Bond Market “Conundrum”: New factors to explain long-term interest rates?

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Abstract

Interest rates behaved highly atypically from 2004 to 2006. While the US central bank raised its policy rate at every meeting, long-term interest rates remained so remarkably stable that former Fed Chairman Alan Greenspan described their behaviour as a “conundrum.” Comparing long-term rates to their theoretical level based on fundamental valuation models, we show that the anomaly was on average 40 bps. Various explanations have been put forward for this, including investors' changed attitude to risk, and the rise in US Treasury purchases by different categories of buyers. We show that, while these variables could theoretically be responsible for the decline in bond risk premiums, they explain less than half of the anomaly when incorporated into a fundamental model of bond yields. However, their recent changing influence could justify their being used for a prospective analysis of bond yields.

**Keywords:** interest rates, central banks, flows of funds, financial markets.

**JEL codes:** E37, E43, E58, G23
1. Introduction

From 2004 to 2006, long-term US interest rates diverged from their fundamentals. The Fed began to tighten policy in June 2004, and the growth rate of the US debt has accelerated since 2002. However, neither of these factors has had any effect on the strikingly low level of long-term interest rates. In a now-famous speech, former Fed Chairman Alan Greenspan described this as a “conundrum.” But is it really a conundrum or merely an extension of the decline in long-term interest rates over the past 20 years? And have we not neglected structural influences on long-term rates that are now resurfacing again? Several “omitted” variables have been cited by the Fed, economists and market participants to explain the anomaly. These include the greater credibility of the monetary authorities, the decline in volatility of macroeconomic variables, and major US Treasury purchases, especially by Asian central banks and pension funds.

Although the academic literature is unanimous in agreeing that a bond market anomaly has existed since 2004, there is less agreement about the explanatory factors. Most authors test only a few items, and results vary greatly according to the methodology used. Rudebusch et al. (2006) tested the influence of the noticeable decline in volatility of major macroeconomic variables on long-term interest rates, and found that this factor explained only a very small part of the anomaly. The impact of US Treasury purchases by Asian central banks was tested by many authors, but the results are far from unanimous. Artus (2005), Warnock and Warnock (2005), and Frey and Moëc (2005) have introduced this variable into an equilibrium model for long-term interest rates and show that it gives a significant explanation of the anomaly, contrary to Rudebusch et al. (2006) using a different model of long-term rates. Other studies show that the link with long-term interest rates is both recent and unstable (McCauley and Jiang (2005), Wu (2005)). We know of no studies that have focused on the impact of other Treasury purchases, for example those made by pension funds, which are nonetheless often cited as a significant explanatory factor for the bond market “conundrum”.

The diversity of results found in the literature and the lack of tests for some of the explanatory factors mentioned above have led us to revisit this question. In this paper, we present a fundamental univariate estimation model, linking long rates to macroeconomic and monetary policy variables but not including the government budget deficit – in contrast to the preceding models – since the inclusion of this variable in long rates is controversial (Mehra (1994), Boulanger (2002)). This model allows us to document the precise size and timing of the
anomaly. We test whether the disconnection could be explained by other economic factors that had been overlooked. These tests shed new light on previously published studies on this topic, as we have examined the two most frequently cited categories of explanations for the bond market anomaly mentioned by investors, as reflected by a Macroeconomic Advisers’ survey: investors’ changed attitude toward risk, with declining macroeconomic volatility as one of its quantifiable causes, and the step-up in US Treasury purchases by various categories of buyers noticeable in the Flow of Funds Accounts. However, we have not limited ourselves to foreign investors, as was the case in all previous studies. To the best of our knowledge, these very detailed data have not so far been tested in an interest rate model. Doing so helps us to understand precisely which purchasers of US Treasuries have a real impact on the level of long-term interest rates.

We show that theoretically, these factors can be responsible for risk premiums decline and for lower bond yields. But when we test them econometrically in a fundamental model, we demonstrate that certain factors such as pension fund purchases, though often cited to explain the bond market anomaly, have had no econometrically quantifiable influence on interest rate levels. Foreign purchases, often cited as well, have played a more important role, although their impact is very unstable and is concentrated in the recent period. Inflation volatility appears to be an explanatory factor in interest rates, but including it in the equation of fundamental determination of 10-year interest rates scarcely changes their equilibrium value. In sum, introducing the most significant factors into the equilibrium model allows us to explain less than half of the anomaly. Nevertheless the recent changing influence of buying factors could justify incorporating them in any prospective analysis of long-term rates.

This paper is organised as follows: in Section 2, we review the bond market “conundrum” and the way it is usually defined. We then present the factors most often cited by investors to explain this anomaly. For factors already tested in the literature, we attempt to explain the differences in the results depending on the methodology used. In Section 3, we present a simple fundamental model of long-term rates based on the Fisher equation, which allows us to document the precise size and timing of the anomaly, and we test econometrically which of the new factors allow us to explain part of the bond market conundrum. Section 4 concludes.
2. The Bond Market “Conundrum” and Its Explanations

In his now-famous address of 16 February 2005, Federal Reserve Chairman Alan Greenspan first spoke of the bond market “conundrum,” thus giving a name to the particularly surprising behaviour of long-term interest rates since June 2004. We have witnessed a period of significant disconnection between long-term US interest rates and their fundamentals. The gradual tightening of key rates by 425 bps, from 1% to 5.25%, by the Fed between June 2004 and June 2006 should have driven long-term rates sharply higher. Instead, they remained relatively inert, rising only 64 bps over the period, from 4.60% to 5.14%. No fundamental economic reason can be advanced to explain such a small move. The United States experienced sustained economic growth over the period, and oil prices soared. The deepening US deficit and resulting large upswing in debt could have provided still more reason for long rates to rise.

2.1. How to define the Bond Conundrum?

A precise definition of the bond “conundrum” is hard to find in the literature. That said, there are two ways of pinpointing and measuring the abnormal behaviour of long-term rates. The first is to compare the reaction of long rates to the Fed’s key rates, to show that, unlike earlier instances of monetary tightening, this lengthy episode of restrictive Fed policy was only very slightly reflected in long rates. The second way is to compare long rates with their equilibrium value, measured using a fundamental model. In the period 2004-2005, long rates remained significantly and persistently below their equilibrium value (Rudebusch et al. (2006), Artus (2005), Frey and Moëc (2005), Warnock and Warnock (2005)). But the degree of disconnection from fundamental value varies with the model selected.

Long term rate behaviour very different from previous periods of monetary tightening

This long-term interest rate behaviour is particularly atypical when compared with previous periods of monetary tightening. In general, a change in key interest rates brings a change in the same direction for long-term bond yields, and the sensitivity of 10 year yields to overnight rates is estimated to average 30%\(^1\). During periods of short-term rate increases, this relationship tends to grow even closer. During preceding periods of monetary tightening in 1988, 1994 and 1999, 10-year yields rose in a relationship of respectively 36%, 67% and

\(^1\) Regression coefficient of 1 month variation in long-term rates on 1 month variation in overnight rate since 1987.
The period of interest rate tightening from 2004 to 2006 is completely different in this way, because long-term yields rose by only 15%.

