intensity, or to minimize the carbon footprint (cf. Equation 2.2) prior to an intended investment into a given industry branch. In the latter case, the investor would focus on the Best-In-Class companies (as regards carbon footprint) of the industry branch under consideration, or whether the investment would increase or decrease his portfolio average carbon footprint. In this context, we want to explicitly introduce our definition of a **Sustainable Investment (SI)**. An investment, or more specific, the investment into a company equity share (the respective equity portfolio position) is sustainable, if for all covered economic entities we have:

1. Revenues from Brown Activities are no more than (for example) 15%. Or equivalently: Green and Grey Activities > 85%.

2. The company adheres to best E/S practices as measured by appropriate ESG subratings.

3. The Governance rating is worst F.

4. No severe Principal Adverse Impact (PAI) is caused by the investment in the company.

   (a) As measured by its CO2 intensity, the economic entity is not part of the (for example) 15% worst in the applicable industry sector.

   (b) No Controversial Weapons involvement of the considered company.

   (c) No Controversies as regards Biodiversity and Pollution and Water.

   (d) No Social controversies, e.g. as measured by applicable Social rating.

Thereby, the first 2 criteria are related to the activities, the third criteria is the good governance practice, and the last criteria is related to DNSH (do not significantly harm) other sustainable targets. This approach combines the definition of SFDR and Taxonomy. Both start with sustainable activities and apply the DNSH principle. While SFDR has its focus on good governance practice, SFDR is more concerned with social safeguards.

[12] The Level 2 regulation of SFDR states: „For the purposes of the assessment of principal adverse impacts by financial market participants, an investment in an investee company or an entity includes direct holdings of capital instruments issued by those entities and any other exposure to those entities through derivatives or otherwise.", cf. EU, n.d.-b, Sec. 4 (3). The carbon emissions referred to in formula 2.2 are scope 1, 2 and 3 emissions.

[13] Among other things, latest AIFMD/UCITS EU Regulation requires the Investment Process of an Asset Manager to take explicitly into account PAI.

2.4 Financial Instrument Valuation and ESG Factors

We consider asset a with observable risk factors $(r_1, \dots, r_\kappa)$ and the valuation formula f_a to calculate the present value $f_a(r_1, \dots, r_\kappa)$ of asset a. For example, a corporate Zero Bond z with one year time to maturity and credit spread cs as well as the credit risk free rate r_f we have

$$f_z(r_f, cs) = \frac{100}{1 + r_f + cs} \tag{2.3}$$

In order to ease the approach and to arrive at an efficient valuation method, given the background to integrate ESG risk, and the lack of precision we face anyway, one could consider a linear approximation of the return distribution, also called Delta-Normal approach. This keeps a normal distribution approach, and accelerates the calculation. The market value changes of each asset in the portfolio are thus approximated by its Total Differential, dependent on the main risk factors of each asset:

$$df_a(r_1, ...r_\kappa) = \frac{df_a(r_1, ...r_\kappa)}{dr_1} \cdot r_1 \cdot \frac{dr_1}{r_1} + ... + \frac{df_a(r_1, ...r_\kappa)}{dr_\kappa} \cdot r_\kappa \cdot \frac{dr_\kappa}{r_\kappa} \tag{2.4}$$

Thereby, infinitesimally small changes or returns dr_i/r_i are considered, that are approximated in reality by daily risk factor returns, and the df_a/dr_i are the relevant Deltas of the respective financial instrument.

In case of substantial non-linear behavior of the portfolio with respect to selected risk factors, e.g. in case of option strategies or overlay management, one could enhance Equation 2.4 by second order derivatives (Gammas), or even use a full valuation. This is at the cost of loosing the normal distribution property of risk factor returns. Thus, the non-linear enhancement calls for quantile approximation like Cornish Fisher or a Monte-Carlo simulation.

There is also a vivid discussion in the financial industry of how to separate and quantify ESG risk from the usual risk factors, like equity prices and interest rates. For explaining purposes only, let's now assume there is a regression, linear explanation respectively, for each daily risk factor return r_i on independent explaining economic variables x_j with $j \in I$. Please note, these will not play an explicit role in the model explained later, because we are of the opinion such a decomposition is purely statistical (without fundamental justification) and thus not stable enough over time.

$$\frac{dr_i}{r_i} = \sum_{j \in I} \beta_j \cdot \frac{dx_j}{x_j} + const \tag{2.5}$$

For the ease of notation, we omit to express the obvious dependence of the sensitivities β and the index set I on the asset under consideration. If ESG factors impact the valuation of the asset then there is an index subset $esg \subset I$ with $x_j \in esg$ the respective explaining variables. Please note, we do not specify the explaining variables by intention, because at the one hand, this is by itself a substantial task, and on the other and – for the purpose of this paper – we do not need to specify these variables.

Having said this, and taking the concrete example of carbon footprint: a possible explaining variable may be the deviation of carbon emissions from the planned trend needed to keep the climate within bearable changes, and the sensitivities of the explaining variables may depend on the branches or classes we have considered earlier. Further, the variables and its sensitivities will be driven by political decisions (which is actually transition risk) and may thus show discontinuous behavior, that should be modelled by a jump process, similar to credit risk.

Concerning the linear decomposition of the risk factors into independent explaining variables we can then separate equation 2.5 as follows:

$$\frac{dr_i}{r_i} = \sum_{j \notin esg} \beta_j \cdot \frac{dx_j}{x_j} + \sum_{j \in esg} \beta_j \cdot \frac{dx_j}{x_j} \tag{2.6}$$

We employ this decomposition just to explain ways to modify the distribution of the risk factors r_i appropriately to account for ESG risk, and how these modifications can be calibrated based on backtesting, cf. Section 3.1 where we modify the Delta-Normal approach, as well as Equation 2.7. We will not try to model these hidden factors explicitly.

2.5 Calibrate The Hazard Rate with ESG Rating

No matter, whether we try to keep a Normal distribution, or whether we want to go for a more advanced approach, the main topic is how to account for ESG risk when fixing the parameter values of the distribution of risk factors. This can depend on the systematic and specific ESG risk driving the risk factor under consideration, and alternatively one could derive the parameters based on the transition risk and physical risk impact, or even combine these two approaches, cf. Section 2.2 "Definition of ESG Risk".

Let us assume available numerical rating scores RSc for the systematic part and for the specific part of ESG risk, or alternatively for transition risk and physical risk. Physical risk impact is (at least in the short and mid term) independent from transition risk, and systematic and specific risk are independent by definition[14]. The reason for the independence of transition- and physical risk is, that in particular the climate reacts very slow and only in the very long term to political decisions that manifest transition risk. This independence is important for the theoretical validity of formula 2.8 below. We then consider a partition of the interval of possible rating scores as defined in Section 2.2.

A ESG rating class with very little ESG risk, for a given company, may even lead to an increased expected value of its daily return, i.e. $\mu(R_{esg}) > 0$ and a decreased variance, i.e. $\sigma^2(R_{esg}) < 0$ given the ESG adjusted risk model. Note thereby, while from a practitioners point of view $\sigma^2(R_{esg}) < 0$ opens an interesting calibration scope, from a modelling point of view this is not possible with the above assumption of independent explaining variables in Equation 2.6, as correlations are then zero. Note, the use of expected value and variance as the only parameters of the distribution restricts the user to a Normal distribution. More generally, this is not at all clear, because given a jump process one has the respective intensity rate being affected by the ESG rating, and this is the root of our proposed approach.

If we take for example the systematic and specific ESG risk view, then the concrete definition and hence the calibration of the risk model would depend on the rating score (rating class) of a given company c taking into account specific as well as systematic ESG risk/ESG rating. For example: the necessary mapping for a Normal distribution

$$M : (RSc_{sys}(c), RSc_{spec}(c)) \longrightarrow (\mu(R^c_{esg}), \sigma(R^c_{esg})) \tag{2.7}$$

can be recalibrated based on backtesting results of the risk model. Likewise, the jump intensity λ of a jump process, can be made up by the systematic and specific component of ESG risk.

$$M : (RSc_{sys}(c), RSc_{spec}(c)) \longrightarrow \lambda_{c,sys} + \lambda_{c,spec} = \lambda_c \tag{2.8}$$

given the independence of systematic and specific ESG risk. Whereby, our starting point is the systematic part of risk, and the specific part plays an offsetting role, that is, decreasing or increasing the hazard rate, dependent on whether the company or entity under consideration is Best-In-Class or rather Worst-In-Class.

[14]Note, to assume the independence of systematic and specific risk is in our framework equivalent with the precondition that environmental risk caused by the product is independent from environmental risk caused by the production process. We argue that this assumption does not violate the reality to an extend that would cause the simulation results to be unreliable.

For our simulation study based on a jump diffusion process (Geometric Brownian Motion enhanced by a Poisson component) we will model the jump intensity based on both, transition risk and physical risk contribution, as well as systematic and specific risk, and compare the parametrization. The synthesis of these two definitions of risk in course of a simulation study is left to a planned future paper, where we intend to investigate a Fixed Income portfolio.

3 Impact On Risk Factor Distribution

Initially, we will briefly consider the case of a Delta Normal distribution approach and then we have a look at the impact on Historical simulation. Finally, and most importantly, we will develop our proposal of a jump diffusion process to analytically model the impact of ESG risk on the return distribution, and fundamentally explain our view, why ESG risk is part of existing risk factors. Thereby, we will be also able to quantify the impact of ESG risk on the overall portfolio risk, more precisely, how ESG rating (via the jump-intensity and company/sector correlations) influences the return distribution of the portfolio. The necessary return distribution will be obtained via Monte Carlo simulation, cf. Section 4. The results of the simulation will also serve to derive some rule of thumb of how to modify ordinary risk figures (given the Average Portfolio Hazard Rate and the Average Portfolio Jump Correlation). Thereby, the intention is to restrict the need of Monte Carlo simulation as far as possible.

3.1 Delta Normal Approach

We keep the Normal distribution based on Equation 2.4, and just modify the expected value and variance. Further, at this stage of the paper, we assume a constant correlation structure of the risk factor returns. More precisely, assume a correction factor that tells how to adjust the expected value and variance of the valuation formula $f_a(r_1, ..., r_\kappa)$ of asset a based on its risk factors $r_1, ..., r_\kappa$. Note, given the linear decomposition of $df_a(r_1, ..., r_\kappa)$ in Equation 2.4, the distribution function of changes is Normal for all financial instruments in the portfolio, and thus, the changes in market value of the portfolio are normally distributed.

A correction factor as mentioned above can be obtained by several approaches, for example:

1. via appropriately calibrated ESG rating, as given by mapping M in Equations 2.7 and 2.8, yielding a factor on asset level.
2. via regression, without and with explaining ESG variables, as given in Equations 2.5 and 2.6, yielding a correction factor on the more granular risk factor level.
3. via a quantile based ratio, as derived in detail by our rule of thumb proposal in Section 4.4, yielding again a factor on asset level.

We want to show in more detail how approach 3 above can be applied. For this consider again the linear decomposition df_a of asset valuation f_a:

$$df_a(r_1, ..., r_\kappa) = \sum_{i=1}^{\kappa} \frac{df_a}{dr_i} \cdot r_i \cdot \frac{dr_i}{r_i}$$

Together with the Standard Normal quantile q^p_ϕ (e.g. the one sided quantile with $p = 0.95$, which gives the 95% confidence VaR) we can write the quantile of the asset return df_a/f_a as[15]:

$$q^p_t(df_a/f_a) = \mu(df_a/f_a) - q^p_\phi \cdot \sigma(df_a/f_a) \cdot \sqrt{t}$$

Given the Value-at-Risk ratio $q_R(\lambda_a, 1)$ from Equation 4.6 for a 1-element portfolio, we obtain the E risk adjusted Value-at-Risk of the return of asset a as

$$q_R(\lambda_a, 1) \cdot q^p_t(df_a/f_a) = q_R(\lambda_a, 1) \cdot \mu(df_a/f_a) - q^p_\phi \cdot q_R(\lambda_a, 1) \cdot \sigma(df_a/f_a) \cdot \sqrt{t}$$

Hence, we can use $q_R(\lambda_a, 1) \cdot E(df_a/f_a)$ and $q_R(\lambda_a, 1)^2 \cdot V(df_a/f_a)$ as the environmental risk adjusted expectation and variance of the Delta-Normal approach. That is for the expected value,

$$\mu_{esg}(df_a/f_a) = \frac{1}{f_a} \cdot q_R(\lambda_a, 1) \cdot E(df_a) = \frac{1}{f_a} \cdot E\left(\frac{df_a}{dr_i} \cdot r_i \cdot \frac{dr_i}{r_i} \cdot q_R(\lambda_a, 1)\right)$$

[15] We omit to show explicitly the dependency on $(r_1, ..., r_\kappa)$

$$= \frac{1}{f_a} \cdot \sum_{i=1}^{\kappa} \frac{df_a}{dr_i} \cdot r_i \cdot E\left(\frac{dr_i}{r_i}\right) \cdot q_R(\lambda_a, 1)$$

and for the variance:

$$\sigma_{esg}^2(df_a/f_a) = \frac{1}{f_a^2} \cdot q_R(\lambda_a, 1)^2 \cdot V(df_a) = \frac{1}{f_a^2} \cdot V\left(\sum_{i=1}^{\kappa} \frac{df_a}{dr_i} \cdot r_i \cdot \frac{dr_i}{r_i} \cdot q_R(\lambda_a, 1)\right) \tag{3.1}$$

$$= \frac{1}{f_a^2} \cdot \left(\frac{df_a}{dr_1} \cdot r_1, ..., \frac{df_a}{dr_\kappa} \cdot r_\kappa\right) \cdot \begin{pmatrix} \sigma_1^2 \cdot q_R(\lambda_a,1)^2 & \cdots & \sigma_{1\kappa} \cdot q_R(\lambda_a,1)^2 \\ \vdots & \ddots & \vdots \\ \sigma_{\kappa 1} \cdot q_R(\lambda_a,1)^2 & \cdots & \sigma_\kappa^2 \cdot q_R(\lambda_a,1)^2 \end{pmatrix} \cdot \begin{pmatrix} \frac{df}{dr_1} \cdot r_1 \\ \vdots \\ \frac{df_a}{dr_\kappa} \cdot r_\kappa \end{pmatrix}$$

As our paper, is focused on equity portfolios, we now consider n-many equity positions $i \in \{1, ..., n\}$ and portfolio weights $w = (w_1, ..., w_n)^T$. In this case the valuation formula collapses to $f_a(r_1, ..., r_\kappa) = f_a(r_i) = r_i$, where frisk factor r_i is the equity share price S_i of position i. Similarly, df_a/f_a becomes dS_i/S_i, which is the equity return. Thus, the E adjusted equity portfolio expected return and variance of return are:

$$E(r_{ptf,esg}) = E\left(\sum_{i=1}^{n} w_i \cdot \frac{dS_i}{S_i} \cdot q_R(\lambda_i, 1)\right) = \sum_{i=1}^{n} w_i \cdot E\left(\frac{dS_i}{S_i}\right) \cdot q_R(\lambda_i, 1) \tag{3.2}$$

$$V(r_{ptf,esg}) = V\left(\sum_{i=1}^{n} w_i \cdot \frac{dS_i}{S_i} \cdot q_R(\lambda_i, 1)\right) = w^T \cdot \tag{3.3}$$

$$\begin{pmatrix} \sigma_1^2\left(\frac{dS_1}{S_1}\right) \cdot q_R(\lambda_1,1)^2 & \cdots & \sigma_{1n} \cdot q_R(\lambda_1,1) \cdot q_R(\lambda_n,1) \\ \vdots & \ddots & \vdots \\ \sigma_{n1} \cdot q_R(\lambda_n,1) \cdot q_R(\lambda_1,1) & \cdots & \sigma_n^2\left(\frac{dS_n}{S_n}\right) \cdot q_R(\lambda_n,1)^2 \end{pmatrix} \cdot w$$

Where the environmental risk adjusted covariance between equity return i and equity return j is

$$\sigma_{ij}^{esg} = \sigma_{ij} \cdot q_R(\lambda_i, 1) \cdot q_R(\lambda_j, 1)$$

This keeps the correlation structure of the portfolio constant, because we have

$$corr_{ij}^{esg} = \frac{\sigma_{ij} \cdot q_R(\lambda_i, 1) \cdot q_R(\lambda_j, 1)}{\sigma_i \cdot q_R(\lambda_i, 1) \cdot \sigma_j \cdot q_R(\lambda_j, 1)} = corr_{ij}$$

Obviously, one can also modify the correlation structure appropriately, if backtesting results of the portfolio risk may deem this reasonable.

For the general case of a not only equity portfolio, we suggest to use the vector of modified asset volatilities (cf. Equation 3.1) together with the correlation matrix, instead of the covariance matrix and portfolio weights. This representation deals better with the complexity of the formula. We finally obtain the variance of portfolio return, using again the correlation matrix, which we keep invariant. Note, the correlation structure can also be modified appropriately, as mentioned above:

$$V(r_{ptf,esg}) = V\left(\sum_{a=1}^{n} w_a \cdot \frac{df_a}{f_a} \cdot q_R(\lambda_a, 1)\right) = \tag{3.4}$$

$$\left(w_1 \cdot \sigma_{esg}\left(\frac{df_1}{f_1}\right), \cdots, w_n \cdot \sigma_{esg}\left(\frac{df_n}{f_n}\right)\right) \cdot \begin{pmatrix} 1 & \cdots & corr_{1n} \\ \vdots & \ddots & \vdots \\ corr_{n1} & \cdots & 1 \end{pmatrix} \cdot \begin{pmatrix} w_1 \cdot \sigma_{esg}\left(\frac{df_1}{f_1}\right) \\ \vdots \\ w_n \cdot \sigma_{esg}\left(\frac{df_n}{f_n}\right) \end{pmatrix}$$

Finally, we note, that Equations 3.1 to 3.4 that cover the term $df_a/f_a \cdot q_R(\lambda_a, 1)$ also give a hint, of how to modify the risk factor returns of a Historical simulation in order to obtain an environmental risk adjusted Value-at-Risk from Historical simulation – instead of modifying the Historical Value-at-Risk figure ex post.

3.2 Historical Simulation Approach

From a conceptional point of view, there is no need to modify the approach of Historical Simulation, because historical return movements should already incorporate ESG risk. Moreover, the emphasis on more recent returns with Decay Factors is common to current approaches.

On the other hand, we argue that historical returns do not sufficiently reflect ESG risk as its impact is new, developing very fast and a rather rare but relatively extreme event, in particular at the beginning and during the transition phase to a more sustainable economy. When the transition is about to be finalized, the events can become more likely with diminishing impact each[16]. Currently, ESG risk will be to a substantial degree a transition risk, properly modeled by a jump process. Implying, that one should modify historical returns, or the risk figures calculated from historical returns, appropriately. Alternatively, one could just try to modify the result from Delta-Normal approach.

What we propose, when modifying Historical VaR or Delta-Normal VaR, is to employ a factor derived from the **Weighted Average Hazard Rate** and the **Jump Correlation Structure** of the portfolio under consideration. We infer this **rule of thumb** by comparing the equity return distribution obtained by jump diffusion Monte Carlo simulation with a Monte Carlo simulation that restricts to a pure Geometric Brownian Motion, and consider the ratio of the two resulting Value-at-Risk figures. This will also mitigate an imanent drawback of historical simulation, as it fails to be able to model the correlation behaviour dedicatedly, because historical simulation is restricted to the historically observed correlation. Cf. Section 4.4 for details on the rule of thumb.

3.3 Monte Carlo Simulation Approach

3.3.1 Framework of the Jump Diffusion Model

It was already outlined that we suggest to view ESG risk as embedded in observable market risk factors. The reason is quite straightforward: in case ESG risk affects a company under consideration, its share prices, credit spreads and other observable risk factors will move accordingly.

In this section, we want to formalize analytically how ESG risk can be embedded explicitly into a given risk factor. Thereby, we will focus on equity portfolios, that is to say, the single equities, more precisely their equity returns, are by themselves the risk factors. Further, we apply one-dimensional Geometric Brownian Motion (GBM) to a single equity share and embed ESG risk via an appropriately parametrized jump component, more precisely, a Poison process with its hazard rate. In addition, we model with a deterministic function to which extent the jumps occurs, i.e. whether there is a 100% loss triggered by the jump or some percentage loss smaller than 100%.

Other risk factors, like credit spread, may call for another diffusion part, e.g. a mean-reversion process, but the main principle of combining a continuous process with a jump will stay the same. It is worth to mention, that within our approach of Monte Carlo simulation of risk factors that are explicit part of fundamental valuation formulas – our next step of research on **Bond Portfolios** will try to discount expected future cash flows (under the original probability measure) by a factor

[16] From a model point of view this would require higher hazard rates with lower Jump-Given-Event.

that covers the stochastic evolution of the risk free rate and the credit spread[17]. Where the E risk of the risk free rate is related to the respective government/country, and the E risk of the credit spread is related to the issuing company. This way we bring the hazard rate via the SDE of the credit spread explicitly into the pricing formula. This Reduced Form market risk factor approach differs substantially from the Structural Approach of Agliardi and Agliardi, 2021. In their model, which is in the spirit of Merton, 1974, the default of the company (and its issued bonds) occurs as soon as the company value crosses a certain threshold. Thus, they express the bond price as a function of the company value (or equity price, under additional assumptions), together with the hazard rate that influences the equity price behavior under the jump diffusion model. Using this structural framework and the generalized Lemma of Ito they then derive explicit valuation formulas for bonds and equities, that incorporate the hazard rate.

At this point, it is important to discuss the **general framework of the parameter estimation** we follow. We use **(1)** historical return time series to estimate the drift, volatility and correlation of the continuous part of the process and then add **(2)** a jump component – mainly parametrized by fundamental and logic considerations. With this approach we assume that currently ESG risk - and in particular the E part - is still only marginally reflected in historical portfolio returns, as transition risk and the perception of Adverse Impact risk is new and developing very fast (cf. Section 2.2 and Section 3.2). If this assumption were violated, we would need to introduce a compensator (e.g. a volatility and correlation reduction) to properly estimate drift, volatility and correlation of the continuous part, in order to avoid double-counting of risk. As regards the approach of **calibration** based on the notion of **systematic and specific risk**, we touched already in Equation 2.8 the possibility to derive the hazard rate of the jump process from available ESG rating data, with a mapping function:

1. An economic entity (cf. Section 2.1 "Definition of a Company") is element of a sector/branch whose products are equally exposed to systematic ESG risk. What we consider the systematic component λ_{sys}. In this sense, they form an equivalent class. The loss, the share price of the company may experience – given an adverse ESG event – is assumed to equal the share $Q(c_s)$ of the economic entity c_s of company c dedicated to sector/branch s, multiplied by a factor ξ representing the magnitude of the jump. The easiest case would be a factor of 1. The magnitude could also be factor ≤ 1, depending on the ESG rating of the company.

2. Within this class, the economic entity of the company may compensate the systematic hazard rate, in case it is Best-In-Class, by using resources optimally during the production process or in a perfectly sustainable manner. We denote this the specific component λ_{spec}. For example: a mining company that meets its energy demand completely by renewable energy. Moreover, being Best-In-Class may even lead to reduced correlation with other companies in the class, easing the diversification effect of this company.