**TABLE 1: Impact of increases in key rates on 10-year bond yields in the United States during periods of monetary tightening**

<table>
<thead>
<tr>
<th>Date of monetary tightening</th>
<th>Increase in Fed policy rate (bps)</th>
<th>Maximum increase in 10-year yield (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb. 88 - June 89</td>
<td>331</td>
<td>121</td>
</tr>
<tr>
<td>Feb. 94 - June 95</td>
<td>300</td>
<td>202</td>
</tr>
<tr>
<td>July 99 – Dec. 00</td>
<td>175</td>
<td>94</td>
</tr>
<tr>
<td>July 04 – Aug. 06</td>
<td>425</td>
<td>64</td>
</tr>
<tr>
<td>Mean</td>
<td>320</td>
<td>112</td>
</tr>
</tbody>
</table>

**Long-term interest rates disconnected from their fundamental values**

Another way to define the bond market conundrum is to compare interest rate levels to their equilibrium value as established through fundamental estimation. Of all those who have examined the conundrum, only Artus (2005), Frey and Moëc (2005), Warnock and Warnock (2005) and Rudebusch et al. (2006) made use of a fundamental model linking long rates (10-year US Treasury yields) to macroeconomic factors. There are two types of models: (1) univariate (Artus (2005), Frey and Moëc (2005), Warnock and Warnock (2005)), which uses an OLS regression, and may take the explicit form of an error-correction model; and (2) multivariate (a VAR or latent-variable model, for Rudebusch et al. (2006)). The extent of the disconnection depends critically on the model selected. This makes it difficult to compare models, because the differences lie not only in the econometric methodologies but also in the explanatory variables.

For Artus (2005), the equation of fundamental determination of long-term US yields incorporates the Fed’s key rate, the current account deficit, the budget deficit, the GDP growth rate and the level of long-term yields, with a lag. Frey and Moëc (2005) estimate a model that takes into account the expected US budget deficit related to GDP and the 3-month Treasury Bill interest rate. Finally, the explanatory variables for the Warnock and Warnock (2005) model are the Fed’s key rates, inflation and growth expectations, the budget deficit in relation to GDP, and interest rate volatility.

Rudebusch et al. (2006) selected a multivariate framework, which has the advantage of recognising the two-way relationships between macroeconomic variables and interest rates.

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2 Maximum increase in 10-year rates during tightening / increase in Fed policy rates.
They used two models. The first, by Bernanke et al. (2005), uses five determining factors for changes in the yield curve: the fed funds rate, employment, inflation, one-year inflation expectations, and eurodollar forward rates. The second, by Rudebusch and Wu (2004), is based on multivariate modelling of rates and includes two latent variables: inflation and the output gap.

Of all the authors cited, only Rudebusch et al. (2006) used their model to rigorously measure the amount of disconnection. Using the fundamental estimates of the Bernanke et al. (2004) model, they showed that the amplitude of the residual values was much greater during 2004-2005 than at earlier periods, even in comparison with other periods of disconnection of long rates from their fundamental estimates. For example, residuals were only about 1/10 of the level of long rates in 1984-1985 and 1/15 in 1997-1998, but 1/6 in 2004-2005. In addition, the earlier periods of disconnection were much shorter.

2.2. **New Explanatory Factors?**

Many explanatory factors have been put forward by Fed officials, economists and market participants to justify this bond market anomaly. A recent survey of investors by *Macroeconomic Advisers*\(^3\) summarises these explanations. The first factor cited is purchases by foreign central banks, which explain (according to the survey respondents) a decline of 21 bps in yields. Next come pension fund purchases, investors’ tendency to “search for yield”\(^4\) and decreased inflationary risk, accounting for about 10 bps. Finally, improved Fed transparency, the world savings glut and the low volatility of economic growth explain less than 8 bps. Detailed results of this survey are shown in Appendix 2.

From investors’ point of view, it is therefore possible to identify two main categories of explanatory factors for the anomaly: those relating to intensified buying by certain categories of investors (mostly Asian central banks and pension funds) and those linked to investors’ changing attitude to risk (such as greater Fed credibility and decreases in inflationary risk and macroeconomic volatility).

**Intensification of US Treasury bond buying**

Some buyers have intensified their purchases of US Treasury bonds in recent years, especially Asian central banks seeking to avoid an appreciation on their currencies, Japanese investors taking advantage of the interest rate differential between US and Japanese bonds, and pension

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\(^3\) “Monetary Policy Insights on Long-Term Interest Rates,” 8 March 2005.

\(^4\) Because of a low-interest-rate environment.
funds. Ageing populations mean that pension funds automatically have an increasing amount of money to invest, and anyway they are now legally obliged to put a larger part of their funds into bonds\(^5\). Thus, despite the deepening US budget deficit and the consequent growth in the supply of US Treasuries, the flood of new issues has largely been offset by a surge in demand. The impact of Asian and other foreign central bank purchases is the factor that has received the most attention in the academic literature as an explanation of the bond market anomaly. Artus (2005), Frey and Moëc (2005) and Warnock and Warnock (2005) add to a fundamental valuation model for long-term bond yields the variable of foreign purchases of US Treasuries, to measure the extent to which these purchases affect fundamental value. First, they show that purchases are a significant variable in their equation. Then they determine, using this new fundamental equation, what the equilibrium level of rates would have been if purchases had remained at their 1999 level or if they had suddenly dropped to zero. Warnock and Warnock (2005) show that if the flow of Treasury bond purchases by the non-resident public sector had remained at its January 1999 level, long-term US yields would be 95 bps higher. Frey and Moëc (2005) make the same assumption and demonstrate that, without this intensification of flows since 1999, yields would have been 115 to 125 bps higher. Artus (2005) shows that if foreign central bank had stopped their purchases, US 10-year yields would have risen 150 bps. Unfortunately, these results are based on a debatable assumption about the level of US Treasury purchases, set at zero or at the 1999 level (without justification for choosing that year).

In a different vein, McCauley and Jiang (2005) regress weekly changes in 10-year yields onto purchasing flows by foreign central banks and find only a very weak relationship between long US yields and these flows over the long term, with only recent significance. Similarly, Wu (2005) compares the residual of a fundamental equation for determining interest rates\(^6\) with the flow of US Treasury purchases and finds that a relationship has existed only since 2002. Before that time, the effect of these purchases on yield levels was the opposite.

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\(^5\) The need to rebalance pension fund portfolios is linked to the recent changes in pension reform legislation in the US (especially for defined benefit pension funds rather than for defined contribution funds). Pension funds now have to face three main problems: underfunding (mismatching problem between present value of assets and liabilities), duration gap (heavy pension fund investment in equities during the past decades increased the problem of underfunding) and demography (because of the aging of the population and the retirement of baby-boomers the pension funds will have to face high commitments in future years). The problem of retirements and pension balances is also important in Europe, but the European legislation is very different among countries. The United Kingdom has already faced this problem. In the Eurozone, the Netherlands has the most advanced process of pension reform.

\(^6\) Unfortunately not precisely defined in the paper, but depending on inflation expectations and macroeconomic factors.
Rudebusch et al. (2006) decompose the residuals of two models (VAR model and neo-Keynesian model with latent variables) into different fractions, corresponding to the contribution of the added variables. They show that non-resident official purchases of US Treasuries explain a very small fraction of the models’ residuals.

Studies that introduce purchases into a model for fundamental determination of long rates produce contrasting results and, moreover, are difficult to compare because of the variety of methodologies and explanatory variables used. The models that find a significant impact of purchases on long rates are those of Artus (2005), Frey and Moëc (2005) and Warnock and Warnock (2005). They share two common features: they are univariate models, and all of them use the budget deficit as an explanatory variable for long rates. In the next section, we shall further address this assertion which, according to Mehra (1994), is debatable. The only models that do not use this variable are those used by Rudebusch et al. (2006), but they employ two multivariate models. Therefore, it is difficult to determine whether the contrasting results stem from the difference in the variables chosen to explain the level of long rates or from the modelling choice.

In addition, purchases by pension funds, insurance companies, the Fed and others may have influenced the level of rates, but these possibilities have scarcely been examined. To our knowledge, only Kuttner (2006) studies the topic and points to the importance of the Fed’s open market operations on the Treasury market. He shows that these have a significant impact on the bond term premium. We know of no study that has analysed all flows of Treasury bond purchases by category of purchaser.

Investors’ Changing Attitude to Risk

A second category of explanation for the bond market anomaly has also been proposed, namely the idea that investors’ changing attitude to risk has led to a decrease in the bond risk premium. Several reasons can be advanced, including the lower volatility of economic variables – especially growth and inflation – and the greater credibility of the monetary authorities, particularly the Fed as an inflation-fighter.

Decreasing economic volatility has been abundantly documented (Blanchard and Simon (2001), Ahmed et al. (2004), Kahn et al. (2002)) and traced to a combination of factors: weaker economic shocks, changes in the composition of production in favour of more stable components such as services, and companies’ use of technologies for managing inventories more efficiently, thus limiting imbalances. Finally, the improved conduct of monetary policy
has led to reduced inflation and more stable growth. Fed officials\(^7\) have pinpointed the lesser volatility of economic variables as an explanation for the decline in the risk premium for holding long-term bonds – dubbed “the Great Moderation.” In practice an interest rate such as the 10-year yield can be split into three parts: one component linked to real interest rates, another linked to inflation expectations, and a risk premium that compensates for the risk of fluctuations in the growth rate, inflation and other variables. Thus, declining volatility in macroeconomic variables may lead to a decrease in the associated risk premiums, and if expectations for growth and inflation are stable, these declines will be reflected in a general decrease in interest rate levels. But another effect may cancel the first one. Weaker volatility could reduce the precautionary component of savings, pushing interest rates higher (Lettau et al. (2004)). In practice, the decrease in macroeconomic volatility has shown a greater tendency to cause yields to decline. Rudebusch et al. (2006) show that it may even provide an explanation of the bond market anomaly.

Another frequently offered explanation for the low level of long-term interest rates is that central banks, particularly the Federal Reserve, have become more credible as inflation-fighters. Some economists have shown that, under these conditions, the markets expect weak and stable inflation. As this expectation further lowers the risk premium they require to cover inflation fluctuations, it loosens the link between short-term and long-term rates. This phenomenon, noted as an anomaly by Carlstrom (1995)\(^8\), stems from the greater credibility of monetary authorities. Unfortunately, it is difficult to gauge the Fed’s credibility, and therefore we know of no precise test that has been carried out in this area.

### 3. Test of New Explanatory Factors for Long-Term Interest Rates

Given the diversity of results in the current literature on the bond market conundrum, we present another equilibrium model for long rates using a univariate approach, but without including the budget deficit, since its value as explanatory variable is doubtful. Within this framework, we test all the factors – Treasury bond purchases and macroeconomic volatility – that might explain the abnormally low level of long-term interest rates.

\(^7\) Fed Chairman Ben Bernanke, speech on 20 March 2006, and Vice Chairman Roger Ferguson, speech on 15 November 2005.

\(^8\) In fact, in this case, monetary policy loses its power, since it is no longer capable of influencing long-term interest rates, which affect companies’ demand for investment.
3.1. **Equilibrium Model of Long Term Rates**

Several models of long-term rates have been developed, mainly based on Fisher’s decomposition of interest rates (Boulanger (2002), Wu (2005), Warnock and Warnock (2005)). The nominal long-term interest rate can be split into a real rate, a component representing the long-term inflation expectation, and a risk premium reflecting investors’ uncertainty about future real interest rates and inflation:

$$TxN_{i,t} = TxR_{i,t} + \pi^e_{i,t} + p_{i,t} \tag{1}$$

with $TxN_{i,t}$ as the nominal rate at maturity $i$, $TxR_{i,t}$ as the real rate at maturity $i$, $\pi^e_{i,t}$ as the inflation expectation for horizon $i$, and $p_{i,t}$ as the risk premium for horizon $i$.

The real long-term interest rate may be seen to depend on expectations of real economic growth (Warnock and Warnock (2005), Wu (2005)), or alternatively it may be split into a real short-term rate and a term premium (Mehra (1994), Boulanger (2002)). We have used the latter decomposition. Thus, by regressing long-term rates onto these two variables: real short-term interest rates and the long-term inflation expectation, we can estimate an equilibrium value for interest rates.

The real short-term interest rate is calculated as the difference between the key interest rate set by the Federal Reserve (overnight rate) and a short-term inflation expectation. Short-term (one-year) and long-term inflation expectations used in the equation are economists’ forecasts from the “Survey of Professional Forecasters” provided by the Philadelphia Fed. In theory, it would have been more appropriate to use market’s expectations, which are now available thanks to inflation-linked bonds\(^9\). But this would have reduced the sample period too much, because these data are not available pre-1997. Another problem would have been the impact of high liquidity premiums on inflation-linked bonds during the first years of the series (Shen (2006)). Among the available series of long inflation expectations\(^10\), the Philadelphia Fed provides inflation expectations for 10 years ahead, thus perfectly matching the maturity of long-term bond yields.

This decomposition implicitly assumes that the constant and residual of the regression capture the bond term premium and risk premium. It is noteworthy that long-term rates, real rates and inflation expectations are all non-stationary over the period of the study (Table 2 in Appendix

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\(^9\) We could have used directly “inflation breakevens”, measured as the difference between long term nominal yields and long term real yields quoted in indexed linked bonds.

\(^10\) 5 years inflation expectations from the Federal Reserve of Michigan and long term expectation from Consensus Economists are also available.
The estimate is therefore made in the form of a cointegration equation, estimated with a simple OLS regression, with Engle and Granger (1987) methodology in two steps\(^{11}\). The following equation defines the long term cointegration relationship.

\[
T_{xN,t} = \beta_0 + \beta_1 T_{xRc,t} + \beta_2 \pi_{e,t} + \varepsilon_t
\]

with \(T_{xRc,t}\) as the real short-term interest rate.

One of the main differences between this univariate equilibrium model and the models presented in Section 2 is that we have chosen to exclude the budget deficit as a variable. According to the findings of Mehra (1994), the impact of the budget deficit on interest rates depends on the measure used to represent inflation expectations: the impact is very significant for one-year inflation expectations but not for longer-term expectations, even though that they should be closer to those actually included as a component in long rates. According to Mehra (1994), this finding is due to the fact that the link between the deficit and long rates is spurious. Deficits influence long rates through their impact on the long-term behaviour of currency and inflation expectations. Therefore, it is preferable to use a measure of long-term inflation expectations and to omit the budget deficit variable.