3. The disjoint classes may impact each other. For example, an environmental risk from the oil industry can negatively impact the fishing sector. This is an example of a uni-directional dependency, cf. also Adenot et al., 2022. We suggest to model this via the hazard rate. Others are mutual dependencies and we suggest to model these via the correlation structure.

4. We further postulate a strong economic (not environmental) correlation between economic entities of the same company.

As regards the applied approach of **calibration** based on the notion of **transition and physical risk**, the above points 3 and 4 are identically applicable, while we need to replace 1 and 2 by simply

[17] Alternatively, we will apply a widely used linear approximation, using Key Rate Durations and the Spread Duration to calculate the change of the bond price.

1. The probability of a transition risk event an economic entity is exposed to, as measured by the Best-In-Class E rating and modelled via the **transition hazard rate** λ_{Trs} is increased by the independent

2. Physical risk impact, that is the probability of adverse physical risk impact, modelled via the **physical hazard rate** λ_{Phy}, to become the **compound hazard rate**.

3. As the risk figure is made up by probability of event and impact of event, we incorporate the loss the share price of the company may experience given an adverse ESG event. It is equal to the share $Q(c_s)$ of the company c dedicated to sector/branch s under consideration, multiplied by a factor ξ representing the magnitude of the jump.

It is worth to note, that here we do not require the mathematical independence of economic entities, as each economic entity is in this sense a random variable, whose dependencies with other entities is modeled via the correlated simulation.

3.3.2 Hazard Rate Modelling

In order to comply with the requirements outlined above, we model each equity share price in a given portfolio with a jump diffusion process, with Geometric Brownian Motion, that is enhanced by a sum of Poisson processes. Each jump process relates to the share of a company that is dedicated to a sector/branch, and these economic entities are added up, cf. Eq. 2.1.

Given a pure equity portfolio, we have the special case of an asset a_c of company c, whose valuation formula $f_a(r_1, ..., r_\kappa)$ depends on only one risk factor $r_c = S_c$, which is the equity share price. Further, we have the identity $f_a(S_c) = S_c$. For this reason, and because the notation of S_c to model the equity price behavior is very common, we use $S_{c,t}$ for the equity price of company c at time $t \geq 0$.

The above mentioned separation of a company into economic entities is reasonable if one wants to model the jump correlation even down to the entity level or if economic entities of a company act in completely different industry sectors. Given the current data availability and status of economic transition we argue that being able to model the dependency between companies - in view of environmental transition risk – is already a very good step into the correct direction. On a single company level, we collapse the sum of single processes into one jump component, whose hazard rate is then determined by applicable company ESG rating. It is important to note, that a simple addition of Poisson components (the entities) without correlated simulation, would require their independence which is not given in general.

In order to account for unidirectional dependencies we consider the index set $UD(c_i)$. This means, that all companies/sectors that uni-directionally influence the economic entity c_i of company c in sector $i \in \{1, ..., k\}$ are covered by this index-set[18]. There are two possibilities to account for such unidirectional impact: One is to force a jump of the impacted entity in case a jump occurs with the impacting company and not to increase the hazard rate of the impacted entity. The other approach is just to increase the hazard rate of the potentially impacted sector/entity, in order to avoid a "manual" intervention into the flow of the stochastic process. The second approach is what we propose below.

$$dS_{c,t} = \mu_c \cdot S_{c,t} \cdot dt + \sigma_c \cdot S_{c,t} \cdot dW_{c,t} - S_{c,t} \cdot \sum_{i=1}^{k} Q(c_i) \cdot dJ_{c_i,t} \tag{3.5}$$

[18]For example, an oil producing company whose sea-platform would pollute the sea, has potential adverse impact on the fishing sector in this area. Another example is air transportation: if airlines need to increase prices due to increased carbon emission cost, possibly caused by carbon tax, this would impact travel agencies.

with [19]

$$dW_{c,t} \sim N(0, dt)$$

$$dJ_{c_i,t} = \sum_{j=1}^{dN_{c_i,t}} \xi(\lambda_{c_i}) = dN_{c_i,t} \cdot \xi(\lambda_{c_i})$$

$$dN_{c_i,t} = \begin{pmatrix} \geq 1 & with & p_{c_i} = \left(\lambda_{c_i} + \sum_{j\in UD(c_i)} \lambda_j\right) \cdot dt + o^2(\lambda dt) \\ 0 & with & 1 - p_{c_i} \end{pmatrix}$$

where the jump size $\xi(\lambda_{c_i})$ is a deterministic function that increases monotonically with the hazard rate[20]. Further,

$$E(dN_{c_i,t}) \approx \left(\lambda_{c_i} + \sum_{j\in UD(c_i)} \lambda_j\right) \cdot dt, \quad \sum_{i=1}^{k} Q(c_i) = 1$$

Given the formula above, it is obvious that with each jump, the company equity share price does not necessarily go to zero completely, but is reduced in its maximum by the share that is contributed by the jump size $\xi(\lambda_{c_i})$ of the economic entity where the jump occurred. A natural extension of this approach would be: not to take the jump magnitude as a deterministic function of λ, but to consider a real compound Poisson process, where the altitude of the jump follows a continuous random variable.

Concerning the definition of the hazard rate λ_{c_i} of economic entity c_i that acts in industry sector i we have for both of our parametrization alternatives:

$$\lambda_{c_i} = \lambda_{c_i,sys} + \lambda_{c_i,spec} \geq 0 \quad with \quad \lambda_{c_i,sys} \geq 0 \wedge \lambda_{c_i,spec} > -\lambda_{c_i,sys}$$

$$\lambda_{c_i} = \lambda_{c_i,Trs} + \lambda_{c_i,Phy} \geq 0 \quad with \quad \lambda_{c_i,Trs} \geq 0 \wedge \lambda_{c_i,Phy} \geq 0$$

For our simulation study we will assume $k = 1$ and $UD(c_i) = \emptyset \quad \forall c_i$. This is: each company acts only in one industry sector, and it is not influenced unidirectionally, i.e. Equation 3.5 simplifies to[21]

$$dS_{c,t} = \mu_c \cdot S_{c,t} \cdot dt + \sigma_c \cdot S_{c,t} \cdot dW_{c,t} - S_{c,t} \cdot \xi(\lambda_c) \cdot dN_{c,t} \tag{3.6}$$

As already mentioned, we face the situation that currently available ESG ratings focus on the hybrid Best-In-Class approach, which is a relative measure. Thus, the user may need to establish by himself an inhouse rating for what we call the systematic part of ESG risk, in case he wants to apply this approach. For this paper we focused on the environmental part of ESG using the **Average Scope 1-3 Carbon Emission** of the industry sector the company belongs to, to derive the systematic rating score in Table 2. In detail, the logic given by Table 1 applies. A similar rationale is used by Adenot et al., 2022 in their Section 3.2.

[19] Cf. Appendix A.1 for an explanation of $o^2(\lambda dt)$.

[20] We went for an increasing function, as we wanted to emphasize the impact of environmental risk in course of a transition phase that is at its beginning. At the end of a transition phase or after the transition is more or less completed, one would rather consider higher hazard rates with smaller jumps, and lower hazard rates with bigger jumps. Implying some smoothing of the jump component. This does not change our approach but just the deterministic function of jump size we apply.

[21] Cf. Appendix A.3 for a quantitative discussion of this model and why it was choosen.

Average CO_2 intensity	RSc_{sys}
< 0.5	A
< 2	B
< 4	C
< 10	D
< 20	E
< 40	F
≥ 40	G

Table 1: Systematic ESG Rating Derivation

If we assume sufficiently many members for an arbitrarily given industry sector, we are able to eliminate the single company (or specific) risk impact, by using the Average Carbon Emission over all members, because single company impact on industry emissions is then negligible. Unfortunately, we had no access to full Scope 3 data that covers the carbon emission of the product, which is due to a lack of data availability. But this would be important to cover the post-production environmental harm of the product. A notable exception, where the data were available in principle, is the Automobile sector. The future emissions of the cars this industry sector produces are relevant as regards our environment (PAI risk), and thus causes (political) transition risk. This brings us to the **Product Carbon Footprint**, which is a measure that covers the carbon emissions over the whole life of a product, starting with Upstream, incorporating the production process as well as Downstream, which is: distribution, use and disposal of the product[22].

Before we come to the specific E rating, we would like to shade some light on our understanding of the terms Upstream, Downstream and Scope123 emissions. **Upstream** refers to the material inputs needed for production, while **Downstream** is the opposite end, where products that have been produced are distributed. **Scope 1 emissions** are directly caused by Downstream activities. **Scope 2 emissions** are indirect and caused by the Upstream activities use of Energy, which is the collection of necessary material. In contrast, **Scope 3 emissions** for Upstream is everything not covered by Scope 2 AND Indirect Downstream emissions. These indirect Downstream emissions should per definition cover the emissions of the product and what is needed at the and of product life.

The logic for the specific environmental rating we use for the single equity positions is given in Table 2, cf. also the Table 10 of static portfolio data in the Appendix.

[22]Cf. Funk C M, n.d., p.3.

RSc_{sys}	λ_{sys}	RSc_{spec}	λ_{spec}
A	$\frac{0.05}{250}$	A	-0.75 $\times$ λ_{sys}
B	$\frac{0.1}{250}$	B	-0.25$\times$ λ_{sys}
C	$\frac{0.25}{250}$	C	0
D	$\frac{0.5}{250}$	D	$0.25 \times \lambda_{sys} + \lambda_{sys,D}$
E	$\frac{1}{250}$	E	$1 \times \lambda_{sys} + \lambda_{sys,E}$
F	$\frac{2}{250}$	F	$3 \times \lambda_{sys} + \lambda_{sys,F}$
G	$\frac{4}{250}$	G	$5 \times \lambda_{sys} + \lambda_{sys,G}$

Table 2: SysSpec Hazard Rate Derivation

We find it intuitive to link the ESG rating/rating score to the **Expected Number of Events per Time**. More precisely, note Table 2, of how ESG rating scores and the expected number of adverse events can be linked. In order to derive this table as a possible calibration example, we used $dt = 1day$ and thus, the expected number of adverse events during 1y time horizon – implied by systematic ESG risk – should approximately equal $\lambda_{sys} \cdot 250d$. Concretely, for a G rating, we expect 4 events during 1y. The specific hazard rate is modelled as a negative or positive offset, based on the company specific transition risk rating[23]. In case, one uses 10d time-steps the above systematic hazard rates need to be multiplied by 10. Note, Table 2 is just an example and its values may vary with the industry sector under consideration, more precisely the impact of the systematic and specific hazard rate may change. We refer to our considerations in Section 2.1, where we cited examples of industry sectors, for which the main part of ESG risk is with the production process and not the product itself.

Using transition and physical risk the starting point of calibration could look like the Table 3:

RSc_{Trs}	λ_{Trs}	RSc_{Phy}	λ_{Phy}
A	$\frac{0.05}{250}$	A	$\frac{0.02}{250}$
B	$\frac{0.1}{250}$	B	$\frac{0.04}{250}$
C	$\frac{0.25}{250}$	C	$\frac{0.125}{250}$
D	$\frac{0.5}{250}$	D	$\frac{0.25}{250}$
E	$\frac{1}{250}$	E	$\frac{0.5}{250}$
F	$\frac{2}{250}$	F	$\frac{1}{250}$
G	$\frac{4}{250}$	G	$\frac{2}{250}$

Table 3: TrsPhy Hazard Rate Derivation

[23]This is the transition risk rating whose availability we postulate in Section 2.2. It measures the exposure of a company to transition risks and its ability to manage such risks

One can easily recognize, while both approaches have the same starting point with identical transition or systematic hazard rate, Table 2 applies -– for weak ratings – a more aggressive start to model the hazard rate. As it attributes a higher event probability to weak rating quality, while Table 3 proposes a more equalized and moderate impact of physical risk. Further, specific ESG hazard rate plays a relative offsetting role, while physical E(SG) hazard rate is an increasing component.

3.3.3 Modelling ESG Dependencies with the Correlation Matrix

We now turn to the dependency structure, correlation structure of the proposed model, respectively. For this, we consider an investment universe that covers our "Financial World". This can be an equity share index, like S&P500, or any arbitrary fixed set of equities. At this stage such equities can be listed economic entities or companies.

The correlation structure of the continuous part, reflected by the Wiener Process dW_t, is standard and well explained in the literature. In summary, this is the correlated simulation of a vector of normally distributed variables based on Cholesky decomposition or the Gauss algorithm. Thereby, the correlations can be derived from historic return time series analysis[24]. In this paper, we focus on the correlation structure of the jump part and how it can be modeled with a **Gaussian Copula**. Nevertheless, for the sake of completion, the generation of the continuous part is

1. Consider a vector Z of i.i.d standard normal variables that is $Z \sim N(0, Id)$, and a given Covariance Matrix Σ_{gbm}.

2. From Σ_{gbm} construct via Cholesky decomposition the lower triangle matrix A_{LT}, with $\Sigma_{gbm} = A_{LT} \cdot A_{LT}^T$.

3. Finally, use $X_{gbm} = (\mu + A_{LT} \cdot Z) \sim N(\mu, \Sigma_{gbm})$

As concerns the jump part, in the most general case of our model, we have $n \cdot k$ many economic entities $e \in \{1, ..., n \cdot k\}$. The algorithmic approach is as follows:

1. Consider a vector Z of i.i.d. standard normal variables, that is $Z \sim N(0, Id)$, and a given covariance matrix Σ_J[25].

2. From Σ_J construct via Cholesky decomposition the lower triangle matrix A_{LT}.

3. Set $Y = A_{LT} \cdot Z \Longrightarrow Y \sim N(0, \Sigma_J)$, with CDF $\Phi_{\Sigma_J}(y)$

4. For step 6, note the distribution functions F_e of univariate Poisson random variables $X_{Je} \sim F_e(x_e, \lambda_e)$ with $e = 1, ..., n \cdot k$.

5. Set $(u_1, ..., u_{n \cdot k}) = (\Phi_1(y_1), ..., \Phi_{n \cdot k}(y_{n \cdot k})) \in [0, 1]^{nk}$ - where $\Phi_e(y_e) \in [0, 1]$ is the normal distribution function of $N(0, \sigma^2(y_e))$.

6. To finally obtain a correlated vector of Poisson random variables set

   (a) $X_J = (X_{J1}, ..., X_{Jk \cdot n}) = (F_1^{-1}(U_1), ..., F_{n \cdot k}^{-1}(U_{n \cdot k}))$

[24] As far as the possible double counting of risk is concerned we refer to our respective outline in Section 3.3.1

[25] It is important to note, the needed covariance matrix is obtained by multiplying the correlation matrix with the two volatility vectors, whereby the volatility vectors are made up by the volatility of the Poisson variables and not the volatility of the continuous part, the historical return time series, respectively. The needed 10-day or 1-day volatility for each asset equals the square root of the 10-day or 1-day hazard rate, because the hazard rate is expected value and variance of the poisson distribution. For our simulation performed here, this is the 10d hazard rate, cf. Section 4

Note, the Gaussian Copula function $C(*)$ is not explicitly visible in the above algorithm. In our case we formally have with the searched multivariate Poisson distribution function $F(*)$ and its marginal distribution functions $F_1, ..., F_{nk}$[26]:

$$\begin{aligned} F(x_1, ..., x_{nk}) &= C(F_1(x_1), ..., F_{nk}(x_{nk})) \\ \text{with} \quad C(u_1, ..., u_{nk}) &= \Phi_{\Sigma_J}(\Phi_1^{-1}(u_1), ..., \Phi_{nk}^{-1}(u_{nk})) \end{aligned}$$

As we are of the opinion that current historic correlation is not an appropriate source for the covariance matrix Σ_J. We now want to show, how the dependence structure can be derived, based on fundamental and logic considerations. For this, we fix two arbitrary companies $c, d \in \{1, ..., n\}$ and two arbitrary economic sectors $i, j \in \{1, ..., k\}$. Then we can identify 4 economic entities $e \in \{1, ..., n \cdot k\}$ as $e(c, i) = c_i$, $e(c, j) = c_j$, $e(d, i) = d_i$ and $e(d, j) = d_j$. We further denote by S_i the set that covers all economic entities acting in sector i, i.e. we have $c_i, d_i \in S_i$ and $c_j, d_j \in S_j$. We also consider the 1-dimensional Best-In-Class index sets $BiC_i \subsetneqq S_i$ and $BiC_j \subsetneqq S_j$, as well as the set of Related Sectors of 2-dimensional index sets $(S_i x S_j)_R$, i.e. for all pairwise related sectors we build the respective cross-product. The correlations shown below will be used in course of our simulations as the Moderate Correlation scenario, together with two further scenarios of No Correlation and High Correlation. They are a starting point and subject to adjustment -– in course of backtesting and recalibration. It can be enhanced by reviewing articles on industry sector dependency structure, as it is investigated in more detail by Adenot et al., 2022, who also distinguishes between bilateral and unidirectional dependencies.

Definition 3.1. *Logic of Correlations*

1. *If* $(c_i, d_i \in S_i) \wedge (c_i \notin BiC_i \wedge d_i \notin BiC_i) \Longrightarrow corr(c_i, d_i) = 0.8$
2. *If* $(c_i, d_i \in S_i) \wedge (c_i \in BiC_i \vee d_i \in BiC_i)) \Longrightarrow corr(c_i, d_i) = 0.2$
3. *If* $(c_i, d_j) \in (S_i x S_j)_R \wedge (c_i \notin BiC_i \wedge d_j \notin BiC_j) \Longrightarrow corr(c_i, d_j) = 0.4$
4. *If* $(c_i, d_j) \in (S_i x S_j)_R \wedge (c_i \in BiC_i \vee d_j \in BiC_j) \Longrightarrow corr(c_i, d_j) = 0.1$
5. *If* $c_i, c_j \in c \Longrightarrow corr(c_i, c_j) = 0.9$
   economic entities c_i, c_j *of the same company* c *are highly dependent*
6. *Else* $corr(c_i, d_j) = 0$

The above logic can even be enhanced by the role of a Worst-In-Class index set WiC_i of an arbitrary sector i. Where all economic entities $e(*, i) \in WiC_i$ are subject to increased correlation.

Finishing this reasoning, we address the question of how one can determine the constituents of BiC and WiC. A relative measure would be to use the z-score of deviations from the average rating score of the sector. A possible absolute measure could be carbon emissions, or more generally: **Green House Gas** emissions, in $tCO_2/year$ or $tGHG/year$ of the economic entities.

Having established the conceptional framework of the Monte Carlo simulation, in particular how the hazard rates depend on available exogenous ESG rating, and how we model the dependency structure of industry sectors via the correlation matrix, we could now turn to the actual simulation. But before we do this, we want to outline some analytical measures of risk factor sensitivities that are relevant for the equity prices we simulate. This is because, investment professionals often use sensitivities and risk decomposition to assess financial instruments quickly, and because the main principle of this paper is to be very close to data and concepts that are familiar to the investment front office.

[26]Cf. Roncalli, 2020, pp. 715, for a more detailed outline on Copulas.

3.4 Analytical Properties of the Model

In this section we will give some analytical properties of the model, as concerns the sensitivity of expected return to changes in the hazard rate, as well as a decomposition of portfolio risk values into ordinary market risk and ESG risk contribution down to the position level. As for our simulation, we again assume $k = 1$ and $UD = \emptyset$, that is, the company acts in only one industry sector and there is no unidirectional influence by other industry sectors.

3.4.1 Single Asset Analytics

Under the real world probability measure P the jump process of Equation 3.6 for the equity value $S_{c,t}$ of company c has the closed form solution given by Equation 3.7, where we have now additionally allowed for dividends denoted by the dividend return q. Please be referred to Appendix A.3 for the proof and discussion of this closed form solution.

$$S_{c,t} = S_{c,0} \cdot e^{(\mu_P - q - \frac{1}{2}\sigma^2)\cdot t + \sigma \cdot W_t + ln(1-\xi(\lambda))\cdot N_t} \tag{3.7}$$

Concerning the analytical results, we are interested in a so called **Greek- or Delta-Value**, that shows equity price sensitivity with respect to the hazard rate, by only using factors directly observable at the market and without trying to explain the equity return over the risk free rate by an extended CAPM[27], or statistically by other explaining economic variables.

Above Equation 3.7 is a Forward Measure for simulation needs. What we need is a **Backward Measure** that calculates back to the current price, starting from a given expected end value of the equity share price, or alternatively the expected return. For the expected return we consider[28]:

$$ln\left(\frac{S_{c,t}}{S_{c,0}}\right) = \mu_P t - qt - \frac{1}{2}\sigma^2 t + \sigma W_t + ln(1-\xi(\lambda))N_t \tag{3.8}$$

and apply the expectation operator under the **real probability measure** P:

$$\Longrightarrow E_P(r(S_{c,t})) = E_P\left(\frac{\Delta S_{c,t}}{S_{c,t}}\right) = E_P\left(\frac{S_{c,t} - S_{c,0}}{S_{c,0}}\right) \tag{3.9}$$

$$\approx E_P\left(ln\left(\frac{S_{c,t}}{S_{c,0}}\right)\right) = \mu_P t - qt - \frac{1}{2}\sigma^2 t + \lambda ln(1-\xi(\lambda))t$$

because $E_P(N_t) = \lambda t$. Note, under the **risk neutral** probability measure Q we would consider a modified (but equivalent) SDE using a **Compensated Poisson Process** $\hat{N}_t = N_t - \lambda t$ with the drift term corrected respectively, together with the risk free rate r_f instead of μ_P, that is:

$$dS_{c,t} = (r_f - q - \xi(\lambda)\cdot\lambda)\cdot dt + \sigma \cdot dW_t - \xi(\lambda)\cdot d(N_t - \lambda t)$$

and thus

$$ln\left(\frac{S_{c,t}}{S_{c,0}}\right) = r_f t - qt - \frac{1}{2}\sigma^2 t + \sigma W_t + ln(1-\xi(\lambda))N_t$$

The risk neutral expectation under Q then is

$$E_Q\left(ln\left(\frac{S_{c,t}}{S_{c,0}}\right)\right) = r_f t - qt - \frac{1}{2}\sigma^2 t + \lambda ln(1-\xi(\lambda))t$$

Instead, the variance is invariant, i.e. identical for both probability measures, and depends on the hazard rate level of environmental rating, where we define for the ease of notation the parameter function $g_\xi(\lambda) = ln(1-\xi(\lambda))$:

$$V(r(S_{c,t})) = V\left(ln\left(\frac{S_{c,t}}{S_{c,0}}\right)\right) = \sigma^2 \cdot t + \lambda g_\xi(\lambda)^2 \cdot t = V_{gbm}(r(S_{c,t})) + V_{esg}(r(S_{c,t})) \tag{3.10}$$

[27] Cf. Karydas and Xepapadeas, 2019 and Roncalli et al., 2020
[28] C.f. Wikipedia, n.d., explanation on compound Poisson process.

$$\Longrightarrow \frac{dV(r(S_{c,t}))}{d\lambda} = \frac{dV_{esg}(r(S_{c,t}))}{d\lambda} = \left(g_\xi(\lambda)^2 + 2\lambda \cdot \frac{d}{d\lambda} g_\xi(\lambda)\right) \cdot t$$

We will now derive the important **Hazard Rate Sensitivity** of the expected equity return $E_P(r(S_{c,t})) = E_P(\Delta S_{c,t}/S_{c,0})$.

$$\frac{d}{d\lambda} E_P(r(S_{c,t})) = \frac{d}{d\lambda}(\lambda g_\xi(\lambda) \cdot t) = \left(g_\xi(\lambda) + \lambda \cdot \frac{d}{d\lambda} g_\xi(\lambda)\right) \cdot t \tag{3.11}$$

In Section 4 we use for the jump size a piecewise linear function in course of the simulation $\xi(\lambda) = Min(m \cdot \lambda + \xi_{min}, \quad \xi_{max})$ which is:

$$\xi(\lambda) = \begin{cases} m \cdot \lambda + \xi_{min}, & \text{if } \leq \xi_{max} \\ \xi_{max}, & \text{Otherwise} \end{cases}$$

With this definition of the jump size, the hazard rate sensitivity in Equation 3.11 becomes

$$\frac{d}{d\lambda} E_P(r(S_{c,t})) =$$
$$\begin{cases} (ln(1 - (m\lambda + \xi_{min})) - \frac{m\lambda}{1-(m\lambda+\xi_{min})}) \cdot t, & \text{if } \lambda \leq \frac{\xi_{max}-\xi_{min}}{m} \\ ln(1 - \xi_{max}) \cdot t, & \text{if } \lambda > \frac{\xi_{max}-\xi_{min}}{m}) \end{cases}$$

As one can expect, the sensitivity of the time discrete equity return becomes linearly more negative with increasing hazard rate and time, as long as $\lambda \leq (\xi_{max} - \xi_{min})/m = \Delta\xi/m$. Increasing λ and t reduce the expected return. Which implies that investors will require a respective risk premium to compensate. For a dedicated analysis of this risk premium we refer to Karydas and Xepapadeas, 2019 and Roncalli et al., 2020 for two different CAPM related approaches. Afterwards $\lambda > \Delta\xi/m$, in the area of constant maximum jump size that is independent from λ, a further increase of the hazard rate itself is irrelevant, but the sensitivity becomes still more negative with increasing time horizon t.