It would have been interesting to study the longest possible period. But monetary policies have changed over the past 30 years, and the relationship between long-term interest rates and macroeconomic variables has not been stable over time (Fuhrer (1996), Clarida et al. (2000)). A structural change in the Fed’s monetary policy conduct became evident with the start of Chairman Paul Volcker’s tenure in the early 1980s. We therefore used the period of July 1981 until 2006 for purposes of our estimate. The results are shown in Table 3 in Appendix 1. The tests of robustness, notably the CUSUM squared test, and the N-forecast probability test, detect a limited period of instability in 1985 and 1986 (see Figures 10 and 11 in Appendix 1). In any case, the equation presents no problem of stability after that period nor in the most recent period under investigation. Reducing the sample period (starting from 1987, which corresponds to Greenspan’s appointment as Fed chairman) makes the regression more stable, but the differences in the long-run estimates of 10 year rates are negligible in the period under review. Since the short period of instability is at the beginning of our estimation, we use the

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\(^{11}\) This methodology has many advantages for the purpose of our article, especially insofar as it allows to focus on the model’s long-term relationship, which gives an estimate of the equilibrium value for interest rates. Note that the conditions of use of this model have been verified: non-stationarity, exogeneity of explanatory variables (reasonable assumption with regard to target rates, slightly less so as regards inflation expectations), uniqueness of the cointegration relationship, sufficient number of data points).
longer sample, which gives us six more years of history. The equation’s residuals are indeed stationary, indicating that the cointegration relationship is valid.

As can be seen in Table 3 in Appendix 1, the coefficients of regression have the expected positive sign\(^{12}\) and the impact of inflation expectations is much more important than that of real rates on the equilibrium estimate. As a rule of thumb, the coefficient values of the OLS regressions tell us that if expected short term real rates increase by 100 bps, the long-run value of 10-year rates increases by roughly 32 bps, and if long-term inflation expectations increase by 100 bps (e.g. from 2.5% to 3.5%) the long-run value of 10-year rates increases by roughly 188 bps.

The purpose of this study is to concentrate on the long-run equation only, so we do not go into detail about the short term dynamic of adjustment of the long rates to their equilibrium value. However, the results of the short-term equation can be found in the appendices (Table 4 in Appendix 1).

3.2. Measurement of the anomaly

Figures 1 and 2 below show the results of our estimates since 1981. The model correctly estimated yield levels, with periods of over- or undervaluation of long-term yields in relation to their fundamental value. Using a cointegration model, we assume the existence of an error correction mechanism through which the discrepancy between the fair value and the effective value disappeared in a certain period of time.

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\(^{12}\) Note that the regression coefficients \(\beta_1\) and \(\beta_2\) need not necessarily be equal to 1. In fact Wu (2003) shows that if monetary tightening leads to higher short-term rates, long rates include long-term expectations of changes in short rates. So long-term rates should subsequently fall, given the anti-inflationary effects of the monetary policy in place. Thus the coefficient \(\beta_1\) linking real short rates to nominal long rates should logically be positive but less than 1. The coefficient \(\beta_2\) is not necessarily equal to 1, notably because economists’ inflation expectations may differ from the market’s.
FIGURES 1 and 2: 10-year yield and fundamental value (equilibrium level of 10-year yield from regression)

The current “conundrum” can be identified using our model, started in August 2004. It lasts until the end of the model's estimation period (June 2006), i.e. 23 months. The average deviation from fundamental value during the period was 40 bps, with a maximum of 83 bps and a minimum of 4 bps.

Table 5 in Appendix 1 presents descriptive statistics (length and size) of the prolonged periods of underestimation detected by the model, in order to compare them with the “conundrum” period. As the table shows, we have encountered in the past other prolonged periods of underestimation. The amplitude of each anomaly should be related to the level of interest rates, which was broadly declining during the study period. A difference of 40 bps in today’s market does not have the same meaning as it did in the 1980s. Viewed in relation to the level of rates, only three periods of anomaly were of greater amplitude than the current one: (1) 1985-1987, (2) 1998-1999, and (3) 2002-2003. The first of these was relatively long (18 months), but the second and third lasted only six and seven months respectively. The current anomaly is certainly atypical from this point of view, as it is the longest period of underestimation: 23 months.

3.3. **Test of new explanatory factors**

Our fundamental model of long-term rates implicitly assumes that term and risk premiums can be captured by the constant and residual of the regression, leaving them unexplained by macroeconomic factors. In this context, it will be of great interest to test whether the two

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13 Periods longer than the half-life of the equation, this is approximately 6 months.
categories of “new” explanatory factors cited by market participants, not previously taken into account in these fundamental models, could theoretically be linked to the risk premiums and whether they can be proved econometrically to be responsible for their recent decline.

**Flow of Funds Data**

We used data from the Federal Reserve’s Flow of Funds Accounts\(^\text{14}\), which has the advantage of providing detailed information on net purchases by all domestic and foreign investors in the US Treasury market\(^\text{15}\). We related these flows to the debt balance\(^\text{16}\). As is usual in the literature, we have considered the sum of purchases over 12 rolling months in order to smooth the purchase data, which are very volatile. All data are available on the Fed’s website and also through Datastream as a quarterly series. In order to limit the variables to be tested\(^\text{17}\), we grouped the data by aggregating the most closely related categories. We studied 11 categories of participant in the Treasury bond market, as described in Appendix 3. The bar chart below shows the pattern of flows in each of these categories over the periods 1998 to 2001 and 2002 to 2006.

**FIGURE 3**: Average purchases by each of 11 categories of market participants during the periods 1998-2001 and 2002-2006 (in USD millions)

![Bar chart showing average purchases by each category of market participants.]

Source: Flow of Funds data, authors’ calculations

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\(^{14}\) [http://www.federalreserve.gov/releases/z1/](http://www.federalreserve.gov/releases/z1/)

\(^{15}\) All maturities are combined, that is, Treasury bills (maturing in less than one year), Treasury Notes (from one to 10 years) and Treasury bonds (maturity beyond 10 years).

\(^{16}\) For this, we used the outstanding marketable debt of Treasury bills, notes and bonds available in the market.

\(^{17}\) The Flow of Funds data includes 21 large categories of participants.
Most noticeable in the 2002-2006 period is a sharp increase in purchases by the “Rest of the World,” which bought on average USD 237 billion of Treasuries although it was a minor seller (USD 12.7 billion) in the 1998-2001 period. In the recent period, these purchases were much greater than those of other participants. Two other categories appear relatively important, although their purchases amounted to much less than those of the Rest of the World: “State & Local Governments” and the Federal Reserve (“Monetary Authority”).

**Treasury International Capital System (TICS) Data**

The “Rest of the World” category in the Flow of Funds data represents all foreign investors together. To analyse more specifically the impact of purchases by foreign central banks, cited as the main explanatory factor for the bond market anomaly, we used figures for net purchases of US bonds by non-residents as supplied by the Treasury International Capital System, or TICS\(^\text{18}\), which distinguishes official sectors (including foreign central banks) from unofficial ones. These data are derived from estimates of purchases and sales of US instruments with maturity over one year (Notes and Bonds). We related these purchases to the corresponding debt\(^\text{19}\). All these data are available as monthly series on Datastream.