The interesting part is where the jump size does no longer grow with λ. As the jump size needs to stay below 1, it is clear that $\xi(\lambda)$ needs to converge to some $\xi_{max} \leq 1$. The piecewise linear function is just a sharp pragmatic approach to model this. If we had taken a smoother, completely differentiable jump size like[29]

$$\xi(\lambda) = \xi_{min} + \Delta\xi \cdot (1 - e^{-m\lambda}), \quad \text{with} \quad \Delta\xi = \xi_{max} - \xi_{min}$$

we would have received

$$\frac{d}{d\lambda} E_P(r(S_{c,t})) =$$

$$\left(ln(1 - \xi_{max} + \Delta\xi e^{-m\lambda}) - \frac{m\lambda \cdot \Delta\xi \cdot e^{-m\lambda}}{1 - \xi_{max} + \Delta\xi e^{-m\lambda}}\right) \cdot t$$

and if we let the hazard rate go to zero and infinity:

$$\lambda \longrightarrow \infty : \quad \frac{d}{d\lambda} E_P(r(S_{c,t})) = ln(1 - \xi_{max}) \cdot t$$

$$\lambda \longrightarrow 0 : \quad \frac{d}{d\lambda} E_P(r(S_{c,t})) = ln(1 - \xi_{min}) \cdot t$$

[29]For this function the exponential distribution function was appropriately transformed.

3.4.2 Portfolio Analytics

For the portfolio analytics we will consider the covariance matrix and thus the portfolio variance and its decomposition into ordinary market risk contribution and ESG risk contribution, as given by the jump part. We will further consider the additive decomposition of portfolio variance and portfolio Value-at-Risk on position level. In summary, we are able to decompose additively to position level and each position level into its risk contributors: Continuous Market Risk and ESG Market Risk.

For this we start with the following definition of covariance between equities S_c and S_d of companies c, d. More precisely, we consider the covariance between returns $r(S_{c,t})$ and $r(S_{d,t})$ with:

$$r(S_{c,t}) = \frac{\Delta S_{c,t}}{S_{c,0}} = \frac{S_{c,t} - S_{c,0}}{S_{c,0}} \approx ln\left(\frac{S_{c,t}}{S_{c,0}}\right).$$

The same holds for $r(S_{d,t})$, and we thus analyse

$$Cov(r(S_{c,t}), r(S_{d,t})) = E\left([r(S_{c,t}) - E(r(S_{c,t}))] \cdot [r(S_{d,t}) - E(r(S_{d,t}))]\right)$$

We can directly insert the values of Equations 3.8 and 3.9 for $r(S_{c,t})$ and $r(S_{d,t})$ and their expected value. After some basic rearrangements, we obtain our **main risk decomposition result**[30]:

$$Cov(r(S_{c,t}), r(S_{d,t})) = \tag{3.12}$$

$$\underbrace{\sigma_c\sigma_d \cdot Corr(W_{c,t}, W_{d,t}) \cdot t}_{\text{GBM Market Risk}=:Cov^{gbm}_{c,d}(t)} + \underbrace{\sqrt{\lambda_c\lambda_d} \cdot ln(1-\xi(\lambda_c)) \cdot ln(1-\xi(\lambda_d)) \cdot Corr(N_{c,t}, N_{d,t}) \cdot t}_{\text{Jump by ESG Risk}=:Cov^{esg}_{c,d}(t)}$$

Thereby, in course of our calibration, $Corr(W_{c,t}, W_{d,t})$ and $Corr(N_{c,t}, N_{d,t})$ are the historical return correlations and expert guess correlations explained in Section 3.3.3 and Definition 3.1, cf. also Table 4 in Section 4. We note, that the hazard rate as well as the jump size increase the covariance of equity returns. We also note, that Equation 3.12 becomes the single asset variance in Equation 3.10 if $c = d$.

In the next subsection we will use the term $Cov^{gbm}_{c,d}(t)$ and $Cov^{esg}_{c,d}(t)$ for the additive decomposition of each covariance matrix entry into the ordinary GBM and jump ESG component as given by Equation 3.12, and $\Sigma^{gbm}_{c,d}(t), \Sigma^{esg}_{c,d}(t)$ for the respective matrices.

3.4.3 Additive Portfolio VaR and Volatility Decomposition

As concerns the Value-at-Risk we suggest to use the **Incremental VaR** for the additive portfolio risk decomposition to position level. The Incremental VaR is defined as follows:

$$IncrVaR(S_c) = \frac{VaR(ptf) - VaR(ptf \setminus \{S_c\})}{\sum_{d=1}^{n}(VaR(ptf) - VaR(ptf \setminus \{S_d\}))} \cdot VaR(ptf) \tag{3.13}$$

The additive decomposition then reduces to a n-many application of the Monte Carlo simulation for each equity asset S_c, $c \in \{1, ..., n\}$, i.e. to calculate the portfolio VaR as if equity position c was liquidated and its value distributed pro rata to the remaining positions.

As we have done our simulations in a way that allows to separate the GBM return component from the jump part return multiplicatively (cf. Equation 4.3), and because we have done simulations for the NoCorr (independent) case and the GBM case, we have in fact two possibilities to further decompose each position level VaR contribution into its jump part and what is caused by non zero correlation. The correlation part is a measure of industry sector interdependencies, for example

[30] For this we just used $E(W_{*,t}) = 0, E(N_{*,t}) = \lambda t$ and $Cov(X, Y) = E(XY) - E(X)E(Y)$ for $(X, Y) = (W_{c,t}, W_{d,t})$ and $(X, Y) = (N_{c,t} - \lambda_c t, N_{d,t} - \lambda_d t)$.

caused by the value chain. That is to say, firstly (1) we could go to the roots and use multiplicative manipulation of the returns as given by equation 4.3, and secondly (2) we can use the formula given by Equation 3.13. For this, we can extend trivially to decompose $VaR \coloneqq VaR(ptf)$, and $VaR(S_c) \coloneqq VaR(ptf \setminus \{S_c\})$ as follows:

$$VaR = \underset{\text{GBM Risk}}{VaR_{gbm}} + \underset{\text{Jump Corr Risk}}{(VaR - VaR_{NoCorr})} + \underset{\text{Pure Jump Risk}}{(VaR_{NoCorr} - VaR_{gbm})}$$

$$VaR(S_c) = \underset{\text{GBM Risk}}{VaR_{gbm}(S_c)} + \underset{\text{Jump Corr Risk}}{(VaR(S_c) - VaR_{NoCorr}(S_c))} + \underset{\text{Pure Jump Risk}}{(VaR_{NoCorr}(S_c) - VaR_{gbm}(S_c))}$$

now in order to infere $\Delta VaR(S_c) = VaR - VaR(S_c)$ define:

1. The Market Risk Contribution:

   $$\Delta VaR_{gbm}(S_c) = VaR_{gbm} - VaR_{gbm}(S_c)$$

2. The Jump Correlation Risk Contribution:

   $$\Delta VaR_{\Sigma_J} = VaR - VaR_{NoCorr}, \quad \Delta VaR_{\Sigma_J}(S_c) = VaR(S_c) - VaR_{NoCorr}(S_c)$$

3. The Pure Jump Risk Contribution:

   $$\Delta VaR_J = VaR_{NoCorr} - VaR_{gbm}, \quad \Delta VaR_J(S_c) = VaR_{NoCorr}(S_c) - VaR_{gbm}(S_c)$$

Then the trivially extended risk contribution, which is the **Single Position Incremental VaR** $IncrVaR(S_c)$ of Equation 3.13, can be decomposed into components (1) Market Risk (GBM), (2) Risk by Jump Correlation, and (3) Risk by the Hazard Rate (Jump) only, as follows:

$$\left(\frac{\Delta VaR_{gbm}(S_c)}{\sum_{d=1}^{n} \Delta VaR(S_d)} + \frac{\Delta VaR_{\Sigma_J} - \Delta VaR_{\Sigma_J}(S_c)}{\sum_{d=1}^{n} \Delta VaR(S_d)} + \frac{\Delta VaR_J - \Delta VaR_J(S_c)}{\sum_{d=1}^{n} \Delta VaR(S_d)} \right) \cdot VaR \tag{3.14}$$

As regards the **Portfolio Volatility**, we know by Equation 3.12 that we can write the Portfolio Variance for any $t \geq 0$ as

$$\sigma^2_{ptf}(w,t) \coloneqq V_{ptf}(w,t) =$$

$$w^T \cdot \begin{pmatrix} V_{gbm}(r(S_{1,t})) + V_{esg}(r(S_{1,t})) & \cdots & Cov^{gbm}_{1,n}(t) + Cov^{esg}_{1,n}(t) \\ \vdots & \ddots & \vdots \\ Cov^{gbm}_{n,1}(t) + Cov^{esg}_{n,1}(t) & \cdots & V_{gbm}(r(S_{n,t})) + V_{esg}(r(S_{n,t})) \end{pmatrix} \cdot w \tag{3.15}$$

$$= w^T \cdot \Sigma^{gbm}_{c,d}(t) \cdot w + w^T \cdot \Sigma^{esg}_{c,d}(t) \cdot w$$

$$\Longleftrightarrow \sigma^2_{ptf}(w) = \sum_{c=1}^{n} w_c \sum_{d=1}^{n} w_d (Cov^{gbm}_{c,d}(t) + Cov^{esg}_{c,d}(t))$$

$$= \sum_{c=1}^{n} \sum_{d=1}^{n} w_c w_d \cdot Cov^{gbm}_{c,d}(t) + \sum_{c=1}^{n} \sum_{d=1}^{n} w_c w_d \cdot Cov^{esg}_{c,d}(t) = \sigma^2_{gbm}(w,t) + \sigma^2_{esg}(w,t)$$

We now consider the first derivative of the portfolio return variance for the above two components *gbm* and *esg* with respect to a single equity weight $w_q, q \in \{1, ..., n\}$:

$$\frac{d\sigma^2_{ptf}(w,t)}{dw_q} = 2 \cdot \sigma_{gbm}(w,t) \cdot \frac{d\sigma_{gbm}(w,t)}{dw_q} + 2 \cdot \sigma_{esg}(w,t) \cdot \frac{d\sigma_{esg}(w,t)}{dw_q}$$

After some rearrangements and summing up over all possible w_q (done separately for the *gbm* and *esg* part) one can show the following formula of **Additive Volatility Decomposition**[31]:

$$\sigma_{ptf}(w,t) = \underbrace{\sum_{c=1}^{n} w_c \cdot \frac{d\sigma_{gbm}(w,t)}{dw_c}}_{\text{GBM Market Risk}} + \underbrace{\sum_{c=1}^{n} w_c \cdot \frac{d\sigma_{esg}(w,t)}{dw_c}}_{\text{ESG Jump Risk}} \tag{3.16}$$

Which gives the separation of portfolio return volatility on position level, and each position level into its risk types *gbm* and *esg*.

For the needed first derivative of $\sigma_*(w,t)$ (* stands for *esg* or *gbm*) we consider the matrix representation in Equation 3.15 and calculate for each $q \in \{1,...,n\}$:

$$\frac{d\sigma_*(w,t)}{dw_q} \approx \frac{\sigma_*(w_1,...,w_q,...,w_n,t) - \sigma_*(w_1,...,w_q - \Delta w_q,...,w_n,t)}{\Delta w_q}$$

as a numerical approximation.

After these results on risk decomposition on position level and level of risk type (Ordinary GBM Market Risk and ESG Market Risk) we now turn to our Monte Carlo simulation, where we investigate in particular the impact of different hazard rates and correlation scenarios between industry sectors and companies.

[31] Cf. Davis and Menchero, 2010 for an ordinary portfolio position decomposition, not covering the separation into a *gbm* and *esg* component

4 Simulation Study - EuroStoxx50 Equity Portfolio

For the simulation study we consider 49 member companies $c \in \{1, ..., 49\}$ of the EuroStoxx50 index. In order to simplify the equation structure, we assume all companies c as only comprising of one economic entity, and thus acting in exactly one economic sector i, that is: $e(c,i) = c$.

As concerns Equation 3.5, we introduce for each given equity share price S_c the deterministic jump magnitude $(\xi_c | dN_{t,c} = 1)$[32].

$$\frac{dS_{c,t}}{S_{c,t}} = \mu_c \cdot dt + \sigma_c \cdot dW_{c,t} - dJ_{c,t} \tag{4.1}$$

$$dJ_{c,t} = \sum_{j=1}^{dN_{c,t}} \xi_c = \underline{\underline{dN_{c,t} \cdot \xi_c}}$$

With $dS_{c,t} = S_{c,t+dt} - S_{c,t}$ we obtain the recursive formula for our computational simulation

$$S_{c,t+dt} = S_{c,t} \cdot (1 + \mu_c \cdot dt + \sigma_c \cdot dW_{c,t} - dJ_{c,t})$$

$$\iff S_{c,t+dt} = S_{c,t} \cdot \left(1 + \underbrace{E(r_{t \to (t+dt),c}) + \sigma(r_{t \to (t+dt),c}) \cdot \omega_{c,t}}_{\bar{x}_{c,t}} - \underbrace{dN_{c,t} \cdot \xi_c}_{\hat{x}_{c,t}} \right),$$

with $\omega_{c,t} \sim N(0,1), \quad dt = 10d, \quad \bar{x}_{c,t} \sim N(E(r_{10d,c}), \sigma^2(r_{10d,c})) \quad dN_{c,t} \sim Po(\lambda_c^{1d} \cdot dt)$

$$\iff S_{c,t+dt} = S_{c,t} \cdot (1 + \bar{x}_{c,t} - \hat{x}_{c,t}) \tag{4.2}$$

Where the 1-day λ is the compound hazard rate, cf. Table 2, Table 3, as well as Tables 11 and 10. The vector $(\omega_1, \ldots, \omega_{49})_t$ is based on historical return correlations, while the vector $(dN_1, ..., dN_{49})_t$ relies on expert guess correlations, as given by the logic of Definition 3.1, cf. also Equation 3.12 of covariance decomposition. The last equation is just an easier notation and tells how we implemented the recursive formula in the VBA-Excel simulation approach. Note, one can in an approximate way consider the following multiplicative separation of Equation 4.2 which eases the simulation and improves comparability between different scenarios, as it allows to keep the GBM component once simulated and just varying the jump part:

$$S_{c,t+dt} = S_{c,t} \cdot (1 + \bar{x}_{c,t} - \hat{x}_{c,t})$$

$$\overset{\bar{x}_{c,t} \cdot \hat{x}_{c,t} \approx 0}{\approx} \underline{S_{c,t} \cdot (1 + \bar{x}_{c,t}) \cdot (1 - \hat{x}_{c,t})} = S_{c,t} \cdot (1 + \bar{x}_{c,t} - \hat{x}_{c,t} \cdot (1 + \bar{x}_{c,t})) \tag{4.3}$$

The last term shows the approximation error: $\hat{x}_{c,t} \cdot (1 + \bar{x}_{c,t})$. Thus, there is only in case of a jump $(\hat{x}_{c,t} \neq 0)$ a biasing impact, that slightly increases the jump size. As $\bar{x}_{c,t} \sim N(E(r_{10d,c}), \sigma^2(r_{10d,c}))$ we consider the highest expected value of our EuroStoxx50 portfolio which is 0.008 rounded up. This implies a maximum expected bias of 1.008, which is very moderate and does not flaw the simulation results

If we define with $R_{s,c,t} = 1 + \bar{x}_{c,t} - \hat{x}_{c,t}$ the above return of equity c for Scenario $s \in \{1, ..., n\}$ during the time interval $[t, t + dt]$ for $t \in \{10, 20, ..., 250\}$, then our simulation approach yields 25-many matrices

$$\begin{pmatrix} R_{11,t} & \cdots & R_{1k,t} \\ \vdots & \ddots & \vdots \\ R_{n1,t} & \cdots & R_{nk,t} \end{pmatrix} \in \mathbb{R}^{nxk}$$

[32] Cf. also Roncalli, 2020, pp. 827, for a discussion of the technical aspects of a Monte Carlo simulation for a 1-dim jump diffusion.

of n-many scenarios for k-many equities in the portfolio. In our case we have $n = 1000$ and $k = 49$. This is aggregated with the vector of portfolio weights as follows, for each simulation day $t \in \{10, 20, ..., 250\}$, resulting in a simulation matrix of 25 column vectors on portfolio level:

$$\begin{pmatrix} R_{11,t} & \cdots & R_{1k,t} \\ \vdots & \ddots & \vdots \\ R_{n1,t} & \cdots & R_{nk,t} \end{pmatrix} \cdot w \quad \xrightarrow{t \in \{10,20,...,250\}} \quad \begin{pmatrix} R^{ptf}_{1,t=10} \\ \vdots \\ R^{ptf}_{n,t=10} \end{pmatrix}, \begin{pmatrix} R^{ptf}_{1,t=20} \\ \vdots \\ R^{ptf}_{n,t=20} \end{pmatrix}, \cdots, \begin{pmatrix} R^{ptf}_{1,t=250} \\ \vdots \\ R^{ptf}_{n,t=250} \end{pmatrix}$$

Thereby, we will base our analysis of the Transition Density, the Value-at-Risk impact and the derivation of the rule of thumb on $t \in \{10, 50, 150, 250\}$.

The jump size $\xi(\lambda)$ we employ for our simulation is a deterministic function dependent on the hazard rate and evolves increasing linearly with the hazard rate[33]:

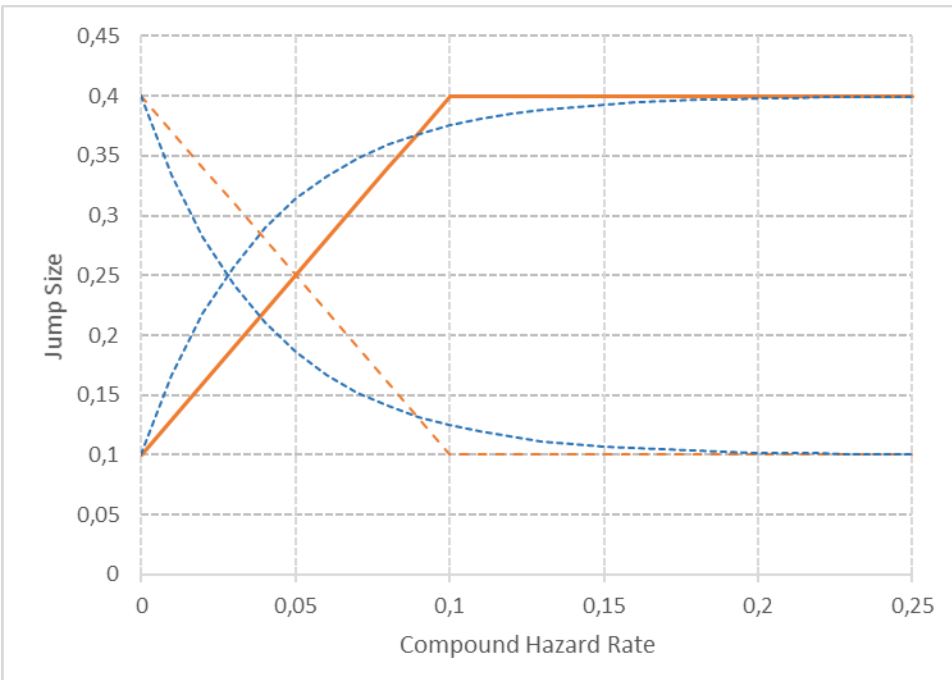

Figure 1: Possible Functions of Jump Size:
Upward: $\xi_c(\lambda_c) = Min(3 \cdot \lambda_c + 0.1, 0.4)$, $\quad \xi_c(\lambda_c) = 0.1 + 0.3 \cdot (1 - e^{-25\lambda_c})$
Downward: $\xi_c(\lambda_c) = Max(0.4 - 3 \cdot \lambda_c, 0.1)$, $\quad \xi_c(\lambda_c) = 0.4 - 0.3 \cdot (1 - e^{-25\lambda_c})$

Please note, for the simulation we decided to go for $dt = 10d$ instead of $dt = 1d$. This is because we not only want to cover the usual market risk time horizon of 20d Value-at-Risk, but up to 250d in order to cover the more credit risk like stochastic behavior of environmental risk. Having such a long time horizon, we decided to reduce the time steps in order to keep the computational effort reasonable. Thus, one needs to multiply the daily hazard rates – in the columns of systematic hazard rates and transition hazard rates in Tables 2, 3 - by 10, and accordingly one needs to use 10-day values for the expected return and volatility of the continuous part, cf. the static data in Tables shown by Figures 11 and 10 in the Appendix. The necessary time steps then reduce from

[33] This is because we are of the opinion, that the current transition process to a sustainable economy is at its beginning. When this process approaches its end – or later stages – it is also possible to assume the opposite: a linearly decreasing relationship $\xi(\lambda) = Max(-m\lambda + \xi^{max}, \xi^{min})$. This would lead to a smoother behavior. That is, big jumps with low probability and more small jumps. Note, such a change in the paradigm would have significant impact on the rule of thumb regression function.

250 to 25, at the cost of a less smooth empirical transition density. Note further, that *we kept the portfolio weights constant after each 10d time step*, which implies that for example losses that were caused by adverse jumps are reinvested as such to keep the portfolio weights constant.

Below we will investigate the impact of the current level of hazard rates as well as the impact of the stressed level, and both for three scenarios of correlations: No Correlation, the Moderate Level of Correlation – as suggested by Definition 3.1, and High Correlations - between industry sectors.

We apply the correlation scenarios shown in Table 4, given the Dependency Type of companies, that is: whether they belong to the same industry sector, whether they are Best-In-Class within their sector, whether two industry sectors are related (e.g. via the Supply Chain), and finally whether they are Best-In-Class of two related sectors, or whether there is no special relation ("Else"). The correlation matrices of Figure 12 and 13 in the Appendix show the moderate and high correlation dependency structure for the EuroStoxx50 portfolio we investigate in this paper[34].

DependencyType	NoCorr	ModerateCorr	HighCorr
Sector	0	0.8	0.9
SectorBiC	0	0.2	0.6
Related	0	0.4	0.75
RelatedBiC	0	0.1	0.5
Else	0	0	0.4

Table 4: Correlation Scenarios

4.1 Simulation Based on the Notion of Systematic and Specific Environmental Risk

In Figure 10 in the Appendix, we show our process parameterization depending on the sustainable rating, as concerns the implied hazard rates by systematic risk and specific risk for: normal and stressed conditions. The level of stress, or the likelihood of an adverse jump, respectively, is indicated by green, yellow, amber or even red color. How the stress scenario is generated out of our pragmatic systematic E rating approach, based on the Average Sector CO2 Intensity, and for the specific E rating, is described by Table 5:

Avg Sector CO2 Intensity	Stressed Avg Sector CO2 Intensity	Implied Sys E-Rating	BiC Spec E-Rating	Stressed BiC Spec E-Rating
< 0.5	$< 0.5/7$	$\to A$	A	C
< 2	$< 2/7$	$\to B$	B	D
< 4	$< 4/7$	$\to C$	C	E
< 10	$< 10/7$	$\to D$	D	F
< 20	$< 20/7$	$\to E$	E	G
< 40	$< 40/7$	$\to F$	F	G
≥ 40	$\geq 40/7$	$\to G$	G	G

Table 5: Logic of Stress Scenario

[34] In the Appendix we differentiate 4 related Blue Sectors, and 3 related Green sectors - with covered BiC companies. In addition we have highlighted there the sector of Materials, but we did not establish a relation of this industry sector with other sectors.

Based on the given rating, as defined by the logic in Table 5, the application of the relationship defined by Table 2 gives the hazard rates. Thereby, the logic is, that under stressed conditions it is more difficult for a company to stay below the CO2 Intensity levels that have been reduced by factor 7. The concrete choice of factor 7 was somehow arbitrarily taken by us, to provide a substantial stress scenario.

Concerning the level of stressed hazard rates, one can recognize by comparison of the Static Data tables in the Appendix, given by Figures 10 and 11, that we have here more aggressive and more red colored hazard rates, compared with the approach based on transition and physical risk[35].

It is worth to note, that what we have not considered so far, is to distinguish by **industry sector specific stress scenarios**, because some products and production processes may be more exposed to the sources of risk: transition risk and physical risk, depending on the industry sector. We will address this topic in the next section, where we outline the Monte Carlo simulation based on the notion of transition and physical risk. Actually, future research will consider a synthesis of the two notions of risk, by separating industry sector transition risk scenarios by their systematic (product impact) and specific (production process impact) component, cf. the table of Figure 5.

Before we assess the impact on the Transition Density over days 10d, 50d, 150d and 250d - separated for normal and stressed hazard rates. We want to show, for an eased comparison, how the 99%-Value-At-Risk changes over the time horizon - for the two hazard rate scenarios, the three different correlation scenarios and ordinary Brownian Motion. First with a table giving the concrete figures and afterwards we depict a respective VaR chart.

Value-at-Risk for SysSpec Hazard Rates				
Stressed Hazard Rates				
Time	Continous_Return	NoCorr	NormalCorr	HighCorr
10	9,7	19,4	22,3	38,7
50	20,0	48,9	54,5	77,5
150	29,8	82,7	83,6	92,0
250	35,9	93,1	93,9	98,2
Current Hazard Rates				
10	9,7	10,3	10,5	11,2
50	20,0	23,7	23,2	24,5
150	29,8	37,2	37,3	38,1
250	35,9	46,6	47,3	48,4

Figure 2: Value at Risk: Scenario Overview vs GBM

It can be seen - for the stressed hazard rates - that our moderate correlation proposal in Definition 3.1 gives a notable but not big increase in VaR, while a high correlation environment makes a substantial difference, compared to no correlation. The VaR curve converges to the complete portfolio loss, implying that differences due to higher correlation become smaller with higher hazard rates. One can also conclude that changes in correlation have more impact in a high hazard rate environment, while under moderate hazard rates the impact is small even for high correlation. **In both cases GBM underestimates the VaR substantially, but for moderate hazard rates only from a time horizon of 50 days on**.

[35] Cf. also the discussion at Tables 2 and 3.

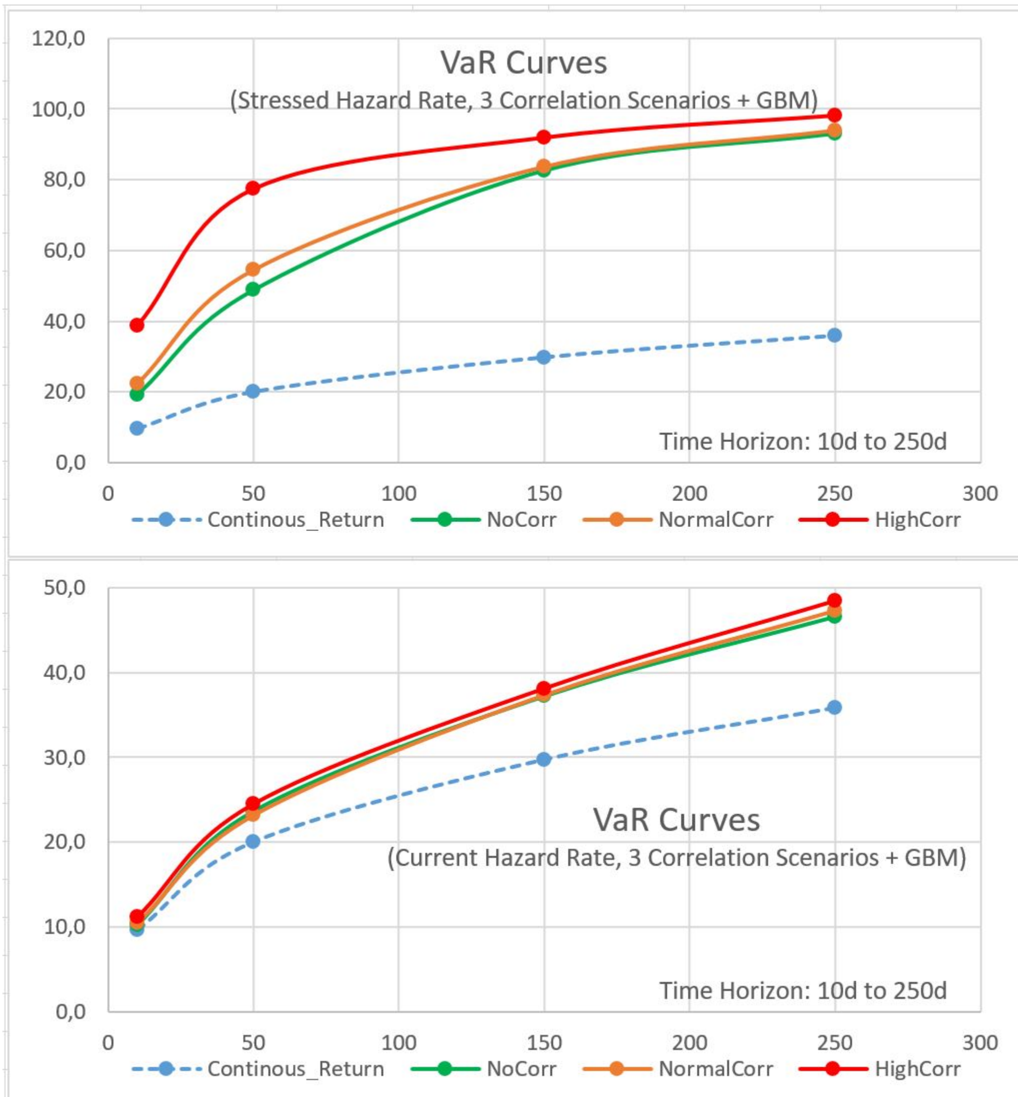

Figure 3: Value at Risk: Scenario Overview vs GBM

4.1.1 Current Market Hazard Rates with Correlation Scenarios

Under the current ESG ratings and respective moderate hazard rates, there would be a notable - even though not substantial - impact of sustainability risk. That is, just a few jumps in the process, as can be seen from the comparison of Transition Density, cf. the density charts given in Appendix C. The more jumps impact the equity portfolio return, the more goes the Transition Density to the left, relative to the GBM case. As expected, the effect becomes remarkably emphasized for longer time horizons. Again, as already expected from the VaR chart, **correlation has only a small influence on the Transition Density for the moderate current hazard rates**.

4.1.2 Stressed Hazard Rates with Correlation Scenarios

Given the stressed ESG rating scenario and respective higher hazard rates, the **picture changes dramatically**: there would be a substantial impact of sustainability risk, that is a remarkably increased amount of jumps in the process, compared to the current level of hazard rates, as can be inferred from the comparison of the Transition Densities in Appendix C and the upper part of the VaR chart in Figure 3. The correlation too, has now a substantial impact – even for short time horizon of 10d and 50d.

4.2 Simulation Based on the Notion of Transition and Physical Environmental Risk

With Figure 11 in the Appendix we show our process parametrization depending on the sustainable rating, as concerns the implied hazard rates by transition risk and physical risk for: normal and stressed conditions. The level of stress, or the likelihood of an adverse jump, respectively, is again indicated by green, yellow, amber or even red color. The modified levels of environmental and physical rating we need for the stress scenario are based on the current rating level and derived as explained below in detail.

The **current rating level** for transition risk is obtained by using the Best-In-Class environmental sub-rating. The current rating level of physical risk is obtained by applying the physical risk score[36]. This physical risk score is then mapped to a physical rating as shown by Table 6. In contrast to the previous Section on systematic and specific environmental risk simulation, we engage here **industry sector specific stress scenarios**, more precisely:

The **stressed physical risk rating** depends (1) on whether the main location of the company is a high physical risk or low physical risk country, as well as (2) whether the applicable industry sector has high physical risk exposure or low. For this, we apply a two stage stress factors ≤ 1 to the ordinary physical rating score[37], in order to make it more difficult to obtain the corresponding rating, as shown by Table 6.

The **country stress** factor depends on two sets of countries within the EuroStoxx50 country scope: $\{IT, ES, NL\}$ for countries with higher physical risk exposure and the rest with lower exposure. Our **industry sector stress** factor is admittedly very pragmatic and just considers insurance companies that are quite straightforwardly impacted by their client physical risk. Future work should be dedicated to extend the respective sector coverage. Thus, only insurance companies face a high industry sector specific stress factor, all others are low.

Physical Risk Score	Implied Current Physical Rating	Physical Risk Country Stress Factor High/Low	Physical Risk Industry Sector Stress Factor High/Low
< 0.15	A	1/3, 2/3	3/4, 1
< 0.3	B	1/3, 2/3	3/4, 1
< 0.45	C	1/3, 2/3	3/4, 1
< 0.6	D	1/3, 2/3	3/4, 1
< 0.75	E	1/3, 2/3	3/4, 1
< 0.9	F	1/3, 2/3	3/4, 1
$\in [0.9, 1]$	G	$\in [0.3, 1]$, $\in [0.6, 1]$	3/4, 1

Table 6: Logic of Physical Scenario, depending on Country and Industry Sector Physical Risk Exposure

For the **stressed transition risk rating** (measured by the environmental rating), we compared the carbon price impact data on industry sectors given by Adenot et al., 2022 in Figure 3 and Table 5 of their paper, with the sensitivity of the industry sector to the Brown-Minus-Green Index

[36] Cf. Section 2.2, where we have explained the ESG rating availability we require for this study

[37] first factor is by country and then by industry sector.

(BMG Index) of Roncalli et al., 2020 as measured by the β_{bmg}-sensitivity of their extended CAPM model, as given by Figure 24 of their paper, to assess the respective environmental/transition risk exposure. What we inferred from this comparison is aggregated in Table 7, that distinguishes 3 impact classes of Moderate, Significant and Intensive impact of transition risk stress scenario on industry sector.

The right hand column of Table 7 shows how we mapped the EuroStoxx50 industry sectors to the three classes of transition risk exposure, based on the information given by the carbon price impact and β_{bmg} sensitivity. An industry sector that faces Intensive exposure will experience a 3 Notch downgrade under the stress scenario, while sectors with Significant and Moderate exposure will experience a 2 Notch, 1 Notch downgrade, respectively, as shown by Table 8. Cf. also the table of Static Simulation Data given by Figure 11 in the Appendix, where we have highlighted the application of this principle by red, orange and green colored company names in the respective industry sectors.

Stress Impact Classes	Carbon Price Impact	Environmental β_{bmg} Impact	Our EuroStoxx 50 Sector Classification
Moderate	Health Care Information Tech Communication Financials Real Estate	Health Care Information Tech Communication Financials	Health Care Consumer Serv Diversified Finan Banks Insurance Pharma/Biotech Semiconductors SoftwareServies TeleCommnunication
Significant	Consumer Descret Consumer Staples Industrials	Consumer Descret Consumer Staples	Consumer Durables Food Staples Retailing Automobiles/Components Household/PersGoods Retailing Transportation
Intensive	Utilities Materials Energy	Industrials Utilities Real Estate Materials Energy	Capital Goods Utilities Real Estate Materials Energy Food/Breverage/Tobacco

Table 7: Stress Scenario Impact On Industry Sectors

Normal E-Rating Score	Stressed E- Rating Industry Sector **Intensively Exposed**	Stressed E-Rating Industry Sector **Significantly Exposed**	Stressed E-Rating Industry Sector **Moderately Exposed**
A	D	C	B
B	E	D	C
C	F	E	D
D	G	F	E
E	G	G	F
F	G	G	G
G	G	G	G

Table 8: Logic of Industry Sector Specific Transition Risk Scenario, depending on Industry Sector Transition Exposure

We admit, this provides only little granularity to distinguish how exposed an industry sector or a country is to transition risk or physical risk. But it gives the basic principle as a starting point and is sufficient for the simulation study to show the *Nature of ESG Risk*. A more granular classification could be based on Figure 8 of Adenot et al., 2022, where they give the sector carbon intensity based on the World-Input-Output Database (WIOD) sectors, that are more granular than what we use in this paper.

4.2.1 Current Market Hazard Rates with Correlation Scenarios

As the current level of hazard rates together with the correlation scenarios of the transition risk and physical risk approach equals what we have already investigated in the previous section for systematic and specific environmental risk, we do not outline the same results here again, but refer to the results of Section 4.1.1. Instead we focus on the industry sector specific stress testing in the next Section.

4.2.2 Stressed Hazard Rates with Correlation Scenarios

From a stress level point of view, what we use here – as measured by the weighted avg portfolio hazard rate of $\lambda_{ptf} = 0.11$ – is roughly in the middle between the intensive stress scenario of the previous section on systematic and specific risk with $\lambda_{ptf} = 0.19$, and the current level of $\lambda_{ptf} = 0.02$. This is confirmed by the VaR chart of Figure 4 that gives a remarkable but less pronounced result compared to what is shown by Figure 3 for the systematic/specific case. As concerns the Transition Densities (cf. respective density charts in Appendix D) also for this hazard rate Level there is a substantial shift to the left – compared to the GBM case – from a time horizon of 50 days on, for all levels of correlation. While only the high correlation level makes a notable (but not substantial) difference from a 100 day time horizon on, which is also confirmed by the VaR chart of Figure 4.

Value-at-Risk for TransPhy Hazard Rates				
Stressed Hazard Rates				
Time	Continous_Return	NoCorr	NormalCorr	HighCorr
10	9,7	11,8	12,0	14,1
50	20,0	28,7	30,4	32,6
150	29,8	48,4	50,9	56,8
250	35,9	63,1	64,3	68,4

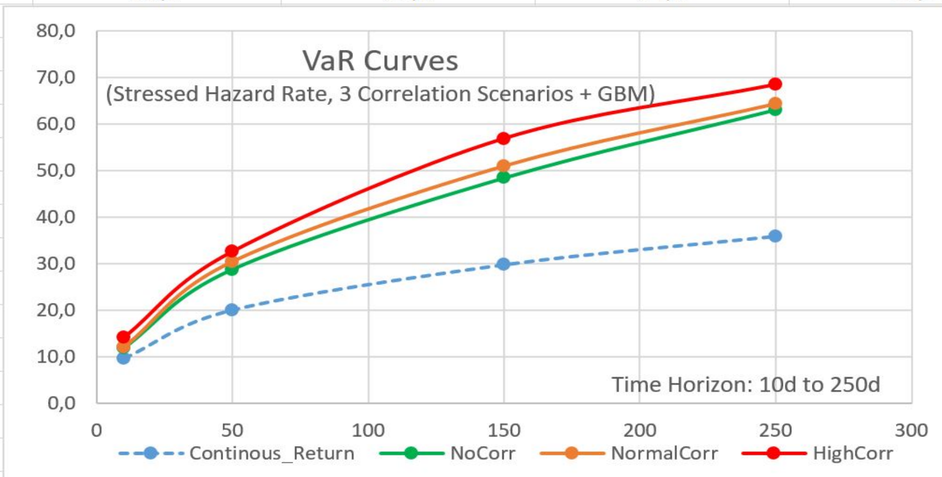

Figure 4: Value at Risk: Scenario Overview vs GBM for Transition - Physical Risk View

4.3 Synthesis of Systematic and Specific Environmental Risk with the Transition Risk Concept

Concluding the parameterization and simulation we want to explain, how the transition risk driven view of environmental risk can be combined with the systematic/specific separation of environmental industry sector risk. This is, separation by product and production process. To illustrate, we decompose Table 7 – that covers the industry sector specific stress scenarios of transition risk - into product related and production process related risk.

E-Risk Type	E-Risk SubType	Risk Level	Industry Sector
Transition Risk	Systematic (Product)	*Moderate*	Transport (Product is just Movement), Software Service, Telecommunication, Health Care, Semiconductors, Retailing, Pharma/BioTech, Food Staples, Utilities (just Electricity)
		Significant	Banks (Brown vs Green Financing), Insurance (Brown vs Green Financing), Diversified Financial, Materials, Food/Breverage/Tobacco, Household Personal Goods
		Intensive	Automobile (CO2 Emission or Energy Use), Energy (Gas/Oil), Capital Goods, Consumer Durables (Energy Use of Electric Gadgets), Real Estate (40% of Worlds Energy Use)
	Specific (Production Process)	*Moderate*	Software Service, Telecommunication, Banks, Insurance, Diversified Financials, Health Care, Semiconductors, Pharma/BioTec, Consumer Durables
		Significant	Automobile, Energy, Capital Goods, Real Estate, Food Staples, Retailing, Household Personal Goods
		Intensive	Transport (Emission or Energy Use), Utilities (CO2 Emissions for Electricity), Materials (Energy Use), Food/Breverage/Tobacco (Tobacco, high Water/Energy use for Meat, Packaging)

Figure 5: Decomposed Stress Scenario Impact

In order to highlight the impact of our decomposition we have underlined some industry sectors where the transition risk changes, based on whether we consider product related or production related transition impact. We have further added in brackets some hints as to why certain sectors where classified respectively.

4.4 Rule Of Thumb Proposal

In light of the simulation results, we now investigate the following ratio of Value-at-Risk figures, given a portfolio of n-many equity positions,

$$\rho^{VaR}(\xi, \lambda_{ptf}, \Sigma_J, d) = \frac{VaR^{\xi,\Sigma_J}_{Jump.GBM,99}(d, \lambda_{ptf})}{VaR_{GBM,99}(d)}$$

over the whole time horizon of $10d, \ldots, 250d$, as a multiplicative measure of how a VaR figure that is based on ordinary risk measurement changes, given additional ESG impact – as measured by the jump diffusion. Thereby, we indicate by the superscript ξ, Σ_J that Loss-Given-Jump, and industry sector dependency – as measured by the jump part correlation matrix $\Sigma_J \in \mathbb{R}^{nxn}$ (cf. Table 4 and the simulation algorithm in Section 3.3.3) – influence the upper VaR in addition to the main driver, which is the Weighted Average Portfolio Hazard Rate λ_{ptf} and the Time Horizon d.

Figure 6 shows the evolution of the proposed ratio over the time horizon of 250d – for selected representative portfolio hazard rates, and the Moderate correlation scenario. We can see that for all hazard rates the ratios are stable around the median with a slight upward trend. For the High correlation scenario in Figure 7, we can see that for low and mid hazard rate level the volatility around median is again small, while for high hazard rate we have higher deviation from median and a clear negative slope. The latter is due to the fact, that for high hazard rate and high correlation, the VaR curve increases at the beginning (low hazard rates) very steeply compared to GBM, while it has to flatten strongly afterwards, as both need to converge to a max loss of 100%, cf. Figure 3.