As Warnock and Warnock (2005) point out, the TICS data for official-sector purchases may underestimate the actual amount bought. Because the status of the owner of the instrument is not given in the TICS survey, it is possible that private-sector banks are buying securities on behalf of the official sector, so that these purchases are recorded in the private sector’s accounts. In addition, the TICS survey deals with banks and brokers, and does not cover transactions between non-residents. To take this criticism into account, we examined both official and unofficial purchases of Treasuries. We see a definite acceleration of purchases starting in 2002, confirming our observations concerning the Flow of Funds data. The massive purchases by non-residents peaked in June 2004 (7.7% of the debt amount, counting official purchases, and 15.6% counting all purchases by non-residents) but decelerated sharply thereafter. This decrease may be almost entirely attributed to the official sector.

**Volatility of Growth and Inflation**

From the quarterly series of GDP growth rates and monthly series of inflation rates, we calculated volatility as the 5-year rolling standard deviation (annualised). We used three


It should be noted that the Fed produces other, similar data but they have the drawback of being available only for 1996 onward.

\(^{19}\) For this purpose, we used only the amount of Treasury notes and bonds available in the market.
inflation indices: the consumer price index (CPI Headline), the consumer price index excluding food and energy (CPI Core) and the index of personal consumption expenditures excluding food and energy (PCE Core). The GDP and inflation data are available from Datastream. We see a steep decline in the volatility of these macroeconomic variables over the study period, more closely related to a structural change than to a long-term decrease. In the 1980s, this volatility was above 2.6% for GDP growth, 2.4% for CPI Headline, 1.9% for CPI Core and 1.1% for PCE Core. Then, in the late 1980s, it plunged, levelling off in the 1990s at around 1.3% for GDP and between 0.5% and 0.8% for inflation (see Table 6 in Appendix 3). In the recent period, the volatilities of these macroeconomic variables have risen slightly, with the exception of PCE Core, which remains around 1%. Figures 12 to 18 in Appendix 3 show the changes in all variables studied since 1981.

3.4. Results

To discriminate which factors could help us explain the conundrum we decided to add one by one the series in our regression. To do this it was first necessary to identify the order of integration of all the variables of interest. Our two-step Engle and Granger (1987) procedure requires the variables to be of equal order of integration. In our case, not all of our 17 additional variables of interest were I(1). Five of them had to be excluded: two I(2) variables (volatility of “CPI headline” and “CPI core”), and three I(0) flow variables (“Monetary Authority”, “Mutual Funds” and “Brokers and Dealers”), as can be seen Table 7 in Appendix 4. Table 8 in Appendix 4 mentions the coefficients of the variables and the summary statistics of the regressions. Our discrimination criteria were the sign of the coefficient (which should be in line with the expected intuitive sign), its significance (we have used the White option correction for heteroscedasticity, when necessary), the stability of the recursive coefficient and the stability of the whole regression (measured with the CUSUM squared test).

The first candidates to explain the bond conundrum are the growth and inflation volatilities. As can be seen in Table 8, the coefficients for these variables have the expected signs, justifying a positive link between macroeconomic uncertainty and interest rates (an increase in macroeconomic volatility should lead to an increase in the bond risk premium). Regarding the inflation volatility (only PCE core was retained because of stationarity reasons), the

---

20 The presence of stationary variables “I(0)” could lead to inconsistency estimates. The problem could be overcome estimating jointly the long run and the short run equation, such through ARDL models (Boswijk and Doornik (1999), Rahbek, and Mosconi (1999)). Unfortunately this procedure isn’t adapted to our case as these models are not able to estimates separately the long run dynamics of rates. In case of I(2) variables to be added, a multivariate approach would then be preferable (Colombo et al. (2002), Omtzigt (2001)).
relationship between this variable and the 10-year rate is relatively stable\textsuperscript{21}. The fact that PCE Core is the reference series of the Fed in determining its comfort zone of inflation allows us to reinforce the case that stability of this variable may partly capture the improvement of the Fed credibility. The volatility of PCE Core decreases the estimated level of 10-year rates during the conundrum, since it did not increase during the last years. Moreover, the average reduction of the estimated value during the conundrum is 1 bp (maximum reduction is 2 bps), as shown in the graphs below. As a rule of thumb, the coefficient value tells us that if the volatility of PCE Core decreases from, say, 10\% to 9\%, the estimate of 10-year rates decreases by roughly 6 bps. These numbers are obviously not exhaustive to explain the conundrum, especially if we consider the stability of the volatility metric during these last years and the fact that it is still at a very low level. Even if inflation volatility is a significant explanatory variable of long-term rates, it is a poor explanation of the conundrum.

**FIGURES 4 and 5**: Comparison of 10-yr rate equilibrium model with the estimate from inflation PCE core volatility and the realised value.

Concerning the volatility of GDP growth, the impact on rates seems to be significant but quite ambiguous, since the correlation between growth volatility and interest rates is quite unstable. During the last two decades, it became negative, thus increasing the estimated value of long-term rates during the conundrum period.

We now turn to analyzing the second cluster of explanatory variables of the conundrum, the Flows of Funds variables. As argued before, the variables not of first order integrated or with a positive coefficient could be immediately eliminated (intuitively, an increase in net purchases should lead to a decrease in interest rates), such as the purchases from the “Household Sector”, “Commercial Banks”, “Monetary Authority”, “Corporate Sector”,

\textsuperscript{21} No change in the coefficient sign and we accept the CUSUM square test.
“Pension Funds”, “Mutual Funds”, “Brokers and Dealers” and the “Others” category. We were surprised to find that the “Pension Funds” category presents a positive coefficient, since this is the second most important cause of the “conundrum” according to the Macroeconomic Advisers survey and it is often mentioned as one of the most important factors among market participants. We therefore investigated further and discovered that the coefficient turns out to be negative if we change the starting point of estimation (from 1987). However, it is still not significant and, more importantly, the recursive coefficient became negative only in the second half of 2003. This leads us to reject the Pension Funds category as an explanatory factor of the conundrum, in contrast to the commonly held opinion.

In the Flows of Funds cluster, two variables have a negative coefficient, but they are not significant and hence not meaningful: Treasury purchases by the Non Financial Corporate Sector and Insurance. This is not really surprising as the amounts of purchases are quite small for these categories compared to some others. The “Rest of The World” category is significant, but only after 2002. Since it is probably the most frequently mentioned explanatory factor for the conundrum (the principal cause according to the abovementioned survey), and the amount of purchases by this category is considerably higher than for the others, we investigated further. We tried to rely on the other official series of foreign purchases, the TICS data. They have the advantage of monthly frequency, they can be decomposed in official purchases and unofficial purchases, and they consider purchases of long-term instruments separately (“Notes” and “Bonds”), which in our case is probably more suitable to explain the behaviour of long-term rates. As expected, looking at the usual regression, TICS data have better explanatory power than Flows of Funds data. Nevertheless, these variables are not statistically significant before 2002, and the recursive coefficient is unstable (it was positive during the Asian crisis, for example). This degree of instability rules it out as an adequate variable for a “standard” cointegration relationship, but we recognise that it has undoubtedly had a statistically significant impact on long-term rates recently and is therefore useful for our purposes.