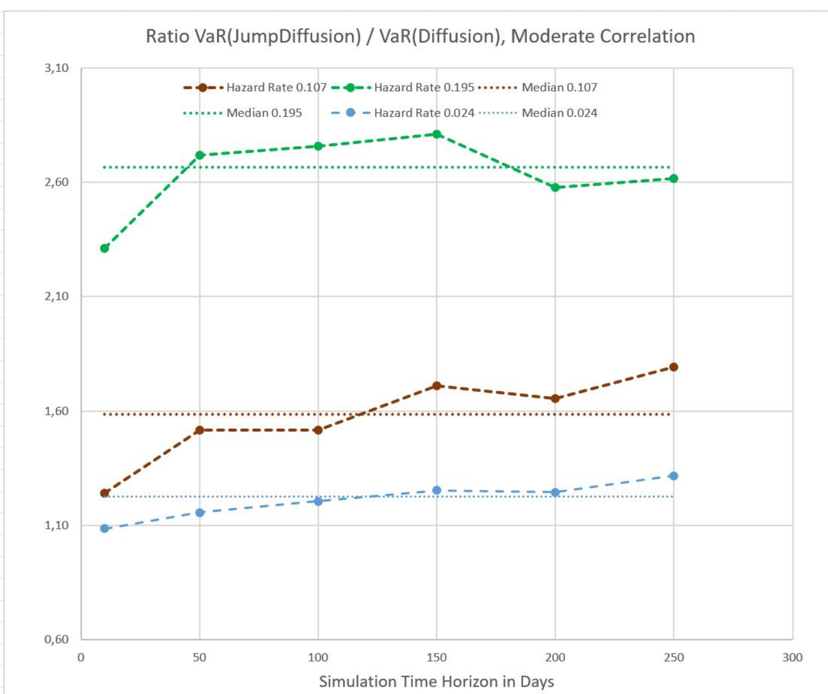

Figure 6: Median Ratio over Time for selected Hazard Rates and given Moderate Jump Correlation

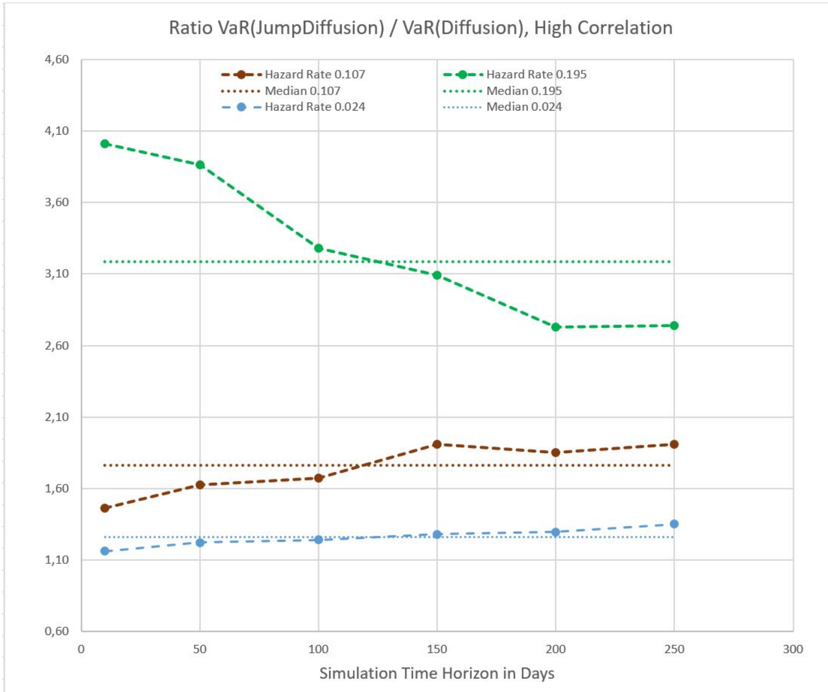

Figure 7: Median Ratio over Time for selected Hazard Rates and given High Jump Correlation

This suggests to apply the following measure as a multiplicative rule of thumb factor to modify ordinary VaR figures appropriately, to obtain an ESG-adjusted VaR value, depending on the **Weighted Average Hazard Rate of the Portfolio**.

$$\rho^{VaR}(\xi, \lambda_{ptf}, \Sigma_J) = \underset{d=10,\ldots,250}{Median}\,(\rho^{VaR}(\xi, \lambda_{ptf}, \Sigma_J, d)) \tag{4.4}$$

$$= \underset{d=10,\ldots,250}{Median}\left(\frac{VaR^{\xi,\Sigma_J}_{Jump.GBM,99}(d, \lambda_{ptf})}{VaR_{GBM,99}(d)}\right)$$

In Figure 8 and Figure 9 we show this median measure depending on the Weighted Average Portfolio Hazard Rate and find a high explanatory power for a quadratic evolution, also to extrapolate for even higher hazard rates. No matter, whether we consider the Zero, Moderate or High correlation level, there is a high quadratic explanatory power of $R^2 \geq 98\%$. On the other hand, the **influence of correlation** as a second explaining variable is not negligible. This is obvious from the notable distance between the solid blue and dotted grey and dotted orange line of Figure 8. They show the quadratic regression of our defined median measure for Zero and Moderate correlation, while the solid blue line is the regression function for the High correlation level[38].

[38]In order to ensure the polynomial regression to start exactly at a ratio of 1 for a hazard rate of zero (=GBM) we had to perform a restricted regression. That is, forcing the coefficient b_0 to 1, cf. Appendix A.2 for details.

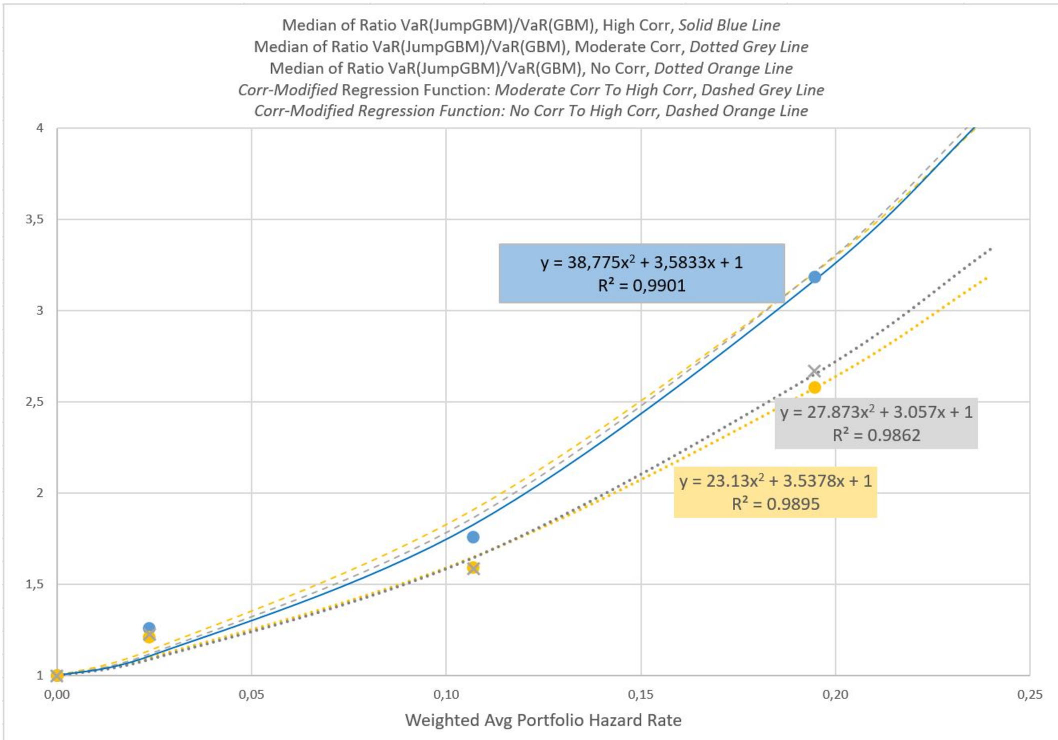

Figure 8: Median Ratio Regression dependent on Weighted Avg Portf Hazard Rate and Weighted Avg Portf Correlation

	VaR-Modification Factors VaR(JumpGBM)/VaR(GBM) to calculate JumpGBM VaR from GBM/Nomal Distribution Based VaR					
		Correlation Scenarios →				
	Wghtd Avg Ptf Hazard Rate	Median of HighCorr Scenario d=(10,...250)	Median of ModerateCorr Scenario d=(10,...,250)	Median of NoCorr Scenario d=(10,...,250)	Modified Regression NoCorr → HighCorr	Modified Regression ModerateCorr → HighCorr
Hazard Rate Scenarios →	0,00	1,00	1,00	1,00	*1,00*	*1,00*
	0,02	1,26	1,23	1,21	*1,14*	*1,12*
	0,11	1,76	1,59	1,59	*1,91*	*1,86*
	0,19	3,18	2,67	2,58	*3,21*	*3,21*
	0,24	NN	NN	NN	*4,08 (extrapolation)*	*4,13 (extrapolation)*

Figure 9: Table of Applicable VaR-Ratios to calculate JumpGBM VaR from Normal GBM VaR for the EuroStoxx50 portfolio

In addition, Figure 8 shows that one can even obtain from a NoCorr simulation and its regression function (orange dotted line) the simulation results under the HighCorr scenario – over all levels of hazard rates, as shown by the orange dashed line, that is very close to the solid blue line. The dashed orange line was obtained from the orange dotted line, by modifying the NoCorr regression function by a appropriately defined adjustment factor, as derived below.

In order to capture the **impact of the correlation level** in addition to the hazard rate level, we modify the NoCorr regression function as such to arrive as good as possible at the Moderate and High correlation VaR regression functions, using the **Weighted Average Portfolio Jump**

Correlation, cf. last two right hand columns of Figure 9. For this, let

$$w_+^T = \sqrt{\frac{2}{(n-1)}} \cdot (\sqrt{w_1}, ..., \sqrt{w_n})$$

be the vector of modified portfolio weights. These weights are choosen to ensure that multiplying this vector with the modified correlation matrix below gives a well-defined Weighted Average Portfolio Correlation. For this it is important to note, that the modified correlation matrix $C_{J,*}$ has $n^2/2 - n/2$ many non-trivial entries. Further, we note that portfolio weights go quadratic into the applied vector-matrix multiplication $w^T C_J w$, and for this reason we apply the square root to the vector entries w_i. More precisely, we have

$$\frac{n^2}{2} - \frac{n}{2} = \frac{(n-1)n}{2}$$

With only n-many portfolio weights w_i, that replace the scaling with $1/n$ that would be done for an ordinary statistical mean calculation, the necessary scaling to be performed, in addition to what the weights contribute as a scaling is:

$$\frac{n}{\frac{(n-1)n}{2}} = \frac{2}{(n-1)}$$

What is left, is to define the modified correlation matrix[39]

$$C_{J,*} = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ Corr(ij) & \cdots & 0 \end{pmatrix}$$

That is, we set the diagonal and upper triangle of the correlation matrix to zero, in order to avoid later double and redundant counting of correlation figures. The place holder "*" stands for the No Corr (nc), Moderate Corr (mc) or High Corr (hc) level. **We can now introduce** our **Measure of the Weighted Avg Ptf Correlation** as:

$$\kappa(w, C_J) = w_+^T \cdot C_J \cdot w_+ \in [-1, 1] \tag{4.5}$$

Based on this weighted avg correlation value of the portfolio we consider the following ratios:

$$\rho_{nc,hc}^{ptf} = \frac{1 + \kappa(w, C_{J,hc})}{1 + \kappa(w, C_{J,nc})} \quad \text{and} \quad \rho_{mc,hc}^{ptf} = \frac{1 + \kappa(w, C_{J,mc})}{1 + \kappa(w, C_{J,nc})}$$

This definition ensures that $\rho_{nc,nc}^{ptf} = 1 \quad \wedge \quad \rho_{nc,*}^{ptf} \geq 0$[40]. The ratio is applied to the orange dotted line of Figure 8 which is the quadratic regression equation of the No Correlation level: $a_{nc}\lambda_{ptf}^2 + b_{nc}\lambda_{ptf} + c_{nc}$, in order to obtain a good approximation of the high correlation regression function (solid blue line) without the need of a correlated Monte Carlo simulation. For the dashed orange line we have:

$$\begin{aligned} q_R(\lambda_{ptf}, \rho_{ptf}) &= \rho_{nc,hc}^{ptf} \cdot (a_{nc} \cdot \lambda_{ptf}^2 + b_{nc} \cdot \lambda_{ptf}) + c_{nc} \\ &= \rho_{nc,hc}^{ptf} a_{nc} \cdot \lambda_{ptf}^2 + \rho_{nc,hc}^{ptf} b_{nc} \cdot \lambda_{ptf} + c_{nc} \end{aligned} \tag{4.6}$$

In our case we have $\rho_{nc,hc} = 1.403, \quad \rho_{mc,hc} = 1.338$.

In summary, we conclude that taking into account the two main driver of environmental marked risk - Weighted Average Portfolio Hazard Rate and Weighted Average Portfolio Jump Correlation

[39]Cf. 12 in the Appendix for the EuroStoxx50 moderate and high correlation matrix
[40]If the correlation matrix is positive semidefinite then we have $\rho_{nc,*}^{ptf} \geq 1$

we are able to derive a **Stable Rule of Thumb** that can be used to sufficiently approximate jump GBM VaR figures from ordinary GBM over different hazard rates, different time horizons (even up to 250d) and different correlation levels for a given equity portfolio, in our case the EuroStoxx50. It gives also the possibility to define stress scenarios for the portfolio based on the risk factors hazard rate and ESG correlation, as shown by Figure 9.

In general, we assume that the derived modification factors are subject to changes, even though they proved to be very stable for the EuroStoxx50, over different time horizons and levels of hazard rate and correlation. We thus, propose to use for all portfolios that are mapped to a Benchmark the **Benchmark Portfolio** in the NoCorr and GBM case and calculate the modification factors as outlined above, e.g. on a quarterly basis, instead of a regular daily Monte Carlo simulation. For Absolute Return portfolios we suggest to set up a **Model Portfolio** and then to go for a similar process.

From an **algorithmic point of view**, this is, instead of performing a correlated Copula Monte Carlo simulation: Consider the ordered sequence of p-many selected hazard rates $(\lambda_1^{ptf}, ..., \lambda_p^{ptf})$ that build our regression support of the independent variable dimension. For these do a NoCorr[41] Monte Carlo simulation over the needed time horizon e.g. $d \in H = \{10, 50, 100\}$ Days to get the calculation basis. Then the regression points to be explained are given with the corresponding sequence of medians

$$\Big(\lambda_1^{ptf}, \rho^{VaR}(\xi, \lambda_1^{ptf}, Id)\Big), ..., \big(\lambda_p^{ptf}, \rho^{VaR}(\xi, \lambda_p^{ptf}, Id)\big)$$

Finally, the obtained quadratic regression function $q_R(\lambda_{ptf}, \rho_{ptf})$ is parameterized as outlined by Equation 4.6, based on Weighted Avg Ptf Hazard Rate λ_{ptf} and Weighted Avg Ptf Jump Correlation Factor ρ_{ptf}, to get the portfolio-customized **modification factors** that correspond to what is shown by the table of Figure 9. Thus, the needed modification factors for ordinary, Normal distribution based, VaR figures are given by the quadratic regression function $q_R(\lambda_{ptf}, \rho_{ptf})$.

[41] NoCorr implies $\Sigma_J = Id \in \mathbb{R}^{nxn}$

5 Conclusion and Further work

We tried to outline a possible approach of how to integrate ESG risk into existing common fundamental market risk measurement approaches. The main principle is that ESG risk is implicitly embedded in observable market risk factors, like share prices and credit spreads, and interprets ESG risk as an additional jump component explaining the future behavior of these risk factors. Mathematically, this paper extends continuous Brownian Motion by a sum of jump components, and models dedicatedly the interdependency of economic entities over different industry sectors by using two correlation matrices for the continuous and jump part. These are used by a Gaussian Copula to generate respective correlated equity return movements over time. Further, hazard rates of possible jumps are taken as exogenously given, directly derived from environmental rating data. Thereby, the hazard rate and the mapped E rating carry both the interpretation of the **Expected Number of Adverse Jumps during 250 trading days**.

As the JGBM model has an explicit solution, we are able to provide in Section 3.4 some analytical properties of the model on single equity return Level as well as portfolio level. These are primarily: Hazard Rate Sensitivity, and Additive Risk Decomposition on risk type (GBM Market Risk vs Environmental Jump Risk) and position level.

In order to calibrate the model, and to reflect market reality properly, we propose to clearly distinguish between systematic (the product) and specific (the production process) ESG risk. Further, we view a company as the sum of its economic entities, where the activities of each entity belongs to one single economic sector. Each economic entity stands for a jump component in the jump diffusion model. For the simulation study of this paper, we assumed that each company is made up of exactly one economic entity.

As concerns the currently observable Best-In-Class rating approaches there is no clear separation between systematic and specific ESG risk, but Green Product information is taken into account directly as an additional variable (depending on the industry sector) in course of an otherwise production process oriented rating methodology. We thus investigated two approaches to calibrate the model: (1) systematic and specific risk, and (2) transition and physical risk – as an alternative calibration approach, that is more in line with current ESG rating methodology.

In view of this, it is worth to note that we apply **industry sector specific stress scenarios** based on the notion of transition and physical risk. In contrast, we have not yet differentiated between sectors as concerns systematic and specific environmental risk. In fact, future research will have a look at the separation of transition risk into its systematic product driven component and its specific component related to the production process, as proposed by the table in Figure 5.

The results show that on our 250 trading day horizon environmental risk — as measured by the hazard rate and the implied jump process — is on diversified portfolio level only relevant for longer time horizons ($> 50d$), or in case of stressed scenarios. As measured by the Transition Density of portfolio returns, and depending on how severe the hazard rates can become. In contrast, on a single stock level, ESG risk is relevant under normal hazard rates on shorter time horizon already. Our simulations indicate, that **an exclusion list** of the worst rated issuers and **clear limits on exposure** to bad rated companies would already do the job of managing or – more precisely – efficiently restricting current ESG risk. Whether this also covers the management of regulatory restricted Principal Adverse Impact (PAI), depends on how the ESG rating is obtained. In case the focus is on Asset Return risk, an additional limit framework for PAI is required.

As concerns an easy **rule of thumb** to modify current risk figures instead of performing regularly a Monte Carlo simulation, a quadratic regression function depending on the median of Weighted Average Portfolio Hazard Rate and modified by a correction factor based on the Weighted Average

Portfolio Correlation showed very promising results in our study, cf. Section 4.4. This approach proved to be very stable over different time horizons, hazard rates and correlation levels. The obtained modification factors can also be used to derive **stress scenarios** applied to ordinary VaR figures **based on scenarios for hazard rate and correlation**, or even to modify historical returns to perform an environmental risk adjusted Historical VaR simulation.

Next steps will be to investigate this proposal in light of a respective simulation for a Fixed Income portfolio. In addition, the split of transition risk into a systematic and specific environmental risk component, together with a respective hazard rate calibration and stress scenario definition is a topic. Thus, the stress scenarios will be not only sector specific but also depend on the level of systematic and specific environmental risk.

A Proofs

A.1 Proof for probabilities of $dN_{i,t}$ at Equation 3.5

$$P(dN_{i,t} \geq 1) = \sum_{i=1}^{\infty} \frac{e^{-\lambda dt}(\lambda dt)^i}{i!} = e^{-\lambda dt} \cdot \underbrace{\sum_{i=1}^{\infty} \frac{(\lambda dt)^i}{i!}}_{\text{exp.Row } = e^{\lambda dt} - 1} = 1 - e^{-\lambda dt} =$$

$$= 1 - \left(1 - \lambda dt + \frac{\lambda^2 dt^2}{2} - \cdots\right) = \lambda dt + o(\lambda dt)^2, \quad o(\lambda dt)^2 < 0$$

A.2 Proof for Restricted Polynomial Regression

Given a sample $x = (x_1, ..., x_n)$ of the explaining variable, and the dependent variable $y = (y_1, ..., y_n)$ that is to be explained, we wish to fit the polynomial regression function $y = b_0 + b_1 x + ... + b_m x^m$ with $b_0 = 1$ and coefficient vector $b = (b_0, b_1, ..., b_m)$. The ordinary quadratic optimization problem without restriction is to solve

$$\underset{b}{Min}((y - X \cdot b)^T \cdot (y - X \cdot b))$$

Thereby, matrix X is defined as follows

$$X = \begin{pmatrix} 1 & x_1 & \cdots & x_1^m \\ 1 & x_2 & \cdots & x_2^m \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^m \end{pmatrix}$$

As we want to fix $b_0 = 1$, we consider the the reduced matrix X_R and the reduced coefficient vector b_R as follows:

$$X_R = \begin{pmatrix} x_1 & \cdots & x_1^m \\ x_2 & \cdots & x_2^m \\ \vdots & \ddots & \vdots \\ x_n & \cdots & x_n^m \end{pmatrix}, \quad b_R^T = (b_1, ..., b_m)$$

Then the restricted optimization problem with b_0 fixed can be written as

$$\begin{aligned} &\underset{b_R}{Min}((y - (\vec{b_0} + X_R \cdot b_R))^T \cdot (y - (\vec{b_0} + X_R \cdot b_R)), \quad \text{where} \quad \vec{b_0}^T = (b_0, ..., b_0) \\ \iff &\underset{b_R}{Min}(((y - \vec{b_0}) - X_R \cdot b_R))^T \cdot ((y - \vec{b_0}) - X_R \cdot b_R)) \\ \iff &\underset{b_R}{Min}((y - \vec{b_0})^T (y - \vec{b_0}) - 2 b_R^T X_R^T (y - \vec{b_0}) + b_R^T \cdot X_R^T X_R \cdot b_R) \end{aligned}$$

Differentiating with respect to b_R, setting this to zero to obtain the necessary condition for a minimum, and rearranging for b_R we arrive at our modified restricted regression, and obtain finally, the searched coefficient vector of the restricted polynomial regression $\hat{b} = (b_0, \hat{b}_R)$:

$$\begin{aligned} &\frac{d}{db_R}\left(-2 b_R^T X_R^T (y - \vec{b_0}) + b_R^T \cdot X_R^T X_R \cdot b_R\right) \\ &= -2 \cdot X_R^T (y - \vec{b_0}) + 2 \cdot X_R^T X_R \cdot b_R = 0 \\ &\iff \hat{b}_R = (X_R^T X_R)^{-1} \cdot X_R^T \cdot (y - \vec{b_0}) \\ &\hat{b} = (b_0, \hat{b}_R)^T = (1, \hat{b}_R)^T \end{aligned}$$

A.3 Proof of Explicit Solution of Jump Diffusion

We will apply the generalized Lemma of Ito (cf. Privault, n.d., Proposition 20.13 and 20.14) to derive the closed form solution in Equation 3.7 from the SDE we use for our Monte Carlo simulation: $dS_t = \mu S_t dt + \sigma S_t dW_t - \xi(\lambda) S_t dN_t$. Note, the Lemma of Ito is the stochastic calculus version of the Chain Rule in ordinary differentiation. It can thus considered piecewiese additively, that is: it can be applied separately for the GBM part and the jump part and added together the two results.

The Lemma of Ito in its differential form is in our case as follows - given a twice differentiable function $f(S_t)$:

$$\begin{aligned} df(S_t) = \mu S_t \cdot f'(S_t) \cdot dt + \frac{1}{2}(\sigma S_t)^2 \cdot f''(S_t) \cdot dt + \sigma S_t \cdot f'(S_t) \cdot dW_t \\ +(f(S_t) - f(S_{t-dt})) \cdot dN_t \end{aligned}$$

with jump size: $f(S_t) - f(S_{t-dt})$ and and number of jumps: dN_t.

In order to derive the formula of Equation 3.7 we consider $f(S_t) = ln(S_t)$, then the stochastic calculus chain rule of the famous Ito Kiyoshi delivers

$$\begin{aligned} dln(S_t) = \mu S_t \cdot \frac{1}{S_t} \cdot dt - \frac{1}{2}\sigma^2 S_t^2 \cdot \frac{1}{S_t^2} \cdot dt + \sigma S_t \cdot \frac{1}{S_t} \cdot dW_t \\ +(ln(S_t) - ln(S_{t-dt})) \cdot dN_t \\ \Longleftrightarrow dln(S_t) = \mu \cdot dt - \frac{1}{2}\sigma^2 \cdot dt + \sigma \cdot dW_t + ln\left(\frac{S_t}{S_{t-dt}}\right) \cdot dN_t \end{aligned}$$

and with $S_t = S_{t-dt} - \xi \cdot S_{t-dt} = (1-\xi) \cdot S_{t-dt}$ we obtain:

$$ln(S_t) = ln(S_{t-dt}) + \mu \cdot dt - \frac{1}{2}\sigma^2 \cdot dt + \sigma \cdot dW_t + ln(1-\xi) \cdot dN_t$$

Repeated iterative application of the last difference equation yields:

$$ln(S_t) = ln(S_0) + \mu \cdot t - \frac{1}{2}\sigma^2 \cdot t + \sigma \cdot W_t + ln(1-\xi) \cdot N_t$$

We apply the exponent to both sides:

$$S_t = S_0 \cdot e^{\mu \cdot t - \frac{1}{2}\sigma^2 \cdot t + \sigma \cdot W_t + ln(1-\xi) \cdot N_t} \qquad \square$$

Or alternatively:

$$\begin{aligned} S_t = S_0 \cdot e^{\mu \cdot t - \frac{1}{2}\sigma^2 \cdot t + \sigma \cdot W_t} \cdot e^{ln\left((1-\xi)^{N_t}\right)} \\ \Longleftrightarrow S_t = S_0 \cdot e^{\mu \cdot t - \frac{1}{2}\sigma^2 \cdot t + \sigma \cdot W_t} \cdot (1-\xi)^{N_t} \qquad \square \end{aligned}$$

The jump version of the Lemma of Ito can also be used to check how our SDE of JGBM should have been "equivalently" defined, to avoid the above natural logarithm. For this we apply the following jump version of Ito's Lemma. Let $f(t, W_t, N_t) = S_t$ again a twice differentiable function in W_t and differentiable in t, N_t, then:

$$df(t, W_t, N_t) = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial W_t} dW_t + \frac{1}{2}\frac{\partial^2 f}{\partial W_t^2} dW_t^2 + \frac{\partial f}{\partial N_t} dN_t$$

thereby note: $\quad dW_t^2 \longrightarrow dt$

and: $\frac{\partial f}{\partial N_t} = \frac{f(N_t)-f(N_{t-dt})}{N_t-N_{t-dt}} = \frac{f(N_t)-f(N_{t-dt})}{1} = f(N_t) - f(N_{t-dt})$

$$\Longrightarrow df(t,W_t,N_t) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial W_t}dW_t + \frac{1}{2}\frac{\partial^2 f}{\partial W_t^2}dt + (f(N_t) - f(N_{t-dt}))dN_t$$

We now consider the following closed form solution: $f(t,W_t,N_t) = S_t = S_0 \cdot e^{(\mu-\frac{1}{2}\sigma^2)t+\sigma W_t - \hat{\xi}\cdot N_t}$. Then we have:

$$\begin{aligned}
\frac{\partial f}{\partial t}\cdot dt &= S_0 \cdot e^{(*)} \cdot (\mu - \frac{1}{2}\sigma^2)\cdot dt = S_t \cdot (\mu - \frac{1}{2}\sigma^2)\cdot dt \\
\frac{\partial f}{\partial W_t}\cdot dW_t &= S_0 \cdot e^{(*)} \cdot \sigma \cdot dW_t = S_t \cdot \sigma \cdot dW_t \\
\frac{1}{2}\frac{\partial^2 f}{\partial W_t^2}\cdot dt &= S_0 \cdot e^{(*)} \cdot \frac{1}{2}\sigma^2 \cdot dt = S_t \cdot \frac{1}{2}\sigma^2 \cdot dt
\end{aligned}$$

In summary the GBM part becomes:

$$df(t,W_t,N_t) = S_t = \mu S_t \cdot dt + \sigma S_t \cdot dW_t$$

As concerns the Jump part it follows:

$$\frac{\partial f}{\partial N_t}dN_t = \left(S_0 \cdot e^{(\mu-\frac{1}{2}\sigma^2)t+\sigma W_t-\hat{\xi}\cdot\mathbf{N_t}} - S_0 \cdot e^{(\mu-\frac{1}{2}\sigma^2)t+\sigma W_t-\hat{\xi}\cdot N_{t-dt}}\right)\cdot dN_t$$

and because the term in brackets is the jump size - given a jump, we can rewrite to:

$$\begin{aligned}
df(N_t) &= \left(S_0 \cdot e^{(\mu-\frac{1}{2}\sigma^2)t+\sigma W_t-\hat{\xi}\cdot(\mathbf{N_{t-dt}+1})} - S_0 \cdot e^{(\mu-\frac{1}{2}\sigma^2)t+\sigma W_t-\hat{\xi}\cdot N_{t-dt}}\right)\cdot dN_t \\
\Longleftrightarrow df(N_t) &= S_0 \cdot e^{(\mu-\frac{1}{2}\sigma^2)t+\sigma W_t-\hat{\xi}\cdot\mathbf{N_{t-dt}}} \cdot \left(e^{-\hat{\xi}} - 1\right)\cdot dN_t \\
\Longleftrightarrow df(N_t) &= S_{t-dt} \cdot \left(e^{-\hat{\xi}} - 1\right)\cdot dN_t \\
\Longleftrightarrow df(N_t) &= -S_{t-dt} \cdot \left(1 - e^{-\hat{\xi}}\right)\cdot dN_t
\end{aligned}$$

And overall we receive the searched alternative JGBM to avoid the logarithm:

$$dS_t = \mu S_t \cdot dt + \sigma S_t \cdot dW_t - \left(1 - e^{-\hat{\xi}(\lambda)}\right) S_{t-dt}\cdot dN_t \qquad \square$$

Thereby, $1 - e^{-\hat{\xi}(\lambda)} \in (0,1]$, with $\hat{\xi}(\lambda) \in [0,\infty)$.

We did not go for this version of the JGBM because we wanted to have a clear and direct interpretation of $\xi(\lambda)$ as the jump size in the SDE as well as in its closed form solution. In this alternative version the jump size is given by $1 - e^{-\xi(\lambda)}$.

B Portfolio Information

EuroStoxx50	ISIN	MSCI Sector	CO2 Implied Systematic E-Rating	Specific E-Rating BIC E-Rating	10d Systematic HazardRate	10d Spec HazardRate Offset	10d Compound HazardRate	ptf_weight	Vola(r_10d)	E(r_10d)	CO2 Intensity Sector Average	CO2 Derived Sector Rating	Stressed Systematic E-Rating	Stressed Specific E-Rating	Stressed 10d Sys Hazard Rate	Stressed 10d Spec Hazard Rate	Stressed 10d Compound Hazard Rate
Daimler	DE0007100000	Automobiles & Components	B	C	0,004	0,000	0,004	0,0183	0,0795	0,0078	0,63	B	D	E	0,02	0,060	0,080
Stellantis	NL00150001Q9	Automobiles & Components	B	D	0,004	0,021	0,025	0,0112	0,0798	0,0032	0,63	B	D	F	0,02	0,140	0,160
Volkswagen	DE0007664039	Automobiles & Components	B	D	0,004	0,021	0,025	0,0097	0,0833	-0,0026	0,63	B	D	F	0,02	0,140	0,160
BMW	DE0005190003	Automobiles & Components	B	D	0,004	0,021	0,025	0,0085	0,0678	0,0013	0,63	B	D	F	0,02	0,140	0,160
BNP Paribas	FR0000131104	Banks	A	B	0,002	-0,001	0,002	0,0210	0,0795	0,0006	0,18	A	B	D	0,004	0,021	0,025
Santander	ES0113900J37	Banks	A	C	0,002	0,000	0,002	0,0182	0,0816	-0,0018	0,18	A	B	E	0,004	0,044	0,048
ING	NL0011821202	Banks	A	C	0,002	0,000	0,002	0,0130	0,0912	-0,0010	0,18	A	B	E	0,004	0,044	0,048
Intesa	IT0000072618	Banks	A	C	0,002	0,000	0,002	0,0129	0,0755	-0,0012	0,18	A	B	E	0,004	0,044	0,048
BBVA	ES0113211835	Banks	A	C	0,002	0,000	0,002	0,0123	0,0835	0,0016	0,18	A	B	E	0,004	0,044	0,048
Siemens	DE0007236101	Capital Goods	B	C	0,004	0,000	0,004	0,0341	0,0607	0,0038	1,64	B	E	E	0,04	0,080	0,120
Schneider Electric	FR0000121972	Capital Goods	B	B	0,004	-0,001	0,003	0,0292	0,0538	0,0083	1,64	B	E	D	0,04	0,030	0,070
Airbus	NL0000235190	Capital Goods	B	B	0,004	-0,001	0,003	0,0214	0,0896	-0,0035	1,64	B	E	D	0,04	0,030	0,070
Vinci	FR0000125486	Capital Goods	B	D	0,004	0,021	0,025	0,0194	0,0620	-0,0018	1,64	B	E	F	0,04	0,200	0,240
Safran	FR0000073272	Capital Goods	B	C	0,004	0,000	0,004	0,0142	0,0894	-0,0053	1,64	B	E	E	0,04	0,080	0,120
Kone	FI0009013403	Capital Goods	B	C	0,004	0,000	0,004	0,0074	0,0441	-0,0035	1,64	B	E	E	0,04	0,080	0,120
LVMH	FR0000121014	Consumer Durables & Apparel	B	C	0,004	0,000	0,004	0,0571	0,0585	0,0068	1,35	B	D	E	0,02	0,060	0,080
EssilorLuxottica	FR0000121667	Consumer Durables & Apparel	B	D	0,004	0,021	0,025	0,0164	0,0572	0,0032	1,35	B	D	F	0,02	0,140	0,160
Kering	FR0000121485	Consumer Durables & Apparel	B	C	0,004	0,000	0,004	0,0148	0,0682	-0,0002	1,35	B	D	E	0,02	0,060	0,080
Adidas	DE000A1EWWW0	Consumer Durables & Apparel	B	C	0,004	0,000	0,004	0,0136	0,0677	-0,0057	1,35	B	D	E	0,02	0,060	0,080
Flutter Entertainmen	IE00BWT6H894	Consumer Services	A	D	0,002	0,021	0,023	0,0066	0,0863	-0,0001	0,14	A	B	F	0,004	0,092	0,096
Deutsche Boerse	DE0005810055	Diversified Financials	A	C	0,002	0,000	0,002	0,0101	0,0508	0,0026	0,16	A	B	E	0,004	0,044	0,048
TotalEnergies	FR0000120271	Energy	E	C	0,040	0,000	0,040	0,0445	0,0679	-0,0003	15,18	E	G	E	0,16	0,200	0,360
Eni	IT0003132476	Energy	E	B	0,040	-0,010	0,030	0,0121	0,0647	-0,0005	15,18	E	G	D	0,16	0,060	0,220
Ahold	NL0011794037	Food & Staples Retailing	B	D	0,004	0,021	0,025	0,0106	0,0404	0,0035	0,70	B	D	F	0,02	0,140	0,160
Anheuser Busch	BE0974293251	Food, Beverage & Tobacco	D	D	0,020	0,025	0,045	0,0167	0,0692	-0,0050	6,26	D	G	F	0,16	0,560	0,720
Pernod Ricard	FR0000120693	Food, Beverage & Tobacco	D	D	0,020	0,025	0,045	0,0135	0,0434	0,0029	6,26	D	G	F	0,16	0,560	0,720
Danone	FR0000120644	Food, Beverage & Tobacco	D	B	0,020	-0,005	0,015	0,0121	0,0465	-0,0065	6,26	D	G	D	0,16	0,060	0,220
Philips	NL0000009538	Health Care Equipment & Services	B	B	0,004	-0,001	0,003	0,0095	0,0581	-0,0077	0,56	B	C	D	0,01	0,023	0,033
Loreal	FR0000120321	Household & Personal Products	B	C	0,004	0,000	0,004	0,0302	0,0440	0,0049	1,46	B	E	E	0,04	0,080	0,120
Allianz	DE0008404005	Insurance	A	A	0,002	-0,002	0,001	0,0298	0,0608	0,0002	0,09	A	B	C	0,004	0,000	0,004
Axa	FR0000120628	Insurance	A	A	0,002	-0,002	0,001	0,0179	0,0658	0,0016	0,09	A	B	C	0,004	0,000	0,004
MunichRe	DE0008430026	Insurance	A	B	0,002	-0,001	0,002	0,0121	0,0614	-0,0017	0,09	A	B	D	0,004	0,021	0,025
Linde	IE00BZ12WP82	Materials	F	C	0,080	0,000	0,080	0,0483	0,0470	0,0072	36,64	F	G	E	0,16	0,200	0,360
Air Liquide	FR0000120073	Materials	F	D	0,080	0,040	0,120	0,0248	0,0394	0,0034	36,64	F	G	F	0,16	0,560	0,720
BASF	DE000BASF111	Materials	F	D	0,080	0,040	0,120	0,0180	0,0631	-0,0035	36,64	F	G	F	0,16	0,560	0,720
CRH	IE0001827041	Materials	F	B	0,080	-0,020	0,060	0,0105	0,0693	0,0020	36,64	F	G	D	0,16	0,060	0,220
Sanofi	FR0000120578	Pharmaceuticals Biotec & Life Sciences	C	B	0,010	-0,003	0,008	0,0393	0,0389	-0,0001	2,57	C	E	D	0,04	0,030	0,070
Bayer	DE000BAY0017	Pharmaceuticals Biotec & Life Sciences	C	E	0,010	0,050	0,060	0,0195	0,0606	-0,0037	2,57	C	E	G	0,04	0,360	0,400
Vonovia	DE000A1ML7J1	Real Estate	C	C	0,010	0,000	0,010	0,0116	0,0470	-0,0010	2,04	C	E	E	0,04	0,080	0,120
Prosus	NL0013654783	Retailing	A	D	0,002	0,021	0,023	0,0165	0,0788	-0,0069	0,46	A	C	F	0,01	0,110	0,120
Inditex	ES0148396007	Retailing	A	B	0,002	-0,001	0,002	0,0088	0,0629	-0,0077	0,46	A	C	D	0,01	0,023	0,033
ASML	NL0010273215	Semiconductors & Semiconductor Equipment	C	D	0,010	0,023	0,033	0,0831	0,0689	0,0138	2,86	C	E	F	0,04	0,200	0,240
Infineon	DE0006231004	Semiconductors & Semiconductor Equipment	C	C	0,010	0,000	0,010	0,0132	0,0781	0,0063	2,86	C	E	E	0,04	0,080	0,120
SAP	DE0007164600	Software & Services	B	C	0,004	0,000	0,004	0,0395	0,0579	-0,0031	0,64	B	D	E	0,02	0,060	0,080
Adyen	NL0012969182	Software & Services	B	D	0,004	0,021	0,025	0,0159	0,0917	0,0161	0,64	B	D	F	0,02	0,140	0,160
Deutsche Telekom	DE0005557508	Telecommunication Services	B	C	0,004	0,000	0,004	0,0202	0,0462	0,0026	1,25	B	D	E	0,02	0,060	0,080
Deutsche Post	DE0005552004	Transportation	B	D	0,004	0,021	0,025	0,0154	0,0568	0,0049	1,93	B	E	F	0,04	0,200	0,240
Iberdrola SA	ES0144580Y14	Utilities	E	C	0,040	0,000	0,040	0,0207	0,0482	0,0007	14,32	E	G	E	0,16	0,200	0,360
Enel	IT0003128367	Utilities	E	C	0,040	0,000	0,040	0,0164	0,0548	-0,0044	14,32	E	G	E	0,16	0,200	0,360

Figure 10: Static Data for Simulation based on Systematic and Specific E-Risk

EuroStoxx50	ISIN	MSCI Sector	Transition Risk (BIC E-Rating)	Physical Risk	10d Phy HazardRate	10d Trans HazardRate	10d Compound HazardRate	ptf_weight	Vola(r_10d)	E(r_10d)	Magnitude of Jump	Stressed Transition Rating	Stressed Physical Rating	Stressed Transition Risk	Stressed Physical Risk	Stressed 10d Compound Hazard Rate	Stressed Magintude of Jump
Daimler	DE0007100000	Automobiles & Components	C	C	0,005	0,010	0,015	0,0180	0,0795	0,0078	0,15	E	E	0,04	0,020	0,060	0,280
Stellantis	NL00150001Q9	Automobiles & Components	D	E	0,020	0,020	0,040	0,0111	0,0798	0,0032	0,22	F	G	0,08	0,080	0,160	0,400
Volkswagen	DE0007664039	Automobiles & Components	D	D	0,010	0,020	0,030	0,0096	0,0833	-0,0026	0,19	F	E	0,08	0,020	0,100	0,400
BMW	DE0005190003	Automobiles & Components	D	D	0,010	0,020	0,030	0,0084	0,0678	0,0013	0,19	F	E	0,08	0,020	0,100	0,400
BNP Paribas	FR0000131104	Banks	B	D	0,010	0,004	0,014	0,0207	0,0795	0,0006	0,14	C	E	0,01	0,020	0,030	0,190
Santander	ES0113900J37	Banks	C	D	0,010	0,010	0,020	0,0180	0,0816	-0,0018	0,16	D	G	0,02	0,080	0,100	0,400
ING	NL0011821202	Banks	C	D	0,010	0,010	0,020	0,0128	0,0912	-0,0010	0,16	D	G	0,02	0,080	0,100	0,400
Intesa	IT0000072618	Banks	C	D	0,010	0,010	0,020	0,0127	0,0755	-0,0012	0,16	D	G	0,02	0,080	0,100	0,400
BBVA	ES0113211835	Banks	C	D	0,010	0,010	0,020	0,0121	0,0835	0,0016	0,16	D	G	0,02	0,080	0,100	0,400
Siemens	DE0007236101	Capital Goods	C	D	0,010	0,010	0,020	0,0336	0,0607	0,0038	0,16	F	E	0,08	0,020	0,100	0,400
Schneider Electric	FR0000121972	Capital Goods	B	D	0,010	0,004	0,014	0,0288	0,0538	0,0083	0,14	E	F	0,04	0,040	0,080	0,340
Airbus	NL0000235190	Capital Goods	B	D	0,010	0,004	0,014	0,0211	0,0896	-0,0035	0,14	E	G	0,04	0,080	0,120	0,400
Vinci	FR0000125486	Capital Goods	D	D	0,010	0,020	0,030	0,0191	0,0620	-0,0018	0,19	G	E	0,16	0,020	0,180	0,400
Safran	FR0000073272	Capital Goods	C	D	0,010	0,010	0,020	0,0139	0,0894	-0,0053	0,16	F	F	0,08	0,040	0,120	0,400
Kone	FI0009013403	Capital Goods	C	D	0,010	0,010	0,020	0,0073	0,0441	-0,0035	0,16	F	F	0,08	0,040	0,120	0,400
LVMH	FR0000121014	Consumer Durables & Apparel	C	D	0,010	0,010	0,020	0,0563	0,0585	0,0068	0,16	E	F	0,04	0,040	0,080	0,340
EssilorLuxottica	FR0000121667	Consumer Durables & Apparel	D	D	0,010	0,020	0,030	0,0162	0,0572	0,0032	0,19	F	F	0,08	0,040	0,120	0,400
Kering	FR0000121485	Consumer Durables & Apparel	C	D	0,010	0,010	0,020	0,0146	0,0682	-0,0002	0,16	E	F	0,04	0,040	0,080	0,340
Adidas	DE000A1EWWW0	Consumer Durables & Apparel	C	D	0,010	0,010	0,020	0,0134	0,0677	-0,0057	0,16	E	F	0,04	0,040	0,080	0,340
Flutter Entertainment	IE00BWT6H894	Consumer Services	D	B	0,002	0,020	0,022	0,0065	0,0863	-0,0001	0,16	E	C	0,04	0,005	0,045	0,235
Deutsche Boerse	DE0005810055	Diversified Financials	C	D	0,010	0,010	0,020	0,0099	0,0508	0,0026	0,16	D	F	0,02	0,040	0,060	0,280
TotalEnergies	FR0000120271	Energy	C	D	0,010	0,010	0,020	0,0438	0,0679	-0,0003	0,16	F	F	0,08	0,040	0,120	0,400
Eni	IT0003132476	Energy	B	E	0,020	0,004	0,024	0,0120	0,0647	-0,0005	0,17	E	G	0,04	0,080	0,120	0,400
Ahold	NL0011794037	Food & Staples Retailing	D	D	0,010	0,020	0,030	0,0105	0,0404	0,0035	0,19	F	G	0,08	0,080	0,160	0,400
Anheuser Busch	BE0974293251	Food, Beverage & Tobacco	D	D	0,010	0,020	0,030	0,0164	0,0692	-0,0050	0,19	G	F	0,16	0,040	0,200	0,400
Pernod Ricard	FR0000120693	Food, Beverage & Tobacco	D	D	0,010	0,020	0,030	0,0133	0,0434	0,0029	0,19	G	F	0,16	0,040	0,200	0,400
Danone	FR0000120644	Food, Beverage & Tobacco	B	D	0,010	0,004	0,014	0,0119	0,0465	-0,0065	0,14	E	E	0,04	0,020	0,060	0,280
Philips	NL0000009538	Health Care Equipment & Services	B	D	0,010	0,004	0,014	0,0094	0,0581	-0,0077	0,14	C	G	0,01	0,080	0,090	0,370
Loreal	FR0000120321	Household & Personal Products	C	D	0,010	0,010	0,020	0,0298	0,0440	0,0049	0,16	E	F	0,04	0,040	0,080	0,340
Allianz	DE0008404005	Insurance	A	D	0,010	0,002	0,012	0,0294	0,0608	0,0002	0,14	B	G	0,004	0,080	0,084	0,352
Axa	FR0000120628	Insurance	A	D	0,010	0,002	0,012	0,0177	0,0658	0,0016	0,14	B	G	0,004	0,080	0,084	0,352
MunichRe	DE0008430026	Insurance	B	D	0,010	0,004	0,014	0,0119	0,0614	-0,0017	0,14	C	G	0,01	0,080	0,090	0,370
Linde	IE00BZ12WP82	Materials	C	E	0,020	0,010	0,030	0,0476	0,0470	0,0072	0,19	F	G	0,08	0,080	0,160	0,400
Air Liquide	FR0000120073	Materials	D	D	0,010	0,020	0,030	0,0244	0,0394	0,0034	0,19	G	F	0,16	0,040	0,200	0,400
BASF	DE000BASF111	Materials	D	D	0,010	0,020	0,030	0,0178	0,0631	-0,0035	0,19	G	E	0,16	0,020	0,180	0,400
CRH	IE0001827041	Materials	B	D	0,010	0,004	0,014	0,0103	0,0693	0,0020	0,14	E	E	0,04	0,020	0,060	0,280
Sanofi	FR0000120578	Pharmaceuticals Biotec & Life Sciences	B	D	0,010	0,004	0,014	0,0387	0,0389	-0,0001	0,14	C	E	0,01	0,020	0,030	0,190
Bayer	DE000BAY0017	Pharmaceuticals Biotec & Life Sciences	E	C	0,005	0,040	0,045	0,0192	0,0606	-0,0037	0,24	F	E	0,08	0,020	0,100	0,400
Vonovia	DE000A1ML7J1	Real Estate	C	C	0,005	0,010	0,015	0,0114	0,0470	-0,0010	0,15	F	D	0,08	0,010	0,090	0,370
Prosus	NL0013654783	Retailing	D	E	0,020	0,020	0,040	0,0162	0,0788	-0,0069	0,22	F	G	0,08	0,080	0,160	0,400
Inditex	ES0148396007	Retailing	B	C	0,005	0,004	0,009	0,0087	0,0629	-0,0077	0,13	D	G	0,02	0,080	0,100	0,400
ASML	NL0010273215	Semiconductors & Semiconductor Equipment	D	D	0,010	0,020	0,030	0,0819	0,0689	0,0138	0,19	E	G	0,04	0,080	0,120	0,400
Infineon	DE0006231004	Semiconductors & Semiconductor Equipment	C	D	0,010	0,010	0,020	0,0130	0,0781	0,0063	0,16	D	F	0,02	0,040	0,060	0,280
SAP	DE0007164600	Software & Services	C	D	0,010	0,010	0,020	0,0389	0,0579	-0,0031	0,16	D	F	0,02	0,040	0,060	0,280
Adyen	NL0012969182	Software & Services	D	D	0,010	0,020	0,030	0,0156	0,0917	0,0161	0,19	E	G	0,04	0,080	0,120	0,400
Deutsche Telekom	DE0005557508	Telecommunication Services	C	D	0,010	0,010	0,020	0,0199	0,0462	0,0026	0,16	D	F	0,02	0,040	0,060	0,280
Deutsche Post	DE0005552004	Transportation	D	C	0,005	0,020	0,025	0,0152	0,0568	0,0049	0,18	F	E	0,08	0,020	0,100	0,400
Iberdrola SA	ES0144580Y14	Utilities	C	C	0,005	0,010	0,015	0,0204	0,0482	0,0007	0,15	F	G	0,08	0,080	0,160	0,400
Enel	IT0003128367	Utilities	C	E	0,020	0,010	0,030	0,0162	0,0548	-0,0044	0,19	F	G	0,08	0,080	0,160	0,400