The graphs of the fair value estimate with each of the two series (see below) allow us to show that total foreign purchases are necessary to explain the biggest part of the conundrum, not only official foreign purchases (especially during last year). Official foreign purchases decrease our equilibrium value by an average 10 bps after August 2004, with the maximum

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22 Whereas Flows of Funds data include also the “Treasury Bills” purchases in the total purchases.
23 95% confidence interval
24 Even it is the recursive coefficient is unstable, the CUSUM squared test is acceptable.
impact of this variable reached in September-October 2004. The series of total foreign purchases decreases our equilibrium value by an average 15 bps during the conundrum (with a peak of 20 bps in November 2004). This is in line with the results of Warnock and Warnock (2005), which underline the importance of considering total purchases, not just official purchases. As a rule of thumb, the coefficient value tells us that if the ratio between total foreign-treasury buying of Notes and Bonds and debt increases from, say, 10% to 11% the estimate of 10-year rates decreases by roughly 2 bps. These numbers are obviously not exhaustive in explaining the conundrum, especially if we consider that foreign buying increased significantly from 2001 to 2004, reaching a peak in May 2004, and then during the conundrum the percentage of foreign buying relative to the debt is decreasing.

FIGURES 6 and 7: Comparison of long-term rate equilibrium model with the estimation coming from TICS data and the realised value.

Finally, one other category of flows presents a significant and negative link with the long-term interest rates: “State & Local Government”. However, its coefficient in the regression is so unstable that the CUSUM square test of stability is unacceptable. The recursive coefficient has become more negative during the last year, emphasising the strengthening of the negative correlation between 10-year rates and State & Local Government Treasury buying. But the instability of the estimation leads us to reject it in our final equation.25

This screening leads us to retain two plausible variables to be added to the final equation (see Table 9 and figures 19 and 20 in Appendix 4): the volatility of PCE Core and total foreign

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25 The instability we detected in the coefficient will be “a priori” enough to exclude this variable from the final screening, but to eliminate all uncertainties we estimate a VECM imposing one cointegration relationship. We found that the long run coefficient of the “state and government” variable was positive (contrary to the univariate case), this result confirms our doubts on the reliability of this variable as source of explanation of the conundrum.
purchases of Treasury securities. With these two additional variables, our equilibrium estimate decreases by 16 bps on average, with a peak of 22 bps in December 2004, which allows us to explain less than 50% of the gap between our model and market level after August 2004. It should be stressed that according to our univariate econometric approach, this new equation does not define a new equilibrium model of the 10-year rate because of the instability of the new coefficients, a sign of the varying intensities and importance of the relationships between 10-year rates and the new variables. Be that as it may, they could have a temporary impact in keeping down the level of long-term rates.

FIGURES 8 and 9: Comparison of the equilibrium value of 10-yr rate with the estimate from the final regressions and the realised value.

For completeness, it seems worth stating that most results remain valid if we conduct the estimates with a sample starting from 1987 (the results are not presented in this paper). Three categories ("Insurance", "Rest of The World" and "Pension Funds") have a higher level of significance. The "Pension Funds" category remains insignificant, but "Insurance" now becomes significant, even more than the "Rest of The World" category.

This higher level of significance reveals that the aforementioned categories are probably having a greater influence on the long-term bond yields now than they did before, but the estimate of their impact through a cointegration relationship needs further investigation.

Our results are quite different from the perception of investors, as reflected by the Macroeconomic Advisers survey. We agree on the importance of Foreign Purchases of Treasury Bonds (total and not only official), but only recently (given the instability of their impact). Inflation volatility is also an important factor in determining interest rates, but not growth volatility. We were unable to detect a significant impact of other variables. On the
debate of "Pension Funds" purchases, we stress that, according to our method, the Flows of Funds variable is not significant on the whole study period (even if the relationship with interest rates has turned to be negative and more significant in the last two years). This surprising result could be due to the fact that the available data concern Treasury purchases across all maturities, not totally suitable to explain 10-year rates. But it is true that, despite a sharp increase in recent years, pension fund purchases are still quite small in volume compared with foreign purchases, and this is probably another reason for their insignificant impact on interest rates.

4. Conclusion

Although the bond market conundrum has been universally noted by central bankers and economists, the explanatory factors have certainly not been identified with precision. Among the factors most often cited by investors, not all have yet been tested in the academic literature, and for those tested, results vary considerably from one author to another depending on methodology.

Our aim in this paper was therefore to present a fundamental univariate estimation model, linking long rates to macroeconomic variables and monetary policy, but deviating from the approach of most authors by excluding the budget deficit on the grounds that its usefulness as an explanatory variable for long-terms rate is questionable (Mehra (1994)). By estimating this model, based on the Fisher equation and treating these rates as dependent on inflation expectations and monetary policy, we have been able to document the size and timing of the bond market “conundrum” observable in the United States beginning in August 2004. For the first time in more than 20 years, long-term rates remained persistently below their equilibrium value (by an average of 40 bps) for a period of 23 months, despite stringent monetary tightening that caused key rates to climb by 425 bps.

We have sought to find out whether this anomaly may be explained by some previously neglected factors. Two main categories of explanation have been cited by market participants and taken up by a Macroeconomic Advisers survey: (1) investors’ changed attitude toward risk, as they require reduced risk premiums in the bond market for various reasons (greater credibility of monetary authorities, “searching for yield” in a low-interest-rate environment, decrease in macroeconomic volatility), and (2) the intensification of Treasury purchases by certain categories of participants, especially pension funds and foreign official purchasers.
The latter now have substantial reserves, which they are using to invest massively in US Treasury bonds because of the dollar’s status as a reserve currency. These two types of factors could be theoretically responsible for a decline of the bond risk premium in the recent period.

We have tested precisely all the quantifiable factors cited above in a fundamental valuation model of long-term rates. The Flow of Funds data provides us with an exhaustive look at all purchases of US Treasuries by the various categories of market participant. We have shown that pension fund purchases, though often cited to explain the bond market anomaly, have had no impact on interest rate levels – rather surprisingly. Purchases by foreign investors have played a more important role, but their impact was very unstable and is highly concentrated in the recent period. Investors’ changing attitude to risk is perhaps a more convincing explanation for the bond market anomaly. Unfortunately, this explanation may itself have several causes, not all of them quantifiable. Two of these, the decline in the volatility of both growth and inflation, were tested for this paper. While they turned out to have relevance as explanatory factors for the level of long-term rates, they do not help explain the recent period’s bond market anomaly.

Finally, introducing these two additional factors (i.e. purchases by foreign investors and PCE Core volatility) into our equilibrium model allows us to explain slightly less than half of the bond market anomaly (only 16 bps on the 40 bps average discrepancy between fundamental value and 10 year rates). Nevertheless, tests on a restricted sample reveal that the influence of three categories (pension funds, insurances, and foreigners purchases) has been growing recently, possible contributing to the low level of long-term rates. This would justify further prospective analysis of bond rates. Moreover it is not impossible that non-measurable factors, linked to investors’ changing perception of market risks or monetary authorities’ credibility, may have played a role. Accordingly, the “bond market conundrum” has not been completely resolved.
References


Appendices

Appendix 1

TABLE 2: Stationary tests of the variables in the fundamental valuation model for long-term interest rates, July 1981 – July 2006

<table>
<thead>
<tr>
<th></th>
<th>Levels</th>
<th>1st differences</th>
</tr>
</thead>
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<td></td>
<td>t-stat</td>
<td>t-stat</td>
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<td>Real rate</td>
<td>-3.75</td>
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</tr>
<tr>
<td>Expected inflation 10-year</td>
<td>-2.06</td>
<td>-17.48</td>
</tr>
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</table>

1% Critical level -3.98
5% Critical level -3.42
10% Critical level -3.15


<table>
<thead>
<tr>
<th></th>
<th>Beta coefficient</th>
<th>T-Stat</th>
</tr>
</thead>
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<tr>
<td>Constant</td>
<td>-0.22</td>
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<tr>
<td>Real rate</td>
<td>0.33</td>
<td>12.24</td>
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<tr>
<td>Expected inflation 10-year</td>
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<td>42.13</td>
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<tr>
<td>Adjusted R² of the regression</td>
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<td>S.E. of the regression</td>
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<tr>
<td>Durbin-Watson statistic</td>
<td></td>
<td>0.32</td>
</tr>
<tr>
<td>Stationarity test on residuals (Phillips-Perron T-stat)</td>
<td></td>
<td>-18.83*</td>
</tr>
</tbody>
</table>

* stationarity is accepted.
FIGURES 10 & 11: Results of N-Step Probability test and CUSUM square test on the fundamental valuation model for long-term interest rates, July 1981 – July 2006


<table>
<thead>
<tr>
<th>Dependent Variable d (10 yr rates)</th>
<th>Beta coefficient</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.02</td>
<td>-0.87</td>
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<tr>
<td>D(Real rate)</td>
<td>0.24</td>
<td>4.88</td>
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<tr>
<td>D(Expected inflation 10-year)</td>
<td>0.53</td>
<td>3.08</td>
</tr>
<tr>
<td>ECM(-1)</td>
<td>-0.15</td>
<td>-4.64</td>
</tr>
<tr>
<td>Adjusted R² of the regression</td>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td>S.E. of the regression</td>
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<td>0.33</td>
</tr>
<tr>
<td>Durbin-Watson statistic</td>
<td></td>
<td>1.90</td>
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</table>
TABLE 5: Periods of underestimation of the model

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<th>Periods</th>
<th>N months</th>
<th>Max gap</th>
<th>Min gap</th>
<th>Mean gap (A)</th>
<th>Mean 10 rates (B)</th>
<th>A/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>31/07/1981-31/12/1981</td>
<td>6.00</td>
<td>2.44</td>
<td>0.50</td>
<td>1.36</td>
<td>14.61</td>
<td>0.09</td>
</tr>
<tr>
<td>30/11/1985-30/04/1987</td>
<td>18.00</td>
<td>1.65</td>
<td>0.08</td>
<td>0.79</td>
<td>7.74</td>
<td>0.10</td>
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<tr>
<td>31/10/1987-31/03/1988</td>
<td>6.00</td>
<td>0.61</td>
<td>0.01</td>
<td>0.34</td>
<td>8.63</td>
<td>0.04</td>
</tr>
<tr>
<td>31/10/1988-31/01/1990</td>
<td>16.00</td>
<td>1.51</td>
<td>0.03</td>
<td>0.67</td>
<td>8.54</td>
<td>0.08</td>
</tr>
<tr>
<td>30/09/1990-30/04/1991</td>
<td>8.00</td>
<td>0.39</td>
<td>0.01</td>
<td>0.18</td>
<td>8.25</td>
<td>0.02</td>
</tr>
<tr>
<td>28/02/1993-28/02/1994</td>
<td>13.00</td>
<td>0.87</td>
<td>0.08</td>
<td>0.52</td>
<td>5.82</td>
<td>0.09</td>
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<tr>
<td>31/05/1995-29/02/1996</td>
<td>10.00</td>
<td>0.69</td>
<td>0.08</td>
<td>0.45</td>
<td>6.05</td>
<td>0.07</td>
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<td>31/08/1998-28/02/1999</td>
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<td>0.98</td>
<td>0.29</td>
<td>0.60</td>
<td>4.69</td>
<td>0.13</td>
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<tr>
<td>31/12/2002-30/06/2003</td>
<td>7.00</td>
<td>0.87</td>
<td>0.13</td>
<td>0.44</td>
<td>3.72</td>
<td>0.12</td>
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<tr>
<td>31/08/2004-30/06/2006</td>
<td>23.00</td>
<td>0.83</td>
<td>0.04</td>
<td>0.39</td>
<td>4.41</td>
<td>0.09</td>
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<tr>
<td>Mean</td>
<td>11.30</td>
<td>1.08</td>
<td>0.12</td>
<td>0.57</td>
<td>7.24</td>
<td>0.08</td>
</tr>
</tbody>
</table>

We retained the periods longer than average underestimation period length.

Appendix 2

*Macroeconomic Advisers* Survey results: “Monetary Policy Insights on Long-Term Interest Rates,” 8 March 2005

(1) Demand by foreign central banks (21 bps)
(2) Increased demand by pension funds (11 bps)
(3) “Searching for yield” (10 bps)
(4) Minimal inflation risk (10 bps)
(5) Greater transparency of the Fed (8 bps)
(6) Excess global savings (8 bps)
(7) Low economic growth volatility (7 bps)

Appendix 3

**Definition of 11 categories of US Treasury investors used in the study (aggregation of Flow of Funds data)**

(1) “Household Sector”
(2) “Non-Financial Corporate”
(3) “State & Local Government”
(4) “Rest of the World”
(5) “Monetary Authority”
(6) “Commercial Banks”
(7) “Insurances” (“Property-casualty Insurance” and “Life Insurance”)
(8) “Pension Funds” (“Private Pension Funds,” “State & Local Government Retirement Funds” and “Federal Government Retirement Funds”)
(9) “Mutual Funds” (“Money Market Mutual Funds,” “Mutual Funds,” “Closed-end Funds” and “Exchange-traded Funds”)
(10) “Brokers and Dealers”
(11) “Others” (“Savings Institutions,” “Credit Unions” and “Government-Sponsored Enterprises”).
FIGURES 12 TO 18: Main explanatory factors tested to explain the bond market anomaly, purchases and macroeconomic volatility

Source: TICS data, Flow of Funds, Datastream, authors’ calculations
TABLE 6: Average volatility of macroeconomic variables

<table>
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<th></th>
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<tr>
<td></td>
<td>Vol inflation (CPI headline)</td>
<td>Vol inflation (CPI core)</td>
<td>Vol inflation (PCE core)</td>
<td>Vol GDP</td>
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<tr>
<td>ALL</td>
<td>1.38</td>
<td>0.98</td>
<td>0.68</td>
<td>1.76</td>
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<tr>
<td>1981 - 1989</td>
<td>2.43</td>
<td>1.93</td>
<td>1.17</td>
<td>2.63</td>
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<td>1990 - 1999</td>
<td>0.78</td>
<td>0.55</td>
<td>0.52</td>
<td>1.30</td>
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<tr>
<td>2000 - 2006</td>
<td>0.76</td>
<td>0.36</td>
<td>0.27</td>
<td>1.33</td>
<td></td>
</tr>
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Appendix 4

TABLE 7: Stationary tests of the candidate variables to be inserted in the equilibrium equation of long-term rates, July 1981 – July 2006