Figure 11: Static Data for Simulation based on Transition and Physical E-Risk

		Daim	Stell	Volks	BMW	BNP	Santa	ING	Intes	BBVA	Siem	Schn	Airbu	Vinci	Safra	Kone	LVMH	Essil	Kerin	Adida	Flutt	Deut	Total	Eni	Ahol	Anhe	Pern	Danc	Phili	Lorea	Allia	Axa	Muni	Lind	Air Li	BASF	CRH	Sano	Baye	Vono	Prosu	Indit	ASML	Infin	SAP	Adye	Deut	Deut	Iberd	Enel
		Auto	Auto	Auto	Auto	Bank	Bank	Bank	Bank	Bank	Capit	Capit	Capit	Capit	Capit	Capit	Cons	Cons	Cons	Cons	Cons	Diver	Energ	Energ	Food	Food	Food	Food	Healt	Hous	Insur	Insur	Insur	Mate	Mate	Mate	Mate	Phar	Phar	Real	Retai	Retai	Semi	Semi	Softv	Softv	Telec	Trans	Utilit	Utiliti
Daimler	Automobiles & Components	1	0,2	0,8	0,8	0	0	0	0	0	0,4	0,1	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,4	0,4
Stellantis	Automobiles & Components	0,2	1	0,2	0,2	0	0	0	0	0	0,1	0,1	0,1	0,1	0,1	0,1	0	0	0	0	0	0	0,1	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,1	0,1
Volkswagen	Automobiles & Components	0,8	0,2	1	0,8	0	0	0	0	0	0,4	0,1	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,4	0,4
BMW	Automobiles & Components	0,8	0,2	0,8	1	0	0	0	0	0	0,4	0,1	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,4	0,4
BNP Paribas	Banks	0	0	0	0	1	0,8	0,8	0,8	0,8	0	0	0	0	0	0	0	0	0	0	0	0,4	0	0	0	0	0	0	0	0	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Santander	Banks	0	0	0	0	0,8	1	0,8	0,8	0,8	0	0	0	0	0	0	0	0	0	0	0	0,4	0	0	0	0	0	0	0	0	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ING	Banks	0	0	0	0	0,8	0,8	1	0,8	0,8	0	0	0	0	0	0	0	0	0	0	0	0,4	0	0	0	0	0	0	0	0	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Intesa	Banks	0	0	0	0	0,8	0,8	0,8	1	0,8	0	0	0	0	0	0	0	0	0	0	0	0,4	0	0	0	0	0	0	0	0	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BBVA	Banks	0	0	0	0	0,8	0,8	0,8	0,8	1	0	0	0	0	0	0	0	0	0	0	0	0,4	0	0	0	0	0	0	0	0	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Siemens	Capital Goods	0,4	0,4	0,4	0,4	0	0	0	0	0	1	0,2	0,8	0,8	0,8	0,8	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Schneider Electric	Capital Goods	0,1	0,1	0,1	0,1	0	0	0	0	0	0,2	1	0,2	0,2	0,2	0,2	0	0	0	0	0	0	0,1	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Airbus	Capital Goods	0,4	0,4	0,4	0,4	0	0	0	0	0	0,8	0,2	1	0,8	0,8	0,8	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Vinci	Capital Goods	0,4	0,4	0,4	0,4	0	0	0	0	0	0,8	0,2	0,8	1	0,8	0,8	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Safran	Capital Goods	0,4	0,4	0,4	0,4	0	0	0	0	0	0,8	0,2	0,8	0,8	1	0,8	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Kone	Capital Goods	0,4	0,4	0,4	0,4	0	0	0	0	0	0,8	0,2	0,8	0,8	0,8	1	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LVMH	Consumer Durables & Apparel	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0,8	0,8	0,8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EssilorLuxottica	Consumer Durables & Apparel	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,8	1	0,8	0,8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Kering	Consumer Durables & Apparel	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,8	0,8	1	0,8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Adidas	Consumer Durables & Apparel	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,8	0,8	0,8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Flutter Entertainmen	Consumer Services	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Deutsche Boerse	Diversified Financials	0	0	0	0	0,4	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TotalEnergies	Energy	0,4	0,1	0,4	0,4	0	0	0	0	0	0,4	0,1	0,4	0,4	0,4	0,4	0	0	0	0	0	0	1	0,8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,4	0,4
Eni	Energy	0,4	0,1	0,4	0,4	0	0	0	0	0	0,4	0,1	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0,8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,4	0,4
Ahold	Food & Staples Retailing	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Anheuser Busch	Food, Beverage & Tobacco	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0,8	0,2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Pernod Ricard	Food, Beverage & Tobacco	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,8	1	0,2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Danone	Food, Beverage & Tobacco	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,2	0,2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Philips	Health Care Equipment & Servi	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Loreal	Household & Personal Products	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Allianz	Insurance	0	0	0	0	0,4	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0,4	0	0	0	0	0	0	0	0	1	0,8	0,8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Axa	Insurance	0	0	0	0	0,4	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0,4	0	0	0	0	0	0	0	0	0,8	1	0,8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MunichRe	Insurance	0	0	0	0	0,4	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0,4	0	0	0	0	0	0	0	0	0,8	0,8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Linde	Materials	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0,2	0,2	0,2	0	0	0	0	0	0	0	0	0	0	0	0	0
Air Liquide	Materials	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,2	1	0,8	0,2	0	0	0	0	0	0	0	0	0	0	0	0	0
BASF	Materials	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,2	0,8	1	0,2	0	0	0	0	0	0	0	0	0	0	0	0	0
CRH	Materials	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,2	0,2	0,2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Sanofi	Pharmaceuticals Biotec & Life S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0,2	0	0	0	0	0	0	0	0	0	0	0
Bayer	Pharmaceuticals Biotec & Life S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,2	1	0	0	0	0	0	0	0	0	0	0	0
Vonovia	Real Estate	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Prosus	Retailing	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0,8	0	0	0	0	0	0	0	0
Inditex	Retailing	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,8	1	0	0	0	0	0	0	0	0
ASML	Semiconductors & Semiconduct	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0,2	0	0	0	0	0	0
Infineon	Semiconductors & Semiconduct	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,2	1	0	0	0	0	0	0
SAP	Software & Services	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0,2	0	0	0	0
Adyen	Software & Services	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,2	1	0	0	0	0
Deutsche Telekom	Telecommunication Services	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
Deutsche Post	Transportation	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Iberdrola SA	Utilities	0,4	0,1	0,4	0,4	0	0	0	0	0	0,4	0,1	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0,8
Enel	Utilities	0,4	0,1	0,4	0,4	0	0	0	0	0	0,4	0,1	0,4	0,4	0,4	0,4	0	0	0	0	0	0	0,4	0,4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,8	1

Figure 12: Moderate Correlation Structure between Companies and Industry Sectors

		Daim	Stella	Volks	BMW	BNP I	Santa	ING	Intes	BBVA	Siem	Schn	Airbu	Vinci	Safra	Kone	LVMI	Essilc	Kerin	Adida	Flutt	Deut	Total	Eni	Aholc	Anhe	Pernc	Dano	Philip	Lorea	Alliar	Axa	Muni	Linde	Air Li	BASF	CRH	Sanol	Bayer	Vono	Prosu	Indite	ASMI	Infin	SAP	Adye	Deut	Deut	Iberd	Enel
		Auto	Auto	Auto	Auto	Bank	Bank	Bank	Bank	Bank	Capit	Capit	Capit	Capit	Capit	Capit	Cons	Cons	Cons	Cons	Cons	Diver	Energ	Energ	Food	Food	Food	Food	Healt	Hous	Insur	Insur	Insur	Mate	Mate	Mate	Mate	Pharr	Pharr	Real I	Retai	Retai	Semi	Semi	Softw	Softw	Telec	Trans	Utilit	Utiliti
Daimler	Automobil	1	**0,6**	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,8	**0,5**	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8
Stellantis	Automobil	**0,6**	1	**0,6**	**0,6**	0,4	0,4	0,4	0,4	0,4	**0,5**	**0,5**	**0,5**	**0,5**	**0,5**	**0,5**	0,4	0,4	0,4	0,4	0,4	0,4	**0,5**	**0,5**	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	**0,5**	**0,5**
Volkswage	Automobil	0,9	**0,6**	1	0,9	0,4	0,4	0,4	0,4	0,4	0,8	**0,5**	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8
BMW	Automobil	0,9	**0,6**	0,9	1	0,4	0,4	0,4	0,4	0,4	0,8	**0,5**	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8
BNP Pariba	Banks	0,4	0,4	0,4	0,4	1	0,9	0,9	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Santander	Banks	0,4	0,4	0,4	0,4	0,9	1	0,9	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
ING	Banks	0,4	0,4	0,4	0,4	0,9	0,9	1	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Intesa	Banks	0,4	0,4	0,4	0,4	0,9	0,9	0,9	1	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
BBVA	Banks	0,4	0,4	0,4	0,4	0,9	0,9	0,9	0,9	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Siemens	Capital Goo	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	1	**0,6**	0,9	0,9	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Schneider	Capital Goo	**0,5**	**0,5**	**0,5**	**0,5**	0,4	0,4	0,4	0,4	0,4	**0,6**	1	**0,6**	**0,6**	**0,6**	**0,6**	0,4	0,4	0,4	0,4	0,4	0,4	**0,5**	**0,5**	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Airbus	Capital Goo	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,9	**0,6**	1	0,9	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Vinci	Capital Goo	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,9	**0,6**	0,9	1	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Safran	Capital Goo	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,9	**0,6**	0,9	0,9	1	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Kone	Capital Goo	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,9	**0,6**	0,9	0,9	0,9	1	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
LVMH	Consumer	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,9	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
EssilorLuxo	Consumer	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,9	1	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Kering	Consumer	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,9	0,9	1	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Adidas	Consumer	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,9	0,9	0,9	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Flutter Ent	Consumer	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Deutsche B	Diversified	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
TotalEnerg	Energy	0,8	**0,5**	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,8	**0,5**	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	1	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8
Eni	Energy	0,8	**0,5**	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,8	**0,5**	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,9	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8
Ahold	Food & Sta	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Anheuser E	Food, Beve	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,9	**0,6**	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Pernod Ric	Food, Beve	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,9	1	**0,6**	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Danone	Food, Beve	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	**0,6**	**0,6**	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Philips	Health Car	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Loreal	Household	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Allianz	Insurance	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,9	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Axa	Insurance	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,9	1	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
MunichRe	Insurance	0,4	0,4	0,4	0,4	0,8	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,9	0,9	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Linde	Materials	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	**0,6**	**0,6**	**0,6**	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Air Liquide	Materials	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	**0,6**	1	0,9	**0,6**	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
BASF	Materials	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	**0,6**	0,9	1	**0,6**	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
CRH	Materials	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	**0,6**	**0,6**	**0,6**	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Sanofi	Pharmaceu	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	**0,6**	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Bayer	Pharmaceu	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	**0,6**	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Vonovia	Real Estate	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Prosus	Retailing	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,9	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Inditex	Retailing	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,9	1	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
ASML	Semicondu	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	**0,6**	0,4	0,4	0,4	0,4	0,4	0,4
Infineon	Semicondu	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	**0,6**	1	0,4	0,4	0,4	0,4	0,4	0,4
SAP	Software &	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	**0,6**	0,4	0,4	0,4	0,4
Adyen	Software &	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	**0,6**	1	0,4	0,4	0,4	0,4
Deutsche T	Telecommu	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,4	0,4	0,4
Deutsche P	Transporta	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,4	0,4
Iberdrola S	Utilities	0,8	**0,5**	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,8	**0,5**	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	0,9
Enel	Utilities	0,8	**0,5**	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,8	**0,5**	0,8	0,8	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,8	0,8	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,9	1

Figure 13: High Correlation Structure between Companies and Industry Sectors

Table 9: Scope 123 Carbon Intensity for CO2 Rating Derivation

EuroStoxx50 EQ Position	**ISIN**	**MSCI Sector**	**Intensity in tCO2/M.EUR**	**Avg Sector Intensity**
Daimler	DE0007100000	Automobiles & Components	0,83	0,63
Stellantis	NL00150001Q9	Automobiles & Components	0,67	0,63
Volkswagen	DE0007664039	Automobiles & Components	0,63	0,63
BMW	DE0005190003	Automobiles & Components	0,40	0,63
BNP Paribas	FR0000131104	Banks	0,19	0,18
Santander	ES0113900J37	Banks	0,17	0,18
ING	NL0011821202	Banks	0,11	0,18
Intesa	IT0000072618	Banks	0,12	0,18
BBVA	ES0113211835	Banks	0,34	0,18
Siemens	DE0007236101	Capital Goods	2,11	1,64
Schneider Electric	FR0000121972	Capital Goods	2,33	1,64
Airbus	NL0000235190	Capital Goods	1,41	1,64
Vinci	FR0000125486	Capital Goods	1,89	1,64
Safran	FR0000073272	Capital Goods	0,72	1,64
Kone	FI0009013403	Capital Goods	1,41	1,64
LVMH	FR0000121014	Consumer Durables & Apparel	3,30	1,35
EssilorLuxottica	FR0000121667	Consumer Durables & Apparel	1,11	1,35
Kering	FR0000121485	Consumer Durables & Apparel	0,43	1,35
Adidas	DE000A1EWWW0	Consumer Durables & Apparel	0,58	1,35
Flutter Entertainment	IE00BWT6H894	Consumer Services	0,14	0,14
Deutsche Boerse	DE0005810055	Diversified Financials	0,16	0,16
TotalEnergies	FR0000120271	Energy	20,20	15,18
Eni	IT0003132476	Energy	10,17	15,18
Ahold	NL0011794037	Food & Staples Retailing	0,70	0,70
Anheuser Busch	BE0974293251	Food, Beverage & Tobacco	5,00	6,26
Pernod Ricard	FR0000120693	Food, Beverage & Tobacco	3,76	6,26
Danone	FR0000120644	Food, Beverage & Tobacco	10,01	6,26
Philips	NL0000009538	Health Care Equipment & Services	0,56	0,56
Loreal	FR0000120321	Household & Personal Products	1,46	1,46
Allianz	DE0008404005	Insurance	0,16	0,09
Axa	FR0000120628	Insurance	0,07	0,09
MunichRe	DE0008430026	Insurance	0,05	0,09
Linde	IE00BZ12WP82	Materials	82,98	36,64
Air Liquide	FR0000120073	Materials	37,43	36,64
BASF	DE000BASF111	Materials	8,56	36,64
CRH	IE0001827041	Materials	17,57	36,64
Sanofi	FR0000120578	Pharmaceuticals Biotec & Life Sciences	1,99	2,57
Bayer	DE000BAY0017	Pharmaceuticals Biotec & Life Sciences	3,15	2,57
Vonovia	DE000A1ML7J1	Real Estate	2,04	2,04
Prosus	NL0013654783	Retailing	0,31	0,46
Inditex	ES0148396007	Retailing	0,62	0,46
ASML	NL0010273215	Semiconductors & Semiconductor Equipment	3,48	2,86
Infineon	DE0006231004	Semiconductors & Semiconductor Equipment	2,24	2,86
SAP	DE0007164600	Software & Services	1,18	0,64
Adyen	NL0012969182	Software & Services	0,09	0,64
Deutsche Telekom	DE0005557508	Telecommunication Services	1,25	1,25
Deutsche Post	DE0005552004	Transportation	1,93	1,93
Iberdrola SA	ES0144580Y14	Utilities	11,14	14,32
Enel	IT0003128367	Utilities	17,49	14,32

C Transition Densities From Systematic Risk and Specific Risk

C.1 Normal Market Conditions

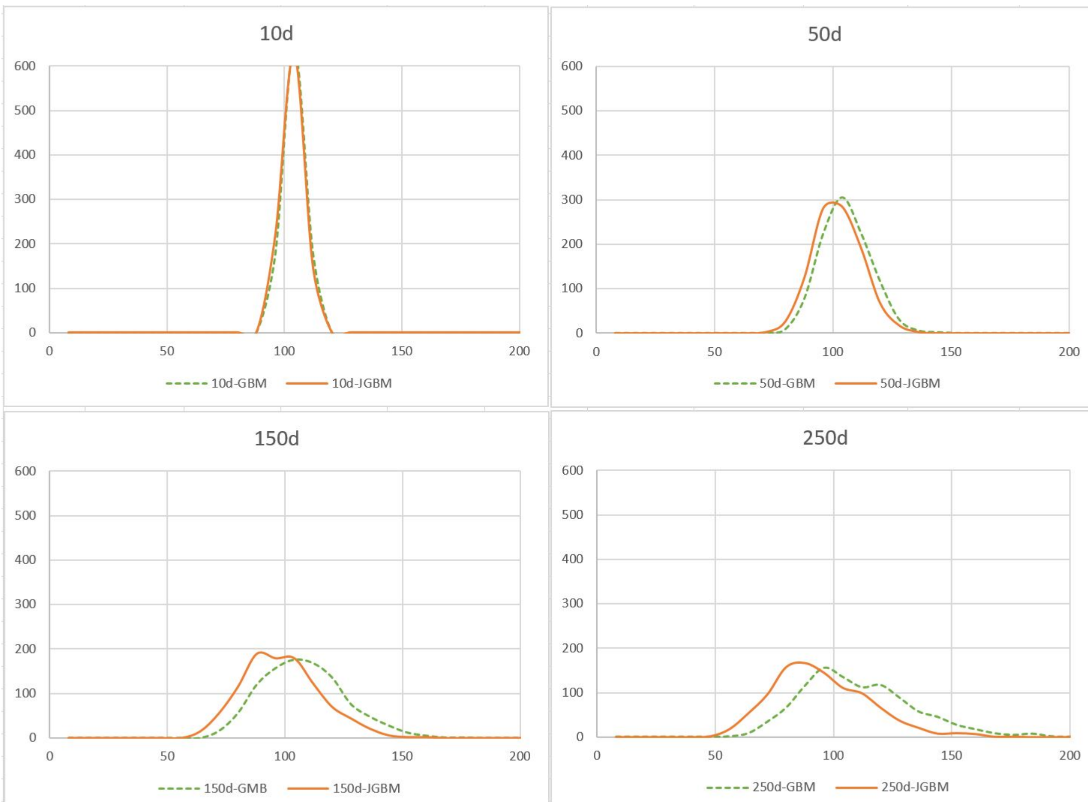

Figure 14: Transition Density over Time: JGBM vs GBM SysSpec Current Risk, Moderate Jmp-Correlation

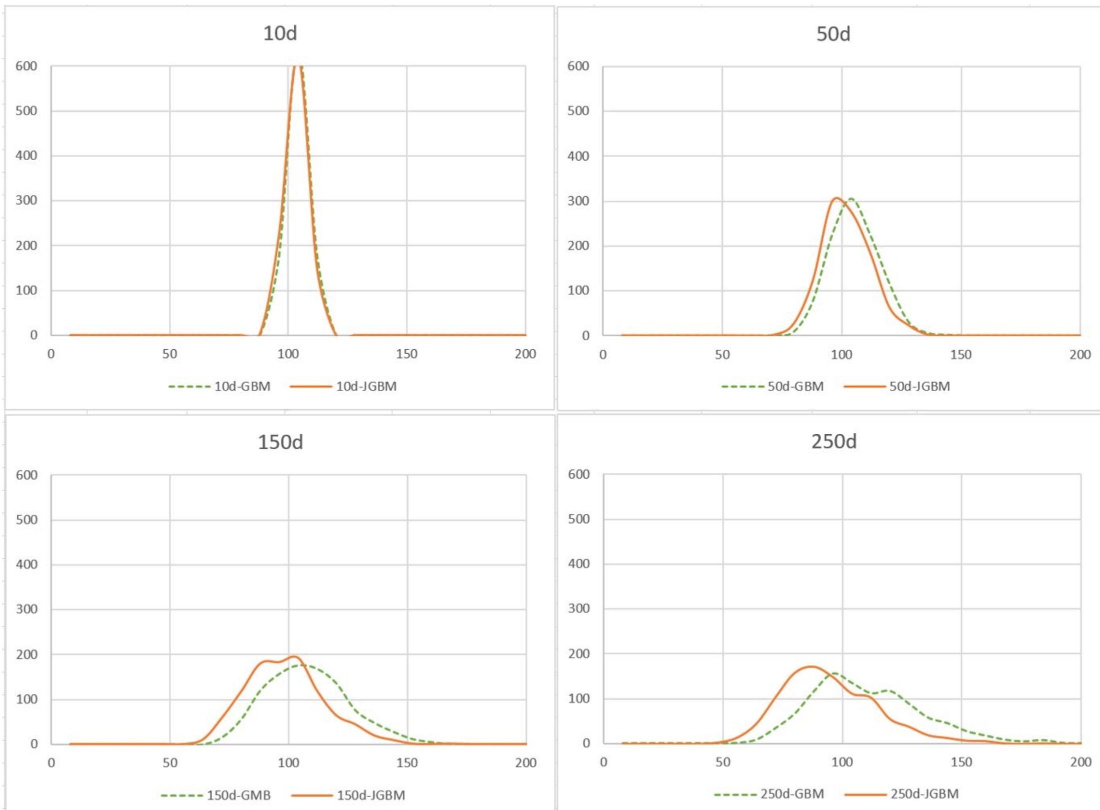

Figure 15: Transition Density over Time: JGBM vs GBM SysSpec Current Risk, No Jmp-Correlation

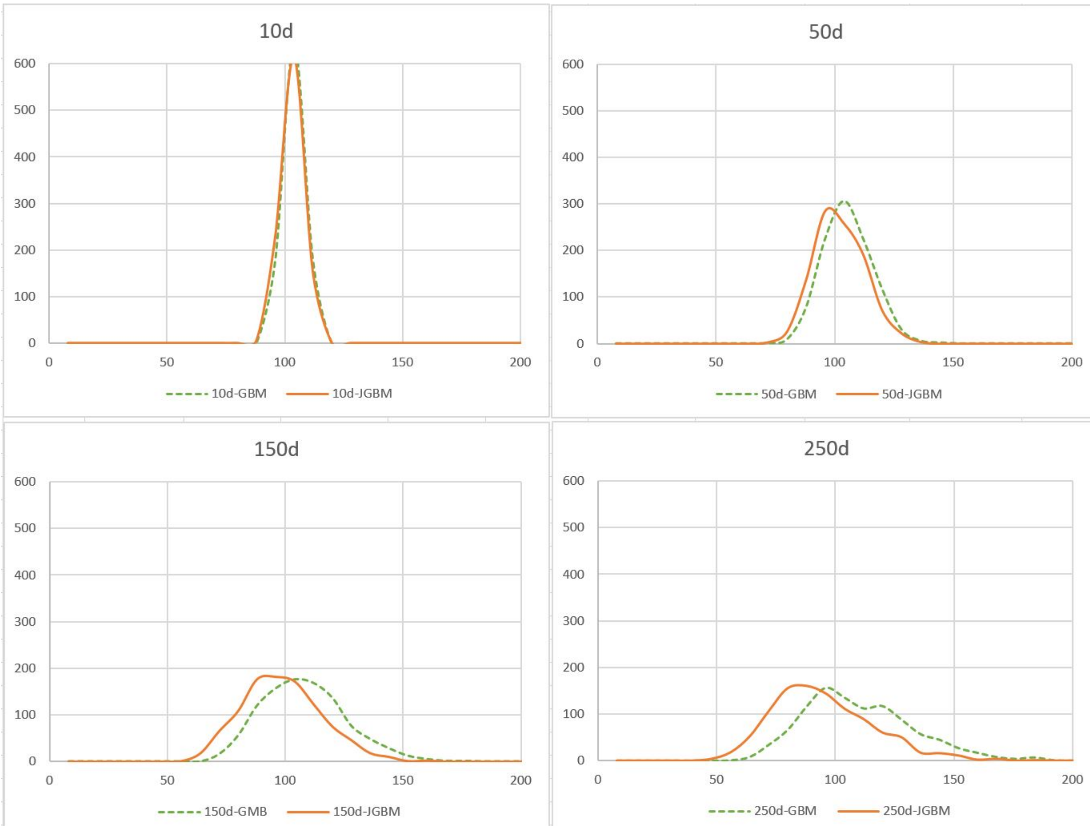

Figure 16: Transition Density over Time: JGBM vs GBM SysSpec Current Risk, High Jmp-Correlation

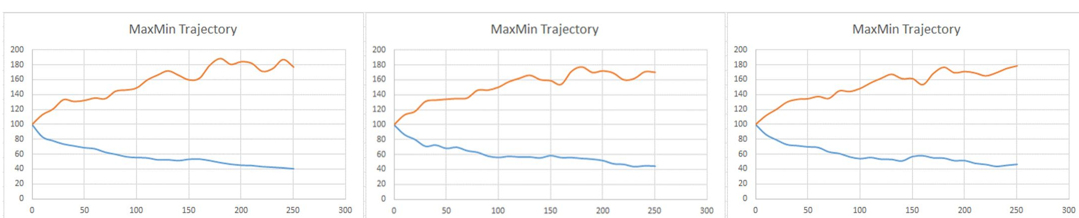

Figure 17: MinMax Trajectories: High, Normal and No Correlation

C.2 Intensively Stressed Market Condition

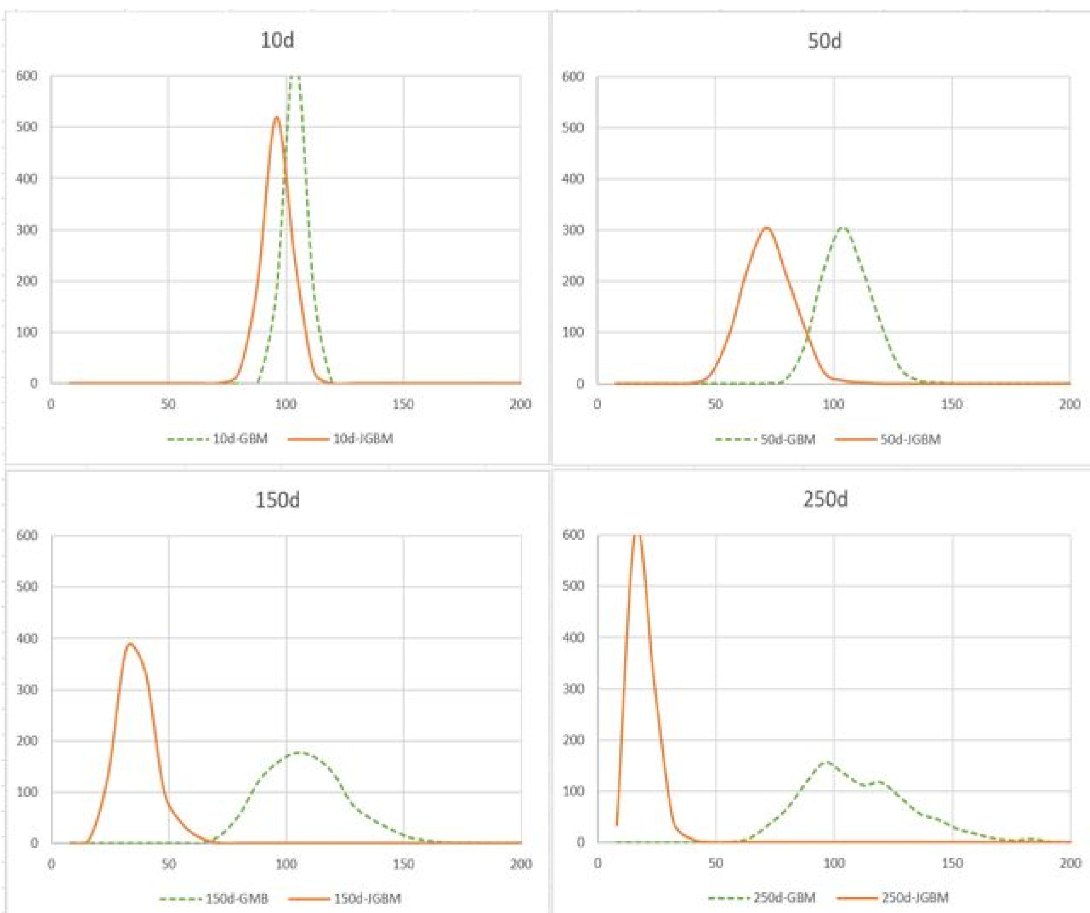

Figure 18: Transition Density over Time SysSpec Intensive Stress Risk: JGBM vs GBM, Moderate Jmp-Correlation

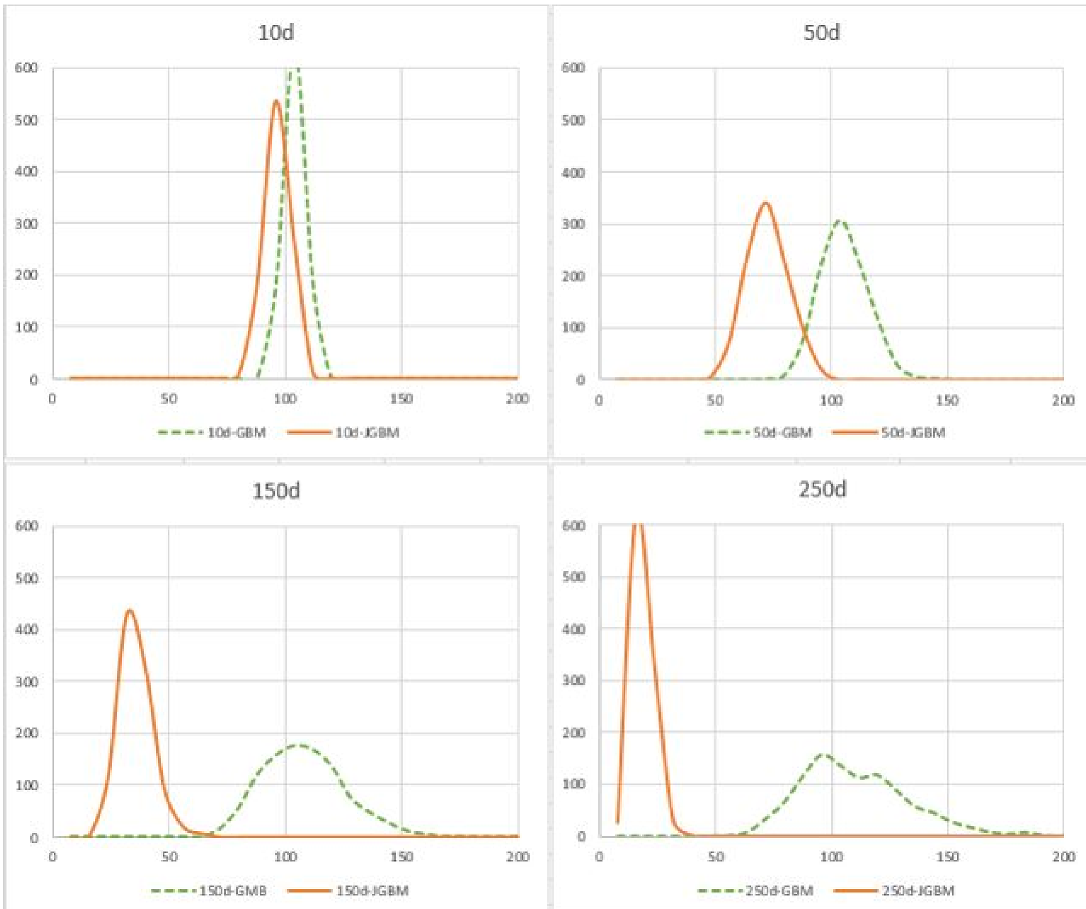

Figure 19: Transition Density over Time: JGBM vs GBM SysSpec Intensive Stress Risk, No Jmp-Correlation

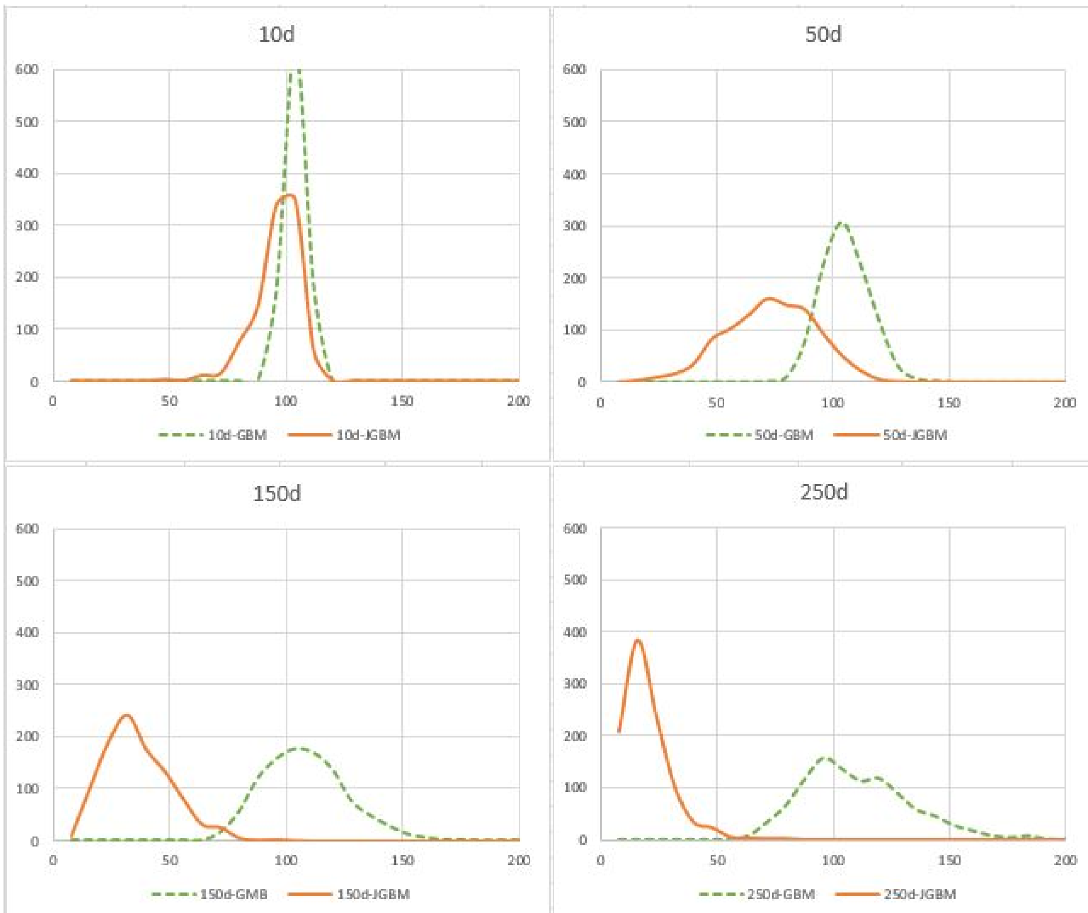

Figure 20: Transition Density over Time SysSpec Intensive Stress Risk: JGBM vs GBM, High Jmp-Correlation

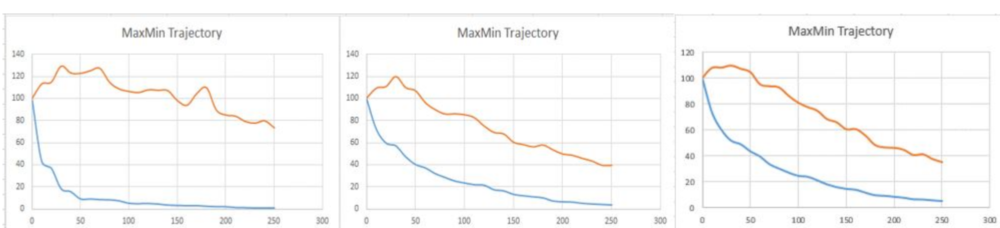

Figure 21: MinMax Trajectories: High, Normal and No Correlation

D Transition Densities From Transition Risk and Physical Risk

D.1 Stressed Market Conditions

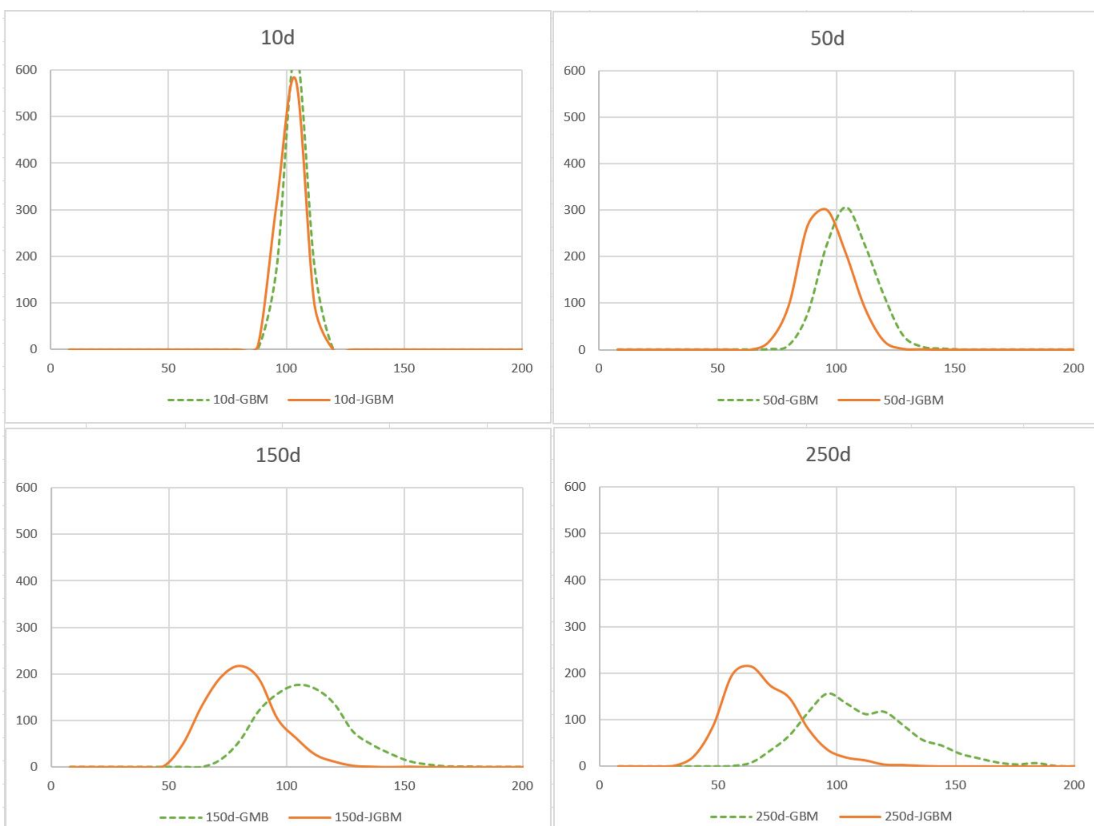

Figure 22: Transition Density over Time TransPhy Stressed Risk: JGBM vs GBM, Moderate Jmp-Correlation

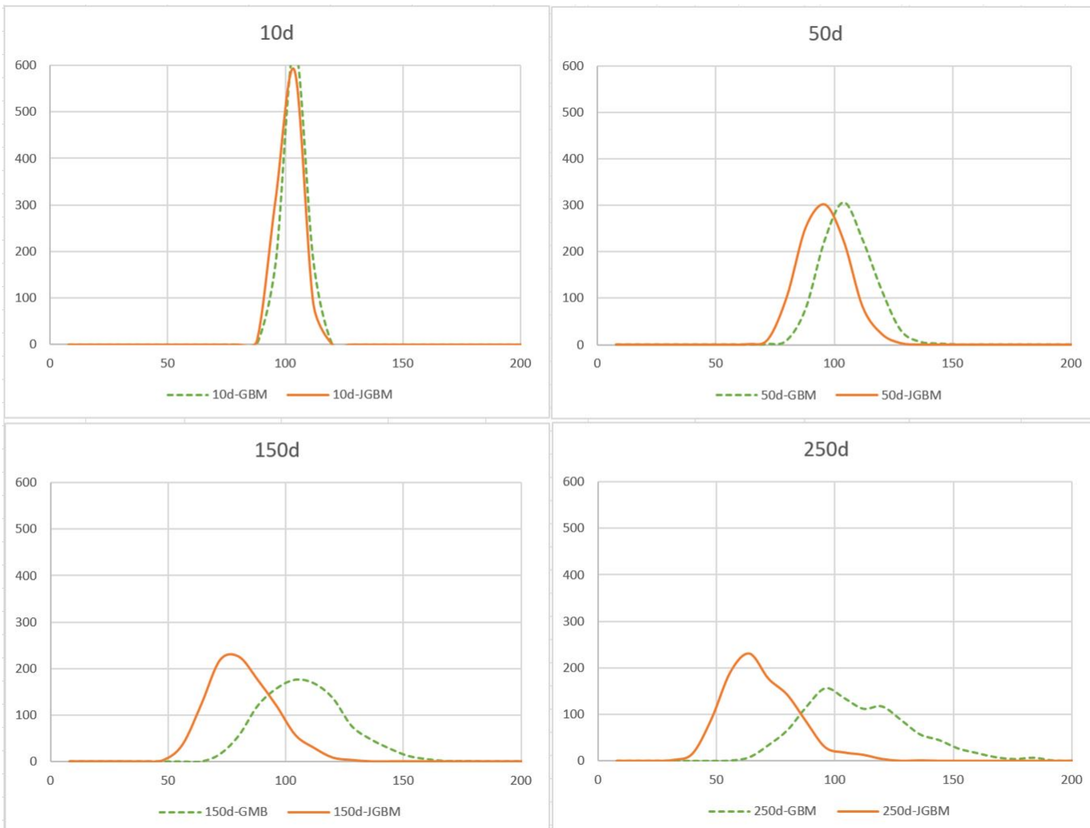

Figure 23: Transition Density over Time TransPhy Stressed Risk: JGBM vs GBM, No Jmp-Correlation

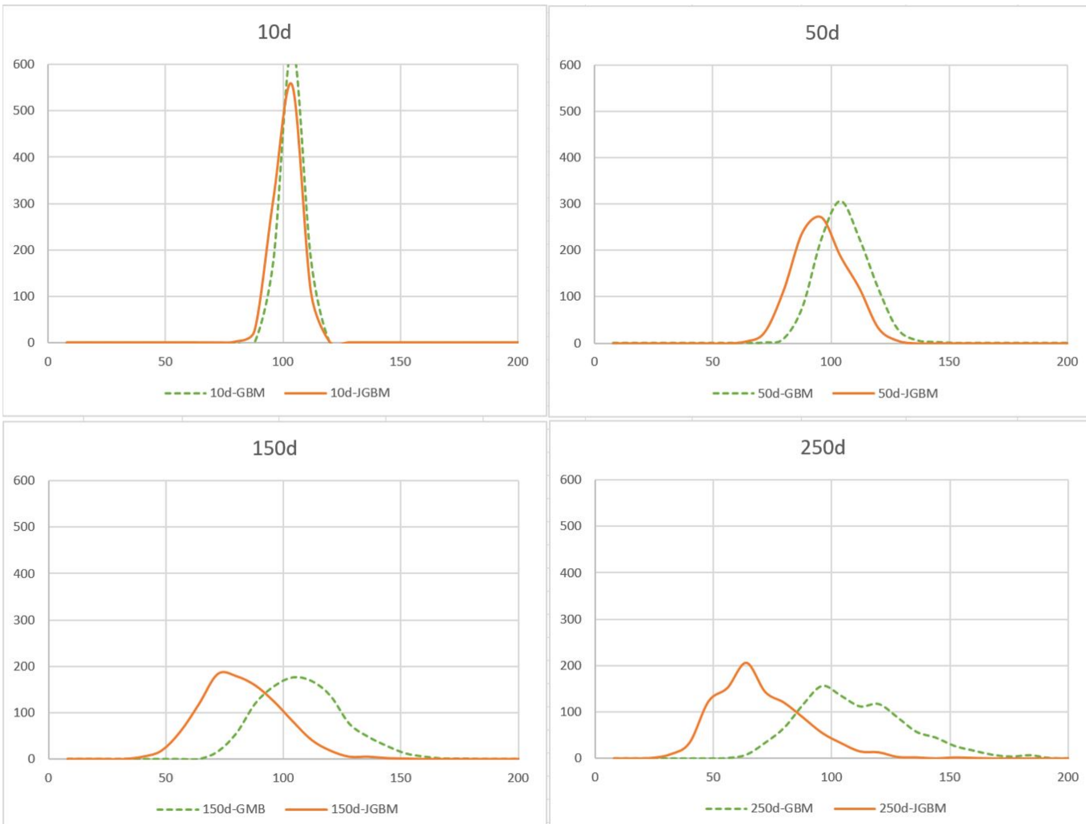

Figure 24: Transition Density over Time TransPhy Stressed Risk: JGBM vs GBM, High Jmp-Correlation

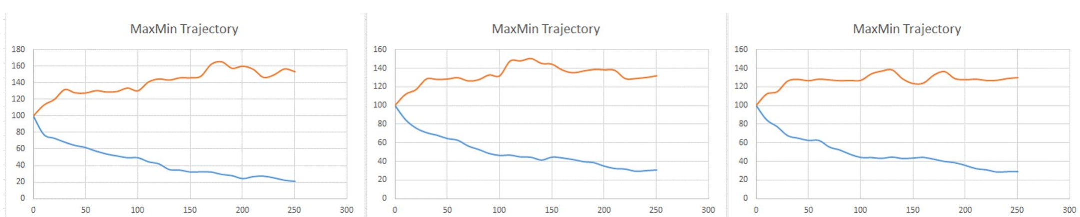

Figure 25: MinMax Trajectories: High, Normal and No Correlation

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Important Information

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