<table>
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<td></td>
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<td>t-stat</td>
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<tr>
<td>Volatility</td>
<td>vol CPI Headline</td>
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<td>-0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>vol CPI Core</td>
<td>-1.15</td>
<td>-1.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>vol PCE core</td>
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<td>-3.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>vol GDP</td>
<td>-1.73</td>
<td>-19.48</td>
<td></td>
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</tr>
<tr>
<td>Flows of Funds</td>
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<td>-17.58</td>
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<td></td>
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<td></td>
<td>Commercial Banks</td>
<td>-3.53</td>
<td>-18.13</td>
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<td></td>
<td>Household sector</td>
<td>-3.16</td>
<td>-17.36</td>
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<td></td>
<td>Non Financial Corporate</td>
<td>-4.62</td>
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<td>Insurances</td>
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<td>Pension funds</td>
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<td>Mutual Funds</td>
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<td>-17.70</td>
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<tr>
<td></td>
<td>Brokers &amp; Dealers</td>
<td>-5.66</td>
<td>-21.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>State &amp; Local govt</td>
<td>-2.31</td>
<td>-17.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rest of World</td>
<td>-2.35</td>
<td>-17.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>-3.50</td>
<td>-17.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TICS data</td>
<td>Foreign Official</td>
<td>-2.55</td>
<td>-13.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Foreign Total</td>
<td>-2.103</td>
<td>-15.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1% Critical level -3.98
5% Critical level -3.42
10% Critical level -3.15
## TABLE 8: Main statistics of the regressions adding one by one each variable on our equilibrium model, July 1981 – July 2006

<table>
<thead>
<tr>
<th>Variable added</th>
<th>Constant (t stat)</th>
<th>( \beta ) (t stat) for real short term rates</th>
<th>( \beta ) (t stat) for 10 inflation expectations</th>
<th>Beta coefficient (t-stat) of variable added</th>
<th>Adjusted R2</th>
<th>SE of regression</th>
<th>DW test</th>
</tr>
</thead>
<tbody>
<tr>
<td>vol PCE Core</td>
<td>-0.01 (-0.05)</td>
<td>0.31 (12.58)</td>
<td>1.72 (31.75)</td>
<td>0.58 (4.31)</td>
<td>0.95</td>
<td>0.63</td>
<td>0.35</td>
</tr>
<tr>
<td>vol GDP</td>
<td>-0.13 (-0.91)</td>
<td>0.36 (13.93)</td>
<td>1.71 (25.66)</td>
<td>0.26 (3.02)</td>
<td>0.94</td>
<td>0.65</td>
<td>0.33</td>
</tr>
<tr>
<td>Monetary Authority</td>
<td>-0.42 (-2.22)</td>
<td>0.36 (11.48)</td>
<td>1.85 (43.11)</td>
<td>0.22 (2.44)</td>
<td>0.94</td>
<td>0.66</td>
<td>0.34</td>
</tr>
<tr>
<td>Commercial Banks</td>
<td>0.12 (0.74)</td>
<td>0.39 (12.53)</td>
<td>1.73 (29.88)</td>
<td>0.13 (3.51)</td>
<td>0.94</td>
<td>0.65</td>
<td>0.34</td>
</tr>
<tr>
<td>Household sector</td>
<td>0.41 (1.99)</td>
<td>0.37 (13.87)</td>
<td>1.68 (25.70)</td>
<td>0.09 (5.04)</td>
<td>0.94</td>
<td>0.64</td>
<td>0.36</td>
</tr>
<tr>
<td>Non Financial Corporate</td>
<td>-0.29 (-1.94)</td>
<td>0.32 (11.72)</td>
<td>1.91 (39.60)</td>
<td>-0.13 (-1.33)</td>
<td>0.94</td>
<td>0.67</td>
<td>0.32</td>
</tr>
<tr>
<td>Insurances</td>
<td>-0.28 (-1.34)</td>
<td>0.31 (*.42)</td>
<td>1.91 (23.89)</td>
<td>-0.04 (0.39)</td>
<td>0.94</td>
<td>0.67</td>
<td>0.32</td>
</tr>
<tr>
<td>Pension funds</td>
<td>0.53 (2.58)</td>
<td>0.33 (14.61)</td>
<td>1.60 (22.99)</td>
<td>0.32 (4.58)</td>
<td>0.95</td>
<td>0.63</td>
<td>0.34</td>
</tr>
<tr>
<td>Mutual Funds</td>
<td>-0.34 (-2.37)</td>
<td>0.31 (11.30)</td>
<td>1.96 (46.94)</td>
<td>-0.18 (-4.77)</td>
<td>0.94</td>
<td>0.64</td>
<td>0.35</td>
</tr>
<tr>
<td>Brokers &amp; Dealers</td>
<td>-0.17 (-1.33)</td>
<td>0.35 (14.13)</td>
<td>1.84 (41.81)</td>
<td>-0.15 (-4.72)</td>
<td>0.94</td>
<td>0.64</td>
<td>0.37</td>
</tr>
<tr>
<td>State &amp; Local govt</td>
<td>-0.31 (-2.01)</td>
<td>0.33 (11.99)</td>
<td>1.92 (40.43)</td>
<td>-0.07 (-3.09)</td>
<td>0.94</td>
<td>0.66</td>
<td>0.34</td>
</tr>
<tr>
<td>Rest of World</td>
<td>-0.09 (-0.55)</td>
<td>0.31 (10.37)</td>
<td>1.88 (42.34)</td>
<td>-0.03 (-2.49)</td>
<td>0.94</td>
<td>0.66</td>
<td>0.32</td>
</tr>
<tr>
<td>Others</td>
<td>0.07 (0.41)</td>
<td>0.37 (14.92)</td>
<td>1.75 (33.34)</td>
<td>0.32 (3.87)</td>
<td>0.94</td>
<td>0.65</td>
<td>0.34</td>
</tr>
<tr>
<td>Foreign Official</td>
<td>-0.16 (-1.11)</td>
<td>0.32 (11.65)</td>
<td>1.88 (42.86)</td>
<td>-0.04 (-2.02)</td>
<td>0.94</td>
<td>0.66</td>
<td>0.33</td>
</tr>
<tr>
<td>Foreign Total</td>
<td>-0.06 (-0.34)</td>
<td>0.32 (11.79)</td>
<td>1.86 (41.32)</td>
<td>-0.02 (-2.86)</td>
<td>0.94</td>
<td>0.66</td>
<td>0.33</td>
</tr>
</tbody>
</table>

TABLE 9: Results of estimation of the final equation model for long-term interest rates, July 1981 – July 2006

<table>
<thead>
<tr>
<th></th>
<th>Beta coefficient</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.16</td>
<td>0.96</td>
</tr>
<tr>
<td>Real rate</td>
<td>0.31</td>
<td>12.12</td>
</tr>
<tr>
<td>Expected inflation 10-year</td>
<td>1.71</td>
<td>30.94</td>
</tr>
<tr>
<td>vol pce core</td>
<td>0.58</td>
<td>4.33</td>
</tr>
<tr>
<td>Foreign Total</td>
<td>-0.024</td>
<td>-3.03</td>
</tr>
<tr>
<td>Adjusted R² of the regression</td>
<td></td>
<td>0.95</td>
</tr>
<tr>
<td>S.E. of the regression</td>
<td></td>
<td>0.63</td>
</tr>
<tr>
<td>Durbin-Watson statistic</td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>Stationarity test on residuals (Phillips-Perron T-stat)</td>
<td>-18.56*</td>
<td></td>
</tr>
</tbody>
</table>

* stationarity is accepted.

FIGURE 19 and 20: Results of N-Step Probability test and the CUSUM square test on the final equation, July 1981 – July 2006
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